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Introductory pricing and subscription as signals*

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Abstract We consider a series of signaling models of experience goods. In the first model, the seller attempts to signal its quality by introductory pricing in the trial phase in the hope of future profit. We identify plausible forms of equilibria by applying the intuitive criterion. Then we expand the model horizon to examine what happens after the trial phase. We show that when the product is durable and requires costly maintenance, the price alone is not effective as a signal of the seller's long-livedness. A subscription scheme is suggested as an effective instrument for ensuring long-term transaction. We also discuss interaction between the two phases. These models can illuminate on recent business practices, e.g. in the mobile applications market.

Keywords signaling model, perfect Bayesian equilibrium, intuitive criterion

JEL Classification D43, D83, L15

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1. INTRODUCTION

Our motivating story in this paper concerns computer softwares or mobile applications ("*Apps*"). The product is an experience good Nelson (1970) in that the buyer needs to actually use it to determine its quality. It is durable since the buyer can continue to use it once purchased. But for effective future usage, the seller typically needs to maintain post-purchase services (e.g. fixes and updates) at some cost.

Casual observations suggest that many sellers of *Apps* offer a trial product at very low prices. Some *Apps* expire at the end of the trial and require the buyer to permanently purchase it in order to continue using it. Others do not expire but ask for further payments (in-*App* purchases) to activate additional functions. Moreover, some but not all *Apps* charge the buyer periodically in the form of subscription.

This paper offers a framework to understand and compare this plethora of sales practices. Our general strategy is to suggest that introductory pricing and subscription scheme work as signaling devices. Towards this end, we divide the interaction into two phases (the trial and the post-trial) and model each phase as signaling games.

In the trial phase, the buyer's uncertainty concerns the quality of the product, which introductory pricing can help mitigate. In the post-trial phase, the buyer's uncertainty is whether the seller would continue to offer services in the future ("long-livedness"), which subscription scheme can resolve. Our analysis of the trial phase is not limited to the *Apps* market story but applies to experience goods generally. The post-trial phase is more adapted to the *Apps* market story in that we need all of the above-mentioned features—quality uncertainty, durability and post-purchase maintenance.

Both introductory pricing and subscription have been discussed as sellers' strategies in the literature. Our contribution lies in combining these in a consistent framework. Our analysis may shed some light on the determinants and interactions of these pricing schemes.

To analyze the model, we first identify several perfect Bayesian equilibria. The fact that there are several forms of equilibria correspond to the real world experience of various sales practices. Still, in order to limit our attention to more plausible equilibria, we attempt to narrow down the set of equilibria using the Cho-Kreps intuitive criteria and some modification thereof, because commonly used intuitive criteria are not directly applicable in our context. Also by the parameter conditions under which different forms of equilibria arise, we identify the market circumstances for each practice.

There have been several explanations provided for the practice of introductory pricing in the literature. Bagwell (1987), for example, cites three: signaling of quality, price discrimination, and durability of products. Quality signaling is our main interest so we will discuss some previous work below. Price discrimination occurs by exploiting buyers' heterogeneity in how much they are informed or what their reservation prices are. We abstract away from these considerations, partly because we are mainly concerned about experience goods. Durable goods play some role in our story, but previous work on durable goods is mostly on cyclical sales rather than one-time introductory pricing.

Bagwell (1987) in fact presents an analysis of introductory pricing schematically similar to our trial phase model, arguing via a two-period signaling model that current financial sacrifice lead to repeat business. He also uses an earlier version of the intuitive criterion to refine some of the pooling equilibria. But in his model, the buyer's uncertainty is about hidden costs and not about quality, and it is exogenously assumed that a seller with lower cost will set a lower price in the future. He also assumes that it is costly for a buyer to learn the price charged. In our model, different types of sellers have identical marginal costs, and the buyer can observe prices costlessly, which seem to better represent the Internet environment.

Milgrom and Roberts (1986) argue that introductory prices can signal quality and also employ an earlier version of the intuitive criterion for refinement, but their major concern is on "contentless" advertising (or any seemingly useless but observable expenditures) as the signaling instrument. Moreover, while they acknowledge the relevance of post-introductory pricing path, Milgrom and Roberts (1986) do not elaborate on it and restrict their attention to the initial signaling stage with introductory pricing and advertising. Advertising is an interesting part of a seller's strategies (especially in the *Apps* market), but our analysis is geared towards pricing decisions and also on what happens *after* the initial signaling stage, which Milgrom and Roberts (1986) do not explore.

For durable goods markets, the problem of time-inconsistency and the need for "leasing" are well established since Coase (1972). Desai and Purohit (1987) argue that co-existence of leasing and selling contradict conventional theoretical findings that leasing is more profitable than selling for a durable goods firm; they attempt to resolve this via different depreciation rates of products. While our setting of post-purchase services could be re-interpreted in terms of product depreciation, Desai and Purohit (1987) model seems to be more suitable for "hardware" rather than "software" products. We also simplify by applying a common discount factor.

Zhang and Seidman (2010) discuss the leasing/selling comparison in the context of software licensing, contrasting between perpetual licensing and subscription licensing. They observe that subscription is becoming prevalent in the software market since the 2000s. But Zhang and Seidman (2010) analysis is driven by network externalities and does not consider signaling.

Our analysis is presented in two phases, which can be subsumed within a larger framework. Separate exposition is more transparent because each phase requires a lengthy consideration of different kinds of equilibria. Section 2 considers the trial phase where the post-trial phase outcomes are incorporated as the presence of future surpluses. Section 3 considers the post-trial phase, first with permanent purchase only and then subscription added as an alternative. In Section 4, we present the overall framework that combines the two phases and discuss their interaction.

2. INTRODUCTORY PRICING AS SIGNAL OF QUALITY

2.1. THE TRIAL PHASE MODEL

Throughout the paper, we consider games between one seller ("it") who offers some take-or-leave-it offer and one buyer ("she") who either accepts or rejects the offer.

In the trial phase, a seller offers a product at trial price $p \geq 0$. The seller's marginal cost of providing the product is $c > 0$. A buyer decides whether to try it. The product's value to the buyer is either $v > 0$ (high quality, H) or 0 (low quality, L).[1] The buyer doesn't know the quality before the trial and the prior probability of the product being of high quality is $\mu_0 \in [0,1]$. We make the following assumption so that it is socially optimal for high quality product to be traded.[2]

Assumption: $v > c$

After the trial, the quality is completely revealed to the buyer. If the product is of low quality, then the buyer ends the trial and never uses it again. If the

[1] That v is fixed and known is a strong assumption made for tractability. A referee suggested introducing different distributions of values for each quality with a stochastic order. This is an interesting extension but requires a whole new approach to the problem, so I leave it for future research.

[2] A referee pointed out that optimality is limited to the trial phase, as the overall payoffs involve future surpluses from repeat business (see below). But since the net future surpluses in turn are mostly determined by $v - c$, the assumption can ensure overall optimality as well.

product is of high quality, then the buyer engages in a prolonged transaction with the seller, which yields additional value $B \geq 0$ to the buyer and $S \geq 0$ to the seller. We will later elaborate on how B and S are determined. For the moment, we suppose that there is some potential gain to be shared between the buyer and the seller.

Then this is a two-stage signaling game where two types (H or L) of the seller use the price p as a signal for the quality. The timeline of the game is as follows.[3] Since we intend to embed this model in a long-term framework later, we distinguish between "stage" and "period". Hence, our two-"stage" model is essentially contained in a single "period", after which the next phase of the model continues. This next phase is subsumed into the payoff structure of the current game.

- stage 0 (pre-game): Nature determines the type (H or L) of the seller, with probability $\mu_0 \in [0,1]$ for type H.
- stage 1 (period 0): The seller observes its own type and chooses a trial price p
- stage 2 (period 0): The buyer observes the price p, forms the posterior belief μ_p on being H-type and decides whether to try the product
- payoffs (period 1 and on):
  - If the buyer chose not to try in stage 2, everyone's payoff is 0.
  - If the buyer chose to try and the quality is low, then the buyer's payoff is $-p$ and the seller's payoff is $p-c$.
  - If the buyer chose to try and the quality is high, then the buyer's payoff is $v-p+B$ and the seller's payoff is $p-c+S$.

We will look for perfect Bayesian equilibria (PBE) and attempt to apply the Cho-Kreps intuitive criteria to refine them. A PBE consists of (a) the seller's signaling (i.e., pricing) strategy, (b) the buyer's posterior beliefs on the product's quality on the equilibrium path and (c) the buyer's trial strategy. We also need to discuss (d) the posterior beliefs for signals off the equilibrium path. For ease of reference, we will denote the equilibrium conditions by (a)–(d) correspondingly. For simplicity, we will not consider randomized or hybrid strategies, so each type of the seller presents a single price and the buyer either tries or not.

[3] This part of the model is partly inspired by Harrington's (2015, chapter 11) discussion. (Harrington, 2015)

2.2. PERFECT BAYESIAN EQUILIBRIA

We consider three classes of PBE. In the first class (PBE-1), both seller types charge the same price (pooling signal) and the buyer never chooses to try the product. The second class (PBE-2) is also a pooling PBE but the buyer chooses to try in equilibrium. The third class (PBE-3) is a separating PBE where different types choose different prices and the buyer tries only high quality product. The high-quality seller's (low) price may be called an introductory price.

2.2.1 Pooling PBE with no trial

In this equilibrium, both seller types choose the same price p_1. The buyer does not try the product at any price, including p_1. Given the buyer's strategy, the seller is indifferent across all prices so any p_1 is equally optimal. While it is not inconceivable to use a negative price (paying a buyer to try it), we let for simplicity

$$p_1 \geq 0 \tag{1a}$$

We need to check sequential rationality of the buyer's strategy as well as to consider the buyer's posterior beliefs. After observing the equilibrium pooling signal p_1, the buyer's prior beliefs are maintained as the posterior beliefs

$$\mu_{p_1} = \mu_0 \tag{1b}$$

Then the buyer's expected payoff from the trial is $\mu_0(v+B) - p_1$. Since the buyer's equilibrium payoff is zero (no trial), we must have

$$p_1 \geq \mu_0(v+B) \tag{1c}$$

otherwise, the buyer may deviate. This places a lower bound on the no-trial equilibrium price.

If the buyer observes a non-equilibrium price $p \neq p_1$ such that $p \geq v+B$ the buyer would not deviate for any belief μ_p. If $c-S < p < v+B$, the buyer must hold the posterior μ_p such that $p \geq \mu_p(v+B)$ or

$$\mu_p \leq \frac{p}{v+B} < 1 \tag{1d}$$

If not, the buyer will want to try and the seller (at least H-type) will have an incentive to deviate to the price. If $p \leq c-S$, then the seller would never deviate to it, so the buyer may hold arbitrary beliefs. In the extreme case, $\mu_p = 0$ for all $p \neq p_1$ (any deviation is deemed a signal of low quality) works. The pooling PBE is summarized in the following lemma.

Lemma 1 (PBE-1: pooling with no trial)**.** *For the trial game, there are pooling equilibria with no trial such that*

- *Both seller types choose $p_1 \geq \mu_0(v+B)$.*
- *The buyer does not try at any p.*
- *Upon observing p, the buyer's posterior belief on being high quality is μ_0 for $p = p_1$ and μ_p for $p \neq p_1$. We need $\mu_p \leq p/(v+B) < 1$ for $c - S < p < v+B$ and unrestricted otherwise.*

This class of equilibria always exists for *any* values of parameters since p_1 can be arbitrarily high. The equilibrium price is independent of supply-side parameters c or S. The equilibrium is driven by a high price and "pessimistic" beliefs of the buyer about non-equilibrium prices. This equilibrium is uninteresting and will be shown to be implausible in our refinement later.

