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Business Cycles and Limited Participation in Financial Markets: The Case of Korea

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Abstract This paper investigates sources of business cycles in Korea to shed some lights on the role of limited participation in financial market along the line of Christiano and Eichenbaum (1992) and King and Watson (1996). For this purpose, the paper sets up a small open economy model with two agents subject to limited participation in financial markets. Applying Watson (1993)'s measure of fit to evaluate the role of limited participation over Korean business cycles, it finds that the household's limited participation has played an important role in the business cycle in Korea after the Asian financial crisis.

Keywords Business Cycles, Limited Participation, Measure of Fit

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1. INTRODUCTION

In recent, a literature on open economy macroeconomics has documented some interesting stylized facts about business cycles in emerging economies. For example, business cycles in emerging economies are more volatile than business cycles in advanced economies and there is a consumption puzzle, i.e. consumption is more volatile than output in emerging economies. Many authors have explored the source of business cycles in emerging economies that differentiates them from advanced countries.

Some studies point out the poor and immature economic systems or institutions in the emerging economies such as fragile financial systems and weak enforcement of economic contracts as the source of the excess volatility of consumption. Others suggest that the volatile internal or external exogenous shocks such as productivity and country risk premium shocks and the government's inability to moderate these shocks can be associated with the driving forces of volatile business cycles in emerging economies.

The role of productivity shocks and market frictions over business cycles in emerging economies have been addressed through the lens of a representative RBC model. For example, Aguiar and Gopinath (2007) argue that the more volatile productivity shock found in emerging economies is the main driving forces, while Neumeyer and Perri (2005) and García-Cicco *et al.* (2010) assert that the RBC model driven by permanent and transitory shocks does a poor job in generating the observed business cycles. By incorporating the international financial market frictions into the RBC model, García-Cicco *et al.* (2010) assert that the financial frictions are main driving forces of the business cycles in emerging economies.

However, many studies have pointed out a drawback of a representative agent model wherein the direct or intertemporal substitution effect dominates the indirect or income effect in the fluctuation of consumption. Furthermore, households can participate in the financial markets to smooth their consumption profiles over time in the representative agent model. This kind of framework is at odds with the fact that the large fraction of households merely consume their current income even in the advanced economies as in Campbell and Mankiw (1989). Many studies using asset holdings data also point out that only a small fraction of the US and EU population holds assets.

This paper departs from the representative agent model by introducing constrained households or Hand-to-Mouth (HtM hereafter) households into the model with unconstrained households. In this paper, the unconstrained households are also limited in the participation of the financial market as in Christiano and

Eichenbaum (1992) in the sense that they need to pay some resources to purchase goods and services. In the extended two agent model with limited participation frictions, not only constrained households but also unconstrained households face some financial market frictions along the lines of Christiano and Eichenbaum (1992) and King and Watson (1996). Both unconstrained and constrained households need money to purchase goods and services as in Christiano and Eichenbaum (1992), King and Watson (1996), and Lucas (1990). Constrained or HtM households just consume their labor income paid in cash by firms as in Bilbiie (2008). However, unconstrained households can use their labor income and cash with some financial frictions. That is, unconstrained households also need some portfolio adjustment costs to purchase goods and services as in the limited participation model of Christiano and Eichenbaum (1992) and King and Watson (1996). On the production side, furthermore, firms need to borrow from financial intermediary to pay the labor cost to workers.[1]

We will explore the role of a limited asset market participation over business cycles in emerging economies along the line of Christiano and Eichenbaum (1992) and Bilbiie (2008). Specifically, we will evaluate the explanatory power of the two agent model with financial frictions as well as the representative agent model with limited participations over business cycles using the spectrum power in frequency domain as in King and Watson (1996), Stock and Watson (1999), and Watson (1993). It is well documented in the U.S. business cycles in frequency domain that the real macroeconomic variables such as output, consumption, and investment have common, hump-shaped growth rate spectra. Because the height of the spectral density of the selected variable at each frequency indicates the contribution of the corresponding frequency to the variance of the selected variable, the variance of the selected variable occurring between any two frequencies can be represented by the areas under the spectrum between those two frequencies. Therefore, the comparison of the spectra associated with the model and the ones with the data can improve our understanding how the model performs well in matching the volatile movements of the selected variables in the data in a specific frequency band.

We will address whether the two agent model with limited participation can generate the dynamics of the selected variables at low and high frequencies in addition to business cycle frequencies. To evaluate more precisely the performance of the two agent model augmented with limited participation relative to the representative agent model, we will also utilize Watson (1993)'s measures of fit, i.e. the spectrum of the error required to reconcile the model and the data

[1] Kim *et al.* (2020) explore the role of credit supply on business cycles in Korea

by comparing the spectral densities of the selected variables calculated from the models with those of the data and Watson (1993)'s RMSAE.

The main findings of this paper can be summarized as follows.

First, both the two agent model with limited participation and the representative agent model with limited participation can generate a hump-shaped spectrum of some selected real variables. The spectra of the selected real variables associated with the model have a peak at the business cycle frequencies as in the data. However, there is a substantial difference between the spectrum of nominal interest rate of the models and the spectrum of the corresponding variable of the data.

Second, the two agent model with limited participation performs better than the representative agent model in that the former is successful in generating the volatile consumption fluctuation, i.e. the consumption puzzle in Korea after the Asian financial crisis. Constrained households who cannot have access to financial markets have difficulty in smoothing out their consumption over time to exogenous shocks. This implies that a substantial fraction of households cannot smooth their consumption by having access to financial market after the Asian crisis.

Finally, the volatilities of consumption and output in Korea are larger after the Asian financial crisis than before the crisis, while the volatility of investment is smaller after the crisis than before the crisis. This implies the fact that firms have given up expanding their capacities with external finance and the end of the Korean seniority-based wage with an early retirement system has made households more susceptible to be influenced by external shocks after the Asian financial crisis.

This paper is composed as follows. Section 2 discusses the features of the data. Section 3 specifies the benchmark two agent model with financial frictions, and discusses the properties of the equilibrium. Section 4 discusses the equilibrium and Section 5 presents the quantitative implications of the model by applying Watson (1993)'s method. Section 6 contains concluding remarks.Features of Business Cycles in Korea

In this section, features of business cycles in Korea are examined, focusing on the time series relationship among gross domestic product, consumption, investment, interest rate, and trade balance. The statistical relationships presented in this section will be used to evaluate the performance of the limited participation model with financial frictions.

2. FEATURES OF BUSINESS CYCLES IN KOREA

In this section, features of business cycles in Korea are examined, focusing on the time series relationship among gross domestic product, consumption, investment, interest rate, and trade balance. The statistical relationships presented in this section will be used to evaluate the performance of the limited participation model with financial frictions.

