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Customer Loyalty and Multi-stop Shopping*

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Abstract This paper studies how customer loyalty impinges on the existence of a multi-stop shopping equilibrium. A particular focus lies on the pricing decisions of firms in two distinct modes of retail, the department store and the shopping mall. We show that a multi-stop shopping equilibrium exists, but the required level of customer loyalty exceeds the total traveling costs. This seemingly unnatural gap arises only to discourage the individual retailer of the shopping mall from raising the price independently, knowing that the loyal customers would not leave him easily.

Keywords Multiproduct Firms, Spatial Competition, Two-stop Shopping, Customer Loyalty

JEL Classification L13, R12, R21, R32

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1. INTRODUCTION

In July 2017, *USA Today* and *NBC News* published articles reporting that an increasing number of consumers nowadays visit multiple stores in different locations on the same trip to obtain the goods on the shopping list.[1] This so-called *multi-stop shopping* or *multiple-store shopping* (henceforth MSS) is not only a recent trend in consumer behavior but also a well-documented behavior in the marketing literature. According to Gijsbrechts *et al.* (2008), about 75% of all grocery shoppers regularly visit more than one store each week, and similar numbers are reported in Fox and Hoch (2005) and in Drèze and Vanhuele (2006). As MSS incurs an additional traveling cost, it seems puzzling as to why customers behave against the temptation to buy everything on their shopping list in one store, thus saving on the traveling cost.

The typical view in the literature (Fox and Hoch, 2005; Lal and Rao, 1997) regards MSS to arise from an opportunistic cherry-picking incentive, i.e. the incentive to search for the lowest available price in each product category. The economic literature on MSS (Brandão *et al.*, 2014; Kim and Serfes, 2006; Thill, 1985, 1986, 1992) also takes the similar view, thereby comparing the traveling cost and the price advantage of making another trip to other stores within a multi-product variant of the celebrated Hotelling model. As Gijsbrechts *et al.* (2008) point out, however, the stability and the regularity of MSS patterns do not square well with cherry picking customers hunting for temporary price promotions, and Urbany *et al.* (1996) report that such customers account for only 10 to 35 percent of the whole group of MSS customers.

In this paper, we investigate a role of customer loyalty as one of the most crucial non-cherry-picking motives for MSS behaviors. By customer loyalty, we mean psychological benefits that a customer expects to derive from her purchasing decision, thus reflecting the concerns for branding, reputation, quality, and loyalty program.[2] This definition allows a customer to feel loyal to a particular store for

[1]Meyer, Z. (2017, July 19). Grocery shopping is no longer a one-stop experience. *USA Today*, Retrieved from https://www.usatoday.com/story/money/2017/07/19/grocery-shopping-no-longer-one-stop-experience/471484001/, White, M.C. (2017, July 28). Americans no longer want one-stop grocery shopping. *NBC News*, Retrieved from https://www.nbcnews.com/business/consumer/americans-no-longer-want-one-stop-grocery-shopping-n787166.

[2]Our definition distinguishes customer loyalty from the customer's preference, which is basically defined over the products. This is consistent with the celebrated work of Jacoby and Kyner (1973), arguing that customer loyalty involves psychological processes in which a customer compares and evaluates products/stores in various dimensions and may choose a certain product/store even when the prices dictate otherwise. Therefore, customer loyalty according to our definition is distinct from two closely related concepts, repeated purchasing behaviors and customer preferences.

one product, but not for the other products. According to Zhang *et al.* (2017), customer loyalty is a store- and product category-specific trait, and this feature of customer loyalty may give rise to MSS. To illustrate, consider a customer now in a Dunkin' store. She may choose to take a shopping trip to a nearby Starbucks store for coffee, although the Dunkin' store is also selling coffee. For donuts, however, the same customer would like to make a purchase in Dunkin'. This customer would decide to drop by both stores if her loyalty is deemed to outweigh the total traveling costs. As the customer loyalty refers to psychological benefits that are not susceptible to a sudden change, her MSS behavior would exhibit the stability and the regularity.

Nevertheless, we may not jump to a conclusion that customer loyalty simply implies the existence of MSS behaviors. The illustration we have just provided overlooks how the presence of customer loyalty itself affects firms' pricing decisions. Dunkin' may set the price for coffee low enough to steal some customers from the Starbucks store. If this is the case, the customer's behavior would not exhibit MSS even with her loyalty to Starbucks for coffee. Even with this possibility, MSS behaviors may appear again if Dunkin' raises the price of donuts, believing that the loyal customer would not leave easily for its opponent. It becomes more challenging when multi-product firms consist of individual retailers like a shopping center. The overall effect of the customer loyalty may depends on whether the pricing decisions of these retailers are coordinated or not. The retailers coordinate their product prices and share the profit if they are organized in the form of a *department store* like Macy's. If organized in the form of a *shopping mall* like Tysons Corner Center, the individual retailers set their own product prices independently. All these issues are left unsettled in the previous studies on customer loyalty and the existence of MSS.

The goal of this paper is thus to study how customer loyalty impinges on the existence of MSS, especially through its effect on the firms' pricing decisions. Particularly, we attempt to answer the following questions: Is the mere presence of customer loyalty sufficient for the existence of MSS? What conditions does the customer loyalty have to satisfy in order to give rise to MSS? What is the effect of customer loyalty on the firms' behaviors that lies behind the condition? Does the effect rely on whether a firm is a department store or a shopping mall? The first two questions are about the existence of a MSS equilibrium while the remaining two are about the effect of customer loyalty on the firms' price-setting behaviors.

Specifically, customer loyalty is not defined in purely behavioral terms, thus being distinct from repeated purchasing. It is also distinct from preference because the notion of customer loyalty in this paper contains evaluative processes. See Jacoby and Kyner (1973) for a more detailed discussion about the concept of customer loyalty.

To address these questions, we adapt the celebrated Hotelling model to allow for two products and customer loyalty. Specifically, each store sells two different goods and customers have a unit demand for each good. We also assume that store 1 has customer loyalty in good 1 while store 2 has in good 2. Therefore, a customer gains an additional value when purchasing good 1/good 2 in the store 1/store 2, respectively. Furthermore, both stores consist of two individual retailers, but the two stores differ in the organization of their constituents. One store is a shopping mall and the other is a department store.[3]

Our main result is that there exists a MSS equilibrium, in which multi-stop shoppers visit the stores they are loyal to, although the posted price of such a good is higher, and more importantly, the minimum level of the customer loyalty guaranteeing the existence of such an equilibrium is higher than the level that provides an incentive for customers to visit both stores (Theorem 1). This result answers the two sets of questions we ask previously about the customer loyalty and MSS.

First of all, the mere presence of customer loyalty is not sufficient for the existence of a MSS equilibrium. If a customer cannot gain from customer loyalty more than the traveling cost when visiting both stores, then there is no MSS equilibrium (Lemma 1). In particular, with no customer loyalty, there exists no MSS equilibrium (Corollary 1). This is also an implication of Brandão *et al.* (2014) when the number of goods in their model is set to two. Motivated by this non-existence result, they provide a cautionary warning that analyzing multi-product competition using the Hotelling model may be a substantial restriction when assuming only two goods. Nevertheless, our result indicates that one may disregard the warning if taking account of the customer loyalty. Secondly, and more importantly, our result indicates that there is a gap between the minimum level for guaranteeing the existence of a MSS equilibrium and the minimum level for covering the expense of customers to do MSS, and we shall refer to it as *the loyalty gap*. This seemingly unnecessary gap is required to regulate each store's and its individual retailers' price-setting incentives which we illustrate in our hypothetical example. A close examination of the loyalty gap reveals how the customer loyalty affects each firm's price-setting incentive, depending on different retail modes of a firm.

Surprisingly, the loyalty gap is required only to discourage the individual retailer in the shopping mall, who enjoys the benefit of customer loyalty, from raising his price. In other words, the other retailer in the shopping mall who

[3]Our terminology regarding the two different retail modes of price-setting behaviors of multi-product firms is due to Brandão *et al.* (2014).

has no loyal customers, and the two retailers in the department store would not deviate from the MSS equilibrium even when there is no loyalty gap. In our illustrative example, the coffee seller of Dunkin' would decrease the price in order to steal customers from Starbucks. The donut seller, knowing that customers would not leave readily, may want to raise the price. If they are organized in the form of a department store, the two sellers share the profit, and thus the coffee seller's loss due to the customer loyalty would be the donut seller's gain. However, the resulting gain is larger than the loss. The presence of multi-stop shoppers divides the markets, thus enabling the department store to charge different prices in different markets. Similarly to the prediction of the third-degree price discrimination, the overall profit of the department store with the presence of multi-stop shoppers is higher than the profit without it. Due to these cross-subsidization and the price discrimination effects, these sellers and the department as a whole have no incentive for deviation.

In the shopping mall, with the absence of cross-subsidization, the customer loyalty has differential effects on the individual retailers. When deviating from the MSS equilibrium to serve only the one-stop shoppers, a retailer faces a lower price elasticity of the demand for his product. For example, the coffee seller in Dunkin' has a weaker control over his demand, because the demand is affected not only by his opponent's price, but also by the prices that the donut sellers offer. For this coffee seller, the customer loyalty is a burden. He needs to set the price prohibitively low enough to compensate these customers for the loyalty value they gain by purchasing the coffee from Starbucks. For the donut seller, the customer loyalty has two opposing effects. It favors MSS because the gap between the price actually paid and the price perceived by loyal customers would make it easier to sustain MSS. At the same time, it threatens MSS. As the loyal customers would not switch away, the retailer is seduced to deviate by raising the price. The loyalty gap is exactly the size of customer loyalty that balances these two opposing effects. After the threshold defined by the loyalty gap, the subsidy effect would dominate because charging a higher price would cost an increasingly larger proportion of the demand that comes from the loyal customers.