2.2.2 Pooling PBE with trial

Both seller types choose the same introductory price p_2 in equilibrium. The buyer chooses to try at price p_2. If L-type seller chooses p_2, it gets the payoff $u_L = p_2 - c$. So we need

$$p_2 \geq c \tag{2a}$$

Since H-type seller's payoff is higher by $S \geq 0$, we need not worry about H-type's supply incentives.

The buyer's posterior for p_2 is again same as the prior:

$$\mu_{p_2} = \mu_0 \tag{2b}$$

After observing the equilibrium price p_2, the buyer chooses to try, which requires

$$p_2 \leq \mu_0(v+B) \tag{2c}$$

placing an upper bound on the equilibrium price.

For (2a) and (2c) to be consistent, we must have

$$c \leq \mu_0(v+B) \implies \frac{c}{v+B} \leq \mu_0 \leq 1 \qquad \textbf{(pooling condition)}$$

In other words, the buyer must believe that there is a sufficiently high probability of the quality being high for a pooling (with trial) equilibrium to be possible. Since we assumed $v > c$, equilibria exist.

For a non-equilibrium price $p \neq p_2$, the buyer holds the posterior belief μ_p. If $p > v+B$, the buyer will not try regardless of μ_p. For $p < p_2$, whatever belief the buyer holds, the seller would not want to lower price from the equilibrium. So the buyer's posterior may be arbitrary for these cases. For $p_2 < p \leq v+B$, the buyer will not try only if $p > \mu_p(v+B)$ or $\mu_p < p/(v+B)$. Collecting these considerations, we conclude that the off-the-equilibrium posterior μ_p should be as follows:

$$\begin{cases} 0 \leq \mu_p \leq 1, & \text{for } p < p_2 \text{ or } p > v+B \\ 0 \leq \mu_p < \dfrac{p}{v+B}, & \text{for } p_2 < p \leq v+B \end{cases} \tag{2d}$$

Note that $\mu_p < 1$ for $p_2 < p \leq v+B$. The buyer puts a positive probability on L-type for these prices. The second class of PBE is summarized in the next lemma.

Lemma 2 (PBE-2: pooling with trial). *For the trial game with a prior μ_0, if*

$$\frac{c}{v+B} \leq \mu_0 \leq 1$$

there are pooling equilibria with trial such that

- *Both seller types choose $p = p_2$ where $c \leq p_2 \leq \mu_0(v+B)$.*
- *The buyer tries at $p = p_2$; does not try at $p > p_2$; may or may not try at $p < p_2$ depending on the posterior belief.*
- *The buyer's posterior is μ_0 for $p = p_2$ and μ_p for $p \neq p_2$ where $\mu_p < p/(v+B) < 1$ for $p_2 < p \leq v+B$ and unrestricted otherwise.*

Unlike the no-trial pooling equilibria, this class of pooling equilibria requires a sufficiently high prior μ_0. In words, the buyer must believe that there is a high *ex ante* probability that the product is of high quality. In addition, the buyer must *not* believe that a price higher than the equilibrium price is exclusively from H-type. The upper bound for μ_p increases with p, so the buyer may consider a higher price as a more likely signal of high quality but still must put a positive probability on low quality.

As Figure 1 illustrates, when a non-equilibrium price exceeds the equilibrium price ceiling $\mu_0(v+B)$ so that $p_2 \leq \mu_0(v+B) < p \leq v+B$, the posterior μ_p may be higher than the prior μ_0. (Note that this case doesn't occur if $\mu_0 = 1$.) On the other hand, if a non-equilibrium price falls within the equilibrium price range (Figure 2), μ_p must be lower than μ_0. To sustain the equilibrium, a moderately higher price must *lower* the buyer's belief that it is from H-type. In either case, we must have $\mu_p < 1$.

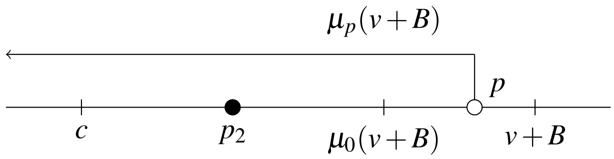


Figure 1: If $p > \mu_0(v+B)$ and $\mu_0 < 1$, the posterior μ_p can be higher than μ_0

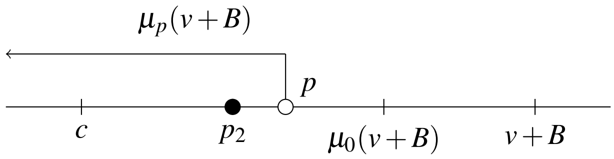


Figure 2: If $p_2 < p < \mu_0(v+B)$, the posterior μ_p must be lower than μ_0

2.2.3 Separating PBE

In a separating equilibrium, H-type seller chooses p_H and L-type seller chooses $p_L \neq p_H$. The buyer chooses to try only at p_H (high quality product).

L-type seller's equilibrium payoff is 0. If L-type imitates H-type by choosing p_H, its payoff will be $u_L = p_H - c$, which should not be positive to sustain the separating equilibrium.

$$p_H \leq c \tag{3a-L}$$

H-type seller's equilibrium payoff is $u_H = p_H - c + S$ so

$$p_H \geq c - S \tag{3a-H}$$

Combining (3a-L) and (3a-H), we have

$$c - S \leq p_H \leq c \tag{3a}$$

The high-quality seller can endure some short-term loss for long-term profit, which the low-quality seller cannot imitate. The maximum short-term loss H-type can endure is S, the future gain from repeat business. (If $S = 0$, then this fixes the equilibrium separating price as $p_H = c$.)

On the other hand, there is no restriction on p_L, other than $p_L \neq p_H$. We have four possible cases of equilibrium ranges for p_L:[4] (i) $p_L > c$, (ii) $p_L = c(\neq p_H)$, (iii) $c - S \leq p_L(\neq p_H) < c$, and (iv) $p_L < c - S$.

[4]I thank a referee for leading me to consider these cases.

The separating signals completely reveal the type to the buyer, so

$$\mu_{p_H} = 1, \quad \mu_{p_L} = 0 \tag{3b}$$

The buyer's payoff from low quality product is $-p_L$, so it is rational not to try for any $p_L > 0$. The payoff from high quality product is $v - p_H + B$ so we need

$$p_H \leq v + B \tag{3c}$$

Since we assumed $v > c$, we have $p_H \leq c < v + B$ hence (3a-L) implies (3c).

For a non-equilibrium price $p \neq p_H$ (and $p \neq p_L$), the buyer holds the posterior belief μ_p. The buyer will not try at too high a price ($p > v + B$) regardless of μ_p so μ_p can be arbitrary. As for $p < p_H \leq c$, again μ_p can be arbitrary: First, note that equilibrium payoffs are $u_H^* = p_H - c + S \geq 0$ for H-type and $u_L^* = 0$ for L-type. If μ_p is high enough so that the buyer tries, H-type's payoff is $p - c + S < p_H - c + S = u_H^*$ and L-type's payoff is $p - c < 0 = u_L^*$. Both types earn less than their equilibrium payoffs. If μ_p is low enough so that the buyer does not try, both types would receive zero, which is not greater than their equilibrium payoffs. Hence, for any posterior μ_p, neither type of seller has an incentive to deviate to $p < p_H$. For $p > p_H$, H-type may want to deviate, so the posterior should be bounded. Therefore,

$$\begin{cases} 0 \leq \mu_p \leq 1, & \text{for } p < p_H \text{ or } p > v + B \\ 0 \leq \mu_p < \dfrac{p}{v+B}, & \text{for } p_H < p \leq v + B \end{cases} \tag{3d}$$

Again note that $\mu_p < 1$ for $p_H < p \leq v + B$.

Lemma 3 (PBE-3: separating)**.** *For the trial game, there are separating equilibria such that*

- *H-type seller chooses $p_H \in [c - S, c]$ and L-type seller chooses $p_L \neq p_H$.*
- *The buyer tries at $p = p_H$; does not try at $p = p_L$; does not try at $p_H < p \leq v + B$; may or may not try at other prices depending on the posterior belief.*
- *The buyer's posterior is 1 for $p = p_H$, 0 for $p = p_L$ and μ_p for $p \neq p_H$ where $\mu_p < p/(v+B)$ for $p_H < p \leq v + B$ and unrestricted otherwise.*

If $S > 0$, we have a continuum of equilibrium prices (for H-type), where p_H is not higher than the marginal cost c. Such a price cannot be imitated by

L-type who does not have a future prospect and must earn now. Such p_H may be called an "*introductory price*". As for the buyer's beliefs, when observing a higher price, the buyer must not believe that this is exclusively from H-type. Since the upper bound for μ_p increases with p, the belief on H-type may increase as long as it doesn't become too high and still puts a positive probability on L-type. Unlike pooling equilibria with trial, separating equilibria do not require any parameter restriction on the prior μ_0.

2.3. REFINEMENT OF PBES

In this subsection, we seek to refine the plethora of equilibria that we derived. A natural way to proceed is to apply the Cho-Kreps intuitive criteria (Cho, Kreps, 1987). But as we will see shortly, the commonly used form of intuitive criteria cannot successfully narrow down the equilibria in our model. So instead we apply a modification of the equilibrium-dominance intuitive criterion, when necessary.

Cho and Kreps (1987) discuss two methods of refinement. The first is by *dominance*: a signal is *dominated* if its maximum possible payoff is less than another signal's minimum possible payoff. The second, referred to as the *intuitive criterion* by Cho and Kreps (1987), instead compares a signal's maximum possible payoff with the payoff from the equilibrium signal—we may say that the signal is *equilibrium-dominated* in this case.

In order to apply *dominance* in our model, we need to compute the maximum and minimum payoffs for signals (prices). There are two possible consequences to any price signal: 'trial' and 'no trial'. The payoff from 'trial' may be positive or negative, depending on the price, while the payoff from 'no trial' is always 0. Hence, if we denote the payoff from the trial as u^t, for any price p, the minimum payoff is $\min\{u^t, 0\} \leq 0$ and the maximum payoff is $\max\{u^t, 0\} \geq 0$. Therefore, for any pair of prices, there can be no dominance relation between them. That is, dominance refinement is inapplicable here.

Unfortunately, a similar consideration applies to *equilibrium-dominance* as well. The seller would sell only if its payoff is non-negative so the equilibrium payoff must be non-negative for both types: $u_L^* \geq 0$, $u_H^* \geq 0$. The maximum payoff from any non-equilibrium price is also non-negative. So in order for *equilibrium-dominance* to have a "bite", we need a strictly positive equilibrium payoff (for one type). But, for example, in PBE-1 (pooling with no trial), the equilibrium payoff is 0 for both types.

Therefore, we opt for a modification of the intuitive criterion by weakening of *equilibrium dominance* (hence strengthening of *intuitive criterion*) as follows.[5] While the definition for weak equilibrium dominance is stated in a general form, the modified intuitive criterion is explicitly given for L-type in our trial phase model for convenience.

