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Complexity and Markov Behavior in Bargaining with Investments

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Abstract This paper invokes complexity considerations to justify Markov behavior in a non-stationary bargaining model of Che and Sákovics (2004) in which the surplus depends on the players' relationship-specific investments. Using the notion of state complexity and the machine game analysis developed by Lee and Sabourian (2007), it is shown that if the players have lexicographic complexity-averse preferences then every Nash equilibrium of the game must be Markov.

Keywords Bargaining, Relationship-Specific Investment, Complexity, Bounded Rationality, Automaton, Markov Strategy

JEL Classification C72, C78

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1. INTRODUCTION

Bargaining theory studies the fundamental determinants of price formation in a decentralized economy. The standard practice in this literature is to assume that the bargaining parties are fully rational who can therefore work through and implement potentially very complex computations and behavior. In this paper, I demonstrate the implications of a small departure from this rationality framework on bargaining behavior in a particular strategic setting.

Specifically, I consider the non-stationary bargaining model analyzed by Che and Sákovics (2004) in which the size of the surplus increases with the bargainers' investments. This model was proposed to understand bilateral exchanges that feature *relationship-specific* investments and hold-up problems. For example, in firm-worker and manufacturer-supplier relationships, the agents often accumulate capital that disproportionately benefit the surplus from their transaction with each another.

In contrast to the previous model, the bargaining players in this paper have a preference for simpler strategies at the margin. The notion of strategic complexity that I invoke is that of *state complexity* proposed by Abreu and Rubinstein (1988), adapted to alternating-offers bargaining setup by Lee and Sabourian (2007). According to this definition, a strategy is more complex than another strategy if the former conditions its behavior more on past histories than the latter.

The introduction of such boundedly rational players results in substantial reduction of the Nash equilibrium set of the bargaining game. The main result of the paper establishes that, with lexicographically complexity-averse players, the non-stationary bargaining game only admits equilibria in which the players use *Markov* strategies that only depend on payoff-relevant aspects of past history, namely, the level of investments accumulated. In their analysis, Che and Sákovics (2004) constructed a Markov perfect equilibrium that attains efficiency asymptotically as common discount factor goes to 1.

Complexity considerations have been invoked in various other dynamic games to select among multiple equilibria (e.g. see the survey of Chatterjee and Sabourian, 2009). In contrast to most of the previous works in this literature, I consider a game that features non-stationarity and derive simple Markov behavior in equilibrium. Also, this paper is related to the recent literature on firm behavior with bounded rational consumers. In particular, consumer loss aversion has been shown to generate optimal pricing that is simpler than with standard rational consumers in various settings (e.g. Heidhues and Kőszegi, 2008; Hahn, Kim, Kim and Lee, 2014). Albeit in a different bargaining context, the present paper demonstrates another channel via which price formation results from simple behavior.

The remainder of the paper is organized as follows. Section 2 describes the bargaining game with investments. Section 3 formally introduces the notions of machine game, strategic complexity and equilibrium refinement. Section 4 contains the main results. Section 5 concludes.

2. THE MODEL

The following describes the bargaining game with investments analyzed by Che and Sákovics (2004). Two players, indexed by $i = 1,2$, bargain over a surplus that itself is a function of their cumulative relationship-specific investments. Time horizon is potentially infinite and indexed by $t = 1,2,\ldots$. The common discount factor is $\delta \in (0,1)$. The players' reservation payoffs, which are independent of their investments, are normalized to zero.

Let $V_1 = [0,\bar{v}_1]$ and $V_2 = [0,\bar{v}_2]$ be the feasible sets of cumulative investments for players 1 and 2 respectively. Let $V \equiv V_1 \times V_2$ and $v = (v_1,v_2) \in V$ denote a pair of cumulative investments. If the players trade in period t, with cumulative investments of v up to, and including, that period, the realized surplus is given by $\phi(v)$ (or $\delta^{t-1}\phi(v)$ in period 1 terms).