The rest of paper proceeds as follows. In Section 2, we describe our model and analyze the optimal pricing behavior of the two stores which differ in their modes of retail. In Section 3, we present our main results and we conclude with a brief discussion in Section 4.

2. MODEL

2.1. ENVIRONMENT

We consider a variant of the model of Brandão *et al.* (2014) which is a multi-product version of the Hotelling model. There are two firms and the spatial distance between them is normalized to unity. As in Hotelling's model, we consider a unit interval $[0,1]$ and each firm is located at the end of this interval. Let L(Left) and R(Right) denote the two firms located at 0 and 1, respectively. Each firm is a shopping center, which consists of two individual retailers selling two different goods, good 1 and good 2. Accordingly, in order to avoid any confusion when referring to a firm, we reserve the term *store* to denote a shopping center as a whole, and refer to its constituents as *retailers*.

There is a continuum of customers uniformly distributed on this unit interval. Each customer has a unit demand for each good, thus she always purchases both goods. All customers share the same preference. Specifically, purchasing both goods yields the intrinsic value of $V > 0$. In addition to this intrinsic value, every customer gains some additional values by choosing to visit a particular store or to buy a particular product. For simplicity, this so-called *customer loyalty* value a customer gains when purchasing good 1 at store L is assumed to be the same as the value when purchasing good 2 at store R, which is denoted by $a > 0$.[4] In other words, every customer conceives good 1 to have a higher value when it is provided by store L, while good 2 is expected to have a higher value when provided by store R. In order to buy a good from store $j = L, R$, a customer whose location is $x \in [0,1]$ (henceforth consumer x) must take a trip to the store by incurring a traveling cost $t > 0$ per distance. To be specific, the traveling cost is tx if she buys from store L, and it is $t(1-x)$ if she buys from store R.

Let p_{ij} be the price of good $i = 1,2$ posted by store $j = L, R$. As each customer's demand for both goods is perfectly inelastic, she chooses among four possibilities: (i) buying both goods from store L, (ii) buying both goods from store R, (iii) buying good 1 from L and good 2 from R, or (iv) buying good 1 from R and good 2 from L. Let L, R, LR, and RL denote each possibility. Hence, the payoff of customer $x \in [0,1]$ in each case can be expressed as follows:

$$u_L(x) = V - tx + a - p_{1L} - p_{2L}, \qquad u_R(x) = V - t(1-x) + a - p_{1R} - p_{2R}$$
$$u_{LR}(x) = V - t + a - p_{1L} + a - p_{2R}, \quad u_{RL}(x) = V - t - p_{1R} - p_{2L}$$

[4] We assume that both the intrinsic value of V and the value of customer loyalty a are exogenously given. This assumption contrasts with the one in Kim (2012) that considers endogenous choice of the vertical attribute, which is interpreted as the gross surplus obtained from purchasing (i.e., quality).

We shall refer to those cases where customer x drops by both stores, as *two-stop shopping*. Accordingly, the other cases are called *one-stop shopping*. Note that the presence of the customer loyalty $a > 0$ reduces the cost, psychologically though, paid by a customer. The customer may find the effective price for good i at store j is not the price p_{ij} on the price tag (set by the store), but $p_{ij} - a_{ij}$. Based on this observation, we shall work with the notion of the effective price of good i at store j, $\mu_{ij} = p_{ij} - a_{ij}$, instead of p_{ij}. Specifically, the effective prices are $\mu_{1L} = p_{1L} - a$, $\mu_{2L} = p_{2L}$, $\mu_{1R} = p_{1R}$ and $\mu_{2R} = p_{2R} - a$.

Now that we use new notations, the customer x's payoffs are expressed as follows:

$$\begin{aligned} u_L(x) &= V - tx - \mu_{1L} - \mu_{2L}, \quad & u_R(x) &= V - t(1-x) - \mu_{1R} - \mu_{2R} \\ u_{LR}(x) &= V - t - \mu_{1L} - \mu_{2R}, \quad & u_{RL}(x) &= V - t - \mu_{1R} - \mu_{2L} \end{aligned}$$

where the payoffs in the last two cases can be further summarized as follows:

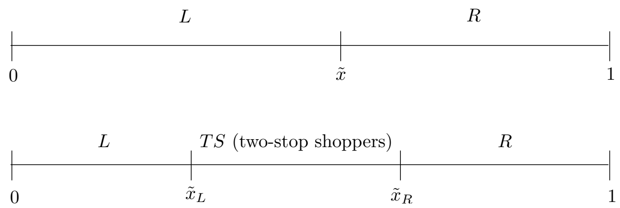
$$u_{TS}(x) = \max\{u_{LR}(x), u_{RL}(x)\} = V - t - [\min\{\mu_{1L}, \mu_{1R}\} + \min\{\mu_{2R}, \mu_{2L}\}].$$

For notational convenience, we shall use $\mu_L = \mu_{1L} + \mu_{2L}$, $\mu_R = \mu_{1R} + \mu_{2R}$ (equivalently, $p_L = p_{1L} + p_{2L}$, $p_R = p_{1R} + p_{2R}$), $\mu_{LR} = \mu_{1L} + \mu_{2R}$, $\mu_{RL} = \mu_{1R} + \mu_{2L}$, and $\mu_{TS} = \min\{\mu_{1L}, \mu_{1R}\} + \min\{\mu_{2R}, \mu_{2L}\}$. Notice that μ_{TS} must be either μ_{LR} or μ_{RL}, otherwise no consumer would travel to both ends.

The demand for store $j = L, R$ selling good $i = 1, 2$ depends upon whether there exist customers who would travel to both stores in search for a cheaper price or for a loyalty value. If there are such customers, we refer to them as *two-stop shoppers*, while calling the other customers who purchase only in one location as *one-stop shoppers*. The two-stop shoppers can further be classified into two different categories, depending upon their consumption-traveling behaviors. If two-stop shoppers buy good 1 from store L, good 2 from store R, they are *LR-type two-stop shoppers* (in short, *LR*-type shoppers). As these two-stop shoppers are seeking to buy goods that bring them with the loyalty values, we shall also refer them as *loyalty-seeking two-stop shoppers*. Otherwise, they are *RL-type two-stop shoppers* or *non-loyalty-seeking two-stop shoppers*. Furthermore, we shall refer to the demand for each store and its constituent retailers in the presence of two-stop shoppers as the demand in the two-stop shopping scenario (henceforth TSS). When there are no two-stop shoppers, we say that the demand under such a situation is in the one-stop shopping scenario (henceforth OSS). These two possible scenarios are illustrated in Figure 1.[5]

[5]One might wonder why there is no demand scenario in which the whole population is divided

Figure 1: One-stop and Two-stop Shopping Demand Scenario



In these scenarios, the demand for each good can be determined by identifying the location of a customer who is indifferent between any two adjacent choices. Specifically, in OSS, the indifferent customer between buying all from L and from R is $\tilde{x} = \frac{1}{2} + \frac{\mu_R - \mu_L}{2t}$ by solving for $u_L(\tilde{x}) = u_R(\tilde{x})$. All the customers lie on the left side of $\tilde{x}$ would buy both goods at L, while those on the right side of $\tilde{x}$ would buy at R. Similarly, in TSS, we can identify the customers $\tilde{x}_L = 1 - \frac{\mu_L - \mu_{TS}}{t}$ and $\tilde{x}_R = \frac{\mu_R - \mu_{TS}}{t}$ by solving for $u_L(\tilde{x_L}) = u_{TS}(\tilde{x_L})$ and $u_R(\tilde{x_R}) = u_{TS}(\tilde{x_R})$ respectively. Note that $\tilde{x}_L < \tilde{x}_R$ must hold in TSS, otherwise the situation goes back to OSS. In other words, the necessary and sufficient condition for the existence of two-stop shoppers is determined by $\tilde{x}_L < \tilde{x}_R$, which is

$$\sum_{i=1,2} |\mu_{iL} - \mu_{iR}| = |\mu_{1L} - \mu_{1R}| + |\mu_{2L} - \mu_{2R}| > t \tag{1}$$

or, equivalently, $|p_{1L} + a - p_{1R}| + |p_{2L} - p_{2R} - a| > t$. Let $\mathscr{P}_{TS}$ denote the set of vectors $\mu = (\mu_{1L}, \mu_{2L}, \mu_{1R}, \mu_{2R}) \in \mathbb{R}^4_+$ satisfying the condition (1), while denoting its complement by $\mathscr{P}_{OS}$. The demand for good i at store L is thus

$$q_{iL} = \begin{cases} \tilde{x} & \text{if } \mu \in \mathscr{P}_{OS} \\ \min(\tilde{x}_R, 1) & \text{if } \mu \in \mathscr{P}_{TS} \text{ and } \mu_{iL} < \mu_{iR} \\ \max(0, \tilde{x}_L) & \text{if } \mu \in \mathscr{P}_{TS} \text{ and } \mu_{iL} > \mu_{iR} \\ \frac{\max(0,\tilde{x}_L) + \min(\tilde{x}_R,1)}{2} & \text{if } \mu \in \mathscr{P}_{TS} \text{ and } \mu_{iL} = \mu_{iR} \end{cases}$$

where we assume that the half of the consumers buy good i at L and the other half

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in alternative ways such as $L - R - TS$ and $TS - L - R$. This is simply due to the transportation cost of each consumer. To see this, consider the case of $TS - L - R$. The consumer located at point 0 is the one who pay the most in terms of transportation costs when buying from store R. Therefore, if she prefers two-stop shopping to one-stop shopping at store L, then any consumer who is located at $x > 0$ must prefer two-stop shopping to one-stop shopping at store L. This is a contradiction to $TS - L - R$.

buys it at R when there is a tie $\mu_{iL} = \mu_{iR}$.[6] The demand for good i at store R is simply $q_{iR} = 1 - q_{iL}$.

