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Seminonparametric Methods for Modeling Conditional Volatility of Exchange Rate

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Abstract We employ a seminonparametric (SNP) methodology in characterizing the conditional density of the exchange rate changes. The model selection procedure based on the BIC is used by moving upward along an expansion path. We find the semiparametric AR(4)-GARCH(2,2) model for the KRW/USD returns and the semiparametric AR(1)-GARCH(2,2) model for the JPY/USD returns as the BIC preferred SNP models. Simulations from the BIC minimizing SNP models seem to appropriately mimic the actual data. The time dependent heterogeneity of the actual data is recognized by the simulations from the semiparametric AR-GARCH-type models and the nonlinear nonparametric AR-GARCH-type models. We show that it is important to take departures from the Gaussianity of the data into account in specifying conditional heterogeneity of the exchange rate returns process. We also provide evidence on the benefits from using the SNP models in estimating the conditional density function via simulations.

Keywords Seminonparametric Methods, Random Restart, Conditional Heterogeneity, Gaussian VAR, Hermite Expansion

JEL Classification C13, C14, C15

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1. INTRODUCTION

Since the seminal work of Engle (1982), the assumption of conditional normality has been the norm in modeling the conditional distribution of asset returns. The success of the GARCH model by Bollerslev (1986) is attributed to the fact that GARCH model and its variant models can be fit to the leptokurtic data that have a higher peak near zero and fatter tails than a normal distribution even under the assumption of conditional normality. Furthermore, Bollerslev (1987) shows the conditional leptokurtosis in the stock market index returns under the assumption of t-distributed disturbances and proves the existence of the serial dependence in the data. Gallant et al. (1991) uncover the sources of conditional leptokurtosis in the exchange rate changes and use a semiparametric method to model conditional heterogeneity in the variance and departures from the normality of the process.

As an alternative approach to conditional heterogeneity of higher moments in explaining fat-tailed and non-normal distributed asset returns, Bollerslev et al. (2013) take note of variance risk premium dynamics. This approach has been taken from the perspective that the variance risk premium is priced at the aggregate market level and that the dynamics of the variance risk premium is time dependent. The sources of the time-varying realized market variance lie in the occurrence of unanticipated market jumps. Therefore, the jumps in the stochastic volatility and their temporal dependence may induce heavy tails and non-normal behavior in asset returns. Similarities in this approach can be found in the SNP methodology of conditional distribution estimation in the sense that semiparametric modeling procedure has been taken due to the lack of agreement on the parametric specification for the jumps in the stochastic volatility.

In exercising the numerical optimization to estimate the parameters of the semiparametric component of nonlinear structural models, it is important to determine a truncation point for a series expansion. Gallant (1981) propose the Fourier flexible form, a Fourier series expansion with a leading quadratic term, and Elbadawi, Gallant and Souza (1983) show that the Fourier flexible form along with deterministic and adaptive rules for choosing the truncation point can consistently estimate the parameters of an unknown function form. Eastwood and Gallant (1991) consider deterministic and adaptive procedures for selecting the truncation point to use a parametric method for estimating a nonlinear multivariate regression model. Gallant and Tuachen (1989) employ a Hermite expansion to test for the misspecification of the intertemporal capital asset pricing model with time separable utility restriction. Gallant, Hsieh and Tauchen (1991) apply a SNP procedure with a truncated Hermite expansion with an ARCH leading term to produce a consistent estimate of the conditional density. Gallant, Rossi and Tauchen (1992) use a SNP method to investigate the joint dynamics of stock price changes and volume, and find empirical regularities concerning the interactions between stock prices and volume. Davidian and Gallant (1993) propose a maximum likelihood method to estimate the parameters of a truncated series expansion of the density due to Gallant and Nychka (1987). Coppejans and Gallant (2002) show efficacy of the hold-out-sample cross-validation strategies compared to maximum likelihood truncation rules such as BIC in locating a truncation point for the SNP density estimator. Yi (2014) applies nonparametric realized volatility model to describe discrete jumps in the US stock market.

It is generally the case in empirical studies that not enough of precise parametric information is specified by the economic theory even though a variety of parametric estimation procedures are developed. The mathematical procedure such as a series expansion methodology can be utilized in order to apply an appropriate parametric procedure to estimate a parametric function. The purpose of this paper is to show a seminonparametric (SNP) methodology in characterizing the conditional density of exchange rate changes.

This paper aims to model the conditional density function of the two exchange rate returns processes, the KRW/USD returns and the JPY/USD returns process. The SNP conditional density is a nonlinear nonparametric model which can accommodate not only the Gaussian component of the Gaussian VAR model and Gaussian ARCH/GARCH model but also deviations from the Gaussianity of the semiparametric VAR model and semiparametric ARCH/GARCH model depending on the value of the tuning parameters of the Hermite polynomial.

The results of this paper show that the GARCH model with conditionally normal, Students t, or the generalized error distribution cannot fully account for the observed leptokurtosis in the exchange rate returns processes. We investigate the SNP method to model conditional heterogeneity and nonlinear dynamics in the returns process, however, our main concern lies in taking note of non-normality which proved to be difficult to capture with GARCH model.

The main contributions of the paper are two-fold. First, we show that it is important to take departures from the Gaussianity of the data into account in specifying conditional heterogeneity of the exchange rate returns process. Second, we provide evidence on the benefits from using the SNP models in estimating the conditional density function via simulations. We evaluate the qualitative performance of the SNP methodology in approximating conditional density functions and reproducing statistical characteristics of observed data. We also investigate whether outliers in returns are observed more frequently than would be implied

