Journal of Economic Theory and Econometrics, Vol. 31, No. 3, Sep. 2020, 23–43

Signaling Valence by Positive and Negative Campaigns*

S. David Kim† Youngse Kim‡

Abstract This paper aims at analyzing both positive and negative political campaigns which affect voters' perception about candidate valence. Consider the situation where candidates know valences each other but the voters cannot observe one candidate's valence. We characterize perfect Bayesian equilibria, which depend on the cost of positive campaign for the candidate and the cost of negative campaign against the opponent. The cost may be interpreted as the campaign budget constraint or the risk from backfire. We show that there always exist a pooling equilibrium and, for a wide range of parameters, a separating equilibrium. If positive campaign costs sufficiently less by the candidate with higher valence, the high valence candidate conducts positive campaign whereas the low valence candidate does not. More important, if negative campaign against the high valence opponent is sufficiently costly or risky due to backfire, the incumbent conducts negative campaign against a low valence challenger but not against a high valence one. We also show that, the smaller the valence difference is between two candidates, the larger the platform divergence becomes on the equilibrium.

Keywords Electoral Competition, Platform, Valence, Signaling, Positive Campaign, Negative Campaign, Perfect Bayesian Equilibrium

JEL Classification C72, D72, D78, D82

---

*This is a much revised and extended version of David's second year paper at Ohio State University. David thanks Yaron Azrieli for valuable discussions. We are grateful to two anonymous referees for helpful comments and suggestions. The usual caveats apply.

†First author. Department of Economics, Ohio State University, Columbus, OH 43210, USA. sdavidkim913@gmail.com

‡Corresponding author. School of Economics, Yonsei University, Seoul 03722, Korea. ykim@yonsei.ac.kr

Received May 11, 2020, Revised September 9, 2020, Accepted September 10, 2020

1. INTRODUCTION

In the 2016 U.S. presidential election, Hilary Clinton seized on the suspicion that Donald J. Trump could have committed income tax evasion of a sizable amount. In the 2018 Ohio governor election campaign, Richard Cordray and Mike DeVine were known to spend a substantial amount of resources on de-advertising each other. Cordray attacked the incumbent DeWine that he had worked for his sponsors at the cost of Ohio residents' welfare. DeWine openly blamed Cordray for his past lenient attitude towards rapists. Expressions were vulgar and sometimes disgusting, but they were aimed at enforcing the competitor's negative image and helping the attacker look relatively better in the eyes of voters.

On the other hand, valence matters in modern politics.[1] The voters reward governments for good governance and punish them for bad performance, based on various information sources, whether retrospective or prospective as well as whether aggregate statistics or personal experiences. Electoral competition provides politicians with incentive to perform and, more importantly, to establish reputation for good governance. This leads to greater incentive for voters to rely on valence issues more than ideological positions.

It matters for an electoral candidate to advertise own competence, conceal own incompetence, and advertise the opponent's incompetence. Often it matters more than to announce to and convince the voters of what exactly are own policy positions. In plain terms, the voters already have a good deal of knowledge about what kind of policies the Democratic would adopt and the Republican would adopt, if elected. By contrast, the voters have uncertainty about who brings about more benefits, with greater integrity and ability, to the constituents. This is consistent with casual observations that so many electoral campaigns in so many countries are full of image ads in favor of oneself and smear ads against the opponent. People lament the missing policy issues and the prevailing valence issues in election, but it has a rational foundation.[2]

In electoral campaign, candidates not only advertise themselves but also endeavor to affect the voters' perception on the competitors. The purpose of this paper is to analyze both positive and negative campaigns which affect voters' perception about candidates' valence. Consider the situation where candidates

[1] Iversen and Soskice (2019) shed an important light on the reason for and mechanism by which valence issues are at the heart of voters' electoral calculations.

[2] Blackwell (2013) applied an empirical method of dynamic causal inference to U.S. statewide elections between 2000 and 2006 to find that negative campaign is an effective strategy for nonincumbents.

know valences each other but the voters cannot observe one candidate's valence.

We characterize perfect Bayesian equilibria, which depend on the cost of positive campaign for the candidate and the cost of negative campaign against the opponent. The cost may be interpreted as the campaign budget constraint or the risk from backfire. We show that there always exist a pooling equilibrium and, for a wide range of parameters, a separating equilibrium. If positive campaign costs sufficiently less by the candidate with higher valence, the high valence candidate conducts positive campaign whereas the low valence candidate does not. More important, if negative campaign against the high valence opponent is sufficiently costly or risky due to backfire, the incumbent conducts negative campaign against a low valence challenger but not against a high valence one. We also show that, the smaller the valence difference is between two candidates, the larger the platform divergence becomes on the equilibrium.

The balance of the paper is organized as follows. Section 2 provides a survey of existing literature and discusses the relations to our paper. In Section 3, we propose a model of bi-partisan electoral competition comprising both positive and negative campaign. Section 4 characterizes the Nash equilibrium in the benchmark case of complete information. Section 5 characterizes a Bayes Nash equilibrium in the case of incomplete information but costly campaigns are banned. Section 6 characterizes perfect Bayesian equilibria in the situation where a challenger's competence is incompletely informed and costly campaign as a signaling device is allowed. Section 7 concludes with suggestions of future research.

2. LITERATURE SURVEY

Among existing literature, the most relevant is Lovett and Shachar (2011). The temporal order and model feature are as follows: (i) candidates choose their campaign budgets, (ii) candidates decide on the allocation between positive and negative campaigns, which affects voters' utility, (iii) voters make their voting choices. They show that, in the unique subgame-perfect equilibrium and for a wide range of parameters, the portion of negative ads increases in both the level of voters' prior knowledge and the endogenously determined size of campaign budget. They present a model of electoral competition in which ads inform voters either of the good traits of the candidate or of the bad traits of his opponent. While we share some major motivations with Lovett and Shachar (2011), the details are so different as to make direct comparison difficult. In our model, campaign budgets are exogenous, if the cost of political ads can be interpreted as spending or budget. We explicitly analyze the platform competition, while Lovett and Shachar (2011) adopt the probabilistic voting function. Most importantly, this paper is based on an conventional model of signaling candidate traits, while Lovett and Shachar (2011) do not consider any information transmission mechanism.