Definition 1 (weak equilibrium dominance)**.** A signal p' is *weakly equilibrium-dominated* for type θ under a proposed equilibrium if type θ's equilibrium payoff u_θ^* is greater than or equal to the maximum possible payoff from p', i.e.

$$u_\theta^* \geq \max u_\theta(p', \cdot)$$

Definition 2 (modified Intuitive Criterion by weak equilibrium-dominance)**.** Let u_L^* and u_H^* be equilibrium payoffs for L- and H-type under a proposed equilibrium. If a non-equilibrium signal p is weakly equilibrium-dominated for L-type and is not weakly equilibrium-dominated for H-type, i.e.

$$u_L^* \geq \max u_L(p, \cdot) = \max\{p - c, 0\}, \quad u_H^* < \max u_H(p, \cdot) = \max\{p + S - c, 0\}$$

then the posterior belief μ_p for p should place probability 0 on L-type.

2.3.1 PBE-1 (pooling with no trial) is eliminated if $S > 0$

Consider PBE-1 where both seller types choose p_1 and the buyer does not try. Since both types fail to get a trial, the equilibrium payoff is 0 for both: $u_L^* = u_H^* = 0$. For a non-equilibrium price $p \neq p_1$, the buyer holds the belief $\mu_p < p/(v+B)$ if $c - S < p < v + B$ and unrestricted otherwise.

L-type's maximum payoff for $p \neq p_1$ is $u_L^{max} \equiv \max\{p - c, 0\}$. If $p > c$, then $u_L^{max} > 0$ so p is not weakly equilibrium-dominated for L-type. The same holds for H-type whose maximum payoff is $u_H^{max} = p + S - c > 0$. Hence, there is no further restriction on posterior beliefs for $p > c$.

On the other hand, if $p \leq c$, then $u_L^{max} = 0$ and p is weakly equilibrium-dominated for L-type: p yields at best the same payoff as the equilibrium, so L-type has little reason to deviate to p.

Now assume that $S > 0$. Then for $c - S < p \leq c$ (meaningful by $S > 0$), we have $u_H^{max} = p + S - c > 0$ so p is *not* weakly equilibrium-dominated for H-type: H-type has a reason to deviate to p. According to the modified intuitive criterion, the posterior belief must put $\mu_p = 1$ for $c - S < p \leq c$. But Lemma 1's PBE-1 (pooling without trial) requires $\mu_p < 1$ for $p > c - S$! Therefore, PBE-1 (pooling with no trial) is eliminated by our modified intuitive criterion, but only if $S > 0$.

[5]Cho and Kreps (1987) note that the intuitive criterion can be modified in this way. Also see Hahn and Kwon (1987) and Hahn and Kwon (1987) for an example of exploiting a different "extension" of the intuitive criterion.

If in a pooling equilibrium the buyer does not try, it is because price is too high. The seller with high quality product can try to entice the buyer by setting the price at or below the marginal cost. The buyer knows that the low-quality seller would never do this, so (correctly) inferring the product to be of high quality, the buyer is willing to try. We can't have a pooling equilibrium where the buyer never tries, if H-type has a positive future surplus.

If $S = 0$, then H and L-type are identical in terms of their own payoff functions (the only difference being the value provided to the buyer). The (modified) intuitive criterion has no bites and the pooling equilibria are sustained.

2.3.2 PBE-2 (pooling with trial) passes the modified intuitive criterion

Now consider PBE-2 where both seller types choose p_2 and the buyer tries in equilibrium. We require $c \leq p_2 \leq \mu_0(v+B)$. L-type's equilibrium payoff is $u_L^* = p_2 - c \geq 0$ and H-type's equilibrium payoff is $u_H^* = p_2 + S - c \geq 0$. (If $S > 0$, we have $u_H^* > 0$.)

Suppose a non-equilibrium price $p > p_2 \geq c$ is observed. L-type's maximum payoff is $p - c > p_2 - c = u_L^*$ so p is not weakly equilibrium-dominated for L. The same goes for H-type: $p + S - c > p_2 + S - c = u_H^*$. Therefore, there is no further restriction on posterior beliefs for $p > p_2$. It is okay to hold $\mu_p < p/(v+B)$ as PBE-2 requires. For a non-equilibrium price $p < p_2$, both types obviously would receive a lower payoff, so such p is equilibrium dominated for both types, again placing no further restriction on beliefs.

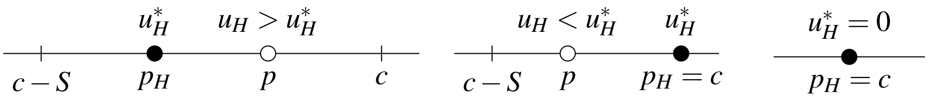
Hence, PBE-2 (pooling with trial) passes our modified intuitive criterion. Recollect that this equilibrium requires the prior μ_0 to be sufficiently high. As long as the probability of high quality is high enough, the buyer is willing to try at a moderately high price.

2.3.3 The modified intuitive criterion selects a "unique" PBE-3 (separating)

The equilibrium price for H-type satisfies the inequalities $c - S \leq p_H \leq c$. H-type's equilibrium payoff is $u_H^* = p_H - c + S \geq 0$, while L-type's equilibrium payoff is $u_L^* = 0$.

If $p > c \geq p_H$, then $u_L^{max} = p - c > 0 = u_L^*$ and $u_H^{max} = p + S - c > p_H + S - c$ so p is not weakly equilibrium-dominated for either type. There is no further restriction on posteriors and it is okay to hold $\mu_p < p/(v+B)$ as required by PBE-3. If $p < p_H \leq c$, neither type would have an incentive to deviate.

If $p_H < p \leq c$, L-type's maximum payoff is $u_L^{max} = 0$ and its equilibrium payoff is also $u_L^* = 0$ so p is weakly equilibrium-dominated for L. On the other hand, H-type's maximum payoff is $p + S - c > p_H + S - c = u_H^*$ so p is *not* weakly equilibrium-dominated for H-type. In other words, $p_H < p \leq c$ leads to the posterior $\mu_p = 1$ (contradicting the equilibrium condition $\mu_p < 1$) and H-type has an incentive to deviate to such p. Hence any $p_H < c$ fails to pass the modified intuitive criterion.



(a) If $p_H < c$, deviation to $p > p_H$ profitable (b) If $p_H = c$, no deviation (c) $S = 0$

Figure 3: Equilibrium price and deviations

We just observed that a deviation can occur for a non-equilibrium price between p_H and c [Figure 3(a)]. In other words, the only p_H that can prevent such a deviation is $p_H = c$ [Figure 3(b)]. Therefore, the "unique"[6] separating equilibrium that passes the modified intuitive criterion is $p_H = c$. This is also the unique equilibrium if $S = 0$ [Figure 3(c)].

By pricing not higher than the marginal cost, H-type seller can successfully distinguish itself to the buyer. Moreover, H-type seller need not price *strictly below* the marginal cost. Then the reasonable posteriors for $p \neq p_H = c$ are as follows:

$$\begin{cases} \mu_p = 1, & c - S \leq p < c \\ 0 \leq \mu_p < \frac{p}{v+B}, & c < p \leq v + B \\ 0 \leq \mu_p \leq 1, & p < c - S \text{ or } p > v + B \end{cases}$$

With p_H having been determined and the posteriors refined, we have yet to examine what values p_L can take in a separating equilibrium. In Section 2.2.3, we identified four possible equilibrium ranges for p_L: (i) $p_L > c$, (ii) $p_L = c(\neq p_H)$, (iii) $c - S \leq p_L(\neq p_H) < c$, and (iv) $p_L < c - S$.

Let us consider each case. The case (i) $p_L > c$ poses no difficulty and we shall later choose this to be the most plausible. Since our refined equilibrium has $p_H = c$, the case (ii) $p_L = c(\neq p_H)$ is eliminated. The case (iii) $c - S \leq p_L < c$ is technically possible but seems implausible because off-the-equilibrium belief

[6] I put quotation marks because p_L is not uniquely determined, although the uniqueness of p_H is more important and interesting. I thank a referee for pointing this out.

puts $\mu_p = 1$ for all $c - S \leq p < c$ except for p_L (where it is $\mu_{p_L} = 0$). A slight change (a "tremble") from p_L would lead the buyer to try, incurring a loss for L-type. So we reject this case, invoking an implicit requirement for the beliefs not to be too "jumpy".[7] For the case (iv), since there is no restriction on beliefs for non-equilibrium $p < c - S$, it is compatible with $p_L < c - S$ being an equilibrium.

So the "unique" separating equilibrium in fact has two sub-classes. One has $p_H = c$ and $p_L > c$ and the other has $p_H = c$ and $p_L < c - S$. If we define "introductory pricing" as a price at or below the marginal cost, then both sub-classes involve introductory pricing. In the first sub-class, high quality product is offered at marginal cost (introductory price), distinguishing itself from more highly-priced low quality product. In the second sub-class, low quality product is offered at an even lower price, which is so low that the buyer understands it to be low quality.

Informally speaking, the first sub-class seems more plausible. In the second sub-class, L-type does not gain anything by choosing $p_L < c - S$, except the non-equilibrium possibility of incurring a loss (if it slips in choosing the price or the buyer slips in forming posteriors). There is a downside risk (with probability zero, of course). But in the first sub-class, L-type in fact may entertain a non-equilibrium possibility of a positive profit (again with probability zero). Later on in discussing possible equilibrium paths, we will focus on the first sub-class where $p_L > c$, ignoring technically possible case of $p_L < c - S$.

We have narrowed down the set of our equilibria. The findings are collected in Proposition 1.

Proposition 1. *The plausible outcomes passing the modified intuitive criterion in the trial phase model are as follows:*

1. *If $S = 0$, the market may collapse: it is possible that the price is too high $[p_1 \geq \mu_0(v + B)]$ and the buyer never tries in equilibrium. On the other hand, if $S > 0$, there is no pooling equilibrium where the buyer doesn't try, so the buyer tries with positive probability for $S > 0$.*

2. *If $\mu_0 \geq c/(v + B)$, then there are pooling equilibria with trial: the single observed price is moderately high $[c \leq p_2 \leq \mu_0(v + B)]$ and the buyer tries in equilibrium. If a higher non-equilibrium price is observed, the buyer puts a positive probability on low quality and doesn't try.*

[7] Here I am appealing very loosely to robustness against small perturbations, used in more refined equilibrium notions such as sequential equilibrium and trembling hand perfect equilibrium.

3. *For any μ_0 and S, there are separating equilibria* (**introductory pricing**)*: H-type seller chooses $p_H = c$ and L-type seller chooses either $p_L > c$ or $p_L < c - S$ and the buyer tries only the H-type's product. If a price below the marginal cost (but above a threshold) is observed, the buyer believes it to be of high quality and would try. For a price higher than the marginal cost, the buyer puts a positive probability on low quality and doesn't try. The equilibrium payoff of the seller is 0 and that of the buyer is positive $v - c > 0$.*

2.4. DISCUSSION OF THE TRIAL PHASE MODEL

Our analysis reveals that there are essentially two classes of plausible outcomes, if future surplus S to H-type seller is positive. Which outcome obtains depends on the level of prior probability μ_0 of high quality. If μ_0 is sufficiently low, the expected outcome (separating equilibrium) is that there are two different prices offered on the market, the lower of them being at the marginal cost. The buyer tries the one priced at the marginal cost. This is introductory pricing.