Figure 1: ACF of KRW/USD returns and squared returns

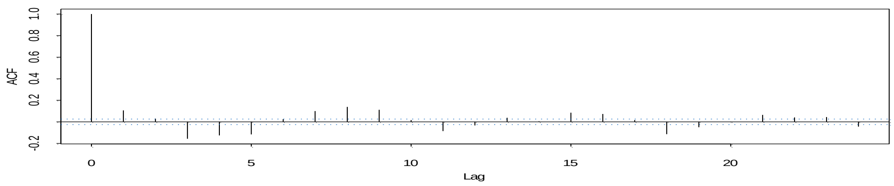
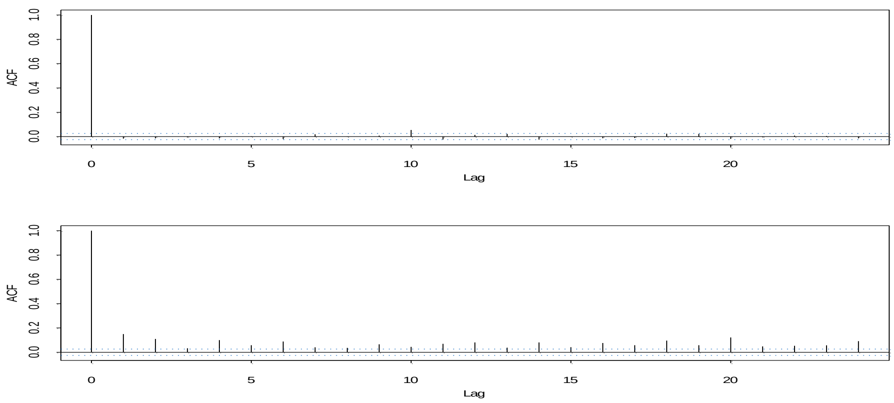


Figure 2: ACF of JPY/USD returns and squared returns



by the ARCH/GARCH type models calibrated to the data. This approach is motivated by the observation that the conditional heterogeneity of time series processes can be described by the SNP conditional density.

The organization of the article is as follows. Section 2 presents statistical tests for the data and the modeling strategies for fitting the SNP models. We also present diagnostic test results on the standardized residuals from the estimated SNP models in this section. Simulation evidence on mimicking the actual data is provided in the section. Section 3 concludes the discussion.

Table 1: Descriptive statistics for the exchange rate changes

	KRW/USD	JPY/USD
A. Index returns		
Mean	0.00	0.00
Standard Deviation	0.01	0.01
Skewness	-0.87	-0.37
Kurtosis	121.61	8.71
Minimum	-0.20	-0.07
Maximum	0.14	0.05
ARCH(20) LM test	1664.26 (0.00)	361.73 (0.00)
Q(20) on the rate changes	836.35 (0.00)	46.29 (0.00)
B. Squared exchange rate changes		
Q(20) on the squared rate changes	4708.22 (0.00)	693.78 (0.00)

Note: The ARCH(20) test reports the Engle (1982) Lagrange multiplier test for ARCH(q) effects for q lags. The entries in the parentheses are the p-values. The Q(20) stands for the modified Ljung-Box Q-statistic for up to twentieth-order serial correlation in the exchange rate changes and the squared exchange rate changes.

2. MODEL AND ESTIMATION RESULTS

2.1. DATA

The data used in the study consist of daily observations of the nominal exchange rate (the Korean won per U.S. dollar and the Japanese yen in terms of the U.S. dollar) and run from August 7, 1991 to January 22, 2013, a total of 5,600 data points. We produce both the autocorrelation functions of the exchange rate changes and of the squared exchange rate changes in Figures 1 and 2. The dotted lines in the figures indicate the Bartlett asymptotic standard errors.

The autocorrelations of the squared returns are significant at least up to lag 24, which indicates that the volatilities of the exchange rate changes are time varying and serially correlated. That is, we expect the autocorrelation function is nonconstant. The descriptive statistics of the data are reported in Table 1. The two returns processes are leptokurtic and negatively skewed, meaning that they are more likely to have extreme negative returns than positive ones. The modified Ljung-Box tests for serial correlation in the returns process and the squared returns process for both exchange rate changes are significant under the 1% significance level, meaning that the statistics indicate first and second order

Table 2: GARCH(1,1) Model $y_t = c + \varepsilon_t, h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$ estimation results

	KRW/USD	JPY/USD
A. Estimation		
$\hat{c}$	0.00 (0.30)	0.00 (0.40)
$\hat{\omega}$	0.00 (0.00)	0.00 (0.00)
$\hat{\alpha}$	0.10 (0.00)	0.05 (0.00)
$\hat{\beta}$	0.91 (0.00)	0.93 (0.00)
$\hat{\omega}/(1-\hat{\alpha}-\hat{\beta})$	-0.000002	0.000048
B. Specification test		
Jarque-Bera	48581 (0.00)	2169 (0.00)
Q(12) on the standardized residuals	48.47 (0.00)	20.86 (0.05)
Q(12) on the sq. standardized residuals	3.56 (0.99)	8.42 (0.75)
TR^2	3.50 (0.99)	8.42 (0.75)

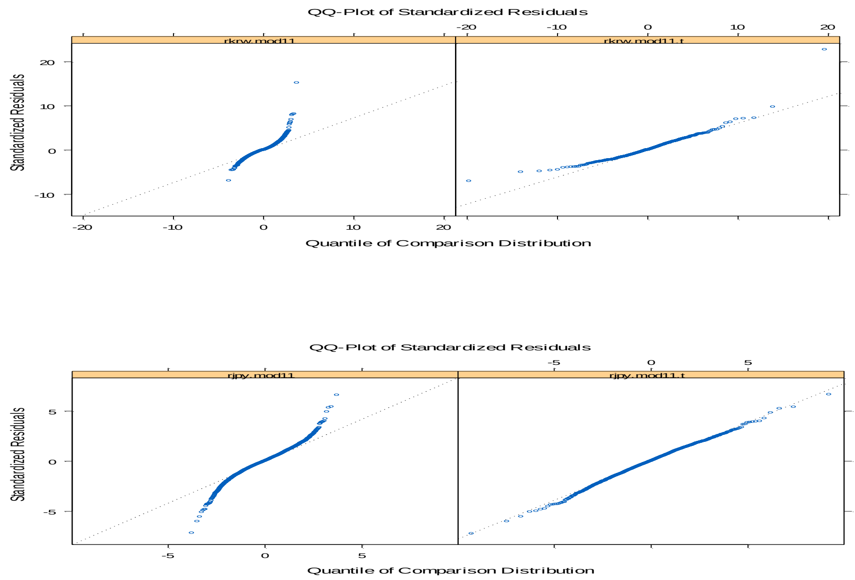
Note: The table shows the GARCH(1,1) model fit to the full sample of data. The entries in parentheses are p-values. The entries in panel A are the estimation results of the GARCH(1,1) model. $\hat{\omega}/(1-\hat{\alpha}-\hat{\beta})$ is an estimate of the unconditional standard deviation of the GARCH residuals. The Q(12) stands for the modified Ljung-Box Q-statistic for up to twelfth-order serial correlation in the standardized residuals and the squared standardized residuals. The diagnostic statistics TR^2 to test for the ARCH effects are reported in panel B.

dependence in the returns process. Engle (1982) Lagrange multiplier test shows strong evidence of ARCH effects.