2.1. POWER SPECTRUM OF SELECTED VARIABLES

The estimated power spectra of growth rates of key macroeconomic variables provide important informations about business cycles (King and Watson, 1996; Watson, 1993). To have some intuitions on the spectral shape of growth rates, it is useful to review key concepts of time series in the frequency domain. Since a covariance stationary variable x_t can be expressed in terms of a summation of period components

$$x_t = \int_0^{\pi} x_t(\omega) d\omega,$$

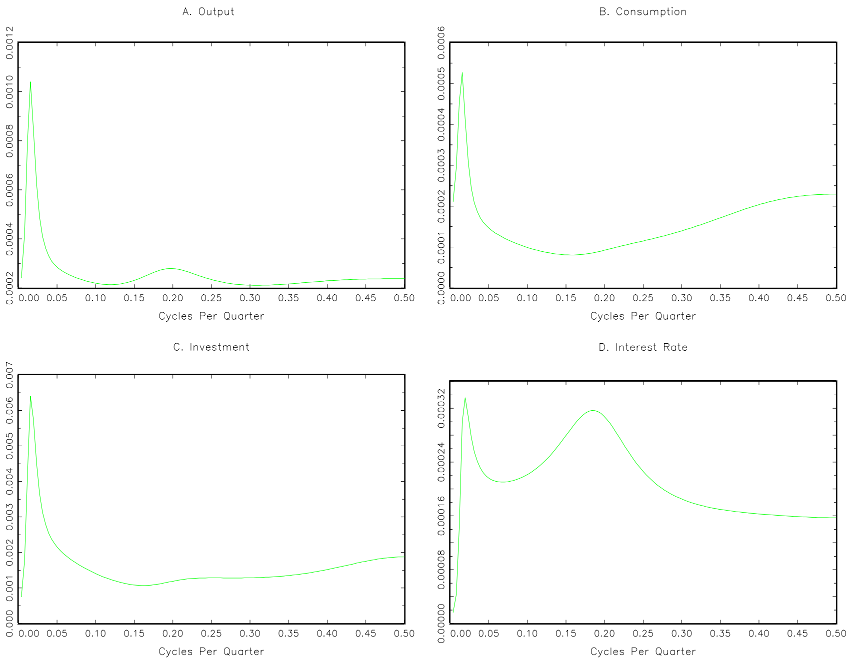
the variance of the corresponding variable can be decomposed as

$$var(x_t) = 2\int_0^{\pi} \tau(\omega) d\omega.$$

where the power spectrum $\tau(\omega)$ is the contribution to the variance of x_t at frequency ω. Figure 1 and 2 show the spectrum of some selected variables before the Asian financial crisis (1976:3Q - 1997:2Q) and after the crisis (1998: 1Q - 2018:3Q), respectively. The height of the spectrum in Figure 1 and 2 at cycles per period $\frac{\omega}{2\pi}$ represents the extent of that frequency's contribution to the variance of the relevant variable.

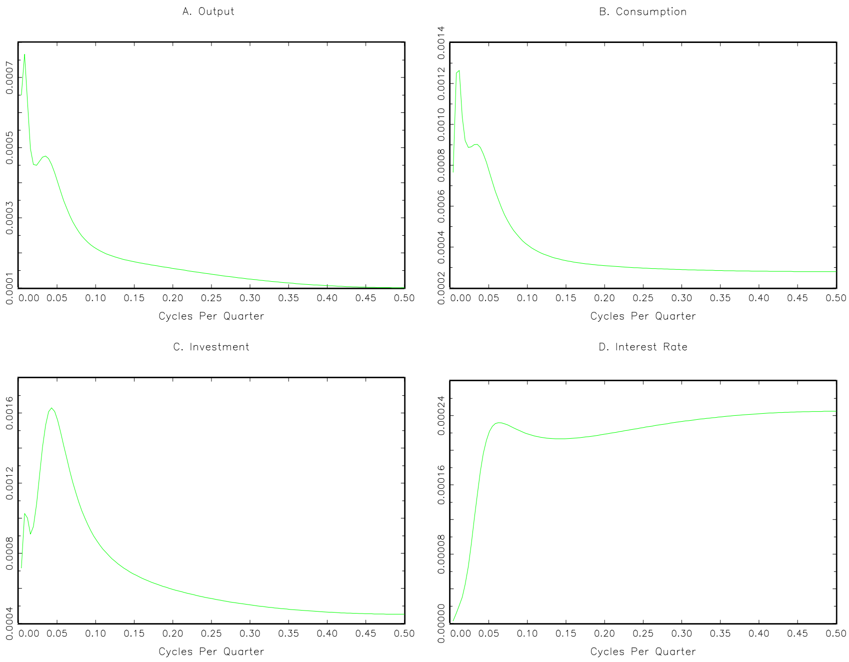
King and Watson (1996) and Stock and Watson (1999) present the estimated spectral density of key macroeconomic variables in the U.S. The growth rate spectrum of the selected variables shows a hump-shape or 'L' shape in the frequency domain. The power spectrum is relatively low at low frequencies, rises to a peak at the frequencies between $0.03(=\frac{1}{32})$ and $0.16(=\frac{1}{6})$, which correspond to the business cycle frequency in Burns and Mitchell (1946). Then, it declines at high frequencies. Furthermore, the estimated spectral density shows that the business cycle interval contains the peak as well as the bulk of the variance of the growth rates of the key macroeconomic variables.

Figure 1: Growth Rate Spectra: Before Asian Financial Crisis



The estimated spectra of the selected real macroeconomic variables in Korea also show the typical shape as the ones in the U.S. as in Figure 1 and 2. The spectral densities of the selected variables have a common hump-shape in frequency domain as in King and Watson (1996) and Stock and Watson (1999). However, the spectra of output, consumption, and investment have a peak at lower frequencies than 0.03, implying that the Korean economy is more vulnerable to permanent shocks than the U.S. economy. Since the height of the spectrum of each variable corresponds to the volatility of the variable, investment is most volatile among the selected variables. It is noteworthy that the height of consumption spectrum after the Asian financial crisis is higher than the one before the crisis, showing that more volatile consumption fluctuations after the crisis. Also Figure 1 and 2 show that the peaks of the selected variables occurs at lower frequency before financial crisis than after the financial crisis, implying that the business cycle in Korea is more frequent and short-lived after the financial crisis.

Figure 2: Growth Rate Spectra: After Asian Financial Crisis



2.2. BUSINESS CYCLE COMOVEMENTS

The comovements of selected variables over business cycles can be compactly represented in terms of cross correlations. Tables 1 and 2 represent second moments of the selected variables calculated from the estimated spectral density matrix with only the business cycle frequencies before and after the Asian financial crisis.

Three key empirical features are evident in Table 2 (1998:1Q - 2018:3Q). First, there occurs a consumption volatility puzzle after the Asian financial crisis. Second, both output and consumption are more volatile after the Asian financial crisis than before the crisis, while investment is much less volatile after the Asian financial crisis than before the crisis. Third, the contemporaneous correlation between trade balance (tb_t) and (log) output (y_t) is negative, but the nominal interest rate procyclically comoves with output, irrespective of the sample period.