2.2. MODES OF RETAIL AND PRICING RULES

Both shopping centers L and R consist of two different retailers, which we refer to as retailer $1L$, retailer $2L$, retailer $1R$ and retailer $2R$. However, these shopping centers differ from each other in their organization, particularly in their modes of retail. Each shopping center may take either of the following modes of retail: *a department store* or *a shopping mall*. In the department store, the two retailers are under the control of one single headquarter and thus they behave like a single firm. In contrast, the two retailers in the same shopping mall behave in a non-cooperative manner: each retailer chooses the price of his product independently of the other retailer. For our later purpose of studying how this difference interacts with the level of customer loyalty, we assume that store L is a department store and store R is a shopping mall. In this subsection, as a preliminary step for it, we shall investigate how a store's pricing behavior differs, depending upon its modes of retail and the different demand scenarios.

Consider firstly the department store L. As its constituent retailers coordinate their prices, the department store's pricing rule is determined by choosing p_{1L} and p_{2L} altogether, given its opponent's prices p_{1R} and p_{2R}. In OSS, its profit maximization problem is $\max_{p_{1L},p_{2L}}(p_{1L} + p_{2L})\tilde{x}$. Expressed in terms of the effective prices, the problem is

$$\max_{\mu_{1L}}(\mu_L + a)\left(\frac{1}{2} + \frac{\mu_R - \mu_L}{2t}\right)$$

and the optimal pricing rule is computed as $\mu_L = \frac{1}{2}\left[t + \mu_R - a\right]$, or equivalently,

$$p_L = \frac{1}{2}\left[t + p_R\right].$$

As the latter expression shows, the customer loyalty has no effect on the pricing behavior. This is not surprising. In OSS, a customer buys all goods at one location, and the department may determine the price for a bundle of two goods by coordinating the prices of its constituent retailers. It is as if the two markets, one for good 1 and the other for good 2, are integrated into one. That is, the department store L is competing against its opponent R in the market for one

[6] This tie-breaking rule has no effect on our results as the same assumption does in Brandão *et al.* (2014).

good, that is to say, a bundle of good 1 and good 2. Consequently, only the bundle prices p_L and p_R do matter. As the values of customer loyalty are equal across stores, their effect on the demand and thus on the pricing behavior vanishes.

In TSS, however, this is no longer true. As there are two different markets (one for good 1 and the other for good 2), the department store's profit relies on the individual prices and their competitive advantage (in terms of the effective prices). This implies that the pricing behavior of the department store also depends on whether the two-stop shoppers are of *LR*-type or of *RL*-type. When the two-stop shoppers are of *LR*-type, the department store's profit maximization problem is

$$\max_{\mu_{1L},\mu_{2L}} (\mu_{1L}+a)\left[\frac{\mu_{1R}-\mu_{1L}}{t}\right]+\mu_{2L}\left[1-\frac{1}{t}(\mu_{2L}-\mu_{2R})\right]$$

and the resulting pricing rules are $\mu_{1L}=\frac{\mu_{1R}-a}{2}$ and $\mu_{2L}=\frac{t+\mu_{2R}}{2}$. That is,

$$p_{1L}=\frac{p_{1R}+a}{2}, \quad p_{2L}=\frac{p_{2R}+t-a}{2}.$$

The above pricing rules witness that the value of customer loyalty matters. When the two-stop shoppers are of *RL*-type, the pricing rules can be obtained as $\mu_{1L}=\frac{\mu_{1R}+t-a}{2}$ and $\mu_{2L}=\frac{\mu_{2R}}{2}$. Equivalently,

$$p_{1L}=\frac{p_{1R}+t+a}{2}, \quad p_{2L}=\frac{p_{2R}-a}{2}.$$

The price of good 1, p_{1L} is increasing in the loyalty value a, while the price of good 2 is decreasing. The presence of two-stop shoppers now renders the department store L compete against its opponent R in two different markets. In the market for good 1, the higher loyalty value attracts more customers to L from R, thus allowing L to raise its price of good 1. On the other hand, in the market for good 2, the department store L cannot enjoy the benefit of customer loyalty when selling good 2. It thus needs to lower the price to attract customers who are loyal to good 2 being sold in its opponent R.

Now, we turn to the pricing rules of the shopping mall R. As the two retailers in the shopping mall set their prices independently, we need to analyze the pricing rule of each retailer $i=1,2$ given the prices (p_{1L},p_{2L}) of the department store L, and the price p_{jR}, $j\neq i$ of the other retailer in the shopping mall R. By adapting Brandão *et al.* (2014) to allow for customer loyalty a, we track down how the demand changes when the price switches back and forth between OSS and TSS. To be specific, we partition the domain of μ_{iR} into five different regions: (D1) all customers buy good i at R; (D2) TSS in which the two-stop shoppers buy good i

at R ($\mu_{iR} < \mu_{iL}$); (D3) OSS; (D4) TSS in which the two-stop shoppers buy good i at L ($\mu_{iR} > \mu_{iL}$); (D5) no customer buys good i at R. Accordingly, the demand for retailer iR selling good i in the shopping mall R, given $(\mu_{iL}, \mu_{jL}, \mu_{jR})$, is

$$q_{iR} = \begin{cases} 1, & \mu_{iR} \in D_1 = [-a(i-1), -t+\mu_{iL}+(\mu_{jL}-\mu_{jR})^+] \\ 1-\tilde{x}_L = \frac{1}{t}(\mu_{iL}-\mu_{iR}), & \mu_{iR} \in D_2 = (-t+\mu_{iL}+(\mu_{jL}-\mu_{jR})^+, -t+\mu_{iL}+|\mu_{jL}-\mu_{jR}|) \\ 1-\tilde{x} = \frac{1}{2}+\frac{1}{2t}(\mu_L-\mu_R), & \mu_{iR} \in D_3 = [-t+\mu_{iL}+|\mu_{jL}-\mu_{jR}|, t+\mu_{iL}-|\mu_{jL}-\mu_{jR}|] \\ 1-\tilde{x}_R = 1-\frac{1}{t}(\mu_{iR}-\mu_{iL}), & \mu_{iR} \in D_4 = (t+\mu_{iL}-|\mu_{jL}-\mu_{jR}|, t+\mu_{iL}+(\mu_{jL}-\mu_{jR})^-) \\ 0, & \mu_{iR} \in D_5 = [t+\mu_{iL}+(\mu_{jL}-\mu_{jR})^-, +\infty) \end{cases}$$

where $(\mu_{jL}-\mu_{jR})^+ = \max\{0, \mu_{jL}-\mu_{jR}\}$ and $(\mu_{jL}-\mu_{jR})^- = \min\{0, \mu_{jL}-\mu_{jR}\}$.

We shall now derive each retailer's optimal pricing rule in the relevant ranges (D_2, D_3, D_4). The profit maximization problem and the corresponding pricing rule of retailer iR can be computed as follows:

D_2: $\max_{\mu_{iR}}(\mu_{iR}+a(i-1))(1-\tilde{x}_L)$ and $\mu_{iR} = \frac{\mu_{iL}-a(i-1)}{2}$, i.e. $p_{iR} = \frac{p_{iL}-a}{2}+a(i-1)$.

D_3: $\max_{\mu_{iR}}(\mu_{iR}+a(i-1))(1-\tilde{x})$ and $\mu_{iR} = t+\mu_L-\mu_R$, i.e. $p_{iR} = t+p_L-p_R$.

D_4: $\max_{\mu_{iR}}(\mu_{iR}+a(i-1))(1-\tilde{x}_R)$ and $\mu_{iR} = \frac{t+\mu_{iL}-a(i-1)}{2}$, i.e. $p_{iR} = \frac{t+p_{iL}-a}{2}+a(i-1)$.

As in the case of the department store L in TSS, the price of each retailer increases in the value of customer loyalty if the product of the retailer has customer loyalty, and the price decreases otherwise.

3. MULTI-STOP SHOPPING EQUILIBRIUM

3.1. EXISTENCE

In this subsection, we investigate whether there exists a two-stop shopping equilibrium in the presence of customer loyalty. Particularly, we focus on the size of customer loyalty a (relative to the unit traveling cost t) that gives rise to a two-stop shopping equilibrium.

We first note that for there to be a two-stop shopper, the total loyalty value she gains from visiting both L and R must be no smaller than the traveling cost incurred by doing so. Otherwise, there would exist no two-stop shoppers.

Lemma 1. *If $2a \leq t$, there exists no two-stop shopping equilibrium.*[7]

[7] When $2a \leq t$, there exists an one-stop shopping equilibrium. As our focus lies in the existence of a two-stop shopping equilibrium, and also it can be easily seen in light of Brandão *et al.* (2014), we shall leave this to the readers.