The fit of the GARCH(1,1) model to the full sample of data is reported in Table 2. From the estimation results, we confirm that the GARCH model provides a good fit to the data. The heterogeneity in the volatility makes a GARCH specification to be appropriate. However, under a conditional normal density, $\hat{\alpha}+\hat{\beta}$ is near one, meaning that the processes are persistent in volatility.

If the GARCH model specification with the normally distributed errors is correct, the estimated standardized residuals and the squared values are serially uncorrelated. To assess the GARCH model fit, we apply the LM test to the residuals from a constant conditional mean equation of the spot exchange rate returns. The Jarque-Bera statistics reject the normality of the standardized residuals. For the KRW/USD returns, the modified Ljung-Box Q-statistics for up to twelfth-order serial correlation in the standardized residuals are significant at the

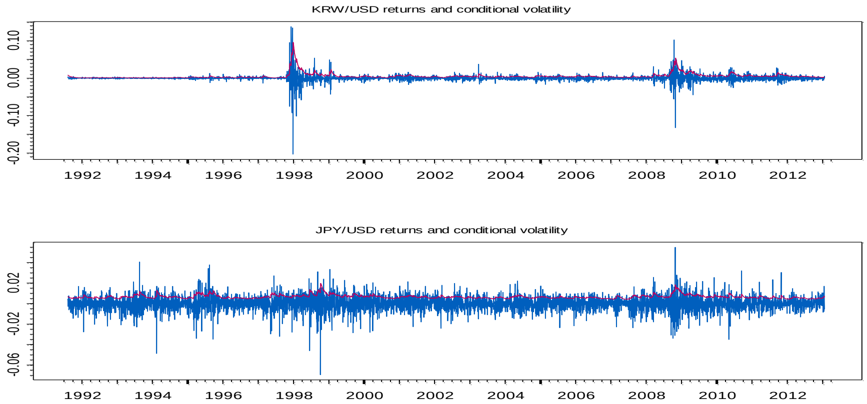
Figure 3: QQ-plot of standardized residuals from GARCH(1,1) with normal errors vs. GARCH(1,1) with Student's t errors



1% level, however, the squared standardized residuals are not significant, meaning that first order serial dependence are not fully captured by the GARCH(1,1) model. For the JPY/USD returns, on the other hand, the statistics indicate any further first and second order serial dependence left in the standardized residuals from the fitted GARCH(1,1) model. However, the normality assumption for the standardized residuals seems to be inappropriate judging not only from the Jarque-Bera statistics but from the qq-plots of standardized residuals in Figure 3 which deviate from the normal qq-line in both tails.

Since the basic GARCH model assumes the normally distributed conditional error, other conditional error distributions have to be introduced to reconcile the diagnostic test results of the standardized residuals from the estimated GARCH models. We extend the GARCH model to allow for the generalized error distribution (GED) in equation (1). The GED subsumes the normal, a fatter-tailed distribution than the normal and a thinner-tailed distribution than the normal as a special case depending on the value of the tail thickness parameter ν. From the estimation of the GARCH(1,1) model with the GED, the estimate of ν is 0.93 for

Figure 4: Exchange rate returns and conditional standard deviation



the KRW/USD returns and 1.28 for the JPY/USD returns, indicating that the distribution of z_t and therefore the conditional distribution of ε_t for both series has significantly fatter tails than the normal. Overall, the fit of the GARCH model with the GED to the series is not favorable, especially for the KRW/USD returns process.

$$f(z) = \frac{\nu\ exp[-(\frac{1}{2})|z/\lambda|]^{\nu}}{\lambda 2^{(1+1/\nu)}\Gamma(1/\nu)}, \quad -\infty < z < \infty, \ \ 0 < \nu < \infty \tag{1}$$

where $\Gamma(\cdot)$ is the gamma function and $\lambda \equiv [2^{(-2/\nu)}\Gamma(1/\nu)/\Gamma(3/\nu)]^{1/2}$.

We investigate the forecasting performance of the estimated GARCH model and the cause for the misspecification. In Figure 4, we show the exchange rate returns with conditional standard deviations superimposed. Throughout the sample period, the conditional volatility of the GARCH model seems to mimic the time-varying volatility well. However, there are some outliers that can only be drawn from the normal or other parametric non-Gaussian error distribution with extremely low probabilities. From the analysis in this section, we find that the GARCH model with conditionally normal, Students t, or the generalized errors cannot fully account for the observed leptokurtosis in the index returns processes. To resolve this issue, we investigate a nonparametric method in this paper.

2.2. THE SNP DENSITY ESTIMATION

In this section, we discuss the SNP methodology based on Galant and Nychka (1987) and Gallant and Tauchen (1989, 2001) to derive the conditional density. We follow the notations used in this section from Gallant and Tauchen (1989). We expect the conditional density to successfully model conditional heteroskedasticity and nonlinear dynamics in the returns process, however, our main concern lies in taking note of non-normality which proved to be difficult to capture with GARCH models in the previous section.

The data $\{y_t\}_{t=-L+1}^{n}$ are an M-dimensional realization from a stationary and ergodic random variable $\{y_t\}_{t=-\infty}^{\infty}$ and have the form: $y_t = \mu_0 + B \cdot x_{t-1} + R \cdot z_t$, where μ_0 is a column vector of length M, B is a vector of L_u lags of y_t, R is an upper triangular matrix, and z_t is a normally distributed disturbance.

The conditional density of y_t is a function of L lagged values of y_t. We denote the L-lagged values of y_t as the state vector $x_{t-1} = (y'_{t-L}, \cdots, y'_{t-1})$. We set up the likelihood as follows:

$$[\prod_{t=1}^{n} h(y_t|x_{t-1})] \int h(y, x_0))dy \tag{2}$$

where $h(y_t|x_{t-1}) = h(y_t, x_{t-1}) / \int h(y_t, x_{t-1})dy$.