If μ_0 is sufficiently high, then an additional possibility arises. The two-price equilibrium (introductory pricing) can still obtain but it may be that a single price not lower than the marginal cost prevails. The buyer tries the product knowing that it may be of low quality but highly likely to be of high quality. For any deviant higher price, the buyer is not convinced that it is of high quality so is not tempted.

Therefore, low quality product may remain in the market via a pooling equilibrium when its market share (as perceived by buyers) is not too high and the prevailing price is above the marginal cost. On the other hand, if buyers believe high quality is relatively rare, then the high quality seller will offer the product at the marginal cost to signal its quality.

We began with the supposition that the buyer and the H-type seller each enjoy potential surpluses $B \geq 0$ and $S \geq 0$ respectively from future repeat business. Do we need strictly positive surpluses for our trial phase outcomes?

The value of B does not affect equilibria significantly: B enters in the upper bounds for equilibrium prices and posteriors. Even if $B = 0$, we still have the above classes of equilibria. For the seller, the gist of introductory pricing is to endure short-term loss for future gain. But our plausible equilibria have H-type seller setting the price either (i) at the marginal cost (separating equilibrium) hence no short-term loss or (ii) at no less than the marginal cost (pooling equilibrium) hence possibly a short-term profit. So, $S > 0$ doesn't seem strictly necessary.

As we saw, if $S = 0$, it is possible for the market to collapse if the seller charges too high and the buyer is pessimistic. Moreover, if the seller requires a positive "overall (trial and post-trial)" profit, e.g. to recover fixed setup cost, then $S \approx 0$ would perhaps preclude the separating equilibrium (introductory pricing) in the trial phase. So one might argue that $S > 0$ is necessary for introductory pricing to occur.

It is time for us to examine how B and S are determined.

3. SUBSCRIPTION AS SIGNAL OF LONG-LIVEDNESS

In the trial-phase model, we posited that there would be some future surplus to be shared between the buyer and the seller (of high quality product) from repeat business. In this section, we now expand the horizon of our model to examine what happens *after* the trial and elaborate on the surpluses.

If the product in question is a perishable good to be consumed periodically by the buyer, the extension of the model is almost trivial. As long as the product offers value v to the buyer that exceeds the marginal production cost c, the buyer and the seller will continue to trade at some price $p \in [c, v]$. Then the buyer's surplus B and the seller's surplus S (discounted to the beginning of the repeat trade) may be written as[8]

$$B = \frac{1}{1-\delta}(v-p) \geq 0, \quad S = \frac{1}{1-\delta}(p-c) \geq 0$$

where $0 < \delta < 1$ is the discount factor. This provides one sufficient background for Section 2.

To make it more interesting, however, let us suppose that the product is durable and requires regular maintenance by the seller. A real-world example that we have in mind here is a computer software or a mobile application that the buyer can continue to use after the purchase. For convenience, we will refer to such products as *Apps*. Once an *App* is sold, the seller typically maintains "free" customer services for fixes and updates at some cost to the seller.

A buyer typically can try such products at zero[9] or very low introductory prices, then choose to permanently purchase it at a higher price or equivalently make an additional payment ("in-*App* purchase") to activate functions that are

[8]Hereafter we denote the total discounted payoffs in the post-trial phase as B and S for the buyer and the seller. Technically, this must be discounted once more for the overall payoffs. This abuse of notations shouldn't cause too much trouble.

[9]The marginal cost to the seller of a user downloading *Apps* is practically zero.

not available at the introductory price. This was the story provided by the trial-phase model.

One concern on the part of the buyer when making the permanent purchase decision is that although the buyer has confirmed the quality of the *App* from the trial, the seller may cease to maintain the service at some point in the future, in which case the total value of the *App* may decrease significantly. Such "exit" events may occur exogenously (the seller is unable to remain in the market due to factors outside its control such as insolvency or acquisition) or endogenously (the seller chooses to stop the service after selling the product). Let us refer to the exogenous exit as *being short-lived* and the endogenous exit as *exit-by-choice*.[10] At the time of purchase, the buyer does not know for certain whether the seller is short-lived or will exit by choice later. To put it in familiar jargon, short-livedness concerns hidden type problem (adverse selection) and exit-by-choice concerns hidden action problem (moral hazard).

If exit-by-choice is allowed, it is in the interest of the seller to stop offering costly services once the product has been sold. If the buyer anticipates this, she will be unwilling to pay for "future" values. So the only equilibrium outcome seems to be as follows: the buyer purchases only if the price doesn't exceed v and the seller exits immediately in the next period. This is undesirable for both the buyer and the long-lived seller. The long-lived seller would like a convincing way to *commit* itself not to exit.

On the other hand, if exit-by-choice is not allowed but a seller may be short-lived, then the buyer has to form a belief as to the seller's lifespan and assess the expected value of the purchase. Then a long-lived seller would want to reveal itself and charge a higher price. The problem is that the price by itself cannot be an effective signal of long-livedness.

In this section, we explore how the long-lived seller can engage in long-term transaction with the buyer, when the buyer fears that the seller may be short-lived and/or may exit by choice. We argue that subscription is a solution, which effectively turn a durable *App* into a series of renewable services with credible warranties. First, we examine why price alone cannot ensure long-term transaction.

3.1. PERMANENT PURCHASE WITHOUT EXIT-BY-CHOICE

If exit-by-choice is allowed and if no commitment device is available, long-term transaction is impossible. So for the moment imagine that a seller never

[10]The original draft of this paper considered exogenous exit only. I thank two anonymous referees for raising the issue of endogenous exit.

willingly exits by choice but may be forced to exit due to exogenous circumstances. We refer to the latter possibility as being (exogenously) short-lived.

As outlined in the timeline of the game in Section 2.1, the period 0 is the trial phase, where its main actions (introductory pricing and trial decision) occur. We now consider periods 1, 2, 3, ..., in which the buyer intends to use the product revealed as high quality and the seller provides maintenance at per period cost of $c > 0$.[11] Let the players apply the common discount factor $0 < \delta < 1$ per period.

The seller may be short-lived but the buyer can't tell when deciding to purchase the product permanently. We can model this as two (further) types among H-type sellers: a long-lived (ℓ-type) seller and a short-lived (s-type) seller. Let $0 \leq \ell_0 \leq 1$ be the prior probability of the seller being ℓ-type. ℓ-type seller operates indefinitely into the future offering value v to the buyer in each period. On the other hand, s-type seller operates for only "one" period.[12] To simplify, we assume that once the short-lived seller stops the maintenance services, the buyer's value drops to zero.

Then the buyer's value of the product from the long-lived seller is $\frac{1}{1-\delta}v \equiv V$, while that for the product from the short-lived seller is only v. Hence, the *ex ante* expected value is

$$\bar{v} \equiv \ell_0 V + (1-\ell_0)v = (\frac{\ell_0}{1-\delta} + 1 - \ell_0)v = \frac{1-\delta(1-\ell_0)}{1-\delta}v$$

On the other hand, the total discounted cost for the long-lived seller is $\frac{1}{1-\delta}c \equiv C$, while of course that for the short-lived seller is c.

Suppose the seller chooses a permanent price q and the buyer, after forming posterior beliefs, decides whether to purchase it permanently. We again look for PBEs in this model. This is easier than in the trial phase model, as we can quickly eliminate some forms of equilibria. We will follow Section 2 in considering (4) pooling with no purchase, (5) pooling with purchase and (6) separating. We will show that the only equilibrium is in the form of pooling with purchase.

3.1.1 Pooling with no purchase is not an equilibrium

First of all, we can rule out pooling equilibria where the buyer never purchases the product. Suppose both ℓ and s-type choose q_1 and the buyer doesn't purchase. No supply incentive condition applies, so we can simply set (4a)

[11] Conceptually this cost may be distinct from c of the model in Section 2. But because there is little direct linkage or comparison between the two costs in our model, we suppress the distinction and simply denote it as $c > 0$.

[12] This is without loss of generality to represent any finite number of operating periods.

$q_1 \geq 0$. The buyer's posterior is identical to the prior (4b) $\ell_{q_1} = \ell_0$. The buyer's expected payoff from purchase is $\overline{v} - q_1$ and she doesn't purchase, so we need

$$q_1 > \overline{v} = \ell_0 V + (1 - \ell_0)v \tag{4c}$$

Note that since $v > c$ and $V > C > c$, (4c) implies $q_1 > c$.

If the buyer holds the posterior belief ℓ_q on ℓ-type for a non-equilibrium price $q \neq q_1$ such that $q \leq V$, we must have $q > \ell_q V + (1 - \ell_q)v$ or

$$0 \leq \ell_q < \frac{q - v}{V - v} \tag{4d}$$

while for $q > V$, the buyer doesn't purchase regardless of the belief, so the posterior may be arbitrary.

The seller's payoff is 0 from choosing q_1. But if s-type deviates to $c < q < v$, (4d) becomes inconsistent and the buyer is willing to purchase it regardless of the belief (even if $\ell_q = 0$). So the equilibrium fails. If the price is too high, the short-lived seller can always lower the price sufficiently to secure a (one-period) transaction yielding a positive profit.

Lemma 4. *In the permanent purchase game without exit-by-choice, there is no pooling equilibrium where the buyer never purchases.*

3.1.2 Pooling PBE with purchase

Both types of sellers choose the same permanent price q_2. From ℓ-type seller's incentives, this must satisfy

$$q_2 \geq C = \frac{1}{1 - \delta} c \tag{5a}$$

This renders a similar condition for s-type seller redundant since $C > c$.

For the equilibrium price q_2, the buyer maintains the prior beliefs (5b) $\ell_{q_2} = \ell_0$ and purchases the product so

$$q_2 \leq \overline{v} = \ell_0 V + (1 - \ell_0)v \tag{5c}$$

For any non-equilibrium price $q \neq q_2$, the buyer forms the posterior belief ℓ_q. For $q < q_2$, the buyer can form any posterior because the seller has no incentives to lower the price. If $v > q > q_2$, for any posterior (even if $\ell_q = 0$) the buyer would purchase it and either type of seller would deviate to it, so the equilibrium price must satisfy a further condition

$$q_2 \geq v \tag{$*$}$$

We need both (5a) and ($*$), so we must have

$$q_2 \geq \max\{C, v\}$$

The lower bound for the pooling price is governed by both (ℓ-type's) total cost and one-period value to the buyer.

For $q > q_2 \geq v$ and the posterior ℓ_q, the buyer must not be tempted to purchase, so we need:

$$\ell_q V + (1-\ell_q)v < q \iff \ell_q < \frac{q-v}{V-v} \tag{5d}$$

Therefore, the pooling PBE is as follows.

Lemma 5 (pooling PBE with purchase)**.** *For the permanent purchase game without exit-by-choice, there are pooling equilibria with purchase such that*

- *Both ℓ and s-type sellers choose q_2 where* $\max\{C, v\} \leq q_2 \leq \bar{v}$.
- *The buyer purchases at $q \leq q_2$ and does not purchase at $q > q_2$.*
- *The buyer's posterior is ℓ_0 for $q = q_2$ and ℓ_q for $q \neq q_2$ where $\ell_q < (q-v)/(V-v)$ for $q > q_2$. In particular, we have $\ell_q < 1$ for $q > q_2$.*

Does the (modified) intuitive criterion help us refine this equilibrium? The equilibrium payoffs are

$$S_\ell^* = q_2 - C \geq 0, \quad S_s^* = q_2 - c > 0$$

For $q < q_2$, the maximum payoff can only be lower so q is equilibrium-dominated for both types. For $q > q_2$, the maximum payoff can only be higher so q is not (weakly) equilibrium-dominated for either type. Hence the intuitive criterion has no "bite" on this PBE.