Using the KLIPS (Korean Labor and Income Panel Study) covering from

Variable	Std. Dev.	Corss	Autocorr.	X_t	with	GDP	Y_{t+k}	(corr	(X_t, Y_{t+k}))	
		k=-4	-3	-2	-1	0	1	2	3	4
y	1.91	0.26	0.49	0.73	0.93	1.00	0.93	0.74	0.49	0.25
c	1.29	0.50	0.66	0.77	0.82	0.78	0.65	0.45	0.24	0.03
i	5.12	0.43	0.62	0.77	0.86	0.84	0.72	0.53	0.31	0.11
tb/y	0.63	-0.27	-0.32	-0.37	-0.41	-0.40	-0.32	-0.18	-0.02	0.13
r	3.09	0.20	0.31	0.33	0.32	0.35	0.15	0.04	-0.09	-0.10

Table 1: Moments of Data (1976:III - 1997:II)

Variable	Std. Dev.	Corss	Autocorr.	X_t	with	GDP	Y_{t+k}	(corr	(X_t, Y_{t+k}))	
		k=-4	-3	-2	-1	0	1	2	3	4
y	2.26	0.25	0.49	0.73	0.93	1.00	0.93	0.74	0.49	0.25
c	3.08	0.16	0.39	0.63	0.82	0.92	0.89	0.75	0.54	0.32
i	4.12	0.35	0.55	0.74	0.86	0.86	0.75	0.55	0.31	0.08
tb/y	2.32	-0.07	-0.23	-0.40	-0.55	-0.63	-0.62	-0.51	-0.36	-0.20
r	2.58	0.25	0.46	0.58	0.57	0.40	0.16	-0.06	-0.19	-0.23

Table 2: Moments of Data (1998:I - 2018:III)

2000 to 2015, we classify cash, saving and demand deposits, and bonds, precautionary insurance as liquid assets. Then, we define the HtM households as the ones who have liquid assets less than half of their monthly income as in Kaplan *et al.* (2014).

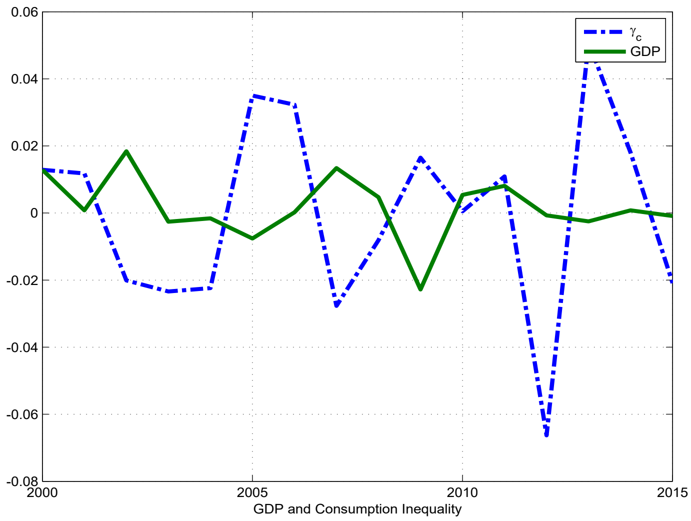
Figure 3 presents the movements of HP-filtered GDP and consumption inequality between unconstrained households and constrained households. The Figure shows that consumption inequality countercyclically comoves with output $(corr(y_t, \gamma_{c,t}) = -0.26)$, where $\gamma_{c,t} \equiv \ln(C_{U,t}) - \ln(C_{K,t})$ is the (log) consumption inequality between unconstrained households $(C_{U,t})$ and constrained households $(C_{K,t})$. That is, constrained households are likely to benefit when the economy expands, while they suffer more than unconstrained households when the economy contracts.

In subsequent section, we will address the role of limited participation in the form of portfolio adjustment costs over the business cycle in Korea using the spectral densities and Watson (1993)'s measure of fit.

3. SMALL OPEN ECONOMY MODEL WITH LIMITED PARTICIPATION FRICTIONS

There are two types of households in the economy: Unconstrained households can participate in financial markets, while constrained or HtM households cannot.

Figure 3: GDP and Consumption Inequality in Korea



3.1. UNCONSTRAINED HOUSEHOLDS

Unconstrained households who can have access to financial market choose their consumption, asset holdings, and labor supply to maximize their expected lifetime utility function subject to a sequence of budget constraints:

$$\mathscr{W}_{U,t} \equiv E_t \sum_{i=0}^{\infty} \beta^i \left[U(C_{U,t+i} - b\widetilde{C}_{U,t+i-1}) - \frac{(N_{U,t+i} + H_{U,t+i})^{1+\nu}}{1+\nu} \right], \; 0 < \beta < 1, \tag{1}$$

where $U(C_{U,t+i}) = \frac{(C_{U,t+i} - b\widetilde{C}_{U,t+i-1})^{1-\sigma} - 1}{1-\sigma}$ for $\sigma \neq 1$, and $U(C_{U,t+i}) = \ln(C_{U,t+i} - b\widetilde{C}_{U,t+i-1})$ for $\sigma = 1$. E_t denotes the expectation operator conditional on the available information at time t Ω_t, and $\widetilde{C}_{U,t}$ represents the time-varying habit level of unconstrained household's consumption in period t. $b \in (0,1)$ is the degree of external habit, and $C_{U,t}$, $N_{U,t}$, and $H_{U,t}$ represent the unconstrained household's consumption, labor hours, and hours of adjusting portfolio holdings in period t, respectively. $C_{U,t}$ is a composite consumption index defined by

$$C_{U,t} = [\theta^{\frac{1}{\eta}} C_{U_H,t}^{\frac{\eta-1}{\eta}} + (1-\theta)^{\frac{1}{\eta}} C_{U_F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}, \; \eta > 0. \tag{2}$$

Here $C_{U_H,t}$ and $C_{U_F,t}$ are indices of domestic and foreign consumption goods consumed by domestic unconstrained households, and θ and $1-\theta$ represent the share of domestic consumption allocated to domestic goods, and imported goods. The indices are given by the following CES aggregator of the quantities consumed of each variety of good:

$$C_{U_H,t} = [\int_0^1 C_{U_H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj]^{\frac{\varepsilon}{\varepsilon-1}},\ C_{U_F,t} = [\int_0^1 C_{U_F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj]^{\frac{\varepsilon}{\varepsilon-1}},\ \varepsilon > 1. \quad (3)$$

Here η and ε measure the elasticity of substitution between domestic and foreign goods, and the elasticity of substitution among goods within each category.