The SNP methodology derives the conditional density $f(z)$ from a truncated Hermite expansion of a Gaussian density:

$$f(z) = \frac{[\sum_{|\alpha|=0}^{K_z} a_\alpha z^\alpha]^2 \phi(z)}{\int [\sum_{|\alpha|=0}^{K_z} a_\alpha z^\alpha]^2 \phi(u) du}, \tag{3}$$

where z is obtained by a location-scale transformation, $z_t = R^{-1}(y_t - \mu_0 - B \cdot x_{t-1})$ and $z^\alpha = \prod_{j=1}^{M} (z_j)^{\alpha_j}$ of degree $|\alpha| = \sum_{j=1}^{M} |\alpha_j|$. The SNP density of the standardized residuals can be expressed as the multiplication of a standard Gaussian density function as the leading term of the expansion and a squared Hermite polynomial in the standardized residuals z . Also, the SNP density integrates to 1. The SNP density of the standardized residuals nests the Gaussian. If $K_z = 0$, the SNP density $f(z)$ is Gaussian. However, if $K_z > 0$, then the SNP density allows for shape departures from the Gaussian density whose shape is modified by the Hermite polynomial in z.

Further, if we define $a_\alpha(x) = \sum_{|\beta|=0}^{K_x} a_{\alpha\beta} x_{t-1}^{\beta}$, where $x^\beta = \prod_{i=1}^{ML} (x_i)^{\beta_i}$ of degree $|\beta| = \sum_{i=1}^{ML} |\beta_i|$. We obtain an approximating conditional transition density of a standardized residual as follows:

$$f_K(z_t|x_{t-1}) = \frac{[\sum_{|\alpha|=0}^{K_z} a_\alpha(x_{t-1}) z_t^\alpha]^2 \phi(z_t)}{\int [\sum_{|\alpha|=0}^{K_z} a_\alpha(x_{t-1}) u^\alpha]^2 \phi(u) du}, \tag{4}$$

where a polynomial in z of degree K_z has polynomial coefficients of degree K_x in x, thus is a polynomial in (z,x) of degree $K_z + K_x$. The number of lags of x on which the coefficients a depend is L_p.

When $K_z > 0, K_x > 0$ under the above normalization, the conditional density for the residuals is a nonlinear nonparametric function which can approximate the conditional heterogeneity. If $K_x = 0$, then the innovation density does not depend on x_{t-1} and is conditionally homogeneous. And if $K_z = 0$ at the same time, then the conditional innovation density $f_K(z_t|x_{t-1})$ is a Gaussian VAR. Further, if $K_z > 0, K_x = 0$, the conditional density is a semi-parametric VAR whose first moment is linearly dependent on x_{t-1} and whose shape is constant with respect to variation in x_{t-1}. We can express the conditional density of y_t given x_{t-1} as:

$$\begin{aligned} h_K(y_t|x_{t-1}, \theta) &= f_K[R^{-1}(y_t - \mu_0 - Bx_{t-1})|x_{t-1}]/det(R) \\ &= \frac{[\sum_{|\alpha|=0}^{K_z} a_\alpha(x_{t-1}) R^{-1}(y_t - \mu_0 - Bx_{t-1})^\alpha]^2 n_M(y_t|\mu_0 + Bx_{t-1}, RR')}{\int [\sum_{|\alpha|=0}^{K_z} a_\alpha(x_{t-1}) u^\alpha]^2 \phi(u) du} \end{aligned} \tag{5}$$

where $n_M(y_t|\mu_0 + Bx_{t-1}, RR')$ denotes the Gaussian density of dimension M. A vector of SNP model parameters θ contains the Hermite polynomial coefficients, the conditional mean μ_0, and the parameters in the location-scale transformation B, R and are estimated by $\hat{\theta}$ that minimizes the sample objective function

$$s_n(\theta) = (-1/n) \sum_{t=1}^{n} ln\ h_K(y_t|x_{t-1}, \theta). \tag{6}$$

The SNP conditional density $h_K(y_t|x_{t-1}, \theta)$ has the flexibility of allowing the shape departures from linearity and Gaussianity to vary with changes in the state vector $x_{t-1} = (y'_{t-L}, \cdots, y'_{t-1})$. The changes in the state vector $x_{t-1} = (y'_{t-L}, \cdots, y'_{t-1})$ result in changes in $a_\alpha(x) = \sum_{|\beta|=0}^{K_x} a_{\alpha\beta} x_{t-1}^\beta$ and, in turn, approximations in the SNP conditional density $h_K(y_t|x_{t-1}, \theta)$ arbitrarily accurately. The SNP conditional density accommodates the conditional heterogeneity.

The polynomial terms in the $a_\alpha(x)$ are to approximate deviations from the leading term. If the conditional mean μ is a linear function μ_x of past observations x, then the leading term is a vector autoregression. If, in addition, R is set to a linear function R_x of the absolute values of past deviations from μ_x, then the leading term is an ARCH model with dimension L_r or a GARCH model with dimension L_g. K_z controls shape departures from a Guassian VAR and K_x controls conditional nonlinearities. We also define the number of lags in the x_{t-1} of the Hermite polynomial as L_p. While the tuning parameters L_u, L_g and L_r determine the location and scale characteristics of the leading term of the expansion, L_p, K_z and K_x determine the characteristics of the polynomial expansion of the conditional innovation process.

We estimate the SNP model with a variety of combinations of tuning parameters L_u, L_g, L_r, L_p, K_z and K_x and choose the model based on the BIC criterion. In moving upward along an expansion path, we initialize the SNP model using a Gaussian autoregressive model. We use 31 pairs of starting values in performing 25 random perturbations of the starting values per each pair of the starting value. Starting from the initial SNP model, we expand the model along the tuning parameters of L_u, K_z and K_x. For the ARCH/GARCH leading term of the series expansion, we consider a GARCH(1,1) along with the GARCH(1,2), GARCH(2,1) and GARCH(2,2) models. We additionally include tuning parameters, I_z and I_x to suppress interactions of specified degree or higher in the Hermite polynomial. For example, if we set $I_z = 1$, we suppress the interaction terms of the highest degree in z of the polynomial.

We put the leading term of the expansion $n_M(y_t|\mu_0 + Bx_{t-1}, RR')$ to a Gaussian GARCH in order to capture conditional heteroskedasticity in the data while we keep the tuning parameter K_z small. The strategy in estimating the SNP conditional densities is to restrict the model with the tuning parameters to improve the computational stability. All of the following computations are implemented with the S-PLUS 8.1.