Also closely related are two earlier papers by Harrington and Hess (1996) and Skaperdas and Grofman (1995). Positive campaign pushes one's own ideology perceived by voters to the opponent's side, which ceteris paribus increases his vote share. Negative campaign pushes the opponent's ideology perceived by voters further to the opposite direction, which decreases her vote share and thus increases the attacker's share. Candidates optimally divide a fixed amount of resource on positive and negative campaign. Underlying their main result that the candidate having lower valence level runs a relatively more negative campaign, the assumption of innate, commonly known, different valences plays a key role. They do not explain the reason why the positive or negative campaign spending changes voters' perception but just assume that. We attempt to explain the reason and mechanism. In particular, candidates' valence may be private information, and campaigns, whether positive or negative, influence voters' perception about candidates' traits. This contrasts to Harrington and Hess (1996), in which campaigns directly affect voters' perception about candidates' policy platforms.

Candidates announce platforms from which the voters infer the true intentions of each candidate. (Kartik and McAfee, 2007; Callander and Wilkie, 2007) In reality, the voters do not usually attempt to infer candidates' policy intentions and valence from announced platforms. While the voters infer intended policies from announced positions, they infer candidates' competence and integrity from other information sources. Most campaigns are advertisements for oneself or attacks against the opponent, especially with respect to competence and integrity.

This paper is related to the literature that formally study the impact of valence on policy platforms. To name a few would be Londregan and Romer (1993), Gersbach (1998), Ansolabehere and Snyder (2000), Groseclose (2001), and Aragones and Palfrey (2002). In those papers, a candidate's valence attribute is assumed observable and hence no signaling element could work. In this paper, some candidate's valence is unobservable to voters and this informational asymmetry motivates both candidates to signal the hidden attribute by campaign tactics.

Campaign finance is not our main interest. However, this paper shares several features in common with the literature that provides informational explanations for campaign finance. Coleman and Manna (2000) find that campaign spending helps voters to know more about the candidates and improve their ability to locate the candidates' ideologies. There are two channels for this information effect. In Prat (2002), interest groups can observe the quality of the candidates, but voters do not. Voters indirectly learn about quality by mapping interest groups' contribution schedule inversely. Prat (2002) shows that an equilibrium exists with informative advertising, even though the ads have no direct informational content. In this paper, politicians can observe the quality of the opponent each other, but voters do not. Voters learn about quality indirectly from candidates' positive and negative campaigns, which play a role as signals.

An alternative mechanism in which campaign finance conveys valuable information to voters is studied by Coate (2004) and Ashworth (2006). They postulate that political advertising contains hard information, such as endorsements, interest-groups ratings and roll-call votes on prominent bills. Lying in ads is risky, since opponents and media have strong incentive to uncover frauds and lies. This may serve as a justification to our maintained assumption that candidates know quality of each other more than often "rationally ignorant" voters.[3]

3. THE MODEL

Two candidates compete for an office. Candidates are perceived as differing in terms of both their ideology and valence. In what follows, we call candidate 1 by masculine pronoun and candidate 2 by feminine pronoun. A candidate's ideology is represented by one's own location in the one-dimensional interval X including 0 and contained in $(-\infty, \infty)$.

An odd number of voters vote for either candidate. Each voter has a single-peaked preference around his ideal point, ω. Voters' ideal points are distributed on X. The median voter's ideal point is normalized to be 0. Other than the realized policy, the quality or competency of the elected candidate also affects voters' welfare. Let z denote the policy of the elected and θ his or her valence.[4]

[3] Caplan (2007) argues that the obstacles to sound economic policies are not politicians' wickedness, special interests or rampant lobbying, but ordinary voters' misconceptions, political apathy and irrational beliefs. Caplan (2007) suggests a bold proposal to make government work better, that is, letting democratic politics do less and allowing the market to do more.

[4] Some traits are observable, such as gender, appearance and past positions, while other traits are hard to know ex-ante, such as competence, leadership and accurate judgment ability. By valence, we keep the latter traits in mind.

A voter with ideal point ω has the utility function as follows:

$$
\begin{aligned}
U_\omega(\theta, z) &= \theta - f(|z-\omega|) \\
f'(\cdot) > 0 &= f(0), f''(0) > 0
\end{aligned}
\tag{1}
$$

Even if the elected candidate intends to faithfully put his pledge into effect, the actual policy may well differ from his intention. The discrepancy between planned and actual policies depends crucially upon competency of the elected as well as uncontrollable stochastic factors. We postulate that the actual policy follows a random distribution around his pledged platform. More specifically, the distribution generated by a highly competent incumbent second-order stochastically dominates the distribution generated by a low counterpart. In other words, the latter distribution is a mean-preserving spread of the former, with the same mean of the pre-election pledge, z. Since a risk-averse voter would prefer the former, the reduced-form payoff function (1) is plausible. This is in the spirit of Bernhardt and Ingberman (1985).[5]

A risk-neutral voter does not care of the spread. Notwithstanding, there are a plenty of reasons that the competence of the elected politician affects voters' welfare. More competent leader can implement the pledge with less cost to the constituents. Less competent leader can implement the pledge only by reciprocating special interest groups and/or pork-barrel politics, which sacrifice the interest of the general public. This is in the spirit of Prat (2006) and Ashworth (2006), although they are interested in explaining campaign finance as a signal of candidate's competence, unlike this paper.