It is assumed that domestic unconstrained households can trade one-period nominal riskless bonds denominated in home and foreign currency, while foreign households trade one-period nominal riskless bonds denominated in foreign currency. It is also assumed that the international trade of foreign currency denominated bonds are subject to intermediation costs as in Benigno (2009).[2] The key element in the limited participation is that unconstrained households with money holdings $M_{U,t}$ at the beginning of period t must decide their holdings of $S_{U,t}$ for goods market transaction and $M_{U,t} - S_{U,t}$ to financial intermediary before the realizations of the monetary and real shocks at time t. Other decisions are made after the realization of the shocks. Unconstrained household's budget at the beginning of the period t is given by

$$M_{U,t+1} + P_t(C_{U,t} + I_{U,t}) + B_{U,t} + \mathscr{E}_t B_{F,t} \leq \Upsilon_t, \quad (4)$$

where $\Upsilon_t \equiv R_t(M_{U,t} - S_{U,t} + B_{U,t-1} + X_t) + R_{K,t}K_{U,t} + W_t N_{U,t} + P_t T_{U,t} + S_{U,t} + S_{U,t} + \mathscr{E}_t \exp(\chi_t) R^*_{t-1} \Xi(\frac{\mathscr{E}_{t-1} B_{F,t-1}}{P_{t-1}}) B_{F,t-1}$ denotes unconstrained household's wealth at time t. $T_{U,t}$, W_t and $R_{K,t}$ are lump-sum taxation/ subsidy, wage rate, and the rental rate of physical capital in period t. X_t is a monetary injection at time t and $B_{U,t}$ and $B_{F,t}$ denote domestic and foreign currency denominated nominal bonds, while R_t, R^*_t and $\mathscr{E}_t$ are the interest rate corresponding to the domestic and foreign bonds, and the nominal exchange rate at time t, respectively. χ_t represents a country spread shock and the function $\Xi(\frac{\mathscr{E}_t B_{F,t}}{P_t})$ incorporates the cost or the risk premium from international borrowings. The risk premium, i.e. $\Xi(\frac{\mathscr{E}_t B_{F,t}}{P_t}) - 1$ is increasing with the country's foreign debt, i.e. $\Xi'(.) > 0$, and it equals zero when the economy is in the steady state, i.e. $\Xi(\mathscr{B}_F) = 1$ in the steady state, where $\mathscr{B}_{F,t} \equiv \frac{\mathscr{E}_t B_{F,t}}{P_t}$.

To have some intuitions on the role of a limited participation in the model, we need to clarify two market frictions. First, some portfolio decisions are made

[2]This intermediation cost assumption is made for technical reasons. See Uribe and Schmitt-Grohé (2016) for alternative assumptions to overcome the stationary problem in a small open economy model.

with incomplete information about the shocks within the period, that is, prior to the actions of the monetary authority. Second, unconstrained households incur some costs to adjust their portfolio positions. Note that unconstrained households divide their money holdings into $S_{U,t}$ for purchase of goods and $M_{U,t} - S_{U,t}$ for deposit to financial intermediaries and they have to pay their consumption expenditure with money carried over period last period and their current labor income paid in cash by firms:

$$P_t C_{U,t} \leq S_{U,t} + W_t N_{U,t}. \tag{5}$$

Following Christiano and Eichenbaum (1992) and King and Watson (1996), assume that there are time costs of adjusting their portfolio holdings $S_{U,t}$ of the form:

$$H_{U,t} = H(\frac{S_{U,t}}{S_{U,t-1}}), \tag{6}$$

where $H > 0, H^{'} > 0$, and $H^{''} > 0$. Since unconstrained households should decide $S_{U,t}$ without knowing the current value of the money stock or exogenous shocks and after the shocks are revealed, the cash-in-advance constraint (5) establishes the value of an additional money to spend.

There is no firm specific capital stock. Only unconstrained household owns capital stock to rent to firms and there is no firm-specific capital stock. Since we do not empirically observe large discrete capital stock adjustments, it is reasonable to introduce an adjustment cost in capital stock installments. If there are costs of installing capital, the capital stock will move more sluggishly. We assume that there are deadweight costs of installing capital stock. To preserve the simple model structure as far as possible, we will adopt the simple form of investment adjustment cost as in Uzawa and Lucas:

$$K_{t+1} = \psi(I_t/K_t)K_t + (1 - \delta_k)K_t, \tag{7}$$

where $\delta_k \in [0, 1)$ is the depreciation are of capital. The increasing convex function ψ introduces an adjustment cost in investment. It is assumed that $\psi = \delta_{k,}$ $\psi^{'} = 1$, and $\psi^{''} > 0$ at the steady state.

The unconstrained households chooses process $\{C_{U,t}, N_{U,t}, H_{U,t}, S_{U,t}, M_{U,t+1}, B_{U,t+1}, K_{U,t+1}, I_{U,t}\}_{t=0}^{\infty}$ to maximize the expected life-time utility function (1) subject to (4), (5), (6), and (7).

3.2. CONSTRAINED HOUSEHOLDS

The HtM or constrained households who cannot have access to the financial market just supply labor $N_{K,t}$ and consume their labor income received from the firms by cash each period:

$$P_t C_{K,t} = W_t N_{K,t} + P_t T_{K,t}, \tag{8}$$

where $C_{K,t}$ and $T_{K,t}$ are HtM household's consumption, the lump-sum tax or transfers in period t, respectively.

HtM households choose their consumption and labor supply to maximize their temporal utility function $(\mathscr{U}_{K_t})$ given by

$$\mathscr{U}_{K_t} \equiv U(C_{K,t} - b\widetilde{C}_{K,t-1}, N_{K,t}) \tag{9}$$

subject to a budget constraint (8). HtM household's optimization conditions are given by

$$U_N(C_{K,t} - bC_{K,t-1}, N_{K,t}) = \frac{W_t}{P_t} U_C(C_{K,t} - bC_{K,t-1}, N_{K,t}), \tag{10}$$

and the budget constraint (8).

3.3. FIRMS

Suppose that there are a continuum of domestic firms producing differentiated goods, and each firm indexed by i, $0 \leq i \leq 1$, produces its product with a constant returns to scale, concave production technology. Each firm i takes $P_{H,t}$ and the aggregate demand as given, and chooses its own product price $P_{H,t}(i)$. Since the input markets are perfectly competitive, the demands for labor and capital are determined by its cost minimization as follows,

$$\begin{aligned} C(W_t, R_{k,t}, Y_t(i)) &\equiv \min_{N_t(i), K_t(i)} \{R_{k,t} K_t(K) + R_t W_t N_t(i)\} \\ s.t. \quad Y_t(i) &\leq A_t F(K_t(i), N_t(i)). \end{aligned} \tag{11}$$

Here $Y_t(i)$ is the output of the ith firm in the home country and the technology shock A_t follows an $AR(1)$ process:

$$\log A_t = \rho_A \log A_{t-1} + \xi_{At}, \quad -1 < \rho_A < 1. \tag{12}$$

where $E(\xi_{At}) = 0$ and ξ_{At} is i.i.d. over time.[3] Note that since firms are required to pay for labor at the start of each period, they should borrow from financial intermediaries as in (11).

From the firm's first order condition,

$$\begin{aligned} R_{K,t} &= MC_t(i) A_t F_1(K_t(i), N_t(i)) \\ W_t R_t &= MC_t(i) A_t F_2(K_t(i), N_t(i)), \end{aligned} \tag{13}$$

The marginal cost of each firm is equal, i.e. $MC_t(i) = MC_t$ for each i as the production function is CRS.

Suppose that the domestic firm i sets its price prior to the realization of shock to maximize its value of cash flow across states of nature as in Christiano and Eichenbaum (1992). Then, the optimal price is given by

$$P_{H,t}(i) = \frac{\varepsilon}{\varepsilon - 1} \frac{E_{t-1}[Q_{t-1,t} P_t^{\varepsilon} Y_t MC_t]}{E_{t-1}[Q_{t-1,t} P_t^{\varepsilon} Y_t]}, \tag{14}$$

where $Q_{t-1,t}$ is a stochastic discount factor at time $t-1$ to the nominal pay-off at time t.