Table 3 reports the optimized model selection results based on the BIC, AIC and the HQ criterion for the KRW/USD returns. Gallant and Tauchen (2010) suggest using the BIC criterion by moving upward along an expansion path. We choose the AR order L_u of the conditional mean equation of the SNP model with up to maximum lag orders $L_u = 4$. For example, we estimate the SNP model from $(L_u, L_g, L_r, L_p, K_z, I_z, K_x, I_x) = (10010000)$ to (40010000) and choose the Gaussian AR(4) model (40010000) to minimize the BIC. We then expand the SNP model for the ARCH and GARCH orders L_r andL_g with maximum orders up to 2 for each term. Based on the BIC, we choose the Gaussian AR(4)-

GARCH(2,2) model (42210000) as the optimal model. In expanding the SNP model, we follow the recommendation by Gallant and Tauchen (2010) and estimate the semiparametric GARCH model from (42214000) to (42219000). In this step of the procedure, we do not consider the SNP models with $K_z < 4$, following the recommendation by Gallant and Tauchen (2010). The BIC is known to prefer parsimonious models and has a tendency to choose sparsely parameterized SNP models with the tuning parameter $Kx = 0$. We confirm that the BIC minimizing semiparametric GARCH model is the semiparametric AR(4)-GARCH(2,2) model with the tuning parameter $K_z = 8$. However, if we base our decision on the AIC or HQ criterion, the nonlinear nonparametric AR(4)-GARCH(2,2) model with the tuning parameters $K_z = 8$ and $K_x = 1$, (42218010) minimizes the information criteria.

We then apply the model selection procedure to the JPY/USD returns and report the results in Table 4. We choose the AR order L_u of the conditional mean equation of the SNP model with maximum lag order $L_u = 4$. For example, we estimate the SNP model from $(L_u, L_g, L_r, L_p, K_z, I_z, K_x, I_x) = (10010000)$ to (40010000) and choose the Gaussian AR(1) model (10010000) as the BIC preferred AR specification. We then expand the SNP model for the ARCH and GARCH leading terms of orders L_r and L_g with maximum orders up to 2 for each term. Based on the BIC, we choose the Gaussian AR(1)-GARCH(2,2) model (12218010) as the optimal model. We expand the SNP model and estimate the semiparametric GARCH model from (12214000) to (12219000). The BIC and HQ criterion minimizing semiparametric GARCH model is the semiparametric AR(1)-GARCH(2,2) model with the tuning parameter $K_z = 8$. However, if we base our decision on the AIC, the nonlinear nonparametric AR(1)-GARCH(2,2) model (12218010) with the tuning parameters $K_z = 8$ and $K_x = 1$, minimizes the information criterion.

When we expand the SNP model to obtain the optimal model, we use the estimated coefficients of the model in the previous step as the starting values. We also implement the random restart procedure proposed by Gallant and Tauchen (2010) to get rid of the possibility of getting local minima. Coppejans and Gallant (2002) suggest using the BIC in model selection procedure due to the fact that integrated squared error falls dramatically at a point where we would like to truncate in SNP methodology. Therefore, we choose the optimal model based on the BIC, however, we also consider the HQ criterion or the AIC minimizing models in the simulation experiment below.

Table 3: Optimized likelihood and information criteria: KRW/USD returns

	BIC	HQ	AIC	Log likelihood
(10010000)	1.4168	1.4156	1.4150	-7899.887
(20010000)	1.4174	1.4159	1.4150	-7898.980
(30010000)	1.4044	1.4025	1.4015	-7822.231
(40010000)	**1.4006**	1.3983	1.3970	-7796.411
(41010000)	0.9258	1.3983	1.3970	-5140.436
(41110000)	0.6404	0.6373	0.6357	-3542.152
(42010000)	0.8389	0.8358	0.8342	-4650.782
(42110000)	0.6387	0.6353	0.6334	-3528.556
(42210000)	**0.6248**	0.6209	0.6188	-3446.227
(42214000)	0.5453	0.5399	0.5370	-2985.324
(42215000)	0.5460	0.5402	0.5371	-2984.492
(42216000)	0.5348	0.5287	0.5253	-2918.063
(42217000)	0.5372	0.5306	0.5271	-2927.024
(42218000)	**0.5302**	**0.5232**	0.5195	-2883.347
(42219000)	0.5311	0.5237	0.5198	-2883.974
(42218010)	0.5346	0.5242	**0.5186**	-2869.209
(42218020)	0.5871	0.5732	0.5658	-3123.855
(42218030)	0.5823	0.5649	0.5556	-3057.934
(42218040)	0.6078	0.5869	0.5757	-3161.386

Note: The table reports the estimation results of the SNP model. Each SNP model is denoted in the first column as $(L_u, L_g, L_r, L_p, K_z, I_z, K_x, I_x)$. The boldface numbers denote the preferred models based on each information criteria or the information criteria values minimized in each category of the SNP models.

Table 4: Optimized likelihood and information criteria: JPY/USD returns

	BIC	HQ	AIC	Log likelihood
(10010000)	**1.4220**	1.4208	1.4202	-7928.650
(20010000)	1.4226	1.4211	1.4202	-7928.071
(30010000)	1.4234	1.4214	1.4204	-7927.930
(40010000)	1.4240	1.4217	1.4205	-7927.337
(11010000)	1.4008	1.3992	1.3984	-7806.162
(11110000)	1.3681	1.3662	1.3652	-7619.427
(12010000)	1.3952	1.3932	1.3922	-7770.451
(12110000)	1.3688	1.3665	1.3653	-7618.981
(12210000)	**1.3667**	1.3640	1.3625	-7602.673
(12214000)	1.3317	1.3275	1.3252	-7390.271
(12215000)	1.3317	1.3271	1.3246	-7385.802
(12216000)	1.3326	1.3276	1.3249	-7386.407
(12217000)	1.3329	1.3275	1.3246	-7384.093
(12218000)	**1.3305**	1.3248	1.3216	-7366.406
(12219000)	1.3314	1.3252	1.3219	-7366.569
(12218010)	1.3340	**1.3247**	**1.3197**	-7346.634
(12218020)	1.3403	1.3276	1.3207	-7343.351
(12218030)	1.3467	1.3305	1.3218	-7340.306
(12218040)	1.3523	1.3326	1.3221	-7332.768

2.3. DIAGNOSTICS

We perform diagnostic analysis on the estimated SNP density function. From the analysis in the previous section, we find that the GARCH model with conditionally normal, Students t, or the generalized errors distribution cannot fully account for the observed characteristics in the exchange rate returns such as leptokurtosis, conditional heteroskedasticity and higher peak than the normal near zero. To resolve this misspecification, we investigate a SNP method in this paper. We expect the conditional density to successfully model conditional heteroskedasticity and nonlinear dynamics in the returns process, however, our main concern lies in modeling non-normality which proved to be difficult to capture with GARCH models.