3.1.3 Separating is not an equilibrium

We can easily see that there is no separating equilibrium. Suppose each type chooses prices q_ℓ and q_s respectively. If separating, the types are revealed. Even for s-type's product, the buyer is willing to purchase if the price is right. So we have $V \geq q_\ell \geq C$ ($> c$) and $v \geq q_s \geq c$. If $q_\ell > q_s \geq c$, then s-type has an incentive to imitate ℓ-type. If $q_s > q_\ell \geq C$, then ℓ-type has an incentive to imitate s-type. Since it doesn't hurt either type to raise the price, no separating equilibrium is possible.

Lemma 6. *In the permanent purchase game without exit-by-choice, there is no separating equilibrium.*

3.1.4 Discussion of the permanent purchase game without exit-by-choice

When exit-by-choice is excluded, if the seller may be exogenously short-lived, it is impossible to signal the long-livedness by the price alone, because the "inferior" short-lived seller has a lower marginal cost. The buyer is willing to trade with the short-lived seller since there is a surplus to be shared and a lower price will only hurt the long-lived seller.

The situation here is somewhat similar to that of the market for "lemons" (Akerlof, 1970). A seller of "peach" (good quality used car) cannot separate itself from a seller of "lemon" (bad quality used car) by price alone since it is not costly to quote a higher price. Akerlof (1970) argues that additional instruments such as warranties and certification can help resolve the problem in case of the used car market.

Similarly, our post-trial phase needs an additional instrument. Warranties may work as in the used car market. While hardware products typically have warranties, it is less so with software products, partly because many *Apps* become essentially new products when they are upgraded to a new version. Some sellers let previous users to freely upgrade while others require new purchases. In any case, we do not expect such warranties to last indefinitely (even for hardware products). Moreover, if we allow exit-by-choice (hence adding a moral hazard challenge as well), we would need a third party, such as the court of law or the governing body of the market platform, to enforce the warranty.

In the next subsection, we will argue that subscription can be a successful instrument for signaling both exogenous long-livedness and endogenous intention of not exiting by choice.

3.2. SUBSCRIPTION AS AN ALTERNATIVE

We now introduce another payment scheme for the product. Under *subscription*, the buyer pays the subscription "fee"[13] $r > 0$ each period. The seller promises to offer its maintenance *for the period*. If the seller is still active in the next period, the buyer can renew it by paying r again. So the transaction can go on indefinitely into the future, as long as the buyer pays r. The buyer can opt out if she wishes but will not do so in equilibrium.

The buyer's payoff if the product continues to be available indefinitely would

[13] For convenience, we use the term "fee" to distinguish it from the permanent "price".

be[14]

$$B = \frac{1}{1-\delta}(v-r)$$

Hence the buyer is willing to subscribe as long as $v \geq r$. On the other hand, two types of the seller face the same per period cost of c so they require $r \geq c$. The equilibrium subscription fee r should then satisfy

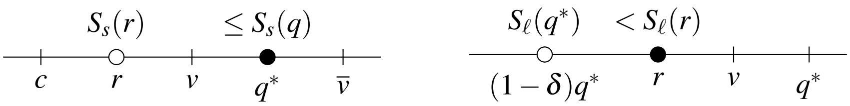
$$c \leq r \leq v$$

The post-trial payoffs (discounted to the beginning of the post-trial phase) for the sellers are

$$S_\ell(r) = \frac{1}{1-\delta}(r-c), \quad S_s(r) = r-c$$

3.2.1 Introducing subscription can destroy the pooling PBE with purchase

In the pooling PBE reported in Lemma 5, the equilibrium permanent purchase price q has the range $\max\{C, v\} \leq q \leq \bar{v}$. The price covers ℓ-type's total cost as well as the one period value v. As we just saw, the equilibrium subscription fee has the range $c \leq r \leq v$. The fee does not exceed v. Therefore $r \leq v \leq q$ and the subscription scheme is weakly dominated for s-type seller (see Figure 4(a) for illustration), so s-type does not deviate to subscription even if it is available. But the same is not true for ℓ-type.



(a) s-type weakly prefers purchase

(b) ℓ-type may prefer subscription

Figure 4: Purchase versus subscription

To see this formally, let q^* be a pooling purchase price equilibrium. So we have $\max\{C, v\} \leq q^* \leq \bar{v}$. Suppose a seller offers a subscription scheme with fee r, satisfying $c \leq r \leq v$. This is an off-the-equilibrium-path situation and the buyer needs to form a posterior belief, call it ℓ_r. Then the buyer's expected payoff from subscription is

$$B(r) = \ell_r \frac{1}{1-\delta}(v-r) + (1-\ell_r)(v-r) = \frac{1-\delta(1-\ell_r)}{1-\delta}(v-r) \geq 0$$

[14] Note that this is basically identical to the case of repeated purchase of perishable product discussed at the beginning of Section 3. Similarly for the long-lived seller's payoff S: see below.

For any posterior $\ell_r \in [0,1]$, we have $B(r) \geq 0$ if $r \leq v$. The buyer is willing to subscribe regardless of the belief. Given the buyer's response, will ℓ-type deviate? The equilibrium payoff is $S_\ell(q^*) = q^* - \frac{1}{1-\delta}c$. By deviating to r, the payoff will be $S_\ell(r) = \frac{1}{1-\delta}r - \frac{1}{1-\delta}c$. For equilibrium, we must have $S_\ell(q^*) \geq S_\ell(r)$ or

$$q^* \geq \frac{1}{1-\delta}r$$

Conversely, if $(1-\delta)q^* < r$, then ℓ-type would want to deviate to r. Since $q^* \leq \bar{v}$, the maximum possible value for $(1-\delta)q^*$ is

$$(1-\delta)q^* \leq (1-\delta)\bar{v} = (1-\delta)\frac{1-\delta(1-\ell_0)}{1-\delta}v = (1-\delta(1-\ell_0))v < v \quad \text{if } \ell_0 < 1$$

Therefore, it is possible to find a fee r such that $(1-\delta)q^* < r \leq v$ if $\ell_0 < 1$ (see Figure 4(b)). If the buyer puts a positive probability on the seller being short-lived, then the long-lived seller would want to deviate to subscription. Therefore, PBE from Lemma 5 is destroyed if subscription is available and if $\ell_0 < 1$.

Lemma 7. *Assume $\ell_0 < 1$. If subscription is available as a strategy for the seller, there is no pooling equilibrium where both the long-lived and the short-lived seller choose the same permanent purchase scheme.*

3.2.2 The post-trial phase model

Now recognizing two different pricing schemes, let us formally write down the post-trial game as follows:

- stage 0 (pre-game): Nature determines the type (ℓ, s) among the high quality seller, with probability $\ell_0 \in [0,1]$ for type ℓ.

  [The trial phase game is played in period 0 and the product is revealed to be of high quality.]

- stage 1 (period 1): The seller (knowing its type) offers either (i) a permanent price q or (ii) a subscription fee r

- stage 2 (period 1): The buyer observes the pricing scheme (q/r), forms the posterior belief $\ell_{q/r}$ on the seller being long-lived and decides to purchase or to subscribe or not to do anything.

- payoffs (period 1 and on):

- If the buyer chose not to purchase or subscribe, everyone's payoff is 0
- If the seller offered a permanent price, the buyer chose to purchase and the seller is ℓ-type, the buyer's payoff is $\frac{1}{1-\delta}v - q$ and the seller's payoff is $q - \frac{1}{1-\delta}c$
- If the seller offered a permanent price, the buyer chose to purchase and the seller is s-type, the buyer's payoff is $v - q$ and the seller's payoff is $q - c$
- If the seller offered a subscription scheme, the buyer chose to subscribe, and the seller is ℓ-type, the buyer's payoff is $\frac{1}{1-\delta}(v - r)$ and the seller's payoff is $\frac{1}{1-\delta}(r - c)$
- If the seller offered a subscription scheme, the buyer chose to subscribe, and the seller is s-type, the buyer's payoff is $v - r$ and the seller's payoff is $r - c$.

In Lemma 7, we established that pooling on permanent purchase is not an equilibrium if $\ell_0 < 1$. Hence, the remaining possibilities for PBE are either separating, or pooling on subscription. We can guess that ℓ-type would choose subscription in a separating equilibrium (if it exists). So let us first consider the subscription-only equilibrium.

3.2.3 A pooling PBE where both types choose subscription

It is conceivable that both types choose subscription but with different fees. But it won't happen. Suppose they offer subscription with fees r_s and r_ℓ. Both fees must be in the range $c \le r \le v$. From the buyer's perspective, subscription is a period-by-period contract so she is willing to subscribe as long as $v \ge r$, regardless of her beliefs on the type. So if $r_s \neq r_\ell$, then whichever type has the lower fee has an incentive to imitate the other type. The only possible subscription-only equilibrium is the pooling equilibrium where both types charge the maximum possible fee $r = v$. Let us denote this as r^*. Note that with fee $r^* = v$, the buyer's payoff is $B(r^*) = 0$ and the seller's payoff is $S_\ell(r^*) = \frac{1}{1-\delta}(v - c)$ and $S_s(r^*) = v - c$.

When the buyer observes a non-equilibrium signal of a permanent purchase contract with price q, the buyer 's posterior is ℓ_q. For pooling on r^* to be an equilibrium, we need the expected payoff from q to be non-positive

$$B(q) = \ell_q V + (1 - \ell_q)v - q \le 0 \iff \ell_q \le \frac{q - v}{V - v}$$

which applies to when $v \le q \le V$. For other non-equilibrium signals, the buyer purchases if $q < v$ and does not if $q > V$, regardless of the belief.

Does the posterior pass the intuitive criterion? For s-type, the maximum payoff from deviating to $q \ge v$ is $S_s^{max} = q - c \ge S_s(r^*)$. Hence q is not equilibrium-dominated (in the original Cho-Kreps sense). For ℓ-type, the maximum payoff from $q \ge v$ is $S_\ell^{max} = q - \frac{1}{1-\delta}c$. It is possible to have $v \le q < \frac{1}{1-\delta}v$ so that q is equilibrium-dominated for ℓ-type. Hence, the intuitive criterion requires $\ell_q = 0$, which is consistent with the PBE's posterior $\ell_q \le (q-v)/(V-v)$. The equilibrium passes the criterion.

Lemma 8 (pooling PBE with subscription for post-trial game)**.** *In the post-trial game where the seller may offer a permanent purchase scheme or a subscription scheme, there are pooling equilibria such that*

- *Both the long-lived and the short-lived sellers choose subscription with fee $r^* = v$.*
- *The buyer subscribes if $v \ge r$ and does not if $v < r$.*
- *For any non-equilibrium subscription fee $r \ne r^*$, the buyer may hold an arbitrary posterior. For a permanent purchase offer with price q, the buyer holds a posterior ℓ_q such that $\ell_q \le (q-v)/(V-v)$ for $v \le q \le V$ and an arbitrary posterior otherwise.*

3.2.4 A separating PBE where long-lived seller chooses subscription

First imagine that ℓ-type chooses a permanent price q_ℓ and s-type chooses a subscription fee r_s. The buyer now knows q_ℓ signals a long-lived seller so the ceiling for the permanent price expands from $\bar{v}$ to $V = \frac{1}{1-\delta}v$. Other considerations are the same as before, so $\max\{C, v\} \le q_\ell \le V$. On the other hand, the short-lived seller can set the subscription fee at $c \le r_s \le v$. But since $r_s \le v \le q_\ell$, the short-lived seller has an incentive to imitate the long-lived seller. This form of separating equilibrium is not sustainable.