3.4. FINANCIAL INTERMEDIARY

A perfectly competitive financial intermediary receives deposits, $D_{U,t} (\equiv M_{U,t} - S_{U,t})$, from the unconstrained households and lump-sum cash injections, X_t, from the monetary authority. Then, the financial intermediary supplies the funds to firms at the gross interest rate, R_t. Firms demand funds in the loan market to finance their wage bill $W_t N_t$.

Loan market clearing condition requires

$$W_t N_t = D_t + X_t \tag{15}$$

At the end of each period, the financial intermediary pays $R_t D_t$ to unconstrained households in return for deposits and $R_t X_t$ in the form of profits.

3.5. MONETARY AUTHORITY

We assume a simple interest rule as in Christiano and Gust (1999) as follows:

$$r_t = b_\pi \pi_t + \varepsilon_{rt}, \; b_\pi > 1, \tag{16}$$

[3] $\rho = 0.95$ and $\sigma_A = 0.007$ are used in the artificial economy.

where $R_t \equiv 1 + r_t$, and π_t is the rate of inflation at time t. ε_{rt} is a normally distributed, mean-zero shock which is serially uncorrelated.

4. EQUILIBRIUM

4.1. AGGREGATION

The aggregate level of any household-specific variable Z_t is given by $Z_t = \int_0^1 Z_t(i)di = (1-\lambda)Z_{U,t} + \lambda Z_{K,t}$. Hence, aggregate consumption and aggregate hours are given by

$$C_t = (1-\lambda)C_{U,t} + \lambda C_{K,t} \tag{17}$$

and

$$N_t = (1-\lambda)N_{U,t} + \lambda N_{K,t}. \tag{18}$$

Aggregate deposit, cash to spend, bond holdings, physical capital, and investment also satisfy

$$D_t = (1-\lambda)D_{U,t}, \tag{19}$$

$$S_t = (1-\lambda)S_{U,t}, \tag{20}$$

$$B_t = (1-\lambda)B_{U,t}. \tag{21}$$

$$K_t = (1-\lambda)K_{U,t}, \tag{22}$$

$$I_t = (1-\lambda)I_{U,t}. \tag{23}$$

Finally, aggregate lump-sum taxes or transfers are also given by

$$T_t = \lambda T_{U,t} + (1-\lambda)T_{K,t}. \tag{24}$$

4.2. EQUILIBRIUM CONDITION

First of all, $N_t(i) = N_t$, $K_t(i) = K_t$, and $Y_t(i) = Y_t$ for all $i \in [0,1]$ in symmetric equilibrium. Turing to the money market equilibrium condition, (5), (8), and (15) imply that

$$P_t C_t = M_t.^4 \tag{25}$$

First order conditions are given by

$$U_N(C_{U,t} - bC_{U,t-1}, N_{U,t} + H_{U,t}) = U_C(C_{U,t} - bC_{U,t-1}, N_{U,t} + H_{U,t})w_t, \tag{26}$$

$$R_t^{-1} = \beta \frac{E_t \Lambda_{U,t+1}}{\Lambda_{U,t}}, \tag{27}$$

$$E[\Lambda_{U,t} - \beta R_t \Lambda_{U,t+1} | \Omega_{b,t}] = 0, \tag{28}$$

$$Q_t = (\psi'(\frac{I_t}{K_t}))^{-1}, \tag{29}$$

$$K_{t+1} = \psi(I_t/K_t)K_t + (1-\delta_k)K_t, \tag{30}$$

$$\frac{R_{k,t}}{R_t W_t} = \frac{F_1(K_t, N_t)}{F_2(K_t, N_t)} \tag{31}$$

$$\Lambda_{U,t} Q_t = \beta E_t[\Lambda_{U,t+1}(R_{k,t} + Q_{t+1}(1-\delta_k))], \tag{32}$$

$$\beta R_t^* \exp(\chi_t) \Xi(\mathscr{B}_{F,t}) E_t[\frac{(C_{U,t+1} - bC_{U,t}, N_{U,t+1} + H_{U,t+1})}{U_C(C_{U,t} - bC_{U,t-1}, N_{U,t} + H_{U,t})} \frac{\mathscr{E}_{t+1} P_t}{\mathscr{E}_t P_{t+1}}] = 1, \tag{33}$$

and constrained household's optimization condition (8), (10), the investment adjustment cost (7), and the budget constraint (4). Here $U_i(.)$ is a partial derivative of U with respect to a variable i. Equation (26) relates the marginal rate of substitution between leisure and consumption to the real wage rate. Equation (27) refers to the decision of bond holdings of unconstrained household.

Note that unconstrained households must select their money holdings for transaction without knowing the period t values of the money stock or technology shock and they decide their financial asset positions after shocks occur.

[4]Note that $M_{t-1} = D_t + S_{U,t}$.

Hence, equation (28) refers to the decision of money holdings for transaction of unconstrained household. Here

$$\begin{aligned}\Lambda_{U,t} =& U_C(C_{U,t}, N_{U,t} + H_{U,t}) \left[\frac{1}{P_t} - \frac{H_t^{'}}{S_{U,t-1}} \right] \\ &+ \beta E \left[\frac{U_C(C_{U,t+1}, N_{U,t+1} + H_{U,t+1}) H_{t+1}^{'} S_{U,t+1}}{S_{U,t}^2} | \Omega_t \right]\end{aligned} \tag{34}$$

denotes a marginal utility of consumption financed by an additional unit of money to spend in goods market in period t. Here the information sets $\Omega_{b,t}$ and Ω_t indicate actions that are taken in the beginning of period t, that is, without knowledge of the shocks that are impinging on the economy within t and at the end of period t.

Equation (29) which is the first order condition with respect to the unconstrained household's investment represents that Tobin's Q equals the inverse of the investment/capital adjustment function derivative. Equation (31) shows that firms should pay cash for labor by borrowing from the financial intermediary. Equation (32) represents the relationship between the rent paid per unit of capital in $t+1$ and the expected return to holding one unit of physical capital from t to $t+1$ with the evolution of Tobin's Q over time. In the limited participation model, households must decide their holdings of $S_{U,t}$ before the realizations of the monetary and real shocks at time t, while other decisions are made after the realization of the shocks. Firms also decide labor demand taking into account the fact that the households face a delay in spending the profits.

Though we need not specify the functional form for adjustment cost function ψ, we should specify three parameters which describe the behavior around the steady state. First, we must specify the steady state value of Tobin's Q and the share of investment in national product. Since the steady state value of Tobin's Q is 1.0, we also set the value of this variable to 1.0 in the steady state. And we will take the same investment share in the steady state as in a model without adjustment cost. Next, we have to specify the parameter which determines the elasticity of marginal adjustment cost function.