A nonparametric BDS test for independently and identically distributed linearity (hereafter, i.i.d.-linearity) is applied to the standardized residuals from the estimated SNP models to test for the goodness of fit and omitted dynamics. If the null hypothesis of i.i.d.-linearity is rejected when tested on the standardized residuals from the fitted SNP model, we can conclude that the series is from non-i.i.d.-linear data generation process and that the SNP model is misspecified. Since the BDS test does not specify the alternative hypotheses to the null, it has good power against a wide variety of alternatives.

We implement the BDS nonlinearity test on the daily KRW/USD returns. We instruct the BDS test to use the embedding dimensions, $m = 2, 3, 4$ and 5 and specifies in 0.5, 1.0, 1.5 and 2.0 units of sample standard deviations the distance threshold in setting up the correlation integral at each embedding dimension m, respectively. In Table 5, the distance threshold values of $\varepsilon/\sigma = (0.5, 1.0, 1.5, 2.0)$ are used in the test. The null hypotheses that the data are independently and identically distributed are strongly rejected for all combinations of m and ε at the 1% significance level for all the SNP models. Since the null hypothesis of i.i.d.-linearity is rejected when tested on the standardized residuals from the fitted SNP model, we can conclude that the estimated standardized residuals are from non-i.i.d.-linear data generation process and that the SNP models are misspecified.

The conditional normality is rejected based on the sample moments of the standardized residuals in Table 6. The LM test for ARCH effects and the modified Ljung-Box test for serial correlation show that the estimated residuals seem to have no further heteroskedasticity. As can be seen from Table 7, the conditional coefficients of kurtosis are 29.30 and 29.14 from the (42218000) and (42218010) models, meaning that the standardized residuals from the SNP models are still leptokurtic. However, they are significantly lower than the conditional coefficient of kurtosis of 92.52 from the Gaussian AR(4) model. The conditional distributions of the standardized residuals of these seimiparametric and nonlinear nonparametric models are positively skewed and leptokurtic. The plots of these conditional densities are given Figure 5 and are compared to the standard normal distribution. The first two plots of the SNP conditional densities are the same as those of the standard normal density because the SNP densities with correspond to the Gaussian AR(4) and the Gaussian AR(4)-GARCH(2,2) model, respectively. On the other hand, the SNP conditional densities from the (42218000) and (42218010) models have tails fatter than the normal and higher peak near zero, which reflects leptokurtosis and non-normality observed in the exchange rate returns processes.

In summary, the semiparametric AR(4)-GARCH(2,2) model with the tuning parameter $K_z = 8$ model, (42218000) and the nonlinear nonparametric model with the tuning parameter $K_z = 8$ and $K_x = 1$ model, (42218010) fit the data relatively well. However, we have to note that conditional leptokurtosis of the observed data is not fully captured in the semiparametric GARCH or nonlinear nonparametric GARCH model.

We apply the BDS nonlinearity test on the daily JPY/USD returns and report the results in Table 8. On the contrary to the results when we apply the BDS nonlinearity test on the KRW/USD returns, the null hypotheses that the data are independently and identically distributed are strongly rejected for all combinations of m and ε at the 1% significance level for the Gaussian GARCH model, the semiparametric GARCH and the nonlinear nonparametric GARCH model. Since the null hypothesis of i.i.d.-linearity is rejected when tested on the standardized residuals from the fitted SNP model, we can conclude that the estimated standardized residuals are from i.i.d.-linear data generation process and that the SNP models are appropriate model for the data.

The LM test for ARCH effects and the modified Ljung-Box test for serial correlation show that the estimated residuals seem to have no further serial correlation and heteroskedasticity. As can be seen from Table 10, the conditional coefficients of kurtosis are 5.79 and 5.56 from the (12218000) and (12218010) model, meaning that the standardized residuals from the SNP models are still leptokurtic but significantly lower than those from the Gaussian AR model and Gaussian GARCH model. The conditional distributions of the standardized residuals of these seimiparametric and nonlinear nonparametric models are slightly negatively skewed and slightly leptokurtic. The plots of these conditional densities are given Figure 6 and are compared to the standard normal distribution. Again, the first two plots of the SNP conditional densities are the

Table 5: BDS statistic for standardized residuals from SNP models: KRW/USD returns

SNP	ε	m=2	m=3	m=4	m=5
(40010000)	$\varepsilon = 0.5\sigma$	35.03 (0.00)	44.26 (0.00)	53.74 (0.00)	65.54 (0.00)
(40010000)	$\varepsilon = 1.0\sigma$	35.40 (0.00)	39.63 (0.00)	42.70 (0.00)	45.25 (0.00)
(40010000)	$\varepsilon = 1.5\sigma$	35.53 (0.00)	39.58 (0.00)	41.15 (0.00)	41.96 (0.00)
(40010000)	$\varepsilon = 2.0\sigma$	33.17 (0.00)	37.40 (0.00)	38.98 (0.00)	39.56 (0.00)
(42210000)	$\varepsilon = 0.5\sigma$	8.37 (0.00)	9.95 (0.00)	10.79 (0.00)	11.78 (0.00)
(42210000)	$\varepsilon = 1.0\sigma$	9.00 (0.00)	9.92 (0.00)	10.13 (0.00)	10.30 (0.00)
(42210000)	$\varepsilon = 1.5\sigma$	9.84 (0.00)	10.24 (0.00)	10.32 (0.00)	10.32 (0.00)
(42210000)	$\varepsilon = 2.0\sigma$	9.71 (0.00)	9.96 (0.00)	10.21 (0.00)	10.28 (0.00)
(42218000)	$\varepsilon = 0.5\sigma$	6.87 (0.00)	9.23 (0.00)	10.58 (0.00)	11.87 (0.00)
(42218000)	$\varepsilon = 1.0\sigma$	7.36 (0.00)	9.09 (0.00)	9.91 (0.00)	10.40 (0.00)
(42218000)	$\varepsilon = 1.5\sigma$	8.16 (0.00)	9.39 (0.00)	10.02 (0.00)	10.28 (0.00)
(42218000)	$\varepsilon = 2.0\sigma$	8.21 (0.00)	9.34 (0.00)	10.10 (0.00)	10.34 (0.00)
(42218010)	$\varepsilon = 0.5\sigma$	6.40 (0.00)	8.79 (0.00)	10.01 (0.00)	11.30 (0.00)
(42218010)	$\varepsilon = 1.0\sigma$	6.45 (0.00)	8.28 (0.00)	8.97 (0.00)	9.48 (0.00)
(42218010)	$\varepsilon = 1.5\sigma$	6.88 (0.00)	8.17 (0.00)	8.66 (0.00)	8.96 (0.00)
(42218010)	$\varepsilon = 2.0\sigma$	6.99 (0.00)	8.13 (0.00)	8.70 (0.00)	8.96 (0.00)