The extensive form is as follows. In period 1, the players first choose *incremental investment* levels simultaneously. Let $w_i \in V_i$ denote player i's investment choice and I assume that this is measured by the associated costs, which are incurred at the time of investment. The players cannot disinvest. Once the investments are sunk and observed publicly, player 1 makes a proposal $x \in \triangle^2$ where $\triangle^2 \equiv \{x = (x_1,x_2) \mid x_1 + x_2 = 1\}$, i.e. how to split the surplus available in the period, which player 2 can either accept (Y) or reject (N). Acceptance ends the game with trade taking place according to the agreed partition of the available surplus; rejection takes the game onto the next period in which the same extensive form is repeated except that player 2 is the proposer.[1]

Let T denote the end of the game and w_i^t player i's investment choice in period t. If the players reach an agreement on partition $z = (z_1,z_2) \in \triangle^2$ in period $T < \infty$, player i's (discounted) payoff in the game is $\delta^{T-1}z_i\phi(v_1,v_2) - \sum_{t=1}^{T}\delta^{t-1}w_i^t$, where $v_i = \sum_{t=1}^{T} w_i^t$. If an agreement is never reached (I describe this by setting $T = \infty$), player i's corresponding payoff is $-\sum_{t=1}^{\infty}\delta^{t-1}w_i^t$.

[1] For their main analysis, Che and Sákovics (2004) actually present a different extensive form in which the identity of the proposer is chosen by nature in each period. However, as they note themselves, their central insights remain unaltered for the alternating-offers case that I assume.

In specifying the players' strategies (and later machines) for this game, I formally distinguish between the different *roles* played by each player every two periods (beginning with an odd one), or in a "stage". He can be either the proposer (p) or the responder (r) in a given period. I index a player's role by k.

In order to define a strategy, I first need to introduce some further notation. I use the following convention. Whenever superscripts/subscripts i and j both appear in the same exposition, I mean $i, j = 1, 2$ and $i \neq j$. Similarly, whenever I use superscripts/subscripts k and l together, I mean $k, l = p, r$ and $k \neq l$.

I refer to e as an outcome that can occur within a period, and this belongs to the set

$$E = \{(w_1, w_2, x^i, Y), (w_1, w_2, x^i, N)\}_{w_1 \in V_1, w_2 \in V_2, x^i \in \triangle^2, i=1,2},$$

where i is the identity of the proposer in the period. Let e^t be the outcome of period t.

I define a partial history, d, within a period as an element in the set

$$D = \{\emptyset, (w_1, w_2), (w_1, w_2, x^i)\}_{w_1 \in V_1, w_2 \in V_2, x^i \in \triangle^2, i=1,2},$$

where, for example, $\emptyset$ refers to the beginning of the period at which the players decide how much incremental investments to make and (w_1, w_2, x^i) is the partial history of investments (w_1, w_2) by the two players followed by player i's offer x^i. Also, let us define

$$D_{ik} \equiv \{d \in D \mid \text{it is } i\text{'s turn to play in role } k \text{ after } d \text{ in the period}\}.$$

Thus, I have

$$\begin{aligned} D_{ip} &= \{\emptyset, (w_i, w_j)\}_{w_i \in V_i, w_j \in V_j} \\ D_{ir} &= \{\emptyset, (w_i, w_j, x^j)\}_{w_i \in V_i, w_j \in V_j, x^j \in \triangle^2}. \end{aligned}$$

I denote the set of actions available to player i in the game by

$$C_i \equiv V_i \cup \triangle^2 \cup Y \cup N \; .$$

Let $C_{ik}(v, d)$ denote the set of actions available to player i given his role k, the level of cumulative investments $v = (v_1, v_2) \in V$ up to the current date and a partial history $d \in D_{ik}$ within the date. Thus, I have

$$\begin{aligned} C_{ip}(v, d) &= \begin{cases} [0, \bar{v}_i - v_i] & \text{if } d = \emptyset \\ \triangle^2 & \text{if } d = (w_i, w_j) \end{cases} \\ C_{ir}(v, d) &= \begin{cases} [0, \bar{v}_i - v_i] & \text{if } d = \emptyset \\ Y \cup N & \text{if } d = (w_i, w_j, x^j). \end{cases} \end{aligned}$$

Let H^t be the set of all possible histories of outcomes at the beginning of period t, excluding those that have resulted in an agreement. Thus, $H^t \subseteq E^{t-1}$ ($t-1$-fold Cartesian product of E) for any $t > 1$ and $H^1 = \emptyset$ is the initial empty (trivial) history. Let $H^\infty \equiv \cup_{t=1}^\infty H^t$.