The above lemma implies that the mere presence of customer loyalty does not guarantee the existence of a two-stop shopping equilibrium. In particular, notice that the case with no customer loyalty ($a = 0$) is consistent with Brandão *et al.* (2014) when there are only two products,[8], which exhibits no two-stop shopping equilibrium. This nonexistence of a multi-stop shopping equilibrium motivates Brandão *et al.* (2014) to warn that analyzing multi-product competition based on the model assuming only two goods may be a substantial restriction. Nevertheless, the following theorem demonstrates that one may disregard it as long as one includes the customer loyalty.

Theorem 1. *For $a/t > \frac{6\sqrt{2}-4}{7}$ (approximately, $a/t > 0.64$), there exists a unique two-stop shopping equilibrium, at which two-stop shoppers purchase each good in pursuit of the additional value from their loyalty (LR-type). In particular, there are two kinds of equilibria depending on the value of a/t.*

(1) $a/t \geq 2$: All customers are two-stop shoppers, $\tilde{x}_L = 0$ and $\tilde{x}_R = 1$.

(a) prices: $p_{1L} = p_{2R} = a - t$ and $p_{1R} = p_{2L} = 0$.

(b) demands: $q_{1L} = q_{2R} = 1$, and $q_{1R} = q_{2L} = 0$.

(c) profits: $\pi_L = a - t$, $\pi_{1R} = 0$, and $\pi_{2R} = a - t$.

(2) $\frac{6\sqrt{2}-4}{7} < a/t < 2$: A proportion of customers are one-stop shoppers, i.e. $0 < \tilde{x}_L < \tilde{x}_R < 1$.

(a) prices: $p_{1L} = p_{2R} = \frac{t+a}{3}$, and $p_{1R} = p_{2L} = \frac{2t-a}{3}$.

(b) demands: $q_{1L} = q_{2R} = \frac{1}{3} + \frac{a}{3t}$, and $q_{1R} = q_{2L} = \frac{2}{3} - \frac{a}{3t}$.

(c) profits: $\pi_L = \frac{(t+a)^2+(2t-a)^2}{9t}$, $\pi_{1R} = \frac{(2t-a)^2}{9t}$, and $\pi_{2R} = \frac{(t+a)^2}{9t}$.

The above theorem shows that a multi-stop shopping equilibrium exists even when assuming two goods, as long as the value of customer loyalty is higher than a certain level, which is $a/t > \frac{6\sqrt{2}-4}{7}$.

The minimum level of customer loyalty $\frac{6\sqrt{2}-4}{7} \approx 0.64$ that supports the existence of a two-stop shopping equilibrium presents a puzzle. By Lemma 1, we see that if $2a \leq t$, there exists no two-stop shopping equilibrium because the two-stop shopping customers cannot cover their traveling expenses by the additional gains from their loyalty. Hence, a natural conjecture would be that a two-stop shopping equilibrium always exists whenever $a/t > 0.5$. Surprisingly, however, it turns out

[8]See Proposition 3 of Brandão *et al.* (2014)

that it is not true. The minimum level for the existence of two-stop shopping, $\frac{6\sqrt{2}-4}{7} \approx 0.64$, is higher than $a/t = 0.5$. This seemingly unnatural result has its relevance to the optimal responses of the individual retailers in the presence of customer loyalty, which we shall elaborate in the next section. Therefore, the gap $\Delta = \frac{6\sqrt{2}-4}{7} - 0.5$ in the value of customer loyalty is required to provide an incentive for the stores and their individual retailers *not* to adjust their product prices further for deviation to OSS. Therefore, we shall refer to this gap as *the loyalty gap* throughout the rest of this paper.

Note that the equilibrium prices in Theorem 1 explains the empirical stability and regularity of multi-stop shopping that the existing literature fails to do. The existing literature regards multi-stop shopping behaviors of the customers to arise from their incentive to search for the lowest available price in each product category. However, this explanation does not square well with the empirical evidences (Gijsbrechts *et al.*, 2008; Urbany *et al.*, 1996), because only a small proportion of multi-stop shopping customers hunt for temporary price promotions. On the contrary, we assume that the multi-stop shopping behaviors of customers are motivated also by their loyalty to a specific product of a specific store. This explains why multi-stop shopping customers do not easily change their behaviors in response to temporary price promotions. Indeed, when $\frac{6\sqrt{2}-4}{7} < a/t < 2$, the posted prices of goods purchased by the multi-stop shoppers in the two-stop shopping equilibrium, p_{1L} and p_{2R}, are higher than the prices of the other goods:

$$p_{1L} = p_{2R} = \frac{t+a}{3} > p_{1R} = p_{2L} = \frac{2t-a}{3}.$$

The multi-stop shoppers buy more expensive goods because they conceive that these goods ($1L$ and $2R$) are actually cheaper because of their loyalty to these goods. In other words, the effective prices that are perceived by these multi-stop shoppers are lower:

$$\mu_{1L} = \mu_{2R} = \frac{t+a}{3} - a = \frac{t-2a}{3} < \mu_{1R} = \mu_{2L} = \frac{2t-a}{3}.$$

3.2. MODES OF RETAIL AND PRICING DECISIONS

In this subsection, we aim to explain the loyalty gap that arises for the existence of a two-stop shopping equilibrium by investigating the price-setting incentives of stores and their constituent retailers. A particular focus lies on the differences between the department store and the shopping mall. This subsection also constitutes the proof of Theorem 1 as well.

First of all, we present the necessary and sufficient condition for the department store L to induce TSS.

Lemma 2. *Suppose that the shopping mall R's effective prices satisfy* $\max\{\mu_{1R}, \mu_{2R}\} < 2t$. *Then, the department store L prefers to induce the two-stop shopping demand scenario instead of the one-stop shopping demand scenario if and only if* $|\mu_{1R} - \mu_{2R}| > t - a$.

The above lemma implies that whenever TSS is feasible (condition (1) holds), the department store L would stick to TSS. In particular, *the department store does not deviate to OSS from a two-stop shopping equilibrium (either LR-type or RL-type) as long as the profile of the equilibrium effective price* $(\mu_{1L}^*, \mu_{2L}^*, \mu_{1R}^*, \mu_{2R}^*)$ *satisfies the condition* (1). Then, we may show easily the following:

Corollary 1. *There exists no RL-type two-stop shopping equilibrium.*

Proof. Suppose otherwise that there exists a *RL*-type two-stop shopping equilibrium. Then, the equilibrium (effective) price profile $(\mu_{1L}^*, \mu_{2L}^*, \mu_{1R}^*, \mu_{2R}^*)$ is $\mu_{1L}^* = \mu_{2R}^* = \frac{2}{3}(t-a)$ and $\mu_{1R}^* = \mu_{2L}^* = \frac{t-a}{3}$. Note that $\max\{\mu_{1R}^*, \mu_{2R}^*\} = \mu_{2R}^* = \frac{2}{3}(t-a) < 2t \iff -a < 2t$ but $\mu_{2R}^* - \mu_{1R}^* = \frac{2}{3}(t-a) - \frac{t-a}{3} = \frac{t-a}{3} \not> t-a$. By Lemma 2, the department store L would deviate to induce OSS. □

As there is no *RL*-type two-stop shopping equilibrium, we may now focus on the price-setting incentives of the department store and the shopping mall in the *LR*-type two-stop shopping equilibrium to check for deviation.

Corollary 2. *If* $2a > t$, *the department store L would not deviate from a two-stop shopping equilibrium. The equilibrium profit of L is higher than the deviation profit under the one-stop shopping demand scenario.*

Proof. It suffices to check whether the equilibrium profile, $\mu_{1L}^* = \mu_{2R}^* = \frac{t-2a}{3}$ and $\mu_{1R}^* = \mu_{2L}^* = \frac{2t-a}{3}$, satisfies the conditions in Lemma 2. Specifically, we first see that $\max\{\mu_{1R}^*, \mu_{2R}^*\} = \mu_{1R}^* = \frac{2t-a}{3} < 2t$. Moreover, we observe that

$$\mu_{1R}^* - \mu_{2R}^* = \frac{2t-a}{3} - \frac{t-2a}{3} = \frac{t+a}{3} > t-a,$$

where the last inequality follows from $2a > t$. □

In other words, as long as the two-stop customers may cover their traveling expenses by the loyalty gains, the department store L would not deviate without being provided any further incentive. To understand the intuition behind this result, we examine more closely the incentives of the individual retailers in the department.

Let π^n_{1L} and π^n_{2L} be the profits of individual retailers in the department store, when they do not share the overall profit of the department store. In the *LR*-type two-stop shopping equilibrium, $\pi^n_{1L} = \frac{(t+a)^2}{9t}$ and $\pi^n_{2L} = \frac{(2t-a)^2}{9t}$. As expected, the increase in the loyalty value a has positive effects on π^n_{1L} and negative effects on π^n_{2L}, $\frac{\partial \pi^n_{1L}}{\partial a} = \frac{2(t+a)}{9t} > 0$ and $\frac{\partial \pi^n_{2L}}{\partial a} = \frac{-2(2t-a)}{9t} < 0$. However, the overall effect on the department store as a whole is positive:

$$\frac{\partial \pi_L}{\partial a} = \frac{2(2a-t)}{9t} > 0$$

where $\pi_L = \pi^n_{1L} + \pi^n_{2L}$ is the profit of the department store. As the individual retailers share this profit, the increase in a is beneficial to both.