Candidate i is endowed with valence index $\theta_i \in \Theta$ and own ideal point in the ideological scale X. Let β_i denote candidate i's ideal policy in X for $i = 1, 2$. Without loss of generality, assume that candidate 1's ideal point is located to the left of the median voter's ideal point and candidate 2's ideal point is located to the right, namely $\beta_1 < 0 < \beta_2$. A candidate's strategy is $s_i : \Theta \times X \to C_i \times X$,

[5]Inspired by Bernhardt and Ingberman (1985)'s seminal paper, many following works have focused on endogenizing the variance between announced and actual policies and hence the credibility of campaign pledge. Banks (1990) and Harrington (1993b) examine whether and to what extent candidates at the campaign stage reveal their true policy intentions if elected. Harrington (1993a) develops a two-period model in which re-election pressures play a key role in motivating the incumbent to fulfill campaign promises. In those papers, candidates or voters have private information about candidates' true policy intentions. In our model, voters have incomplete information about candidates' competency which indirectly affects the implemented policy. Since our interest does not lie in whether and to what extent the elected politician keeps promises, we take the tractable specification as of Bernhardt and Ingberman (1985), Harrington and Hess (1996) and Gersbach (1998).

where C_i denotes the binary set of candidate i's campaigns. Candidate preference is lexicographic: they aim at seeking office and then, amongst the platforms that yield identical chance of winning, choose the policy as close to one's own ideal policy as possible. Recall that the median voter is decisive in selecting the winner. Let S_i^* denote the set of candidate i's strategies that maximizes i's probability of winning:

$$S_i^* := \{x_i | \theta_i - f(|x_i|) \geq \theta_j - f(|x_j|)\}, j \neq i \tag{2}$$

Candidate i's objective is formalized as:

$$\min_{s_i \in S_i^*} |s_i - \beta_i| \tag{3}$$

Candidates know not only one's own valence but also the opponent's valence. Voters do not know either candidate's valence but can observe positive and negative campaign by candidates. Voters infer the valence of candidates based on positive and negative campaigns.

The temporal order is as follows. At the pre-play stage, the Nature selects the type of candidate 2. The information is known to herself and the opponent (candidate 1), but not to voters. At the first stage, candidates simultaneously choose the level of positive campaign for oneself and negative campaign against the opponent. Positive campaign may help signaling one's own high valence. Negative campaign may either successfully jam the opponent's positive campaign or cause a backlash or boomerang effect. At the second stage, candidates simultaneously choose the policy platforms in X. Let x_1 and x_2 denote the policy announced by candidate 1 and candidate 2, respectively. At the terminal stage, each voter votes for either candidate based on both expected competency and announced platforms. We assume that, if a voter is indifferent between two candidates, he votes for the candidate with higher expected valence. This tie-breaking rule is innocuous but helps avoiding irksome arguments due to the continuity of the policy space X.

4. INCUMBENT VS. CHALLENGER FRAMEWORK

We consider the situation in which candidate 1's valence is commonly known as θ_1 whereas candidate 2's valence θ_2 is either high (θ_H) with probability α or low (θ_L) with probability (1-α). We focus on the intriguing case of $\theta_H \geq \theta_1 \geq \theta_L$. One might interpret candidate 1 as the incumbent and candidate 2 as the challenger. Voters, politicians and journalists tend to know well the valence of the incumbent by personality, integrity and, above all, his track of past policies and decisions.

Since θ_1 is common knowledge, candidate 1 has no incentive to send a positive signal for the sake of himself. On the other hand, he may have an incentive to send a negative signal against candidate 2. Candidate 2 may or may not spend on positive campaign for herself, but has no incentive to commit a negative campaign against candidate 1.

In this section we first characterize a Nash equilibrium of a complete information benchmark. In the following section, we characterize a Bayes Nash equilibrium of an incomplete information game in which political campaigns are banned. This analysis would serve as an equilibrium analysis of the subgame starting from the second stage.

Suppose that candidate 2's valence is also commonly known as θ_H. Remind that the ideal point of the median voter is 0 in the policy space X. The median voter selects candidate 1 if and only if[6]

$$\theta_1 - f(|x_1|) > \theta_H - f(|x_2|) \tag{4}$$

Figure 2 depicts the regions in which the median voter is indifferent and, equivalently, which candidate wins depending on the platforms.

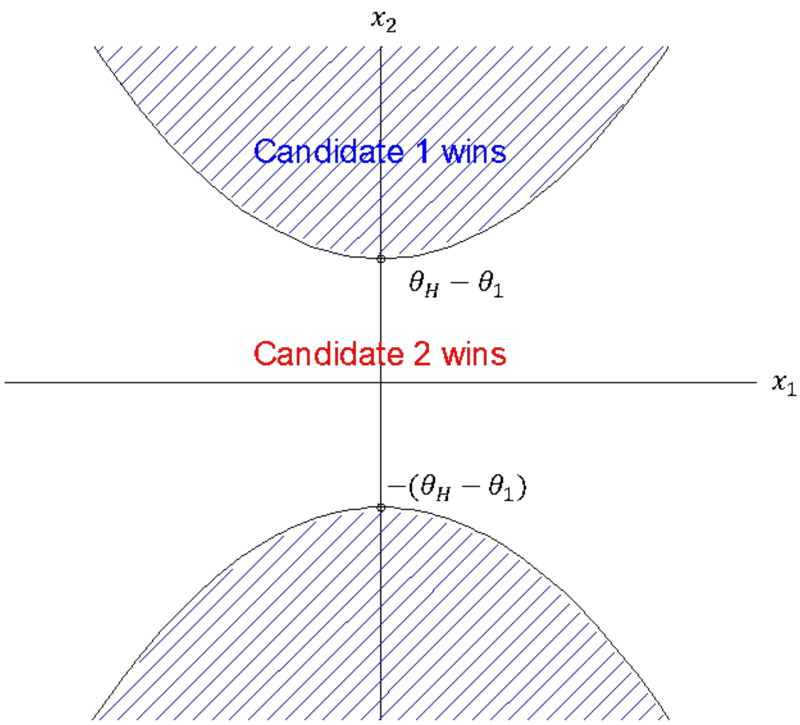
In the current situation where candidate 2 has surely higher valence than candidate 1, the median voter is more tolerable to the departure of candidate 2's platform away from his bliss point. Figure 2 is a version of Stokes (1963, 1992) regions a la Groseclose (2001), although Figure 2 is drawn on the joint policy profiles domain and Stokes region is drawn on the policy/utility domain. Graphically, the boundary of profiles (x_1, x_2) such that the median voter is indifferent cannot intersect with the 45-degree line.