Finally, combining the first order condition of domestic unconstrained household's foreign bond holdings (33) and the corresponding condition of foreign households yields the equilibrium nominal exchange rate determined by

$$\begin{aligned} &E_t\left[\frac{U_C(C^*_{t+1},N^*_{t+1}+H^*_{t+1})}{U_C(C^*_t,N^*_t+H^*_t)}\frac{P^*_t}{P^*_{t+1}}\right] \\ &=\exp(\chi_t)\Xi(\mathscr{B}_{F,t})E_t\left[\frac{U_C(C_{U,t+1},N_{U,t+1}+H_{U,t+1})}{U_C(C_{U,t},N_{U,t}+H_{U,t})}\frac{\mathscr{E}_{t+1}P_t}{\mathscr{E}_tP_{t+1}}\right], \end{aligned} \tag{35}$$

where the variable with asterik (*) denotes the foreign variable corresponding to domestic variable. To consider the international financial market frictions in the model, we have incorporated a country spread shock χ_t with an AR(1) process into (35) as follows

$$\chi_t = \rho_\chi \chi_{t-1} + \xi_{\chi_t},$$

where $\rho_\chi \in [0,1)$ governs the persistence of χ_t and $E(\xi_{\chi_t}) = 0$ and ξ_{χ_t} is i.i.d. over time.

5. QUANTITATIVE EVALUATION OF THE MODEL

5.1. PARAMETER VALUES

All parameter values used in this paper are reported in Table 3. The benchmark model of this paper takes a value of intertemporal elasticity of substitution and Frisch labor supply elasticity to be one, i.e. $\sigma = \nu = 1$. The intratemporal elasticity of substitution between home and foreign goods η is set to 1.5. ψ is calibrated so that the elasticity of Tobin's Q with respect to investment equals 1.0 as the benchmark parameter value as in King and Watson (1996). The depreciation rate and the steady-state real interest rate are calibrated to be 0.025 and 0.015 per quarter.

We also set the financial friction parameters associated with a limited participation as in Christiano and Eichenbaum (1992) and King and Watson (1996). Unconstrained households are assumed to spend 1% of their working hours in portfolio adjustment, i.e. $H_{ss}/N_{ss} = 0.01$ and the initial steady-state annual inflation rate equals 4%. The derivative of portfolio adjustment cost function is assumed to satisfy that a rise in the inflation rate by 4% increases time in financial rearrangement $1.06H_{ss}$ and a decline in the inflation rate by 4% decreases the time to financial adjustment to $0.95H_{ss}$ as in King and Watson (1996).

Parameter	Value	Description of Parameters
α	1/3	steady state capital share
λ	0, 0.3	fraction of constrained households
δ	0.025	rate of depreciation of capital stock
r	0.016	steady state rate of return
σ^{-1}	1	intertemporal elasticity of consumption
ν^{-1}	1	intratemporal elasticity of labor hours
b	0.5	degree of habit persistence in consumption
θ	0.4	degree of openness
η	1.5	elasticity of substitution between home and foreign goods
η_q	1	elasticity of i_t/k_t to Tobin's Q
ρ_A	0.9	persistence of technology shock
ρ_χ	0.9	persistence of premium shock

Table 3: The Calibrated Parameters

5.2. RELATIVE MEAN SQUARE APPROXIMATION ERROR

We take a VAR system as our empirical model as in King and Watson (1996) and Uribe and Schmitt-Grohé (2016):

$$\mathbf{A}\begin{bmatrix}\Delta y_t\\ c_t/y_t\\ i_t/y_t\\ tby_t\\ r_t\\ r_t^*\end{bmatrix}=\mathbf{B}(\mathbf{L})\begin{bmatrix}\Delta y_{t-1}\\ c_{t-1}/y_{t-1}\\ i_{t-1}/y_{t-1}\\ tby_{t-1}\\ r_{t-1}\\ r_{t-1}^*\end{bmatrix}+\begin{bmatrix}\varepsilon_{y,t}\\ \varepsilon_{c,t}\\ \varepsilon_{i,t}\\ \varepsilon_{tb,t}\\ \varepsilon_{r,t}\\ \varepsilon_{r^*,t}\end{bmatrix}, \tag{36}$$

where y_t, c_t, and i_t represent real gross domestic output, consumption, and investment. tby_t, r_t, and r_t^* are the trade balance relative to output, and domestic and foreign nominal interest rate. (36) specifies one stochastic trends by assuming a balanced growth path in y_t, c_t, and i_t.

To evaluate the goodness of fit of the models, we will utilize Watson (1993)'s minimum approximation error representation by consider the error $\mathbf{u}_t$ defined by

$$\mathbf{u}_t=\mathbf{y}_t-\mathbf{x}_t, \tag{37}$$

where $\mathbf{x}_t$ is the evolution of $n\times 1$ vector associated with the economic model, and $\mathbf{y}_t$ is the empirical counterparts of $\mathbf{x}_t$. Suppose that $\mathbf{x}_t$ and $\mathbf{y}_t$ are jointly covariance stationary. Then, the autocovariance generating function (ACGF) of

$\mathbf{u}_t$, $\mathbf{A_u}(z)$, can be found from

$$\mathbf{A_u}(z) = \mathbf{A_y}(z) + \mathbf{A_x}(z) - \mathbf{A_{xy}}(z) - \mathbf{A_{yx}}(z), \tag{38}$$

where $\mathbf{A_x}(z)$ is the ACGF of $\mathbf{x}_t$, $\mathbf{A_{xy}}(z)$ is the cross ACGF between $\mathbf{x}_t$ and $\mathbf{y}_t$ and so forth.

Watson (1993) suggested a bound on the relative mean square approximation error (RMSAE) for the economic model, which is analogous to a lower bound on $1 - R^2$ in a regression, as follows:

$$\mathbf{R}_k(\omega) = \frac{[A_{\mathbf{u}}(z)]_{kk}}{[A_{\mathbf{y}}(z)]_{kk}},\ z = e^{-i\omega}, \tag{39}$$

where $[A_{\mathbf{u}}(z)]_{kk}$, $[A_{\mathbf{y}}(z)]_{kk}$ are the kth diagonal elements of $A_{\mathbf{u}}(z)$, $A_{\mathbf{y}}(z)$,respectively.

Because $\mathbf{R}_k(\omega)$ is the variance of the error relative to the variance of the data for each frequency, it provides an information about how well the economic model fits the data over different frequencies, ω. Since the spectrum of the data, $\mathbf{y}_t$ is not known, we have to estimate it using the unrestricted VAR as in (36).