On the other hand, if deviating to OSS, they earn the profit $\pi^d_L = \frac{t^2}{2t}$ with the bundle price $p^d_{1L} + p^d_{2L} = t$. Notice that $p^d_{1L} + p^d_{2L} = t = p^*_{1L} + p^*_{2L}$ and the demand $\tilde{x}_R - \tilde{x} = \tilde{x} - \tilde{x}_L$. Therefore, the overall demand and the associated price for the bundle are the same, but the profit in TSS is higher. This seems puzzling, but arises naturally according to the logic similar to the theory of third-degree price discrimination. In TSS, there are two markets, one for each good. The department store maximizes its profit in each market, given the prices posted by the shopping mall. In OSS, there is only one market, the market for the bundle that consists of good 1 and good 2. Therefore, in the former, the profit must be higher. The additional profit comes from the difference between the prices posted in the two different markets, $(p^*_{1L} - p^*_{2L})$.[9]

Now, we turn to the incentives of the retailers belonging to the shopping mall *R*. Before analyzing the two retailers one by one, we wish to note the following two facts regarding the retailers of the shopping mall:

(F1) A retailer, when deviating from *LR*-type equilibrium, cannot deviate to *RL*-type TSS. His only choice is to deviate to OSS.

(F2) When switching from TSS to OSS, conditional on the same price profile, an individual retailer of the shopping mall faces a more inelastic demand curve.

The first one is obvious. Suppose that the retailer 1*R* is contemplating to deviate from the *LR*-type equilibrium $(\mu^*_{1L}, \mu^*_{2L}, \mu^*_{1R}, \mu^*_{2R})$. That is, $\mu^*_{1L} < \mu^*_{1R}$ and $\mu^*_{2R} <$

[9] The exact expression for the additional profit is $p^*_{1L}(\tilde{x}_R - \tilde{x}) - p^*_{2L}(\tilde{x} - \tilde{x}_L)$. Using $\tilde{x} - \tilde{x}_L = \tilde{x}_R - \tilde{x}$, we may write it as $(p^*_{1L} - p^*_{2L})(\tilde{x} - \tilde{x}_L)$ (or, equivalently, $(p^*_{1L} - p^*_{2L})(\tilde{x}_R - \tilde{x})$).

μ_{2L}^*. Expecting that the other retailers play accordingly to $\mu_{2R}^* < \mu_{2L}^*$, he cannot induce *RL*-type TSS because it requires $\mu_{2R}^* > \mu_{2L}^*$ which is beyond 1*R*'s control. Similarly, the retailer 2*R*, given $\mu_{1L}^* < \mu_{1R}^*$, has nothing but to induce OSS if he contemplates to deviate.

The second one requires some elaboration. Consider, without loss of generality, the retailer 1*R*. As the price elasticity of the demand is not constant even within each price range D_3 (for OSS) and D_4 (for TSS), arguing that the price elasticity of the demand in TSS is larger than that in OSS would be misleading. What we argue in (F2) is that when the price p_{1R} lies on the border between D_3 and D_4. The price elasticity of the demand at the point goes through a sudden drop when switching from TSS to OSS. Formally, let ε_{iR}^{OS} and ε_{iR}^{TS} be the price elasticity of the demand faced by the retailer *iR* ($i = 1,2$) respectively in OSS and in TSS. For the retailer 1*R*, a simple calculation gives $\varepsilon_{1R}^{OS} = 3p_{1R}/(5t-a-3p_{1R})$ and $\varepsilon_{1R}^{TS} = 3p_{1R}/(4t-2a-3p_{1R})$. When p_{1R} passes the threshold at which TSS switches to OSS, the elasticity drops. That is, at the threshold $p_{1R} = t-a$, $\varepsilon_{1R}^{OS} = 3(t-a)/2(t+a) < \varepsilon_{1R}^{TS} = 3(t-a)/(t+a)$. Similarly, to the retailer 2*R*, at the threshold between OSS and TSS, i.e. $p_{2R} = \mu_{2R} + a = a$, the elasticity drops: $\varepsilon_{2R}^{OS} = 3a/2(2t-a) < \varepsilon_{2R}^{TS} = 3a/(2t-a)$.

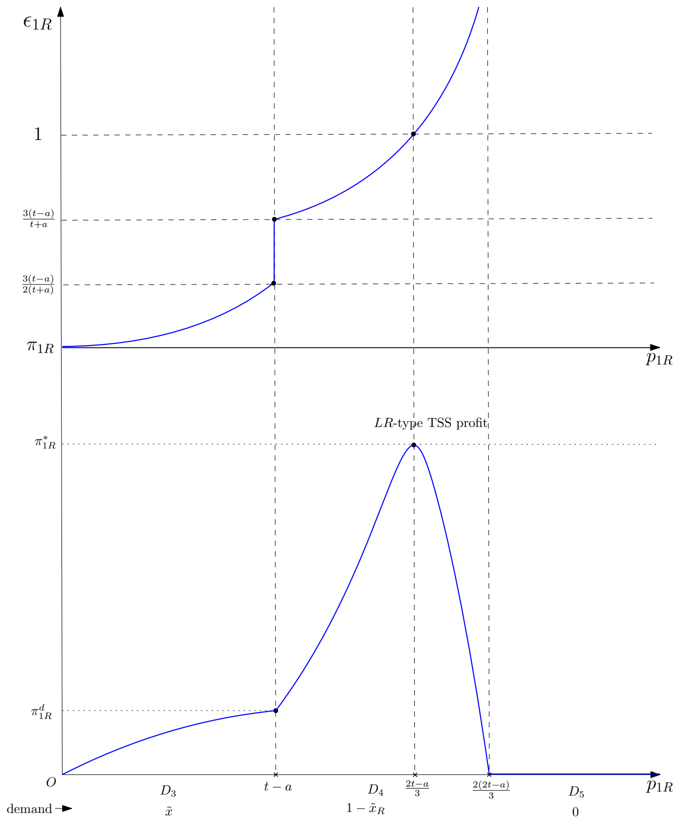
The intuition behind (F2) is as follows: In TSS, there are two different markets, one for each product. On the other hand, in OSS, there is only one market that trades both products as a bundle. An individual retailer thus has a weaker control over the demand, which makes the demand more inelastic. For example, the retailer 1*R* needs to take account of not only his opponent's price (p_{1L}) but also the prices of good 2 posted by 2*L* and 2*R* (p_{2L}, p_{2R}) if switching from TSS to OSS.

These two facts (F1) and (F2) together implies that the loyalty gap does not come from the retailer 1*R*. As the department store does, the retailer 1*R* would stick to TSS as long as the customers would.

Lemma 3. *Suppose that $a/t > 0.5$. The retailer 1R of the shopping mall would not deviate to the one-stop shopping scenario. Specifically, if the deviation occurs, the deviation price would be determined as $\mu_{1R}^d = t-a$ to equate the demand under one-stop shopping scenario with the demand under two-stop shopping scenario ($1-\tilde{x} = 1-\tilde{x}_R$). The resulting profit from deviation (π_{1R}^d) is strictly less than the equilibrium profit (π_{1R}^*).*

The retailer 1*R*, who does not have any loyal customers, has an incentive to set the price low enough to steal the demand from 1*L*. The deviation price must lie in the price range for OSS ($D_3 = [0, t-a]$). Otherwise, it violates (F1). Within D_3, the elasticity decreases as the price falls. This implies that a further decrease in

Figure 2: The price elasticity and profit of the retailer $1R$ when the others charge the equilibrium prices



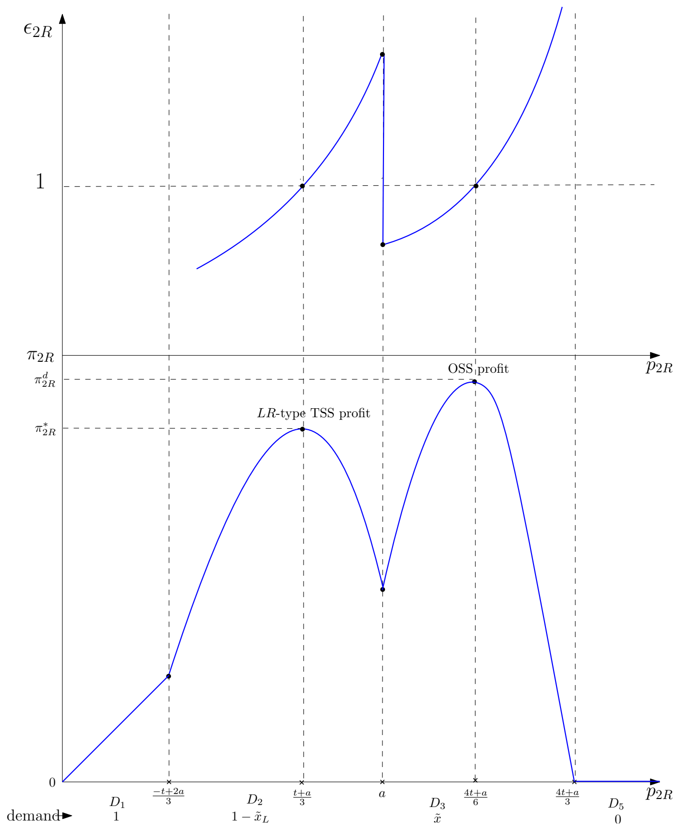
the price from $t-a$ would fail to raise the demand sufficiently. Hence, the retailer $1R$ would decrease the price just to make two-stop shoppers indifferent between OSS and TSS. That is, the deviation price is $t-a$ (See Figure 2).