Lemma 1. *In the case of $\theta_H \geq \theta_1$, it holds that $|x_2| \geq |x_1|$ on any equilibrium.*

Proof. The combinations of (x_1, x_2) along which the median voter is indifferent between candidate 1 and 2 are represented by the equation $\theta_1 - f(|x_1|) = \theta_H - f(|x_2|)$. Consider the first quadrant, that is, $|x_1| = x_1$ and $|x_2| = x_2$. Differentiation yields $\frac{dx_2}{dx_1} = \frac{f'(x_1)}{f'(x_2)}$, which is always greater than unity for $x_2 \geq x_1$ by the

[6] We may make the model more general and realistic by introducing an uncontrollable stochastic factor into the electoral result. Let $\widetilde{\varepsilon}$ be a random variable with a distribution function $F(\cdot)$ having mean 0 and variance σ^2. The preference shock can be interpreted as a scandal, bad news about candidate's health or a partisan swing that is unpredictable in advance. The median voter would select candidate 1 iff $\theta_1 - f(|x_1|) + \widetilde{\varepsilon} > \theta_H - f(|x_2|)$. All following statements could have been stated probabilistically instead of deterministically, but it does not change our results qualitatively.

Figure 1



assumption that $f'' < 0$. Moreover, the intercept at the vertical axis is $(\theta_H - \theta_1)$, which is positive. These facts imply that, in Figure 2, the boundary can never cross the 45-degree line. Applying the same logic to the other three quadrants would yield the desired result. □

Lemma 1 implies that, in the situation where the challenger has surely higher valence than the incumbent, the challenger has no incentive to take the pledge across to the other side of the incumbent's platform. Similarly, in the case of $\theta_L < \theta_1$, candidate 1 has no incentive to take the pledge across to the other side of candidate 2's platform.

Now we characterize the equilibrium. If politicians are purely office-seeking, there exists a plethora of Nash equilibria. Any pledge $x_2 \in [-(\theta_H - \theta_1), (\theta_H - \theta_1)]$ is weakly dominant for candidate 2. If she chooses any one policy in that interval, she surely wins, regardless of the opponent's choice of x_1. Candidate 1 rationally expects this and to surely lose, his choice is irrelevant. Precisely, the set of Nash equilibria would be $X \times [-(\theta_H - \theta_1), (\theta_H - \theta_1)]$.[7]

Since candidate preference is lexicographic, the candidate who is expected to win chooses the policy as close to her ideal point as possible. Let z denote a policy that the winning candidate would implement. Also remind our maintained assumption that $\beta_1 < 0 < \beta_2$, and 0 is the median voter's ideal policy.[8]

Proposition 1. *Assume that candidate 1's valence is θ_1 and candidate 2's valence is $\theta_H > \theta_1$, which is common knowledge. There exists the unique Nash equilibrium outcome:*

(i) If $\beta_2 \geq (\theta_H - \theta_1)$, then $x_1^* = 0$ and $x_2^* = z^* = (\theta_H - \theta_1)$.

(ii) If $\beta_2 < (\theta_H - \theta_1)$, then $x_1^* = \beta_1$ and $x_2^* = \beta_2 = z^*$.

The proof is deferred to Appendix. If the candidate with higher competence has a rather extreme ideology (case (i) above), she chooses a compromising platform so as to guarantee her victory while her opponent flatters the median voter. This echoes Gersbach (1998) who has shown that, while the disadvantaged candidate locates at the median's ideal point, the advantaged candidate moves as close as her ideal point as possible subject to the constraint that she wins the vote of the median.[9] If the candidate with higher valence has modest ideology (case (ii)), she chooses her ideal policy. Whatever the disadvantaged opponent chooses cannot change the electoral outcome and thus makes this candidate indifferent with respect to office-seeking motivation. However, since candidate 1 has a lexicographic preference, he would choose his ideal point.

For the opposite case where candidate 2's competence is commonly known as θ_L, a symmetric argument as above yields the following counterpart to Proposition 1.

Proposition 2. *Assume that candidate 1's valence is θ_1 and candidate 2's valence is $\theta_L < \theta_1$, which is common knowledge. There exists the unique Nash equilibrium outcome: $z^* = x_1^* = \max\{-(\theta_1 - \theta_L), \beta_1\}$.*

---

[7] In a rigorous sense, two points $(0, (\theta_H - \theta_1))$ and $(0, -(\theta_H - \theta_1))$ may be not equilibria, depending on the tie-breaking rule. However, it would not be a problem by introducing the minimal unit of Δ and thus finitizing the model.

[8] Extending our analysis to other three cases would be straightforward and tedious, without adding qualitative insight.

[9] Other than this, Gersbach (1998)'s motivation and model are too distinct to compare with this paper. He assumes that campaigns help reduce the uncertainty about candidates' actual platforms perceived by voters and campaigning resources are financed by donors and interest groups. He then characterizes the political equilibrium strategy profile and donation structure.

A couple of remarks are worth mentioning. First, the Nash equilibrium is reminiscent of the Stackelberg equilibrium in the office-seeking environment. The candidate with higher valence (Stackelberg leader) optimally chooses his or her most preferred platform under the constraint that the opponent (Stackelberg follower) best responds. Second, we obtain a divergence result, in sharp contrast to the standard Downsian theorem. Less competent candidate sticks to the median voter's bliss point on some equilibria. More competent candidate departs away from it by as much as her relative superiority ($|\theta_2 - \theta_1|$) or the distance from her ideal point to the median voter's ideal point ($|\beta_{winner}|$), whichever is smaller.