We conclude that the only separating equilibrium is where ℓ-type chooses a subscription scheme with r_ℓ and s-type chooses a permanent price q_s. For subscription, the posterior belief doesn't affect the subscription decision. This in turn implies the equilibrium subscription fee has to be the highest possible in the feasible range. Hence, we have $r_\ell = v$. For fees $r \ne r_\ell$, the buyer may hold arbitrary beliefs. The buyer doesn't subscribe for a higher fee, and the seller doesn't offer a lower fee.

As for the permanent price q, it reveals the type as short-lived. The equilibrium price range is now reduced to $c \leq q_s \leq v$. For $q < q_s \leq v = r_\ell$, the buyer is willing to purchase for any posterior belief, but either type of seller would not prefer this lower price. If $q_s < q \leq v = r_\ell$, the buyer is still willing to purchase at any posterior belief, but then s-type would find it profitable to deviate while ℓ-type would not. Hence q_s must also be the highest possible for s-type, i.e. $q_s = v$. Finally, for $q_s \leq v < q < \ell_q V + (1-\ell_q)v$, the buyer may want to purchase and s-type seller would want to deviate (and ℓ-type might want to deviate as well). For this not to happen, we must have $\ell_q = 0$ for such q. This belief would pass the intuitive criterion as well because $q > v \geq q_s$ is never equilibrium-dominated for s-type and may or may not be equilibrium-dominated for ℓ-type. If it is equilibrium dominated for ℓ-type, $\ell_q = 0$ is the uniquely plausible belief, while if it is not any posterior is acceptable. Hence the separating equilibrium is essentially unique (up to arbitrary posteriors off the equilibrium path).

Lemma 9 (separating PBE for post-trial game)**.** *In the post-trial game where the seller may offer a permanent purchase scheme or a subscription scheme, there is a separating equilibrium such that*

- *The long-lived seller offers a subscription scheme of per period fee $r_\ell = v$. The short-lived seller offers a permanent price $q_s = v$.*
- *If the buyer observes a subscription fee $r_\ell = v$, the posterior on ℓ is* 1 *and the buyer subscribes. If the buyer observes a subscription fee $r \neq v$, the posterior is arbitrary. If $r > v$, the buyer doesn't subscribe, and if $r < v$, the buyer subscribes.*
- *If the buyer observes a permanent price $q_s = v$, the posterior is* 0 *and the buyer purchases it. If the buyer observes a permanent price $q > v$, the posterior on ℓ is* 0 *and the buyer doesn't purchase while for $q < v$, the posterior is arbitrary and the buyer purchases.*

In this equilibrium, both types seemingly offer the same price. In fact, as seen in Lemma 8, both types offer the same subscription fee $r^* = v$ in the subscription-only pooling equilibrium as well. Then the question arises as to what to make of a subscription offer from the short-lived seller as well as a seemingly identical permanent purchase price offer. Discussion will be given in the next subsection.

The following proposition collects the findings. We also present equilibrium payoffs because these are foundations for the trial phase model.

Proposition 2. *In the post-trial phase, with exogenous short-livedness and without exit-by-choice, possible equilibria are as follows.*

- **(no subscription equilibria)** *If subscription is not available and the seller can only offer a permanent purchase scheme, both types choose the same permanent purchase price q such that*

$$\max\{\frac{1}{1-\delta}c, v\} \leq q \leq \frac{1-\delta(1-\ell_0)}{1-\delta}v = \overline{v}$$

and the buyer purchases in equilibrium. If the buyer observes a higher non-equilibrium price, she puts a positive probability on s-type. The equilibrium payoffs for the sellers are

$$S_\ell(q) = q - \frac{1}{1-\delta}c \geq 0, \quad S_s(q) = q - c > 0$$

The equilibrium payoff for the buyer is

$$B(q) = \overline{v} - q \geq 0$$

- **(purchase or subscription equilibria)** *If the seller can offer either a permanent purchase scheme or a subscription scheme*

  – **(pooling on subscription)** *Both types offer the subscription scheme with per period fee $r^* = v$. The buyer accepts it in equilibrium. If the buyer observes a permanent purchase scheme with price q such that $v \leq q \leq V$, she holds a posterior $\ell_q \leq (q-v)/(V-v)$. The equilibrium payoffs for the sellers are*

$$S_\ell(r^*) = \frac{1}{1-\delta}(v-c) > 0, \quad S_s(r^*) = v - c > 0$$

  The equilibrium payoff for the buyer is $B = 0$.

  – **(separating)** *ℓ-type seller offers a subscription fee v and s-type seller offers a permanent price v. The buyer accepts the offered scheme in equilibrium. The equilibrium payoffs for the sellers are*

$$S_\ell(r) = \frac{1}{1-\delta}(v-c) > 0, \quad S_s(q) = v - c > 0$$

  The equilibrium payoff for the buyer is $B = 0$.

We argued earlier that subscription is weakly dominated for s-type seller. We also checked that ℓ-type has an incentive to deviate from a pooling on purchase equilibrium. A natural question arises to whether each type indeed shows such preferences between these equilibria. It does.

Proposition 3. *The short-lived seller weakly prefers permanent purchase equilibria to subscription equilibria. The long-lived seller strictly prefers subscription equilibria to purchase only equilibria if* $\ell_0 < 1$.

Proof. For any permanent purchase price $q \geq v$, the difference between s-type seller's payoffs from q and from equilibrium subscription fee $r^* = v$ is

$$S_s(q) - S_s(r) = q - v \geq 0$$

The difference between ℓ-type seller's payoffs from equilibrium subscription fee $r^* = v$ and from equilibrium purchase-only price q^* is

$$S_\ell(r^*) - S_\ell(q^*) = \frac{1}{1-\delta}v - q^* \geq \frac{1}{1-\delta}v - \frac{1-\delta(1-\ell_0)}{1-\delta}v > 0$$

since $q^* \leq \bar{v} = (1-\delta(1-\ell_0))/(1-\delta)v$ and $\ell_0 < 1$. ■

The intuition is simple. As long as the buyer perceives a possibility of short-livedness, she must discount the future value she expects to receive so is not willing pay the actual value provided by the long-lived seller. By resorting to subscription, the long-lived seller can instead extract the actual value that it offers to the buyer.

The condition $\ell_0 < 1$ is not too strong, as the case $\ell_0 = 1$ precludes the issue of short-livedness. In fact, if $\ell_0 = 1$, then the price range for permanent purchase-only equilibrium expands to $\max\{C, v\} \leq q^* \leq V$ and we have $S_\ell(r^*) \geq S_\ell(q^*)$ where the equality holds only if $q^* = V$. Hence, ℓ-type weakly prefers subscription equilibria even when $\ell_0 = 1$.

3.3. EXIT-BY-CHOICE AND OTHER DISCUSSIONS ON THE POST-TRIAL PHASE MODEL

We have been vague about what 'permanent purchase' and 'subscription' transactions entail. The explicit terms of transaction were given only for payments. In permanent purchase, payment occurs once at the beginning and in subscription, it occurs every period. In light of our obtained results, we can now attempt to clarify the terms and also consider how to incorporate exit-by-choice in the model.

We have already observed that without a third party enforcement, the seller cannot credibly commit to staying in the market. A warranty (with a third party) legally binds the seller to provide services, but for a limited duration. In fact, what we refer to as "one period" may stand for a finite number of actual time periods[15], so we can think of "one period" as a typical minimal warranty period. In this interpretation, a permanent purchase contract comes with a limited warranty for future service. Due to the presence of short-lived type as well as the incentive for the long-lived type to exit by choice, the only credible warranty is for one period. The short-lived seller fulfills the warranty period and is driven out of the market. The long-lived seller also offers a limited warranty but can stay in the market, on a "goodwill" basis, so to speak.[16] Hence, the distinction between ℓ-type and s-type can be expanded. The key is ℓ_0, which in the buyer's mind represents the proportion of those sellers who will provide service beyond the legally required warranty period. The apparent short-lived seller may have been forced out of the market due to exogenous reasons, but also may have exited by choice. What is important is that when facing a seller (before observing a price signal) the buyer has some prior probability ℓ_0.

When a subscription contract enters, it changes the scene dramatically. When a buyer pays the fee, she is ensured for service for that period. In this sense, a subscription is a way to renew the limited warranty indefinitely into the future. Under a subscription contract, the buyer is not harmed even if the seller is short-lived (although she would have liked to get continued services). Furthermore, the long-lived seller can enjoy a higher profit under subscription, than under permanent purchase where it was subject to the doubt in the buyer's mind that it may be short-lived or exit by choice.

Corollary 1. *Even if exit-by-choice is allowed, the long-lived seller does not exit if it can offer subscription.*

This result is termed a corollary because it doesn't require any new insights. If the seller is allowed to exit for permanent price contract, the equilibrium price is $q = v$ (since the buyer anticipates the exit) and the ℓ-type seller's payoff is only $(v - c)$, which is lower than $S_\ell(r)$. If exit is not allowed for permanent price contract, Proposition 3 shows ℓ-type seller strictly prefers subscription for $\ell_0 < 1$ and weakly prefers subscription for $\ell_0 = 1$. Furthermore, the seller has no incentive to exit in the middle of a subscription contract, as it earns positive profit every period.

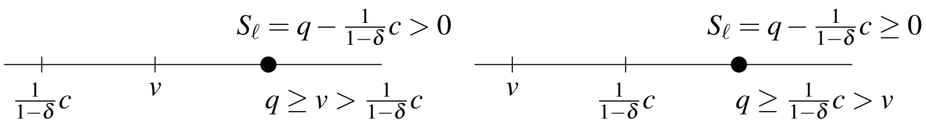
[15] See footnote 12.

[16] Perhaps the seller has reputational concerns, or other related products to sell, etc, which go beyond the confines of our model.

Turning to the real world for a moment, in light of the observation that subscription licensing appeared later than perpetual licensing in the software market (Zhang and Seidman, 2010), we might ask what circumstances changed in the market. Our analysis suggests some explanations. While not as critical as the prior μ_0 in the trial phase, the prior ℓ_0 does have some place in the post-trial phase. Note that ℓ_0 is about buyers' beliefs, not actual market shares, so it may be that in software buyers' minds, most sellers were short-lived in the early days of the market. Then the equilibrium is either pooling-on-purchase (with relatively low purchase price since $\overline{v}$ is low when ℓ_0 is low), or separating but those long-lived sellers offering subscription are rare in the market.

As ℓ_0 rises (because buyers are becoming aware of more long-lived sellers), the long-lived sellers have an incentive to distinguish themselves but equilibrium price can also rise now that the expected value $\overline{v}$ is higher. If there were some friction (from consumer's inertia and/or "menu" costs involved in changes), the long-lived seller may not deem the change of business practice worth the effort.