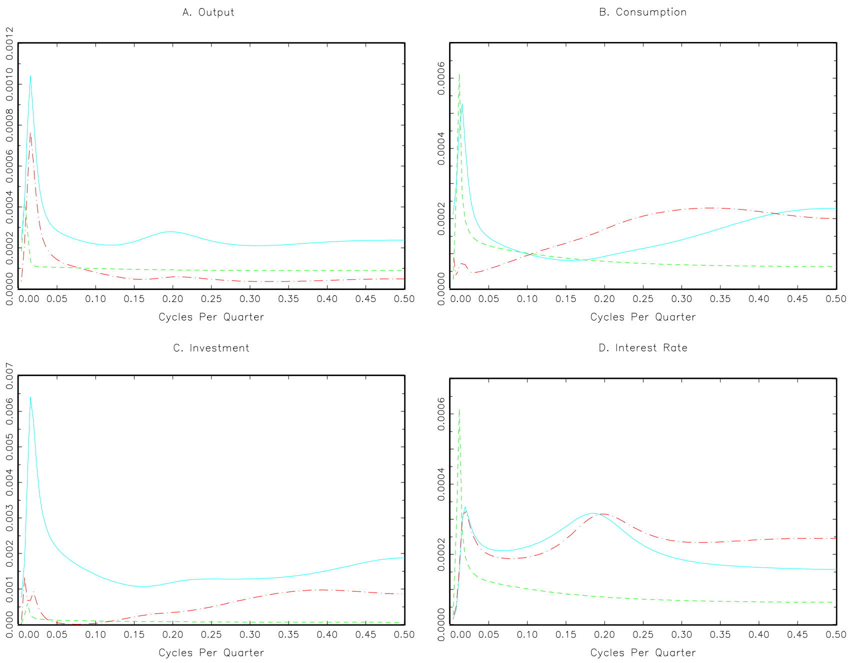
5.2.1 Spectral Density

First of all, we will address whether the spectra of the growth rates of the selected variables such as output, consumption, and investment calculated from the model correspond to the spectra implied by the data.

For each selected variable, Figures 4 presents the spectra of the selected variables associated with a two agent model with limited participation (long dashed lines), the spectrum of the data (solid lines), and the spectrum of the error required to reconcile the model and the data (dotted lines) before the Asian financial crisis, while Figure 5 shows the spectrum of variables of corresponding model and data after the crisis. The error process was chosen to minimize the unweighted trace of the error spectral density matrix subject to the constraint as in Watson (1993). Figures 4 and 5 show that the spectral density of output, consumption, and investment of the model displays a peak at the business cycle frequencies as in the data. The two agent model with limited participation generates the spectrum of consumption higher than the spectrum of output as in the data after the Asian financial crisis. However, the spectra of the selected variables associated with the model are lower than the spectra of the variables in the data and there is a substantial difference between the spectrum of interest rate in the model and the spectrum of interest rate in the data.

Also, note that the heights of spectra of consumption and output at business cycle frequencies are higher after the Asian financial crisis than before the crisis,

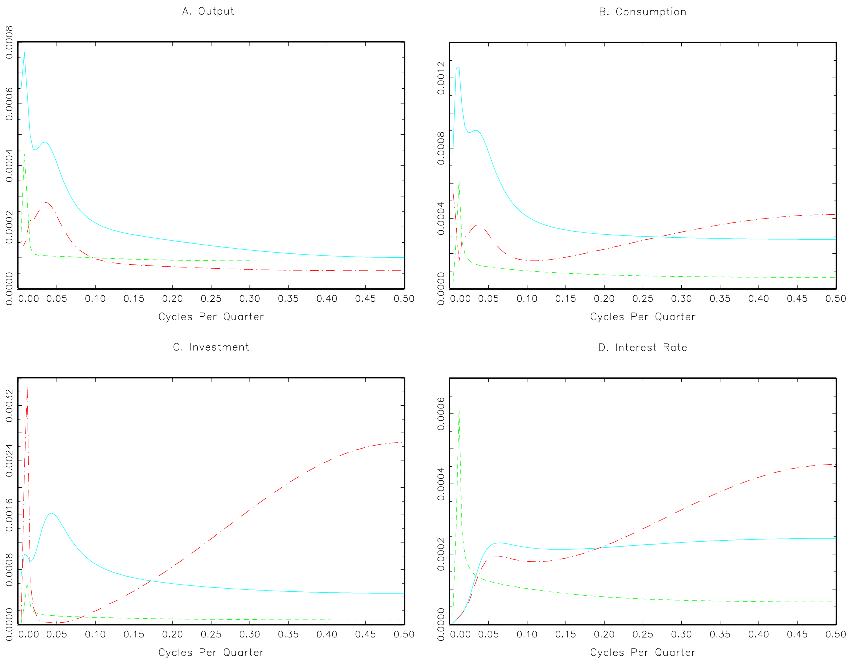
Figure 4: Growth Rate Spectra of Data and Two Agents Model: Before Asian Financial Crisis



while the height of the spectrum of investment is lower after the Asian financial crisis than before the crisis. The largest differences occur at a frequency corresponding to approximately 4 - 20 quarters. This is the most dramatic weakness of the model with limited participation. The weakness of the model can be seen more clearly in the spectrum of the error required to reconcile the model and the data.

Finally, Figures 6 and 7 present the spectra of the selected variables associated with the representative model by setting the fraction of constrained households λ equal to zero. The general shapes of the spectra of the representative model are similar to the ones of a two agent model. The heights of the spectra are lower than the spectra of the variables in the data and there is a substantial difference between the spectrum of interest rate in the model and the spectrum of interest rate in the data.

Figure 5: Growth Rate Spectra of Data and Two Agents Model: After Asian Financial Crisis

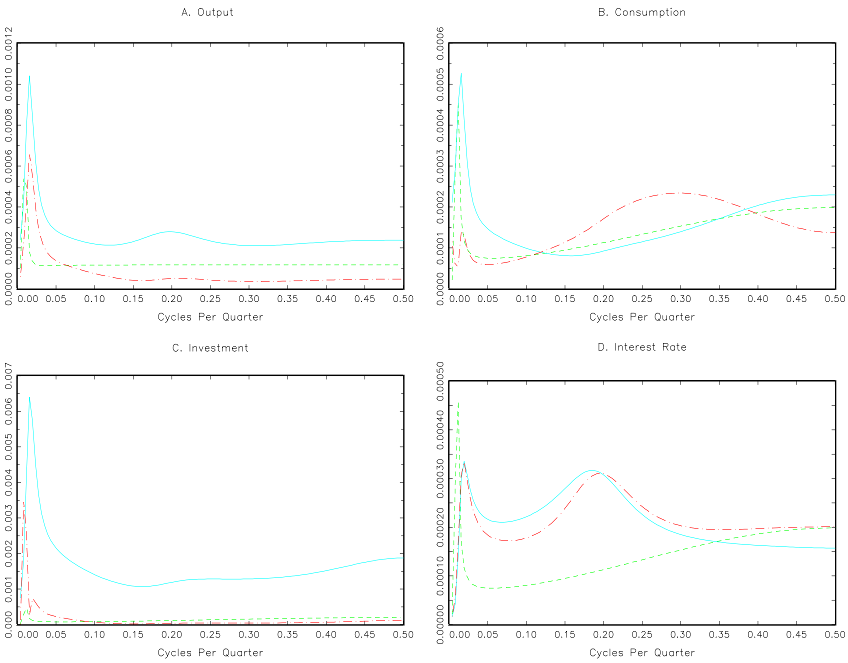


5.2.2 Relative Mean Square Approximation Error

Though the spectral density of the selected variables provides the information about the strengths and weaknesses of the calibrated DSGE models, we can evaluate the success and failure of the relevant model more critically using Watson (1993)'s RMSAE.