Proposition 1 and 2 imply that, when one candidate has a valence advantage over the other, his equilibrium policy choice is uniquely determined at a moderate level whereas the weaker opponent either flatter to the median voter or take his or her own ideal position. The results can read as follows. If the valence advantage is relatively small (in particular, Proposition 1 (i)), the issue positions matter more. If the valence gap is large (Proposition 1 (ii)), the candidates choose whatever they most prefer.

The impact of valence advantage on policy divergence and the winning probability are a very important issue in existing literature. Ansolabehere and Snyder (2000) study the multi-dimensional spatial competition where candidates' valence scores are commonly known and different. They argue that, while the advantaged tends to take moderate stances, the disadvantaged may take either moderate and extreme positions. Our Proposition 1 and 2 provide much sharper predictions, since the office-seeking candidates are also concerned with ideological positions.[10] Groseclose (2001) builds on the Wittman (1977) and Calvert (1985) framework of uncertainty, with an additional feature that candidates care about both office-seeking and ideology in a continuous way. His two main results state that (i) the greater is the valence advantage, the more the candidates diverge; and (ii) the advantaged adopts a more moderate position. Our model does not provide a definite answer to this question, since equilibrium platforms depend not only on the relative size of the valence advantage (Proposition 1 (i) and (ii)) but also on the relative distance of candidates' ideal points to the median voter's ideal point.

[10]We do not claim that this paper generalizes Ansolabehere and Snyder (2000). Our model is more general in terms of informational requirement but more restrictive in the dimension of policy platforms.

5. BAYES GAME WITH COSTLY CAMPAIGNS BANNED

Now consider the situation in which voters do not know candidate 2's realized competency. From the voters' point of view, the expected competency of candidate 2 is $E(\theta_2) \equiv \alpha\theta_H + (1-\alpha)\theta_L$. Remind that the bliss point of the median voter is 0 in the policy space X. The median voter selects candidate 1 (respectively, candidate 2) if $\theta_1 - u(|x_1|) > E(\theta_2) - u(|x_2|)$ (respectively, $<$). By the same arguments as in Section 4, we can derive the following results.

Proposition 3.

(i) If $\theta_1 > E(\theta_2)$, then $z^* = x_1^* = \max\{-(E(\theta_2) - \theta_1), \beta_1\}$ on any Bayes Nash equilibrium.

(ii) If $\theta_1 < E(\theta_2)$, then $z^* = x_2^* = \min\{-(E(\theta_2) - \theta_1), \beta_2\}$ on any Bayes Nash equilibrium.

(iii) If $\theta_1 = E(\theta_2)$, then the unique Bayes Nash equilibrium is $x_1^* = x_2^* = 0$.

Proof. Case (i) and (ii) are simple extensions of Proposition 2 and Proposition 1, respectively. (iii) The median voter votes for candidate 1 (respectively, 2) if $u(|x_1|) > u(|x_2|)$, that is $|x_1| > |x_2|$ (respectively, $<$). The only mutual best response is $(0,0)$. □

Notice that Proposition 1 and 2 assume that candidate 2's valence is commonly known as higher and lower, respectively, than candidate 1. In Proposition 3, the voters do not know whether or not the challenger has a higher valence than candidate 2, but only know the probability distribution.

Proposition 3 (i) analyzes the incumbency advantage situation where an incumbent has a valence advantage of ability over challenger, as postulated by Londregan and Romer (1993). Since, in reality, an incumbent often has turned out to be disappointing, we cannot find a justifying reason why the opposite case be precluded. Case (ii) analyzes the situation where the incumbent valence falls short of the expected valence of a challenger. Case (iii) replicates the standard Downsian model. In this knife-edge case where candidate 1's commonly known competency coincides with candidate 2's expected competency, the median voter theorem emerges and the convergence is the unique equilibrium profile. This is quite robust in the sense that if we replace lexicographic or office-seeking objective function with ideological one, the equivalent result obtains.

6. SIGNALING EQUILIBRIA WITH COSTLY CAMPAIGNS

Candidate 1 knows candidate 2's type. Voters do not know candidate 2's type, but can observe candidate 2's positive campaign (either passive or active) and candidate 1's negative campaign against candidate 2. To abuse notations, let $\oplus, \oplus^x, \ominus$, and $\ominus^x$ denote active positive campaign, passive positive campaign, aggressively negative campaign and no negative campaign, respectively. Formally, the binary set of campaigns are $C_1 = \{\ominus, \ominus^x\}$ and $C_2 = \{\oplus, \oplus^x\}$. Let c_H^+ and c_L^+ denote the marginal cost to positive campaign by high type and low type, respectively, for herself. Let c_H^- and c_L^- denote the marginal cost of negative campaign against candidate 2 who is of high valence and low valence, respectively. The following assumption is a version of standard single-crossing or Mirrlees-Spence condition.

Assumption 1. $c_H^+ < c_L^+$ *and* $c_H^- > c_L^-$

The assumption is intuitively appealing. The first inequality states that positive advertising costs the high type less than it does the low type. It is easier for the candidate of the high type to find merits, strengths and past performances as well as to develop good and persuasive policy agenda. It would be much harder for a candidate of lower valence to do the same. The second inequality also makes sense. It would be harder to defeat by logic or dig into the weak points of the opponent with high valence and dissuade voters to turn around from her. More importantly, negative campaign against high valence candidate 2 is risky in the sense that the offender could be revealed as a liar or a schemer which often backfires. By contrast, negative campaign against bad type entails less risk or no boomerang effect.

Let V_{win} and V_{lose} denote the payoff to the winner and the loser, respectively. Hence, $v \equiv V_{win} - V_{lose}$ is the marginal benefit to winning that this particular campaign attributes to. We restrict the parametric range somewhat.

Assumption 2. $c_H^+ < v$ *and* $c_L^- < v$

Without Assumption 2, no candidate has an incentive to advertise for oneself or exert smear campaign against the opponent. We also rule out the non-generic case $\theta_1 = E(\theta_2)$, without any loss of generality.

Proposition 4. ***Negative campaign***

(1) There always exists the pooling equilibrium in which candidate 1 never chooses negative campaign.