Another consideration may be added regarding the relative sizes between parameters c and v. The floor for the price is $\max\{\frac{1}{1-\delta}c, v\}$ since the price must be acceptable to both the ℓ-type seller and the buyer. If c or δ is low so that $\frac{1}{1-\delta}c < v$, then the equilibrium price would be strictly higher than ℓ-type seller's total cost, leading to a strictly positive payoff [Figure 5(a)]. In other words, if in the early days of the market, the seller's maintenance cost is not too high and the seller doesn't care much about the future, the seller may be quite content with the permanent price scheme (especially if there is some friction in switching). In contrast, if future maintenance costs become a more significant consideration for the seller so the seller recognizes that it is in the business for the "long haul", it will seriously consider switching to a subscription scheme [Figure 5(b)].



(a) when $\frac{1}{1-\delta}c < v$ (b) when $\frac{1}{1-\delta}c > v$

Figure 5: ℓ-type's payoff under pooling permanent price

While the above are merely theoretical speculations, they do provide some potentially testable implications involving the model parameters ℓ_0, c, δ and v and the pricing schemes.

4. COMBINING THE TRIAL AND THE POST-TRIAL GAMES

So far we have considered the trial phase and the post-trial phase separately, although acknowledging the linkage between the phases via the surpluses B and S. What we have in fact is a bigger two-stage game, with each phase corresponding to a stage. Since we have two instances of signaling over the game play, we need to examine the dynamic nature of the game more closely.

We need to face the issue of *when* types are realized and signals sent, particularly for the second stage (post-trial phase). There are two possibilities. First, it may be the case that the long-livedness of the (high quality) seller is revealed after the trial phase. In this case, the seller chooses the first signal (the trial price) without knowing its long-livedness. The game plays out as two ensuing signaling games. This is analogous to the notion of *behavioral strategies* in dynamic games where randomization over pure strategies occurs at each information set when it is reached. Let us refer to this as *sequential signaling*.

Second, another possibility is that the seller, if it is H-type, also learns whether it is long-lived at the beginning of the game. In this case, the seller's signal in the trial phase may contain information on its long-livedness. The seller sends a single signal rather than a series of signals. This is analogous to the notion of *mixed strategies* where randomization (over pure strategies) occurs at the beginning of the game. We shall refer to this as *one-shot signaling*.

Successful implementation of the trial phase may require the players to be aware of the amount of surplus to be shared, so it may be more transparent for players if the pricing scheme for the post-trial phase is presented upfront at the trial stage. This is the case of one-shot signaling. On the other hand, at least some *Apps* do not advertise its purchase price (or subscription fee) at the time of trial. This corresponds to the case of sequential signaling. Moreover, the setup of sequential signaling may be able to capture dynamic nature better.

We will first examine sequential signaling model in more detail, as its dynamic structure is more intuitive and also our previous results can be exploited more easily.

4.1. SEQUENTIAL SIGNALING MODEL

The combined game is obtained by gluing together the two phases. The overall game is as follows:

- stage 0 (pre-game): Nature determines the type (H, L) of the seller, with probability μ_0 for H-type.

- stage 1 (period 0): The seller observes its own type and chooses a trial price p.
- stage 2 (period 0): The buyer observes the price p, forms the posterior belief μ_p on the product being of high quality and decides whether to try the product
  - If the buyer chooses not to try, then the game ends and everyone's payoff is 0.
  - If the buyer chooses to try, the quality is revealed. If the quality is low, the game ends. The buyer's payoff is $-p$ and the seller's payoff is $p-c$.
  - If the buyer chooses to try and the quality is high, the game proceeds to stage 3
- stage 3 (period $0^1/_2$, between 0 and 1):
  - Nature determines the type (ℓ, s) of the (H-type) seller, with probability ℓ_0 for ℓ-type.
  - The seller observes its own type and chooses either a permanent purchase price q or a subscription fee r.
- stage 4 (period 1 and on): The buyer observes the pricing scheme, forms the posterior belief $\ell(q/r)$ on the seller being long-lived and chooses whether to accept the offer
- payoffs:
  - If the buyer chooses not to accept, the buyer's payoff is $v-p$ and the seller's payoff is $p-c$
  - If the buyer accepts a permanent price offer, the buyer's payoff is $v-p+\delta(V^e-q)$ where V^e is the expected sum of the future values, while ℓ-type seller's payoff is $p-c+\delta(q-\frac{1}{1-\delta}c)$ and s-type seller's payoff is $p-c+\delta(q-c)$.
  - If the buyer accepts a subscription offer, the buyer's payoff is $v-p+\frac{\delta}{1-\delta}(v-r)$, while ℓ-type seller's payoff is $p-c+\frac{\delta}{1-\delta}(r-c)$ and s-type seller's payoff is $p-c+\delta(r-c)$.

We are interested in how the post-trial pricing scheme may affect the trial decision phase.

4.1.1 Possible equilibrium paths

From Proposition 1, we know that higher B renders a pooling trial equilibrium more likely ($\mu_0 \geq c/(v+B)$ is the condition) with a higher ceiling for the equilibrium trial price ($p \leq \mu_0(v+B)$ is the range). B also affects the posteriors (see Lemmas 1 through 3). On the other hand, the size of S is not as critical: it doesn't affect pooling trial equilibria at all and $S = 0$ reduces separating equilibria into a "unique" one (which the intuitive criterion also picks out). The only worry is that $S = 0$ might result in a pooling equilibrium with no trial.

From Proposition 2, we know $S > 0$ for most cases in the post-trial equilibria. The only possible case for $S = 0$ is a pooling equilibrium with both ℓ and s-type charging exactly $q = C = c/(1-\delta)$. The probability of this equilibrium obtaining, out of a continuum of pooling equilibria, seems minuscule. Moreover, only ℓ-type seller's S is zero, but ℓ-type has a better alternative of introducing subscription. So while this case is technically possible as an equilibrium, we will henceforth ignore it and deal with $S > 0$ cases only.

We also know from Proposition 2 that $B \geq 0$ in a purchase-only equilibrium and $B = 0$ when subscription is available. Hence, if subscription is not introduced and the buyer expects a big surplus $B(q) > 0$ in the post-trial phase (which can happen if the buyer believes sellers to be mostly long-lived and the seller discounts future heavily), then it is possible that the trial price is higher than the marginal cost (no introductory pricing) and low-quality product remains in the market. On the other hand, if subscription is offered then the buyer doesn't expect a positive surplus and introductory pricing can occur especially if there are many perceived low quality products.

So we can think of 4 possible equilibrium outcome paths[17] as summarized by the following table.

Which path is more likely depends on the buyer's priors μ_0 and ℓ_0. If μ_0 is high, both pooling and separating (introductory pricing) are possible in the trial phase, while if μ_0 is low, introductory pricing would prevail. The permanent purchase scheme is viable if ℓ_0 is sufficiently high and/or δ is sufficiently low (for the ℓ-type seller) especially in the presence of inertia and switching costs from permanent purchase to subscription. If ℓ_0 is low and δ is high, then the subscription scheme prevails in the post-trial phase, which in turn makes it more likely that introductory pricing occurs in the trial phase.

Let us examine each equilibrium outcome path in some detail. For conve-

[17] We ignore pooling with no trial as it requires $S = 0$. We also ignore pooling on subscription, as it is practically indistinguishable from separating and s-type weakly prefers permanent purchase.

Table 1: Possible equilibrium outcome paths for the combined model

			post-trial phase	
			[P] permanent q only (no subscription) $\max\{C,v\} \leq q \leq \overline{v} < V$	**[S]** permanent q, subscription r (subscription) $q = r = v$
trial phase	**[P]** pooling p	condition	$\mu_0 \geq \dfrac{c}{v + B(q)}$	$\mu_0 \geq \dfrac{c}{v}, \quad \ell_0 < 1$
		equilibrium	$c \leq p \leq \mu_0(v + B(q))$	$c \leq p \leq \mu_0 v$
	[S] separating (p_H, p_L)	condition	none	$\ell_0 < 1$
		equilibrium	$p_H = c$	$p_H = c$

where $B(q) = \overline{v} - q \geq 0$

nience, we will refer to the pooling equilibrium in the trial phase as "no introductory pricing" and to the permanent purchase-only equilibrium in the post-trial phase as "no subscription". We will also label each path as "XY" where X = P(ooling) or S(eparating) and Y = P(ermanent purchase) or S(ubscription). So we have 4 paths to consider, namely PP, PS, SP and SS.

Our basic approach is backward induction, relying on equilibrium characterizations given in earlier sections.

4.1.2 No introductory pricing and no subscription (PP) path

Consider PP path where both H and L-type charge a same (relatively high) trial price and both ℓ and s-type offer a same permanent purchase contract.

For backward induction, consider the post-trial phase first. Both ℓ and s-type sellers charge some q^* where $\max\{C,v\} \leq q^* \leq \overline{v}$. The short-lived seller enjoys a high payoff as the price exceeds its one-period cost. The long-lived seller recovers its total cost but the revenue falls short of the value it provides. The buyer faces a risk of not getting the value (when she purchases from s-type). If ℓ_0 is low (short-livedness highly likely), the price is closer to one-period value, lowering the seller's profit.

For any non-equilibrium price $q > q^*$, the buyer holds the posterior $\ell_q < (q - v)/(V - v) < 1$, meaning a higher deviant price does not convince the buyer of the seller's long-livedness. The intuitive criterion cannot refine the beliefs any further, because a higher price (if accepted by the buyer) benefits both types of the seller.

Now consider the trial phase, given what we described above for the post-trial phase. In order for pooling to succeed, L-type seller must imitate H-type seller, who offers one permanent price q^* as well as one trial price p^* where

$c \le p^* \le \mu_0(v+B(q^*)) = \mu_o(v+\overline{v}-q^*)$. This equilibrium is obtained only if μ_0 is sufficiently high: $\mu_0 \ge c/(v+B(q^*)) = c/(v+\overline{v}-q^*)$. The buyer's posterior for a higher price ($p^* < p \le v+B(q^*)$) puts positive probability on S-type: $\mu_p < p/(v+B^*(q)) = p/(v+\overline{v}-q^*) \le 1$.

Since there are many parameters involved, it will be useful to look at some specific (and extreme) cases.

Case (1) $q^* = C > v$

Suppose $C = \frac{1}{1-\delta}c > v$ so that $c < v < \frac{1}{1-\delta}c \approx (1+\delta)c$. This is more likely when δ is high. In this case, we have $q^* = C$ and $B(C) = \overline{v} - C = \frac{1-\delta(1-\ell_0)}{1-\delta}v - \frac{1}{1-\delta}c \ge 0$. Then pooling in the trial phase requires

$$\mu_0 \ge \frac{c}{v + \frac{1-\delta(1-\ell_0)}{1-\delta}v - \frac{1}{1-\delta}c} = \frac{c(1-\delta)}{(2+(\ell_0-2)\delta)v - c} \equiv \underline{\mu}$$

The condition involves several parameters. We can see by inspection that this condition is easier to meet ($\underline{\mu}$ is lower) if ℓ_0 and v are high. We can easily check that $\partial\underline{\mu}/\partial c > 0$ (easier to meet if c is low).[18] Finally, we have $\partial\underline{\mu}/\partial\delta < 0$ if $\ell_0 > c/v$ (easier to meet if δ and ℓ_0 are both high).[19] In this equilibrium, the trial price range is $c \le p^* \le \mu_0(v+\overline{v}-C)$. For concreteness, let's consider numerical examples.