Before turning to the discussion of the model's fit using RMSAE, we compare the consumption volatility relative to output for the representative agent model as well as for the two agent model. The fraction of constrained households in the model is set to 0.3 whose value is taken from Jung and Kim (2019)'s estimate for λ, using the KLIPS (Korean Labor and Income Panel Study) covering from 2000 to 2016. Table 4 presents the volatilities of some selected variables for the representative agent model $(\lambda = 0)$ with limited participation as in Christiano and Eichenbaum (1992) and King and Watson (1996) and the ones for two agent model with limited participation $(\lambda = 0.3)$. Since the volatility of

Figure 6: Growth Rate Spectra of Data and One Agent Model: Before Asian Financial Crisis



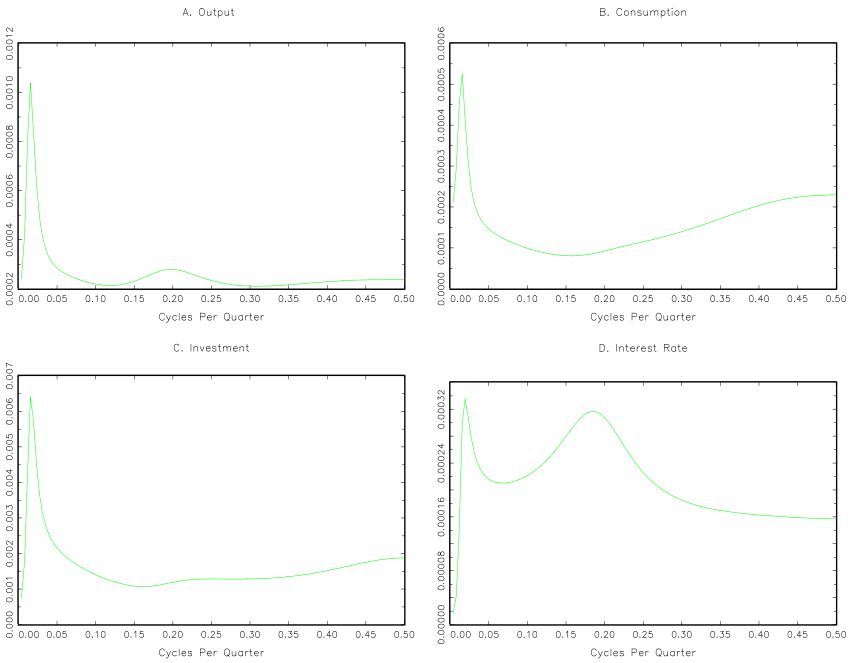
consumption increases with the share of households who cannot have access to financial market, the model with a substantial fraction of constrained households can generate the so-called consumption puzzle as in Table 4.

	One Agent	Two Agent
y	1.26	1.21
c	0.96	1.30
i	3.55	5.10
tb/y	0.85	1.33
r	0.63	0.53

Table 4: Moments of Model

Tables 5 and 6 provide a summary of the relative mean square approximation error (RMSAE) for the selected variables by integrated over business cycle fre-

Figure 7: Growth Rate Spectra of Data and One Agent Model: After Asian Financial Crisis



quencies (6-32 quarters) and by HP filter integrated across all frequencies when the unweighted trace of the spectrum is minimized. The RMSAEs for selected variables except trade balance and interest rate are less than one, either using only business cycle frequencies or HP filter integrated across all frequencies. The disappearance of the Korean seniority-based wage system and the early retirement age without a well-established safety net can be the source of the volatile movements of consumption and output after the crisis.

Finally, Figure 8 presents the consumption inequality and output of the model using the relationship between consumption inequality and trade balance given by

$$tb_t = \theta s_c[\lambda \gamma_{c,t} + (\eta - 1)(2 - \theta)\widehat{\mathscr{T}_t}],$$

where $tb_t \equiv \frac{TB_t}{Y}$, $\widehat{\mathscr{T}_t} \equiv \ln(\mathscr{T}_t/\mathscr{T})$, and $\mathscr{T}_t$ is the terms of trade at time t. The model is partially successful in that it generates a countercyclical consumption

Variable		
	Business Cycle	Frequencies
	One Agent ($\lambda = 0$)	Two Agent ($\lambda = 0.3$)
y	0.54	0.53
c	0.44	0.44
i	0.10	0.03
tb/y	1.36	2.23
r	0.87	0.91
	Hodrick-Prescott	Frequencies
	One Agent	Two Agent
y	0.51	0.52
c	0.55	0.55
i	0.09	0.08
tb/y	1.72	2.71
r	0.92	0.81

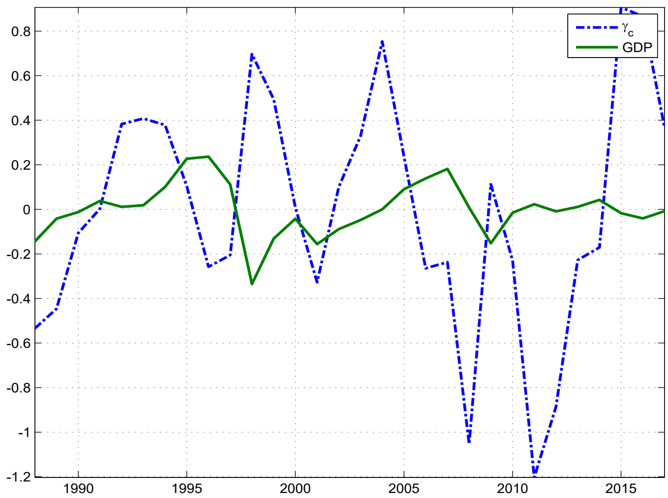
Table 5: Relative Mean Square Approximation Error of Alternative Models (1976:III - 1997:II)

Variable		
	Business Cycle	Frequencies
	One Agent ($\lambda = 0$)	Two Agent ($\lambda = 0.3$)
y	0.58	0.58
c	0.47	0.38
i	0.04	0.08
tb/y	0.45	0.35
r	0.74	0.85
	Hodrick-Prescott	Frequencies
	One Agent ($\lambda = 0$)	Two Agent ($\lambda = 0.3$)
y	0.55	0.57
c	0.48	0.41
i	0.18	0.39
tb/y	0.52	0.40
r	0.87	0.95

Table 6: Relative Mean Square Approximation Error of Alternative Models (1998.I-2018.III)

inequality as in the data $(corr(y_t, \gamma_{c,t}) = -0.42)$, but consumption inequality associated with two agent model is more volatile and countercyclical than the one in the data.

Figure 8: GDP and Consumption Inequality of Model



6. CONCLUSION

The Korean economy has displayed the features of common and unique business cycles relative to the developed countries after the Asian financial crisis. Some authors have argued that the financial frictions associated with international borrowing costs are main driving forces, while others have suggested that the permanent productivity shock processes matter.

In this paper, we have set up a two agent model with limited participation wherein both unconstrained and constrained households need cash to purchase goods and services along the line of Christiano and Eichenbaum (1992) and King and Watson (1996). The paper shows that the limited participation frictions have played an important role in explaining the business cycle in Korea after the 1997 financial crisis by applying Watson (1993)'s measure of fit and spectral density to the model. A more sophisticated model augmented with interaction between financial intermediaries and households and firms is needed to improve our understanding economic fluctuations in Korea.

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