(2) If and only if $v > c_H^-$ and $\theta_1 > E(\theta_2)$, there exists the pooling equilibrium in which candidate 1 chooses negative campaign against any type of candidate 2.

(3) (i) If $v > c_H^-$, there does not exist a separating equilibrium; (ii) If $c_L^- < v < c_H^-$, there exists the separating equilibrium in which candidate 1 chooses negative campaign against the low type candidate 2 only.

Proof. (1) If candidate 1 never chooses negative campaign, the median voter expects candidate 2's valence to be $E(\theta_2)$. Consider the case where $\theta_1 > E(\theta_2)$. Candidate 1 wins and thus obtains the payoff of $(V_{win} - 0)$ irrespective of candidate 2's valence. In the contrary case where $\theta_1 < E(\theta_2)$, candidate 1 loses and thus obtains the payoff of $(V_{lose} - 0)$ regardless of candidate 2's valence. Let us assume the voter's off-the-equilibrium belief as follows: if candidate 1 deviates from $\ominus^x$ to $\ominus$, the median voter believes candidate 2 to be of the higher valence. Candidate 1's payoff to this deviation would be $(V_{lose} - c_H^-)$ (respectively, $(V_{lose} - c_L^-)$) depending on candidate 2's type, which is smaller than the equilibrium payoff.

(2) On the equilibrium, if candidate 1 chooses negative campaign, the median voter expects candidate 2's valence to be $E(\theta_2)$. Consider the case where $\theta_1 > E(\theta_2)$. Candidate 1 wins and thus obtains the payoff of $(V_{win} - c_H^-)$ (respectively, $(V_{win} - c_L^-)$) if candidate 2 has the higher (respectively, lower) valence. Let us assume the voter's off-the-equilibrium belief as follows: if candidate 1 deviates from $\ominus$ to $\ominus^x$, the median voter believes candidate 2 to be of the higher valence. Candidate 1's payoff to this deviation would be V_{lose}, regardless of candidate 2's type, which falls short of the equilibrium payoff. In the contrary case where $\theta_1 < E(\theta_2)$, candidate 1 loses and thus obtains the payoff of $(V_{lose} - c_H^-)$ (respectively, $(V_{lose} - c_L^-)$) if candidate 2 has the higher (respectively, lower) valence. No off-the-equilibrium beliefs and strategies exist to make a deviation to no negative campaign less profitable.

(3) (i) If $v > c_H^-$, not only candidate 1 against the low type candidate 2 but also candidate 1 against the high type candidate 2 has an incentive to choose negative campaign. Hence, a separating equilibrium cannot exist. (ii) The incentive compatibility condition for candidate 1 against the high (respectively, low) valence candidate 2 is $V_{lose} - 0 > V_{win} - c_H^-$ (respectively, $V_{win} - c_L^- > V_{lose} - 0$). Two inequalities coincide with the presumed parametric range $c_H^- > v > c_L^-$.

□

Proposition 4 (1) proposes a babbling equilibrium. Proposition 4 (2) implies that, if the winning stake is sufficiently large and the challenger's expected valence falls short of the incumbent's known valence, there exists the pooling equilibrium in which the incumbent goes negative against the challenger. This result seems to be at odds with the front-runner effect that the trailing candidate tends more to go negative. (Skaperdas and Grofman, 1995; Harrington and Hess, 1996) However, the overall evidence is rather mixed (Damore, 2002; Sigelman and Buell, 2003), and some distinct approaches may well explain the opposite phenomenon (Lovett and Shachar, 2011). Our framework can be interpreted as another channel.

Proposition 5. ***Positive campaign***
(1) If $v > c_L^+$ and/or $\theta_1 > E(\theta_2)$, there exists a pooling equilibrium in which both types of candidate 2 choose active campaign.
(2) If $c_H^+ < v < c_L^+$ and/or $\theta_1 < E(\theta_2)$, there exists a pooling equilibrium in which both types of candidate 2 choose passive campaign.
(3) (i) If $v > c_L^+$, there cannot exist a separating equilibrium; (ii) If $c_H^+ < v < c_L^+$, there exists the separating equilibrium in which candidate 2 with high valence chooses active campaign and candidate with low valence chooses passive campaign.

Proof. (1) If candidate 2 chooses active campaign for herself, the median voter expects candidate 2's valence to be $E(\theta_2)$. Consider the case where $\theta_1 > E(\theta_2)$. The median voter would vote for candidate 1. Candidate 2 loses and thus obtains the payoff of $(V_{lose} - c_H^+)$ (respectively, $(V_{lose} - c_L^+)$) if candidate 2 has the high (respectively, low) valence. Let us assume that the off-the-equilibrium belief is as follows: if candidate 1 deviates from $\oplus$ to $\oplus^x$, the median voter believes candidate 2 to be of the lower valence. Candidate 2's payoff to deviation is identical to the equilibrium payoff, so candidate 2 has no incentive to deviate. In the contrary case where $\theta_1 < E(\theta_2)$, candidate 2 wins and thus obtains the payoff of $(V_{win} - c_H^+)$ (respectively, $(V_{win} - c_L^+)$) if candidate 2 has the higher (respectively, lower) valence. Let us assume the voter's off-the-equilibrium belief as follows: if candidate 1 deviates from $\oplus$ to $\oplus^x$, the median voter believes candidate 2 to be of the lower valence. Candidate 2's payoff to this deviation would be $(V_{lose} - 0)$ depending on candidate 2's type. Candidate 2 has no incentive to deviate to passive campaign only if $v > c_L^+$.

(2) If candidate 2 chooses passive campaign for herself, the median voter expects candidate 2's valence to be $E(\theta_2)$. Consider the case where $\theta_1 < E(\theta_2)$. Candidate 2 wins and thus obtains the payoff of $(V_{win} - 0)$ regardless of her type. Candidate has no incentive to deviate whatsoever. In the contrary case where $\theta_1 > E(\theta_2)$, candidate 2 loses and thus obtains the payoff of $(V_{lose} - c_H^+)$ (respectively, $(V_{lose} - c_L^+)$) if candidate 2 has the higher (respectively, lower) valence.