Note: The BDS test in the table use the embedding dimensions, $m = 2, 3, 4$ and 5 and specifies in 0.5, 1.0, 1.5 and 2.0 units of sample standard deviations the distance threshold in setting up the correlation integral at each embedding dimension m, respectively. The distance threshold values of $\varepsilon/\sigma = (0.5, 1.0, 1.5, 2.0)$ are used in the test. The null hypotheses that the data are independently and identically distributed are rejected for combinations of m and ε at the 1% significance level if the p-values in the parenthesis is less than 0.01.

Table 6: Diagnostic tests for standardized residuals from SNP models: KRW/USD returns

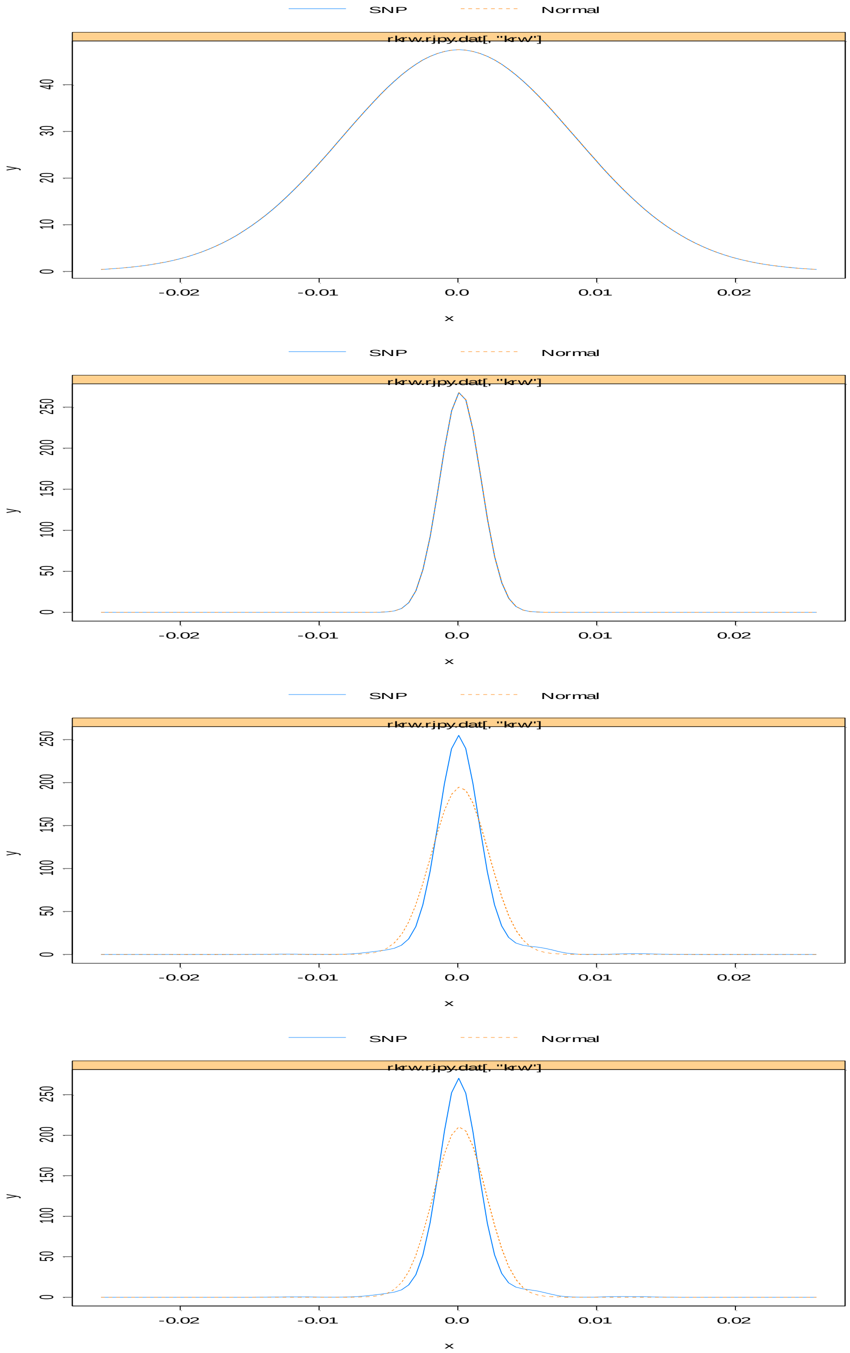
SNP	LM test for ARCH effects	Ljung-Box test
(40010000)	1882.54 (0.00)	177.56 (0.00)
(42210000)	13.37 (0.34)	20.78 (0.05)
(42218000)	8.98 (0.71)	26.35 (0.01)
(42218010)	5.95 (0.92)	20.21 (0.06)

Note: The LM test for ARCH effects and the modified Ljung-Box test for serial correlation are reported in the table. The entries in the parentheses are the p-values.

Table 7: Diagnostic tests for standardized residuals from SNP models: KRW/USD returns

SNP	Mean	S.D	Skewness	Kurtosis
(40010000)	0.00	1.00	0.03	92.52
(42210000)	-0.04	1.00	1.42	21.14
(42218000)	-0.01	0.99	1.84	29.30
(42218010)	-0.00	1.00	1.76	29.14

Figure 5: SNP density: $(40010000), (42210000), (42218000), (42218010)$



same as those of the standard normal density because the SNP densities with $K_z = 0$ and $K_x = 0$ correspond to the Gaussian AR(1) and the Gaussian AR(1)-GARCH(2,2) model, respectively. On the other hand, the SNP conditional densities from the (12218000) and (12218010) models have tails fatter than the normal and higher peak near zero, which reflects leptokurtosis and non-normality observed in the JPY/USD exchange rate returns processes.

In summary, the semiparametric AR(1)-GARCH(2,2) model with the tuning parameter $K_z = 8$ model, (12218000) and the nonlinear nonparametric model with the tuning parameter $K_z = 8$ and $K_x = 1$ model, (12218010) fit the data very well.