Example 1. (1) Let $v = 150$, $c = 100$, $\delta = 0.9$ and $\ell_0 = 0.7$. The permanent purchase price is $q^* = 1000$. Then the trial pooling condition is $\mu_0 \ge \frac{100}{245} \approx 0.41$. Let $\mu_0 = 0.5$. Then the trial price range is $100 \le p^* \le 325$. The trial price can exceed the triple of the marginal cost.

(2) Let $v = 150$, $c = 100$, $\delta = 0.5$ and $\ell_0 = 0.7$. The permanent price is $q^* = 200$. The trial pooling condition is $\mu_0 > 0.49$. Again let $\mu_0 = 0.5$. The trial price range is $100 \le p^* \le 125$. With lower δ, future is more heavily discounted, which can lower the equilibrium trial price. Now the maximum excess over the marginal cost is only 25%. ∎

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[18] $\dfrac{\partial\underline{\mu}}{\partial c} = \dfrac{(1-\delta)(2+(\ell_0-2)\delta)v}{[\cdots]^2} > 0$ since $0 < \delta < 1$, $0 \le \ell_0 \le 1$ and $v > 0$

[19] $\dfrac{\partial\underline{\mu}}{\partial\delta} = \dfrac{c(c-\ell_0 v)}{[\cdots]^2}$

Case (2) $q^* = v > C$

Now suppose $v > C = \frac{1}{1-\delta}c$ so that v is relatively higher than before. This is more likely when δ is lower. (For example, when $v = 150$, $c = 100$ as in Example 1, we need $\delta < 1/3$.) In this case, we have $q^* = v$ and $B(v) = \overline{v} - v = \frac{\delta \ell_0}{1-\delta} v > 0$. The trial pooling condition is[20]

$$\mu_0 \geq \frac{c}{v + \frac{\delta \ell_0}{1-\delta} v} = \frac{c(1-\delta)}{(1-\delta(1-\ell_0))v} \equiv \underline{\mu}$$

Here $\underline{\mu}$ is lower (easier to meet the pooling condition) when ℓ_0 is higher, v is higher, c is lower and δ is higher. The trial price range is $c \leq p^* \leq \mu_0 \overline{v}$.

Example 2. Let $v = 150$, $c = 100$, $\delta = 0.2$ and $\ell_0 = 0.7$. Then $q^* = v = 150$. The pooling condition is $\mu_0 > 0.57$. Let $\mu_0 = 0.6$. Then the trial price range is $100 \leq p^* \leq 112.5$. The trial price is relatively low, because the future is discounted heavily which leads to lower total value $\overline{v}$. ∎

Case (3) $q^* = \overline{v}$

Now we consider the case of the maximum possible permanent price $q^* = \overline{v}$. Then $B(\overline{v}) = 0$. The trial pooling condition is $\mu_0 \geq c/v$. The trial price range is $c \leq p^* \leq \mu_0 v$.

Example 3. Let $v = 150$, $c = 100$, $\delta = 0.9$. The $q^* = 1500$. The pooling condition is $\mu_0 \geq 2/3$. Let $\mu_0 = 0.7$. Then the trial price range is $100 \leq p^* \leq 105$. As the upper bound is $\mu_0 v \leq v$, the trial price is relatively low, although it may be higher than the marginal cost. ∎

4.1.3 No introductory pricing and subscription (PS) path

In PS path, H and L-type sellers charge the same trial price, while (among H-type) ℓ-type offers subscription and s-type offers permanent price. Then the equilibrium path is straightforward. Because we know $q^* = r^* = v$ in the post-trial phase, we have the easy equilibrium characterization as $\mu_0 \geq c/v$ and $c \leq p^* \leq \mu_0 v$. This is in fact identical to that in Section 4.1.2 where we considered the maximum possible permanent price in PP path. See Example 3.

[20] Not wanting to introduce too many notations, we again denote the RHS as $\underline{\mu}$, not to be confused with the previous $\underline{\mu}$.

4.1.4 Introductory pricing and no subscription (SP) path

Now consider SP path, where H-type and L-type choose different trial prices, where $p_H = c$ is the introductory price and $p_L > c$ is rejected by the buyer. When H-type sellers advance to the post-trial phase, both ℓ and s-type offer a same permanent price contract with $\max\{C, v\} \leq q \leq \bar{v}$.

Unlike in 4.1.2 when we examined PP path, we need not worry about the pooling condition.

4.1.5 Introductory pricing and subscription (SS) path

The last path has the seller choosing separating strategies in each phase and all prices are essentially determined in equilibrium. That is, we have $p_H = c$ (accepted), $p_L > c$ (rejected) in the trial phase and $q = v$ and $r = v$ (both accepted) in the post-trial phase.

4.2. ONE-SHOT SIGNALING MODEL

Next, let us briefly consider another model of the combined game. In this game, all types are realized and observed by the seller at the beginning. One signal (covering both trial and post-trial contracts) is sent upfront. Then we have 3 types of the seller. We may need to consider semiseparating equilibria (where two out of three types choose the same signal). But the nature of our problem is such that this structure is not so complicated to analyze. The game is as follows.

- stage 0 (pre-game): Nature determines the type ($H\ell$, Hs, L) of the seller, with probability $\mu_0\ell_0$ for $H\ell$-type and $\mu_0(1-\ell_0)$ for Hs-type.

- stage 1 (period 0): The seller observes its own type and chooses (i) an introductory price p and either (ii-1) a permanent price q or (ii-2) a subscription fee r. Denote a pricing scheme as $(p, q/r)$ where it is understood that either q or r is "null".

- stage 2 (period 0): The buyer observes the pricing scheme $(p, q/r)$, forms the posterior belief $\mu(p, q/r)$ on the product being of high quality and decides whether to try the product

  - If the buyer chooses not to try, then the game ends and everyone's payoff is 0.

- If the buyer chooses to try, the quality is revealed. If the quality is low, the game ends. The buyer's payoff is $-p$ and the seller's payoff is $p-c$.
- If the buyer chooses to try and the quality is high, the game proceeds to stage 3

• stage 3 (period 1 and on): The buyer forms the posterior belief $\ell(q/r)$ on the seller being long-lived and chooses whether to accept the offer

• payoffs:

- If the buyer chooses not to accept, the buyer's payoff is $v-p$ and the seller's payoff is $p-c$
- If the buyer accepts a permanent price offer, the buyer's payoff is $v-p+\delta(V^e-q)$ where V^e is the expected sum of the future values, while $H\ell$-type seller's payoff is $p-c+\delta(q-\frac{1}{1-\delta}c)$ and Hs-type seller's payoff is $p-c+\delta(q-c)$.
- If the buyer accepts a subscription offer, the buyer's payoff is $v-p+\frac{\delta}{1-\delta}(v-r)$, while $H\ell$-type seller's payoff is $p-c+\frac{\delta}{1-\delta}(r-c)$ and Hs-type seller's payoff is $p-c+\delta(r-c)$.

A question immediately arises as to what L-type would offer as a post-trial pricing scheme. This depends on whether the equilibrium is pooling, separating, or semiseparating. In a separating trial equilibrium, since L-type is revealed, what L-type offers for the post-trial phase is irrelevant as it will be ignored by the buyer. In a pooling trial equilibrium, L-type is imitating H-type's strategy, so L-type should imitate H-type's post-trial offer as well, using a randomized strategy if necessary (when ℓ and s separate). Intuitively speaking, PP path involves pooling, SS path involves separating while PS and SP paths involve semiseparating.

4.2.1 Pooling equilibria

In a pooling equilibrium, all three types choose the same signal $(p,q/r)$, i.e., (p,q) or (p,r). To be indistinguishable from H-type, L-type also offers a post-trial contract identical to that of two H-types.

Let us first rule out pooling equilibria where the buyer never tries: such equilibria are possible but the intuitive criterion can eliminate them. Basically, H-type can lower the trial price and convince its type to the buyer. Likewise, we rule out pooling equilibria where the buyer never purchases. Consider a (p,q) pooling equilibrium. The buyer's expected payoff in the post-trial phase is $B(q) = \overline{v} - q$ and the overall expected payoff from trial is

$$\mu_0(v + B(q)) - p = \mu_0(v + \overline{v} - q) - p \geq 0$$

If $v > C$, since $\max\{C, v\} \leq q \leq \overline{v}$,

$$p \leq \mu_0(v + \overline{v} - q) \leq \mu_0(v + \overline{v} - \max\{C, v\})$$

this is basically PP path from the sequential signaling model. Likewise (p,r) is essentially on PS path.

4.2.2 Fully separating equilibria

As was argued above, in a separating equilibrium, L-type is revealed by the signal so what it offers for the post-trial phase is irrelevant. Hence, L-type chooses $p_L > c$ for the trial phase and some arbitrary offer for the post-trial phase. $H\ell$-type offers $(p_H = c, r = v)$ and Hs-type offers $(p_H = c, q = v)$.

4.2.3 Semiseparating equilibria

There are two immediate possibilities for semiseparating, when only *pure* signals are allowed. First, $H\ell$ and Hs are pooled, while S is separated, and second, L is pooled with one of H-subtypes, while one subtype of H is separated. If S is separated from two H-subtypes, then it is simply SP path from the sequential signaling model. New possibilities might arise when L pretends to be one of the H-subtypes while ℓ and s separate. But this doesn't make sense. Whichever type is tied with L must be better off with revealing itself to be H! Another possibility arises when L-type is allowed to randomize. Then we can recover PS path. In other words, L-type offers the same trial price as H-subtypes, and also offers a subscription with probability ℓ_0. Then L-type is indistinguishable from H-type.

5. CONCLUSION

In this paper, we examined long-term pricing strategies of experience goods in terms of signaling. First, we argued that the high quality seller would be willing to set a low price at the trial stage in the hope of ensuring future profit. Such introductory pricing does not actually cause a short term "loss" in our model and is more likely when the share of high quality sellers (more precisely the buyer's belief about it) is not too high. If the buyer puts only a small probability on low quality, then a higher trial price is possible and some low quality product may remain in the market (although for only a short time, because the quality is revealed soon).

Once the trial phase has passed and the buyer has learned of the product quality, another issue arises if the product is durable and requires costly maintenance. The long-lived seller's product incurs higher long-term costs for the seller as well as provides higher long-term values to the buyer, and the seller cannot signal its long-livedness by a price signal alone. If exit-by-choice is allowed, then long-term transaction is impossible in equilibrium.

As the need for signaling its long-livedness increases, the long-lived seller can choose to offer a different pricing scheme, namely subscription. Subscription can completely resolve the buyer's concerns about the future, involving either exogenous short-livedness or endogenous exit-by-choice. Such a switch is more likely when more short-lived sellers flood the market and the long-lived seller cares more about the future. Some actual historical trends in the software market seem to be consistent with our findings.

There are many shortcomings of our analysis. Fixing the buyer's value as a known constant v greatly simplified the analysis, but is unrealistic. A better model should allow heterogeneity in the buyers' valuations. In addition, the dynamic interaction between the trial and the post-trial phase is somewhat simplistic as it focuses on the sizes of future surpluses. In order to face the issues of dynamic signaling better, we may want to extend our setting to where types are slowly revealed over time. A repeated signaling game Kaya (2009) might be relevant here.[21] So ample avenues for future research remain.

[21] I would like to thank a referee for pointing me in this direction.

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