Let us assume the voter's off-the-equilibrium belief as follows: if candidate 1 deviates from $\oplus$ to $\oplus^x$, the median voter believes candidate 2 to be of the lower valence. Candidate 2's payoff to this deviation would be $(V_{lose} - 0)$ regardless of her type. She has no incentive to deviate only when $v < c_L^+$.

(3) (i) If $v > c_L^+$, not only candidate 2 with high valence but also candidate 2 with low valence has an incentive to choose active campaign. Hence, a separating equilibrium cannot exist. (ii) The incentive compatibility condition for candidate 2 with high valence (respectively, low valence) is $V_{win} - c_H^+ > V_{lose} - 0$ (respectively, $V_{lose} - 0 > V_{win} - c_L^+$). Two inequalities coincide with the presumed parametric range $c_H^+ < v < c_L^+$. □

It is straightforward and tedious to show that no other equilibrium exists. Proposition 5 (1) implies that, if $c_H^+ < v < c_L^+$ and $\theta_1 < E(\theta_2)$, no type of candidate 2 has an incentive to choose positive campaign for herself. This is intuitive. If the winning stake is not huge and the challenger has a valence exceeding the incumbent's, the challenger does not have to carry out the costly advertisement for herself. Likewise, Proposition 5 (2) implies that, if the winning stake is sufficiently large and the incumbent's valence exceeds the challenger's expected valence, the challenger has no reason to refrain from active campaign.

Combining Proposition 4 (3) (ii) and Proposition 5 (3) (ii) leads to the following result. For such a range of parameter space, the incumbent's negative campaign against the challenger as well as the challenger's positive campaign for herself co-exist.

Corollary 1. *Suppose that $c_L^- < v < c_H^-$ and $c_H^+ < v < c_L^+$. Candidate 1 exerts negative campaign against the low valence candidate 2 only, while only candidate 2 with high valence chooses active campaign.*

As a traditional belief-based signaling model, this result of this section is plagued by a multiplicity of equilibria. This may be a weakness compared to a non-belief-based model as of Kartik and McAfee (2007) who obtain a unique signaling equilibrium. However, we also provide an intuitive and parsimonious result, if rather trivial pooling equilibria are ruled out.

7. CONCLUDING REMARK

In this paper, we assume that candidate 2's valence is unknown to voters while candidate 1's valence is commonly known. Presumably, a more intriguing case would be both candidates' competences are unknown to voters. It seems to be hard to characterize signaling equilibria in such a model of two-sided incomplete information.

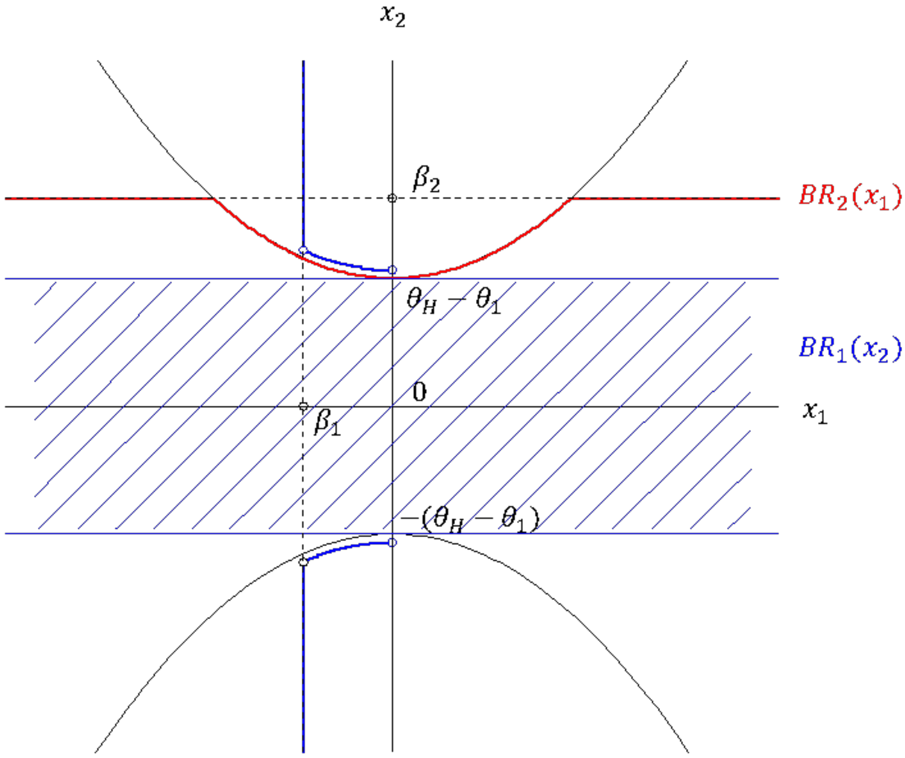
The model can be made tractable by modifying as follows. At the pre-play stage, the Nature selects the type of candidate 2. The information is known to herself and the opponent (candidate 1), but not to voters. At the first stage, candidates simultaneously choose the level of positive campaign for oneself and negative campaign against the opponent. Positive campaign may help signaling one's own high valence. Negative campaign may either successfully jam the opponent's positive advertising with probability $(1-\rho)$ or cause a backlash or boomerang effect with the complementary probability, ρ. At the second stage, candidates simultaneously choose the policy platforms in X. Let x_1 and x_2 denote the policy announced by candidate 1 and candidate 2, respectively.

It would be great if we can show there exist an equilibrium with intriguing features. Candidates both with low valence crossfire against each other and simultaneously shield or jam the opponent's smear campaign by costly positive campaign for oneself. We have been working on it, but solid results await future research. A more ambitious research agenda would be to set up a dynamic framework and study the information revelation process so as to explain Blackwell (2013)'s empirical results.