In contrast to the previous cases, the individual retailer $2R$ would deviate even though $a/t > 0.5$. As the following lemma demonstrates, the loyalty gap, the difference between 0.5 and $(6\sqrt{2}-4)/7$, arises in order to discourage $2R$ from deviating to OSS.

Lemma 4. *If* $0.5 < a/t \leq (6\sqrt{2}-4)/7$, *the retailer* $2R$ *would deviate to the one-stop shopping scenario. Otherwise, if* $a/t > (6\sqrt{2}-4)/7$, *then the retailer* $2R$ *would have no profitable deviation.*

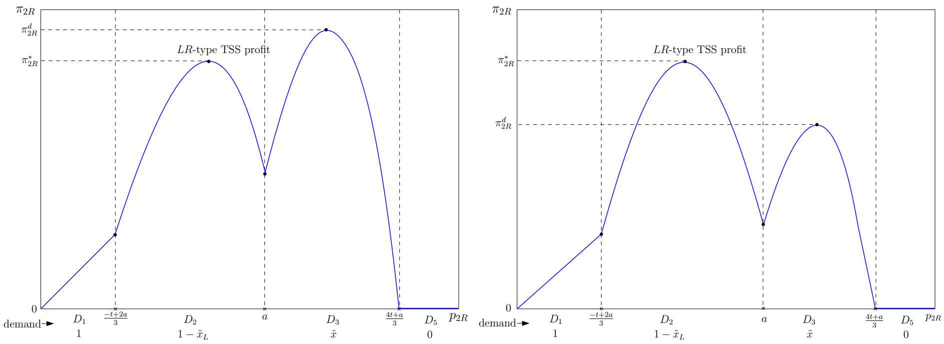
The individual retailer $2R$, in the presence of the customers who are loyal to

Figure 3: The price elasticity and profit of the retailer $2R$ when the others charge the equilibrium prices



him, may attempt to raise the price, because he knows that these loyal customers would not leave him easily. This attempt would be thwarted effectively by a significant reduction in the demand, if the elasticity of the demand increases monotonically in the price. Interestingly, however, the discontinuous fall in the price elasticity of the demand (F2) leaves a room for the successful deviation of $2R$ by creating non-monotonicity. In Figure 3, when the retailer $2R$ deviates by switching from D_2 to D_3, he goes through a sudden drop in the price elasticity at the point where D_2 and D_3 join, thereby rendering the elasticity of the demand non-monotonic at that point. To elaborate on how such a non-monotonicity leads to a profitable deviation, we argue that the deviation price for $2R$ occurs at the point where the demand is unit-elastic (conditional on the equilibrium price profile of the other retailers). Given the equilibrium price profile of the other retailers

Figure 4: The profit function of $2R$ when the others charge the equilibrium prices



(a) $0.5 < a/t \leq (6\sqrt{2}-4)/7$ (b) $a/t > (6\sqrt{2}-4)/7$

(thereby anticipating the residual demand for him), the retailer $2R$ chooses the price (for deviation) as if a monopolist chooses his profit-maximizing price. As the monopolist does, the retailer $2R$ would operate only at the price where the demand is elastic. Otherwise, if he operates at the point where the elasticity is less than one, he may raise the total revenue (and the profit) by setting the price higher. If operating at the point where the elasticity exceeds one, he may also increase his profit by decreasing the price. The same logic indeed applies to the LR-type equilibrium price. Therefore, we may conclude that the price elasticity of the demand at the optimal deviation price would coincide with that of the demand at the LR-type equilibrium price as the unity. As Figure 3 illustrates, the profit that declines as the price approaches to the threshold level for D_3 would rise beyond the threshold before reaching the optimal deviation price. This creates another hump in the profit function.

Accordingly, a successful deviation for $2R$ relies on how quickly (or slowly) the price elasticity of the demand catches up after its precipitous decline. Now, the customer loyalty comes into play. For the retailer $1R$, not having any loyal customers, any increase in the customer loyalty value always lowers his profit. For the retailer $2R$, however, it is a subsidy. The effective price, which is the price perceived by the customers, is lower than the posted price by the amount of the customer loyalty. The customer loyalty, through its role as a subsidy, affects the retailer $2R$'s price-setting incentive in two different directions.

In one direction, as we previously mentioned, the presence of customer loyalty makes it more attractive for the retailer $2R$ to raise the price. As the value of the customer loyalty increases, the retailer $2R$ gets more confident to raise the price further, even further to induce OSS if his desirable level of the price goes beyond the level that the price region for TSS (D_2) can accommodate. The non-monotonicity of the price elasticity of the demand makes it possible for $2R$ to do so. Indeed, the gap in the price elasticity at the point of discontinuity increases in the value of the customer loyalty, thus making deviation more profitable.

In the other direction, as the customer loyalty is a subsidy, it allows the effective price to be negative. Recalling that the posted price cannot be negative, an increase in the value of the customer loyalty expands the range of the prices supporting TSS. In other words, the price region for TSS (D_2) accounts for an increasingly larger proportion than the price region for OSS (D_3), thereby favoring TSS.

Lemma 4 shows that there is a threshold in the value of customer loyalty at which these two opposing forces are balanced. Beyond the threshold, the subsidy effect dominates the other effect (See Figure 4).

4. CONCLUDING REMARKS

In this study, we aim to understand the well-documented phenomenon of multi-stop shopping behaviors of customers through the lens of the customer loyalty. Specifically, we adapt the celebrated Hotelling model to allow for two products and customer loyalty. Moreover, we consider different modes of retail for stores as in Brandão *et al.* (2014).

The innovation that differentiates this study from the vast literatures on customer loyalty or from those on multi-product competition, is our focus on the firms' price-setting behaviors rather than solely on the consumer behavior. Specifically, we show that there exists a multi-stop shopping equilibrium and the minimum level of customer loyalty guaranteeing such an equilibrium, is above the level required to induce customers to make another trip. The key to understanding this gap lies in the optimal responses of firms, knowing that customers are loyal to a particular product in their stores.

In particular, we find that the gap is required to discourage only an individual retailer who has loyal customers when the store is organized to take the form of a shopping mall. In the department store, such a gap is not required because of the cross-subsidization across individual retailers. In the shopping mall, however, each retailer may deviate to face only the one-stop shoppers. This deviation is not profitable for the retailer having no loyal customers, because the presence of customer loyalty puts additional pressure on him to set the price to a prohibitively low level. For the retailer who owns loyal customers, however, the customer loyalty works as a subsidy, and thus makes it more bearable to face lower and more elastic demands from deviation in exchange for a higher price. This subsidy effect does not last as the customer loyalty gets larger by making it costly to give up multi-stop shoppers.

Lastly, we conclude by addressing possible concerns regarding the two assumptions we impose in this study: (i) symmetric loyalty values and (ii) different modes of retail for the two stores. We assume throughout this paper that the loyalty value a customer gains when purchasing good 1 at store L is identical to the value when purchasing good 2 at store R. Our main results about the multi-stop shopping equilibrium carries over to the case of asymmetric loyalty values, only to make the exposition harder because the region of the customer loyalty values for the existence of a multi-stop shopping equilibrium is characterized in two dimensions. The other assumption in this study is that one store is a department store while the other is a shopping mall. This assumption is sufficient to serve our main interest about how the customer loyalty affects the existence of a multi-stop shopping through different retail modes and the individual retailers' price-setting incentives. Specifically, considering the cases in which both stores share the same retail modes are redundant: two-stop shopping equilibrium also arises, although no loyalty gap is required for the case of two department stores.

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APPENDIX

OMITTED PROOFS

We shall compute a candidate two-stop shopping equilibrium for *LR*- and *RL*-type, respectively. By combining the pricing rules in Section 2.2, we obtain the following:

(LR) $\mu_{1L}^* = \mu_{2R}^* = \frac{t-2a}{3}$ and $\mu_{1R}^* = \mu_{2L}^* = \frac{2t-a}{3}$.

(RL) $\mu_{1L}^* = \mu_{2R}^* = \frac{2}{3}(t-a)$ and $\mu_{1R}^* = \mu_{2L}^* = \frac{t-a}{3}$.

where $t > a$ holds in the case of *RL*-type to make sure that $\tilde{x}_L < 1$ and $\tilde{x}_R > 0$. Moreover, in the case of *LR*-type, as the price cannot be negative, $a \leq 2t$. We shall deal with the case where $a \geq 2t$ in the last part of this appendix.

Proof of Lemma 1. Suppose to the contrary that there exists a two-stop shopping equilibrium. Let $\mu^* = (\mu_{1L}^*, \mu_{2L}^*, \mu_{1R}^*, \mu_{2R}^*)$ denotes the equilibrium effective price vector. Such an equilibrium must be either of *LR*-type ($\mu_{1L}^* < \mu_{1R}^*$ and $\mu_{2L}^* > \mu_{2R}^*$) or of *RL*-type ($\mu_{1L}^* > \mu_{1R}^*$ and $\mu_{2L}^* < \mu_{2R}^*$). In each case, the corresponding candidate equilibrium profile must satisfy the following (1):

$$(LR) \quad |\mu_{1L}^* - \mu_{1R}^*| + |\mu_{2L}^* - \mu_{2R}^*| = \frac{2}{3}(t+a) > t; \tag{2}$$

$$(RL) \quad |\mu_{1L}^* - \mu_{1R}^*| + |\mu_{2L}^* - \mu_{2R}^*| = \frac{2}{3}(t-a) > t \tag{3}$$

However, none of the above inequalities hold. In the inequality (2), $\frac{2(t+a)}{3} \leq \frac{2t+t}{3} = t$ due to $a \leq 2t$. In (3), $\frac{2}{3}(t-a) > t$ is equivalent to $-a > t$, contradicting to $a > 0$ and $t > 0$.