For the analysis, I divide H^∞ into two smaller subsets according to the different roles that the players take up in each stage. Let H_{ik}^t be the set of all possible histories over t periods after which player i's role is k. Notice that $H_{ik}^t = H_{jl}^t$. Also, let $H_{ik}^\infty = \cup_{t=1}^\infty H_{ik}^t$. Thus, $H^\infty = H_{ip}^\infty \cup H_{ir}^\infty$ ($i = 1,2$).

The present bargaining game is not stationary since the surplus itself depends on history. The payoff-relevant aspect of a history amounts to the pair of cumulative investments that the two players have incurred up to the period. Thus, it is natural to write a strategy for player i, denoted by f_i, as

$$f_i : (H_{ip}^\infty \times V \times D_{ip}) \cup (H_{ir}^\infty \times V \times D_{ir}) \to C_i$$

such that for any $(h,v,d) \in H_{ik}^\infty \times V \times D_{ik}$ I have $f_i(h,v,d) \in C_{ik}(v,d)$. The set of all strategies for player i is denoted by F_i. Also, I denote by F_i^t the set of player i's strategies in the game starting with role distribution given in period t.

I define a *Markov* strategy in the following way.

Definition 1. *A strategy f_i is Markov if and only if $f_i(h,v,d) = f_i(h',v,d)$ $\forall h,h' \in H_{ik}^\infty$, $\forall v \in V$ and $\forall d \in D_{ik}$ for $k = p,r$. A strategy profile f is Markov if f_i is Markov $\forall i$.*

The behavior induced by such a strategy may depend on the level of cumulative investments up to the period and the partial history within the current period but not on the history of the game up to the period.

3. MACHINES, COMPLEXITY AND EQUILIBRIUM

In order to facilitate the complexity approach, I now consider the "machine game". Extending the approach of Lee and Sabourian (2007), a strategy in the game can be equivalently represented by the following machine (or "automaton") that employs two sub-machines.

Definition 2. *For each player i, a machine (automaton), $M_i = \{M_{ip}, M_{ir}\}$, consists of two sub-machines $M_{ip} = (Q_{ip}, q_{ip}^1, \lambda_{ip}, \mu_{ip})$ and $M_{ir} = (Q_{ir}, q_{ir}^1, \lambda_{ir}, \mu_{ir})$*

where, for any $k, l = p, r$,

> Q_{ik} *is the set of states*;
> q_{ik}^1 *is the initial state belonging to* Q_{ik};
> $\lambda_{ik} : Q_{ik} \times V \times D_{ik} \to C_i$ *is the output function such that*
> $\lambda_{ik}(q_{ik}, v, d) \in C_{ik}(v, d)\ \forall q_{ik} \in Q_{ik}, \forall v \in V$ *and* $\forall d \in D_{ik}$; *and*
> $\mu_{ik} : Q_{ik} \times V \times E \to Q_{il}$ *is the transition function.*

Let Φ_i denote the set of player i's machines in the machine game. I also let Φ_i^t denote the set of player i's machines in the machine game starting with role distribution given in period t.

Each sub-machine in the above machine definition consists of a set of distinct states, an initial state, an output function enabling a player to play a given role and a transition function that takes a state from one sub-machine to a state in the other sub-machine (as roles are reversed). It is straightforward to establish that machines and strategies are equivalent in this setup; see Lee and Sabourian (2007) for a formal discussion on this.

I assume that each sub-machine has to have at least one state.[2] But, notice that I do not assume *finiteness* of a machine; each sub-machine may have any arbitrary, possibly infinite, number of states. This contrasts with Abreu and Rubinstein (1988) and others who consider only finite automata. Assuming that machines can only have a finite number of states imposes a restriction on the players' choice of strategies.

The following defines a *minimal* machine.

Definition 3. *A machine is minimal if and only if each of its sub-machines has exactly one state.*

A minimal machine implements the same actions in every period regardless of the history of the preceding periods, given the level of cumulative investments and the partial history (for each role). Thus, it corresponds to a Markov strategy as in Definition 1. I henceforth refer to a minimal machine (profile) interchangeably as a Markov machine (profile).