APPENDIX

Proof. **(Proposition 1)**
(1) Case of $\beta_2 \geq (\theta_H - \theta_1)$:
Refer to Figure 2. Candidate 1's best response to $x_2, BR_1(x_2)$, is as follows:
$\{\beta_1\}$, for $|x_2| > f^{-1}(u(\beta_1) + (\theta_H - \theta_1))$
$\{-f^{-1}(u(|x_2|) - (\theta_H - \theta_1)) - \varepsilon\}$, for $|x_2| \in [(\theta_H - \theta_1), f^{-1}(f(\beta_1) + (\theta_H - \theta_1))]$
X, for $x_2 \in [-(\theta_H - \theta_1), (\theta_H - \theta_1)]$
Candidate 2's best response to $x_1, BR_2(x_1)$, is as follows:
$\{f^{-1}(f(|x_1|) + (\theta_H - \theta_1))\}$, for $|x_1| < f^{-1}(f(\beta_2) - (\theta_H - \theta_1))$
$\{\beta_2\}$, for $|x_1| \geq f^{-1}(f(\beta_2) - (\theta_H - \theta_1))$
Nash equilibrium is mutual best response is unique: $x_1^* = 0$ and $x_2^* = z^* = (\theta_H - \theta_1)$.
(2) Case of $\beta_2 < (\theta_H - \theta_1)$:
$BR_2(x_1)$ to any $x_1 \in X$ is $\{\beta_2\}$. Candidate 1 rationally expect this fact and that, regardless of his choice, he would lose the election. Since candidate 1 has a lexicographic preference, he would choose his ideal position. Hence, $x_1^* = \beta_1$ and $z^* = x_2^* = \beta_2$. $\square$

Figure 2



REFERENCES

Ansolabehere, S. and J.M. Snyder (2000). "Valence Politics and Equilibrium in Spatial Election Models," *Public Choice* 103(3-4), 327-336.

Aragones, E. and T.R. Palfrey (2002). "Mixed Equilibrium in a Downsian Model with a Favored Candidate," *Journal of Economic Theory* 103(1), 131-161.

Ashworth, S. (2006). "Campaign Finance and Voter with Entrenched Incumbents," *American Political Science Review* 100, 55-68.

Banks, J.S. (1990). "A Model of Electoral Competition with Incomplete Information," *Journal of Economic Theory* 50, 309-325.

Bernhardt, M.D. and D.E. Ingberman (1985). "Candidate Reputations and the 'Incumbency Effect'," *Journal of Public Economics* 27, 47-67.

Blackwell, M. (2013). "A Framework for Dynamic Causal Inference in Political Science," *American Journal of Political Science* 57(2), 504-520.

Callander, S. and S. Wilkie (2007). "Lies, Damned Lies, and Political Campaigns," *Games and Economic Behavior* 60, 262-286.

Calvert, R. (1985). "Robustness of the Multi-dimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence," *American Journal of Political Science* 29(1), 69-95.

Caplan, B. (2007). *The Myth of the Rational Voter: Why Democracies Choose Bad Policies*, Princeton University Press.

Coate, S. (2004). "Pareto-Improving Campaign Finance Policy," *American Economic Review* 94, 628-655.

Coleman, J.J. and P.F. Manna (2000). "Congressional Campaign Spending and the Quality of Democracy," *Journal of Politics* 62(3), 757-789.

Damore, D.F. (2002). "Candidate Strategy and the Decision to Go Negative," *Political Research Quarterly* 55(3), 669-686.

Gersbach, H. (1998). "Communication Skills and Competition for Donors," *European Journal of Political Economy* 14(1), 3-18.

Groseclose, T. (2001). "A Model of Candidate Location when One Candidate Has a Valence Advantage," *American Journal of Political Science* 45(4), 862-886.

Harrington Jr., J.E. (1993a). "The Impact of Re-election Pressures on the Fulfillment of Campaign Promises," *Games and Economic Behavior* 5, 71-97.

Harrington Jr., J.E. (1993b). "Economic Policy, Economic Performance, and Elections," *American Economic Review* 83, 27-42.

Harrington Jr., J.E., and G.D. Hess (1996). "A Spatial Theory of Positive and Negative Campaigning," *Games and Economic Behavior* 17, 209-229.

Iversen, T. and D. Soskice (2019). *Democracy and Prosperity: Reinventing Capitalism through a Turbulent Century*, Princeton University Press.

Kartik, N. and R.P. McAfee (2007). "Signaling Character in Electoral Competition," *American Economic Review* 97(3), 852-870.

Londregan, J. and T. Romer (1993). "Polarization, Incumbency, and the Personal Vote," in *Political Economy: Institutions, Competition, and Representation*, eds., W.A. Barnett, M.J. Hinich, and N.J. Schofield, Cambridge University Press, 355-377.

Lovett, M.J. and R. Shachar (2011). "The Seeds of Negativity: Knowledge and Money," *Marketing Science* 30(3), 430-446.

Prat, A. (2002). "Campaign Advertising and Voter Welfare," *Review of Economic Studies* 69, 999-1017.

Prat, A. (2006). "Rational Voters and Political Advertising," in *Oxford Handbook of Political Economy*, eds., B. Weingast and D. Wittman, Oxford University Press, 50-63.

Sigelman, L. and E.H. Buell Jr. (2003). "You Take the High Road and I'll Take the Low Road? The Interplay of Attack Strategies and Tactics in Presidential Campaigns," *Journal of Politics* 65(2), 518-531.

Skaperdas, S. and B. Grofman (1995). "Modeling Negative Campaigning," *American Political Science Review* 89, 49-61.

Stokes, D. (1963). "Spatial Models of Party Competition," *American Political Science Review* 57, 368-377.

Stokes, D. (1992). "Valence Politics," in *Electoral Politics*, eds., D. Kavanagh, Oxford: Clarendon Press, 141-164.

Wittman, D.A. (1977). "Candidates with Policy Preferences: A Dynamic Model," *Journal of Economic Theory* 14(1), 180-189.