Proof of Lemma 2. For the necessity, we shall prove firstly the claim for the case of *LR*-type. Suppose by way of contradiction that $\mu_{1R} - \mu_{2R} \leq t - a$. To induce TSS, the department store needs to set its product prices so that condition (1) holds. The condition, when inducing *LR*-type TSS, is written as $\mu_{1R} - \mu_{1L} + \mu_{2L} - \mu_{2R} = (\mu_{1R} - \mu_{2R}) + (\mu_{2L} - \mu_{1L}) > t$. To have such a pair (μ_{1R}, μ_{2R}), the department store must sell good 1 at a lower price than good 2, i.e. $\mu_{1L} + a = p_{1L} < \mu_{2L} = p_{2L}$. This implies that $(\tilde{x}_R - \tilde{x})p_{1L} < (\tilde{x} - \tilde{x}_L)p_{2L}$ because $\tilde{x}_R - \tilde{x} = \tilde{x} - \tilde{x}_L$. Equivalently, $\tilde{x}_R p_{1L} + \tilde{x}_L p_{2L} < \tilde{x}(p_{1L} + p_{2L})$. The department store must prefer to induce OSS, and this is a contradiction. Hence, $\mu_{1R} - \mu_{2R} > t - a$ is a necessary condition for the department store to prefer *LR*-type TSS.

Similarly, for *RL*-type, suppose that $\mu_{2R} - \mu_{1R} > t - a$. Again, being combined with the condition (1), $(\mu_{2R} - \mu_{1R}) + (\mu_{1L} - \mu_{2L}) > t$, it must be that $\mu_{1L} - \mu_{2L} > a$, or equivalently, $p_{1L} > p_{2L}$. This in turn implies $(\tilde{x} - \tilde{x}_L)p_{1L} > (\tilde{x}_R - \tilde{x})p_{2L}$, thus leading to the same contradiction as in the previous case.

For the other direction, we first show that if $\mu_{1R} - \mu_{2R} > t - a$, the department store prefers to induce the *LR*-type TSS rather than to induce OSS. Together with the first order conditions $\mu_{1L} = \frac{\mu_{1R} - a}{2}$ and $\mu_{2L} = \frac{t + \mu_{2R}}{2}$, the hypothesis $\mu_{1R} - \mu_{2R} > t - a$ implies the following:

(i) $\tilde{x}_R = \frac{1}{t}(\mu_{1R} - \mu_{1L}) = \frac{1}{2t}(\mu_{1R} + a) \in [0,1] \iff \mu_{1R} + a \in [0, 2t]$, implying $0 \leq \mu_{1R} = p_{1R} \leq 2t - a$.

(ii) $\tilde{x}_L = 1 - \frac{\mu_{2L} - \mu_{2R}}{t} = \frac{1}{2} + \frac{\mu_{2R}}{2t} \in [0,1] \iff \mu_{2R} = p_{2R} - a \in [-t, t]$, which implies $p_{2R} \leq t + a$ $(t - a > 0)$.

(iii) $\mu_{1R} - \mu_{1L} + \mu_{2L} - \mu_{2R} > t \iff \mu_{1R} - \mu_{2R} > t - a$.

(iv) $\mu_{1L} < \mu_{1R} \iff \frac{\mu_{1R} - a}{2} < \mu_{1R} \iff \mu_{1R} > -a$.

(v) $\mu_{2L} > \mu_{2R} \iff \frac{t + \mu_{2R}}{2} > \mu_{2R} \iff \mu_{2R} < t$.

(i) and (iii) are true by hypothesis. (iv) trivially holds, because $\mu_{1R} = p_{1R} \geq 0$ (thus larger than $-a$). (ii) and (v) hold true by (i) and (iii). Therefore, the first-order conditions yield the maximum profit of the department store with two-stop shopping. The resulting profit of the department store is thus $\pi_L^{TS}(\mu_{1R}, \mu_{2R}) = \frac{1}{4t}\left[(\mu_{1R} + a)^2 + (\mu_{2R} + t)^2\right]$. On the other hand, in OSS, $\mu_L = \frac{t + \mu_R - a}{2}$ and the resulting profit function is $\pi_L^{OS}(\mu_{1R}, \mu_{2R}) = \frac{1}{8t}(t + \mu_R + a)^2$. By comparing these two, we obtain

$$\pi^{TS} - \pi^{OS} = \frac{1}{8t}(\mu_{1R} - \mu_{2R} + a - t)^2 \geq 0.$$

Similarly, for *RL*-type TSS, together with the first order conditions, we see that

(i) $\tilde{x}_R \in [0,1] \iff \mu_{2R} \in [0, 2t]$, implying $p_{2R} \leq 2t + a$.

(ii) $\tilde{x}_L \in [0,1] \iff \mu_{1R} + t + a \in [-2t, 2t]$, yielding $0 \leq \mu_{1R} = p_{1R} \leq t - a$.

(iii) $\mu_{1L} - \mu_{1R} + \mu_{2R} - \mu_{2L} > t \iff \mu_{2R} - \mu_{1R} > t + a$.

(iv) $\mu_{1R} < \mu_{1L} \iff \frac{\mu_{1R} + t - a}{2} < \mu_{1R} \iff \mu_{1R} < t - a$.

(v) $\mu_{2R} > \mu_{2L} \iff \frac{\mu_{2R}}{2} > \mu_{2R} \iff \mu_{2R} > 0$.

(i) and (iii) are true by hypothesis, thus implying (ii) and thus (iv). Lastly, (v) holds trivially by (i). Then, the first-order conditions yield the maximum profit of the department store with two-stop shopping. By comparing the profits in TSS and OSS, we obtain the following: $\pi^{TS} - \pi^{OS} = \frac{1}{8t}(\mu_{2R} - \mu_{1R} + a - t)^2 \geq 0$.

Proof of Lemma 3. By plugging the *LR*-equilibrium strategy profile of the other players $(\mu_{1L}^*, \mu_{2L}^*, \mu_{2R}^*)$ for $a < 2t$, we obtain the following expression for the retailer $1R$'s demand function:

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$$q_{1R} = \begin{cases} 1, & \mu_{1R} \in D_1 = \emptyset \\ 1 - \tilde{x}_L = \frac{t-2a-3\mu_{1R}}{3t}, & \mu_{1R} \in D_2 = \emptyset \\ 1 - \tilde{x} = \frac{5t-a-3\mu_{1R}}{6t}, & \mu_{1R} \in D_3 = [0, t-a] \\ 1 - \tilde{x}_R = \frac{4t-2a-3\mu_{1R}}{3t}, & \mu_{1R} \in D_4 = \left(t-a, \frac{2(2t-a)}{3}\right) \\ 0, & \mu_{1R} \in D_5 = \left[\frac{2(2t-a)}{3}, +\infty\right) \end{cases}$$

In order to show that there is no profitable deviation for the retailer $1R$, we need to check first whether the retailer $1R$'s *LR*-type equilibrium price indeed belongs to D_4. That is, $\mu_{1R}^* = p_{1R}^* = \frac{2t-a}{3} \in D_4$, which leads to the following condition:

$$\frac{a}{2} < t < 2a$$

Next, we need to check whether there is a profitable deviation for the retailer $1R$. In particular, we first consider the retailer $1R$'s deviation to D_3, i.e. it decreases his product price in order to induce OSS. This requires us to compare the corresponding profits in the related demand regions. Recall that in the *LR*-type two-stop shopping equilibrium,

$$\mu_{1R}^* = p_{1R}^* = \frac{2t-a}{3}, \quad q_{1R}^* = 1 - \tilde{x}_R = \frac{2t-a}{3t}, \quad \pi_{1R}^* = \mu_{1R}(1 - \tilde{x}_R) = \frac{(2t-a)^2}{9t}.$$

Turning to the retailer $1R$'s deviation to D_3, consider the slope of the corresponding profit maximization problem,

$$slope(\mu_{1R}) \equiv \frac{\partial \pi_{1R}}{\partial \mu_{1R}} = \frac{1}{6t}(5t - a - 6\mu_{1R}),$$

where $\pi_{1R} = \pi_{1R}(\mu_{1R}; \mu_{1L}^*(t,a), \mu_{2L}^*(t,a), \mu_{2R}^*(t,a))$. This expression is decreasing in μ_{1R}, thus achieving its minimum at the right end of the interval D_3. Notice that this expression is positive at its minimum within D_3:

$$\min_{\mu_{1R}\in D_3} slope(\mu_{1R}) = \frac{1}{6t}(-t+5a) > \frac{1}{6t}(-t+a+2t) = \frac{1}{6t}(t+a) > 0.$$

The inequality follows from $2a > t$. As the slope is positive on the interval D_3, we may see that the profit maximization problem of the retailer $1R$ when deviating to D_3 does not have an interior solution. Moreover, the retailer $1R$'s profit-maximizing price under deviation to OSS is the right endpoint of D_3, i.e. $\mu^d_{1R} = \frac{3t-3a}{3} = t-a$. The resulting profit is thus

$$\pi^d_{1R} = \frac{1}{3t}(t-a)(t+a).$$

However, this deviation is not profitable:

$$9t(\pi^*_{1R} - \pi^d_{1R}) = (2t-a)^2 - 3(t-a)(t+a) = (2a-t)^2 \geq 0.$$

It is trivial to see that there is no profitable deviation to D_5 in which the retailer $1R$ earns zero profit.