Let $\|M_i\| = \sum_k |Q_{ik}|$ be the total number of states (or size) of machine M_i. I now formally define the notion of *state complexity* in terms of machines, as

[2]The initial state of the sub-machine that operates in the second period is in fact redundant because the first state used by this sub-machine depends on the transition taking place betIen the first two periods of the game. Nevertheless, I endow both sub-machines with initial state for expositional ease. Also, I could specify a distinct terminal state that a machine enters when an agreement is reached. This is immaterial.

adopted in the literature on repeated games played by automata (Abreu and Rubinstein, 1988).[3]

Definition 4 (State complexity). *M_i' is more complex than M_i if $\|M_i'\| > \|M_i\|$.*

To wrap up the description of the machine game, let us fix some more notational convention. If $M = (M_1, M_2)$ is the chosen machine profile, $T(M)$ refers to the end of the game; $z(M) \in \triangle^2$ is the agreed partition of the final surplus if $T(M) < \infty$; $x^t(M) \in \triangle^2$ is the partition offered in period $t < T(M)$; $v^t(M) = (v_1^t(M), v_2^t(M)) \in V$ is the cumulative investment pair reached at the beginning of period $t \leq T(M)$; $v(M) \in V$ is the final surplus available; $w_i^t(M) \in V_i$ is the (incremental) investment made by i in period $t < T(M)$; and $q_i^t(M)$ is the state of player i's machine appearing in period $t \leq T(M)$ (the state of the active sub-machine in period t).

Similarly, I denote by $\pi_i^t(M)$ player i's (discounted) *continuation payoff* at period $t \leq T(M)$ when the machine profile M is chosen. Thus, I have

$$\pi_i^t(M) = \begin{cases} -\sum_{s=t}^{\infty} \delta^{s-t} w_i^s(M) & \text{if } T(M) = \infty \\ \delta^{T(M)-t} z_i(M) \phi(v(M)) - \sum_{s=t}^{T(M)} \delta^{s-t} w_i^s(M) & \text{if } t \leq T(M) < \infty. \end{cases}$$

I use the abbreviation $\pi_i(M) \equiv \pi_i^1(M)$.

For ease of exposition, the argument in M will sometimes be dropped when I refer to one of these variables that depends on the particular machine profile. For example, π_i^t will mean $\pi_i^t(M)$. Unless otherwise stated, the abbreviated variable will refer to the machine profile in the *claim*.

I now introduce an equilibrium notion that captures the players' preferences for less complex machines at the margin. Complexity enters each player's preferences *lexicographically*.

Definition 5. *A machine profile $M^* = (M_1^*, M_2^*)$ constitutes a Nash equilibrium of the machine game (NEM) if $\forall i$*

(*i*) $\pi_i(M_i^*, M_j^*) \geq \pi_i(M_i', M_j^*)\ \forall M_i'$;
(*ii*) $\nexists\, \tilde{M}_i$ *such that* $\pi_i(\tilde{M}_i, M_j^*) \geq \pi_i(M_i', M_j^*)\ \forall M_i'$ *and* $\|M_i^*\| > \|\tilde{M}_i\|$.

Under this definition, the complexity consideration only enters a player's choice of strategy when there are multiple best responses to his opponent's strategy. By definition, any NEM profile is a Nash equilibrium of the underlying bargaining game. The cost of complexity is in implementation, rather than computation, of a strategy.

[3] Kalai and Stanford (1988) established a formal equivalence between the state complexity notion and the number of *continuation strategies* of the underlying strategy of a machine.

4. NEM RESULTS

Let us present the implications of complexity requirements in the model. The results below are all independent of the discount factor δ.

First, suppose that there exists a state in some player's equilibrium (NEM) machine that never appears on the equilibrium path. Unless the machine is minimal, however, this cannot be possible because this state can be "dropped" by the player to reduce the complexity of his behavior without affecting the outcome and payoff, thereby contradicting the NEM assumption. That is, there exists another machine identical to the original equilibrium machine except that it has one less state which, given the other player's equilibrium machine, will generate the same outcome/payoffs. This argument immediately leads to the following.

Lemma 1. *Suppose that $M^* = (M_1^*, M_2^*)$ is a NEM. Then, I have the following:*

(i) If $T(M^*) \geq 2$, then every state in each player's machine appears on the equilibrium path;

(ii) If $T(M^*) \leq 2$, then M_1^* and M_2^* are minimal.

Second, if an equilibrium induces perpetual disagreement, the equilibrium machines must also be minimal. The reasoning is straightforward. Such an equilibrium generates at most zero payoff and, hence, if an equilibrium machine was not minimal, the corresponding player could adopt another minimal machine which never invests and always demands the whole surplus while rejecting any offer that gives him less than the whole surplus. This deviation guarantees him at least zero payoff while involving less complexity.

Lemma 2. *Suppose that $M^* = (M_1^*, M_2^*)$ is a NEM such that $T(M^*) = \infty$. Then, M^* is minimal.*

Next, since any NEM profile $M^* = (M_1^*, M_2^*)$ is a Nash equilibrium of the underlying game, I have $\pi_i(M_i^*, M_j^*) = \max_{f_i \in F_i} \pi_i(f_i, M_j^*)$ for any i, j, where, with some abuse of notation, $\pi_i(f_i, M_j^*)$ refers to i's payoff in the game played by i and j according to f_i and M_j^*, respectively. More generally, NEM machines must be best responses (in terms of payoffs) *along* the equilibrium path. Formally, for any $M = (M_i, M_j)$, let $v^\tau \equiv v^\tau(M)$ and $q_j^\tau \equiv q_j^\tau(M)$; define $M_j(v^\tau, q_j^\tau)$ as the machine identical to M_j except that it plays the game with cumulative investments v^τ, starting with the sub-machine that would operate in period τ and initial state q_j^τ. Also, with some further abuse of notation, let $\pi_i(f_i, M_j(v^\tau, q_j^\tau))$ be i's payoff in the game that begins with cumulative investments v^τ and role distribution as in period τ, and is played by i and j according to $f_i \in F_i^\tau$ and $M_j(v^\tau, q_j^\tau)$, respectively. I then obtain the following.

Lemma 3. *Suppose that $M^* = (M_1^*, M_2^*)$ is a NEM. Then, $\forall i, j$ and $\forall \tau \leq T(M^*)$, $\pi_i^\tau(M^*) = \max_{f_i \in F_i^\tau} \pi_i(f_i, M_j^*(v^\tau, q_j^\tau))$.*

Proof. Suppose not. Then, for some i and $\tau \leq T(M^*)$, there exists another machine $\tilde{M}_i \in \Phi^\tau$ such that

$$\pi_i^\tau(M^*) < \pi_i(\tilde{M}_i, M_j^*(v^\tau, q_j^\tau)).$$

Consider player i using at the outset of the game another machine $M_i' = \{M_{ip}', M_{ir}'\}$ where, for $k = p, r$, $M_{ik}' = (Q_{ik}', q_{ik}^{1\prime}, \lambda_{ik}', \mu_{ik}')$. This machine is constructed as follows.

Let e^t be the outcome in period t when M^* is chosen. Also, as usual, let $v^t \equiv v^t(M^*)$ and $q_i^t \equiv q_i^t(M^*)$. Then, for every $t < \tau$ and $k = p, r$, fix a distinct state $q_i'(t) \in Q_{ip}' \cup Q_{ir}'$ such that

$$\lambda_{ik}'(q_i'(t), v, d) = \lambda_{ik}^*(q_i^t, v, d) \;\; \forall v \in V, d \in D_{ik}.$$

The transition function of the new machine is such that

$$\mu_{ik}'(q_i'(t), v^t, e^t) = \begin{cases} q_i'(t+1) & \forall t < \tau - 1 \\ \tilde{q} & \text{for } t = \tau - 1, \end{cases}$$

where $\tilde{q}$ is another distinct state such that $M_i'(v^\tau, \tilde{q}) = \tilde{M}_i$.

Thus, M_i' played against M_j^* replicates the same outcome path as M_i^* up to τ, followed by activation of $\tilde{M}_i$ at period τ. It then follows that $\pi_i(M_i', M_j^*) > \pi_i(M_i^*, M_j^*)$. But this contradicts the NEM assumption. □

Now it follows that if a state belonging to a player's equilibrium machine appears twice on the outcome path then the cumulative investments reached at the beginning of the two corresponding periods must be different (and hence the continuation games differ across the two periods); otherwise, the other player's continuation payoffs must be equal to zero at both periods.