Proof of Lemma 4. As in the case of the retailer $1R$, we plug the LR-type equilibrium strategy profile of the other players $(\mu_{1L}, \mu_{2L}, \mu_{1R})$ to obtain the following expression for the retailer $2R$'s demand function:

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$$q_{2R} = \begin{cases} 1, & \mu_{2R} \in D_1 = [-a, -a + \frac{-t+2a}{3}] \\ 1-\tilde{x}_L = \frac{2t-a-3\mu_{2R}}{3t}, & \mu_{2R} \in D_2 = (-a + \frac{-t+2a}{3}, -a+a) \\ 1-\tilde{x} = \frac{4t-2a-3\mu_{2R}}{6t}, & \mu_{2R} \in D_3 = [-a+a, -a+\frac{4t+a}{3}] \\ 1-\tilde{x}_R = \frac{5t-3\mu_{2R}-a}{3t}, & \mu_{2R} \in D_4 = \emptyset \\ 0, & \mu_{2R} \in D_5 = [-a+\frac{4t+a}{3}, +\infty) \end{cases}$$

where $\mu_{2R} = p_{2R} - a$, $1-\tilde{x}_L = \frac{1}{t}(\mu_{2L} - \mu_{2R}) = \frac{2t-a-3\mu_{2R}}{3t}$, $1-\tilde{x} = \frac{1}{2} + \frac{1}{2t}(\mu_L - \mu_R) = \frac{4t-2a-3\mu_{2R}}{6t}$, and $1-\tilde{x}_R = 1 - \frac{1}{t}(\mu_{2R} - \mu_{2L}) = \frac{5t-3\mu_{2R}-a}{3t}$.

We first check whether the retailer $2R$'s LR-type equilibrium price indeed belongs to D_2. That is, $\mu^*_{2R} = \frac{t-2a}{3} \in D_2$, which leads to the following condition:

$$\frac{a}{2} < t < 2a$$

Now turning to check if there is a profitable deviation, we consider the retailer $2R$'s deviation to D_3. Recall again that in the LR-type two-stop shopping equilibrium,

$$\mu_{2R}^* = \frac{t-2a}{3}, \quad q_{2R}^* = \frac{t+a_{2R}}{3t}, \quad \pi_{2R}^* = \frac{(t+a)^2}{9t}.$$

In D_3, the slope of the profit function with respect to μ_{2R} is

$$slope(\mu_{2R}) \equiv \frac{\partial \pi_{2R}}{\partial \mu_{2R}} = \frac{1}{6t}(4t-5a-6\mu_{2R}).$$

At the right endpoint of D_3, when $\mu_{2R} = -a + \frac{4t+a}{3}$, the slope is $\frac{1}{6t}(-4t-a) \leq 0$ because the price p_{2R} at the left endpoint of D_5 cannot be negative. On the other hand, at the left endpoint of D_3, the slope is $\frac{1}{6t}(4t-5a)$ and the deviation price to D_3 depends on its sign: (i) if it is non-positive, the deviation price occurs at the left endpoint of D_3; (ii) if the sign is positive, the deviation price would be an interior solution to the profit maximization based on D_3, i.e. the FOC holds (slope $= 0$).

Case (i) $5a \geq 4t$: The deviation price is $\mu_{2R}^{d(1)} = 0$. The resulting profit is $\pi_{2R}^{d(1)} = \frac{a}{3t}(2t-a)$. Comparing this with the equilibrium profit of the retailer $2R$, we may see that there is no profitable deviation:

$$9t(\pi_{2R}^* - \pi_{2R}^{d(1)}) = (t+a)^2 - 3a(2t-a) = (2a-t)^2 \geq 0.$$

Case (ii) $5a < 4t$: The interior solution $\mu_{2R}^{d(2)} = \frac{4t-5a}{6}$ would be the deviation price and the profit is thus $\pi_{2R}^{d(2)} = \frac{1}{2t}\left(\frac{4t+a}{6}\right)^2$. Then, the deviation is not profitable if $\pi_{2R}^* \geq \pi_{2R}^{d(2)}$, equivalently,

$$[2(\triangle)+1]^2 > 2, \text{ where } \triangle = \frac{2a-t}{2t-a} > 0 \tag{4}$$

Notice that $5a < 4t$ can be rewritten equivalently as $\triangle < 1/2$. Therefore, there is no profitable deviation if

$$\frac{\sqrt{2}-1}{2} < \triangle < \frac{1}{2}.$$

Lastly, we consider a deviation to D_1. The slope of the profit function on D_1 is constant as 1, thus being always positive. Hence, the deviation price occurs at the right endpoint of D_1, thus the profit under this deviation is $\pi_{2R}^d = \frac{-t+2a}{3}$. This deviation is not profitable:

$$\pi_{2R}^* - \pi_{2R}^d = \frac{(t+a)^2}{9t} - \frac{-t+2a}{3} = \frac{(2t-a)^2}{9t} \geq 0$$

Recall that $\frac{\sqrt{2}-1}{2} < \triangle < \frac{1}{2}$ can be explicitly solved for a/t. Note that $\frac{5}{4} < \frac{1}{4}(2+3\sqrt{2}) \approx 1.56$. Therefore, the retailer $2R$ may benefit from his deviation to D_3 when $\frac{1}{4}(2+3\sqrt{2})a \approx 1.56a < t < 2a$. In other words, no-deviation condition for the retailer $2R$ is thus $\frac{1}{2}a < t < \frac{1}{4}(2+3\sqrt{2})a \approx 1.56a$. Equivalently,

$$\frac{6\sqrt{2}-4}{7} \approx 0.64 < \frac{a}{t} < 2 \tag{5}$$

Case for $a \geq 2t$ According to Corollary 1, we may see that the two-stop shopping equilibrium $(\mu_{1L}, \mu_{2L}, \mu_{1R}, \mu_{2R})$, if it exists, must be of *LR*-type. Moreover, from the previous section, we may see that the equilibrium satisfies $\tilde{x}_L \leq 0$ and $\tilde{x}_R \geq 1$. Hence, the *LR*-type equilibrium when $a \geq 2t$ must satisfy at least the following:

(1) non-negative prices: $\mu_{1L}, \mu_{2R} \geq -a$, $\mu_{1R}, \mu_{2L} \geq 0$.

(2) Two-stop shopping scenario: $|\mu_{1L} - \mu_{1R}| + |\mu_{2L} - \mu_{2R}| > t$

(3) *LR*-type two-stop shopping: $\mu_{1L} \leq \mu_{1R}$; $\mu_{2R} \leq \mu_{2L}$

(4) $\tilde{x}_L \leq 0$: $\mu_{2L} - \mu_{2R} \geq t$

(5) $\tilde{x}_R \geq 1$: $\mu_{1R} - \mu_{1L} \geq t$

Note that conditions (4) and (5) together imply (3). Now, we consider the profit maximization motives of the department store and the shopping mall yield conditions for the equilibrium. Firstly, consider the department store. As long as condition (5) holds (the demand is constant as one), it is profit-maximizing for L to raise μ_{1L} instead of abiding by the first-order condition, $\mu_{1L} = (\mu_{1R} - a)/2$. Notice that $\mu_{1R} - t > (\mu_{1R} - a)/2 > -a$ when $a \geq 2t$. Therefore, the profit maximization implies that (5) must hold with equality: $\mu_{1L} = \mu_{1R} - t$. For μ_{2L}, under the condition (4), the demand for $2L$ vanishes, so any choice of μ_{2L} works. On the other hand, for given μ_{2R}, the optimal choice of μ_{2L} according to the first-order condition must satisfy (4). Formally, this gives the condition $\mu_{2L} = (t + \mu_{2R})/2 \geq \mu_{2R} + t$, which yields $\mu_{2L} = 0$ and $\mu_{2R} = -t$. To sustain an equilibrium satisfying (1)-(5), the department store must have no incentive for deviation to one-stop shopping. By Lemma 2 (and the values we obtain), this condition can be simply stated as follows: when $\mu_{1R} < 2t$, $\mu_{1R} > -a$ holds.

Turning to the shopping mall, conditions (4) and (5) lead to $\mu_{1R} \in D_5 = [t + \mu_{1L}, \infty)$ (zero demand) and $\mu_{2R} \in D_1 = [-a, -t + \mu_{2L}]$ (unit demand). Similarly to μ_{1L}, the unit and constant demand for $2R$ implies that $\mu_{2R} = -t + \mu_{2L}$, i.e. (4)

holds with equality. This is consistent with the value we obtained for (μ_{2L}, μ_{2R}). For the retailer $1R$ to have no incentive for deviation to D_4, the marginal profit calculated within D_4 must be positive at $\mu_{1R} = t + \mu_{1L}$, implying $t + \mu_{1L} - 2\mu_{1R} = -(t + \mu_{1L}) \geq 0$. That is, $\mu_{1L} = -t$ and $\mu_{1R} = 0$.

In all, the two-stop shopping equilibrium when $a \geq 2t$ is $\mu_{1L} = \mu_{2R} = -t$ and $\mu_{1R} = \mu_{2L} = 0$. The corresponding profits are $\pi_L = a - t$, $\pi_{1R} = 0$, and $\pi_{2R} = a - t$.