Lemma 4. *Suppose that $M^* = (M_1^*, M_2^*)$ is a NEM. Then, for any i and any $t, t' \leq T(M^*)$, if $q_i^t(M^*) = q_i^{t'}(M^*)$, I have either (i) $v^t(M^*) \neq v^{t'}(M^*)$ or (ii) $v^t(M^*) = v^{t'}(M^*)$ and $\pi_j^t(M^*) = \pi_j^{t'}(M^*) = 0$.*

Proof. Suppose that, for some i and $t, t' \leq T(M^*)$, I have that $q_i^t(M^*) = q_i^{t'}(M^*)$ and $v^t(M^*) = v^{t'}(M^*)$. Let $t' > t$.

Then, by Lemma 3, it follows that $\pi_j^t(M^*) = \pi_j^{t'}(M^*)$, or

$$\pi_j^t = \delta^{t'-t} \pi_j^{t'} - \sum_{s=t}^{t'-1} \delta^{s-t} w_j^s = \pi_j^{t'}.$$

But, since $v^t = v^{t'}$, $w_j^s = 0$ for every integer $s \in [t, t'-1]$ and, therefore, with $\delta < (0,1)$, it must be that $\pi_j^t = \pi_j^{t'} = 0$. $\square$

The next Lemma establishes that, if one of the sub-machines of a NEM machine employs only one state, every other equilibrium sub-machine must also employ only one state.

Lemma 5. *Suppose that $M^* = (M_1^*, M_2^*)$ is a NEM. Suppose also that, for some i and k, $|Q_{ik}^*| = 1$. Then, M^* is minimal.*

Proof. Suppose not. Given Lemmas 1 and 2, I restrict attention to the case of $2 < T < \infty$. Suppose that i plays role k in T, $|Q_{ik}^*| = 1$ and M_j^* is not minimal. (The other cases can be treated similarly to below.) Also, define $t_{ik} = \{t \leq T |\ i \text{ plays role } k\}$.

First, it must be that $v^t \neq v^{t'}$ for any $t, t' \in t_{ik}$. Otherwise, by Lemma 4 above, I have $\pi_j^t = \pi_j^{t'} = 0$ and, hence, $\pi_j \leq 0$. Then, by the same arguments as those behind Lemma 2, M_j^* must be minimal.

Second, since $v^t \neq v^{t'}$ for any $t, t' \in t_{ik}$, it immediately follows that $v^s \neq v^{s'}$ for any $s, s' \in t_{jk} = \{t \leq T |\ j \text{ plays role } k\}$.

Now, instead of M_j^*, consider j using a minimal machine, M_j', constructed in the following way. The two sub-machines, M_{jk}' and M_{jl}', each employ one state, q_k and q_l, such that

- $\lambda_{jk}'(q_k, v^t, d) = \lambda_{jk}^*(q_j^t, v^t, d)\ \forall t \in t_{jk}, d \in D_{jk}$ and $\lambda_{jl}'(q_l, v^t, d) = \lambda_{jl}^*(q_j^t, v^t, d)$ $\forall t \in t_{jl}, d \in D_{jl}$;
- $\mu_{jk}'(q_k, v^t, e^t) = q_l\ \forall t \in t_{jk}$ and $\mu_{jl}'(q_k, v^t, e^t) = q_k\ \forall t \in t_{jl}$.

Since v^t is distinct along the original equilibrium path for every $t \leq T$, the new machine replicates exactly the same outcome as M_j^* played against M_i^*. But, $\|M_j^*\| > \|M_j'\|$. This contradicts NEM. $\square$

I am now ready to present this paper's main result on NEM and Markov behavior. Given Lemmas 1 and 2, it suffices to consider the case in which a NEM induces an agreement in a finite time after the first two periods. My proof uses the following arguments case by case. If the last period occurs beyond the first stage of the game and the NEM profile is not minimal, by Lemma 5, every sub-machine in the profile must itself employ more than one state. Then, one of the two players must be able to drop a state in one of his sub-machines and find another state in that sub-machine to condition his behavior in periods where this dropped state would operate such that the outcome of the game is not affected. This however reduces complexity of the machine.

Proposition 1. *Every NEM profile M^* is minimal and, hence, Markov.*

Proof. Suppose not. So, suppose that M^* is not minimal. Given Lemmas 1 and 2, I focus on the case of $2 < T < \infty$. Let i be the proposer, j the responder and $z \in \triangle^2$ the accepted partition in period T. Also, as before, define $t_{ik} = \{t \leq T |\ i \text{ plays role } k\}$. Given Lemma 5, assume that M^*_{jr} has more than one state.

I consider the following cases in turn.

Case A: $q^T_j \neq q^t_j$ for any $t \in t_{ip}$.

Case A1: $v^T \neq v^t\ \forall t \in t_{ip}$.

But then, consider j using another machine $M'_j = \{M'_{jp}, M'_{jr}\}$ where, for $k = p, r$, $M'_{jk} = (Q'_{jk}, q^{1\prime}_{jk}, \lambda'_{jk}, \mu'_{jk})$ which is identical to M^*_j except that:

- q^T_j is dropped (i.e. $Q'_{jr} = Q^*_{jr} \backslash \{q^T_j\}$);
- the transition function is such that $\mu'_{jp}(q^{T-1}_j, v^{T-1}, e^{T-1}) = q_j \neq q^T_j$ for some arbitrary but fixed $q_j \in Q'_{jr}$ (such q_j exists since I have $|Q^*_{jr}| > 1$ and q^T_j is distinct by assumption);
- the output function is such that $\lambda'_{jr}(q_j, v^T, (w^T_i, w^T_j, z)) = Y$.

Since q^T_j is distinct and the cumulative investment pair v^T is not reached at any period before T on the original equilibrium path when i proposes, playing the new machine against M^*_i does not affect the outcome and payoff. But, q^T_j is dropped and therefore I have $\|M^*_j\| > \|M'_j\|$. This contradicts NEM.

Case A2: $v^T = v^\tau$ for some $\tau \in t_{ip}$.

In this case, first notice that $(w^\tau_1, w^\tau_2, x^\tau) \neq (w^T_1, w^T_2, z)$. Otherwise, since $\delta < 1$, j could improve his payoff by accepting the offer at τ.

Now, let τ^* be the earliest period in t_{ip} at which $v^{\tau^*} = v^T$. Consider j using another machine $M'_j = \{M'_{jp}, M'_{jr}\}$, which is identical to M^*_j except that:

- q^T_j is dropped (i.e. $Q'_{jr} = Q^*_{jr} \backslash \{q^T_j\}$);
- the transition function is such that $\mu'_{jp}(q^{T-1}_j, v^{T-1}, e^{T-1}) = q^{\tau^*}_j$;
- the output function is such that $\lambda'_{jr}(q^{\tau^*}_j, v^T, (w^T_1, w^T_2, z)) = Y$.

Since q^T_j is distinct and $(w^{\tau^*}_1, w^{\tau^*}_2, x^{\tau^*}) \neq (w^T_1, w^T_2, z)$, playing the new machine against M^*_i does not affect the outcome and payoff. But, q^T_j is dropped and therefore I have $\|M^*_j\| > \|M'_j\|$. This contradicts NEM.

Case B: $q_j^T = q_j^\tau$ for some $\tau \in t_{ip}$.

Case B1: $v^T = v^\tau$.

In this case, by Lemma 4, $\pi_i^T = \pi_i^\tau = 0$. Then, I must also have $\pi_i \le 0$ and, therefore, M_i^* must be minimal. The claim follows immediately from Lemma 5.

Case B2: $v^T \ne v^\tau$.

Let $q_j^T = q_j^\tau = q^*$. There are further two sub-cases to consider.

First, consider the case in which there exists some $s \in t_{ip}$ such that $q^* \ne q_j^s \in Q_{jr}^*$ and I have either $v^s \ne v^\tau$ or $e^s \ne e^\tau$. Consider j using another machine $M_j' = \{M_{jp}', M_{jr}'\}$, which is identical to M_j^* except that:

- q_j^T is dropped (i.e. $Q_{jr}' = Q_{jr}^* \backslash \{q^*\}$);
- the transition function is such that

$$\begin{aligned} \mu_{jp}'(q_j^{T-1}, v^{T-1}, e^{T-1}) &= \mu_{jp}'(q_j^{\tau-1}, v^{\tau-1}, e^{\tau-1}) = q_j^s \\ \mu_{jr}'(q_j^s, v^\tau, e^\tau) &= q_j^{\tau+1}; \end{aligned}$$

- the output function is such that

$$\begin{aligned} \lambda_{jr}'(q_j^s, v^T, (w_1^T, w_2^T, z)) &= Y \\ \lambda_{jr}'(q_j^s, v^\tau, (w_1^\tau, w_2^\tau, x^\tau)) &= \lambda_{jr}^*(q^*, v^\tau, (w_1^\tau, w_2^\tau, x^\tau)). \end{aligned}$$

It is straightforward to verify that M_j' generates the same outcome path against M_i^* as M_j^* but it is less complex. This is a contradiction.

Second, consider the remaining case in which, for any $s \in t_{ip}$ with $q_j^s \ne q^*$, I have $v^s = v^\tau$ and $e^s = e^\tau$. In this case, it must be that for all $t \le \tau$, $w_i^t = w_j^t = 0$ and $v^t = 0$. Moreover, since $v^T \ne v^\tau$, it must be that, for some $\tau < t' < T$, either $w_i^{t'} > 0$ or $w_j^{t'} > 0$.

Without loss of generality, suppose that i is such a player and his role in t' is k. By Lemma 5, $|Q_{ik}^*| > 1$. Then, since $\lambda_{ik}^*(q_i^{t'}, v^{t'}, \emptyset) = w_i^{t'} > 0$, there must exist some $s' \in t_{ik}$ such that $q_i^{s'} \ne q_i^{t'} \in Q_{ik}^*$ while $v^{s'} \ne v_i^{t'}$ or $e^{s'} \ne e^{t'}$.

Now, consider i deviating from M_i^*. The deviating machine, M_j', is identical to M_i^* except that:

- $q_i^{t'}$ is dropped (i.e. $Q_{ik}' = Q_{ik}^* \backslash \{q_i^{t'}\}$);
- the transition function is such that, for any $s \in t_{ik}$ such that $q_i^s = q_i^{t'}$,

$$\begin{aligned} \mu_{il}'(q_i^{s-1}, v^{s-1}, e^{s-1}) &= q_i^{s'} \\ \mu_{ik}'(q_i^{s'}, v^s, e^s) &= q_i^{s+1}; \end{aligned}$$

- the output function is such that, for any $s \in t_{ik}$ such that $q_i^s = q_i^{t'}$ and for any $d \in D_{ik}$,
$$\lambda'_{ik}(q_j^{s'}, v^s, d) = \lambda^*_{ik}(q_j^s, v^s, d).$$

It is straightforward to verify that M'_i generates the same outcome path against M^*_j as M^*_i but it is less complex. This is a contradiction. $\square$

5. CONCLUDING DISCUSSION

In this paper, I have invoked complexity considerations to justify Markov behavior in a particular non-stationary environment where two players bargain over a surplus which is itself a function of their investments. Adopting the notion of state complexity, players' preferences for less complex strategies only at the margin, namely, lexicographic preferences, yield the result that every Nash equilibrium of the game must be Markov.

Multiple equilibria may still exist among Markov strategies, however. The set of such equilibria will in general depend on the precise structure of how the players' investments affect the level of surplus, although the derivation of Markov behavior is not affected in any way by this. Che and Sákovics (2004) demonstrated a Markov perfect equilibrium that attains efficiency asymptotically as δ goes to 1.

Several potential research questions remain. In particular, one would like to learn if refinements based on complexity arguments will select Markov behavior also in other non-stationary games. In the bargaining game considered in this paper, a crucial feature is that the players cannot disinvest. This restricts the paths of how the continuation game can evolve and, hence, allows only marginal complexity requirements to deliver the result. Tackling other non-stationary dynamic games, such as the bargaining models of Merlo and Wilson (1995) and Lee and Liu (2013), for instance, poses an interesting direction for future research.

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