We evaluate the performance of the estimated SNP conditional density function in generating the observed data via simulation. In order for the estimated SNP density to work as a score generator during the EMM estimation method, not only the statistical characteristics of the observed data but the dynamic stability of the fitted model has to be guaranteed. Since the SNP density function captures departures of the observed data from the underlying data generating process specified in the leading term, it is difficult to assign statistical meaning to the estimated coefficients of the Hermite polynomial. For this reason, we generate the data from a variety of estimated SNP conditional density functions to see if the fitted SNP density appropriately recognize the pattern of the original data.

We show the original data along with the simulated data from a variety of estimated SNP conditional density functions in Figure 7. The first panel of Figure 7 illustrates the actual returns on the KRW/USD exchange rate. The large spikes correspond to the period of the Asian financial crisis in 1997 and the global financial crisis in 2009. The second panel of Figure 7 illustrates the simulated data from the Gaussian AR(4) model. The bottom three panels show the artificially generated data from the semiparametric AR(4)-GARCH(1,1) model with the tuning parameter $K_z = 8$, the semiparametric AR(4)-GARCH(2,2) model with the tuning parameter $K_z = 8$ and the nonlinear nonparametric AR(4)-GARCH(2,2) model with the tuning parameters $K_z = 8$ and $K_x = 1$ model. Obviously, the Gaussian AR(4) model does not generate the data which depicts the conspicuous conditional heteroskedasticity in the observed returns data. Compared to the Gaussian AR(4) model, the bottom three panels mimic the characteristics of the actual data quite well.

Simulation results from a variety of the estimated SNP models with the JPY/USD returns data are illustrated in Figure 8. The first panel of the graph shows the raw returns on the JPY/USD exchange rate during the sample period.

Simulations from the Gaussian AR(1) and the Gaussian AR(1)-GARCH(1,1) model are illustrated in the second and third panel. Simulations from the semiparametric AR(1)-GARCH(1,1) model with the tuning parameter of the Hermite polynomial $K_z = 8$ are presented in the fourth panel. The fifth panel of the graph shows simulations from the BIC preferred semiparametric AR(1)-GARCH(2,2) model with the tuning parameter of the Hermite polynomial $K_z = 8$. Lastly, simulations from the AIC preferred nonlinear nonparametric AR(1)-GARCH(2,2) model with the tuning parameters of the Hermite polynomial $K_z = 8$ and $K_x = 1$ are presented in bottom panel. As can be easily verified, simulation results from a variety of SNP models are distinctly different. However, the time dependent characteristic of the conditional volatility and the side lobes of the tails or the leptokurtosis of the actual data are appropriately recognized by the simulations from the semiparametric AR-GARCH-type models.

Table 8: BDS statistic for standardized residuals from SNP models: JPY/USD returns

SNP	ε	m=2	m=3	m=4	m=5
(10010000)	$\varepsilon = 0.5\sigma$	7.93 (0.00)	9.23 (0.00)	10.29 (0.00)	12.25 (0.00)
(10010000)	$\varepsilon = 1.0\sigma$	8.50 (0.00)	10.01 (0.00)	11.14 (0.00)	12.83 (0.00)
(10010000)	$\varepsilon = 1.5\sigma$	9.31 (0.00)	11.07 (0.00)	12.20 (0.00)	13.56 (0.00)
(10010000)	$\varepsilon = 2.0\sigma$	10.27 (0.00)	12.31 (0.00)	13.30 (0.00)	14.29 (0.00)
(12210000)	$\varepsilon = 0.5\sigma$	-0.53 (0.59)	-1.48 (0.14)	-2.06 (0.04)	-1.75 (0.08)
(12210000)	$\varepsilon = 1.0\sigma$	-0.25 (0.80)	-1.00 (0.32)	-1.48 (0.14)	-1.23 (0.22)
(12210000)	$\varepsilon = 1.5\sigma$	0.32 (0.75)	-0.17 (0.86)	-0.48 (0.63)	-0.32 (0.75)
(12210000)	$\varepsilon = 2.0\sigma$	1.13 (0.26)	0.96 (0.34)	0.84 (0.40)	0.88 (0.38)
(12218000)	$\varepsilon = 0.5\sigma$	-0.65 (0.52)	-1.55 (0.12)	-1.97 (0.05)	-1.77 (0.08)
(12218000)	$\varepsilon = 1.0\sigma$	-0.38 (0.70)	-1.09 (0.28)	-1.55 (0.12)	-1.32 (0.19)
(12218000)	$\varepsilon = 1.5\sigma$	0.07 (0.94)	-0.46 (0.65)	-0.76 (0.45)	-0.62 (0.54)
(12218000)	$\varepsilon = 2.0\sigma$	0.92 (0.36)	0.60 (0.55)	0.51 (0.61)	0.53 (0.60)
(12218010)	$\varepsilon = 0.5\sigma$	-1.55 (0.12)	-2.24 (0.02)	-2.67 (0.01)	-2.33 (0.02)
(12218010)	$\varepsilon = 1.0\sigma$	-1.59 (0.11)	-1.95 (0.05)	-2.25 (0.02)	-1.84 (0.07)
(12218010)	$\varepsilon = 1.5\sigma$	-1.28 (0.20)	-1.46 (0.14)	-1.53 (0.13)	-1.19 (0.24)
(12218010)	$\varepsilon = 2.0\sigma$	-0.40 (0.69)	-0.41 (0.68)	-0.25 (0.80)	-0.01 (1.00)

Table 9: Diagnostic tests for standardized residuals from SNP models: KRW/USD returns

SNP	LM test for ARCH effects	Ljung-Box test
(10010000)	275.77 (0.00)	27.89 (0.00)
(12210000)	10.48 (0.57)	16.86 (0.16)
(12218000)	9.63 (0.65)	19.22 (0.08)
(12218010)	8.80 (0.72)	18.73 (0.10)

Table 10: Diagnostic tests for standardized residuals from SNP models: KRW/USD returns

SNP	Mean	S.D	Skewness	Kurtosis
(10010000)	0.00	1.00	-0.39	8.77
(12210000)	-0.01	1.00	-0.25	5.95
(12218000)	-0.01	1.00	-0.30	5.79
(12218010)	0.00	1.00	-0.23	5.56

Figure 6: SNP density: $(10010000), (12210000), (12218000), (12218010)$

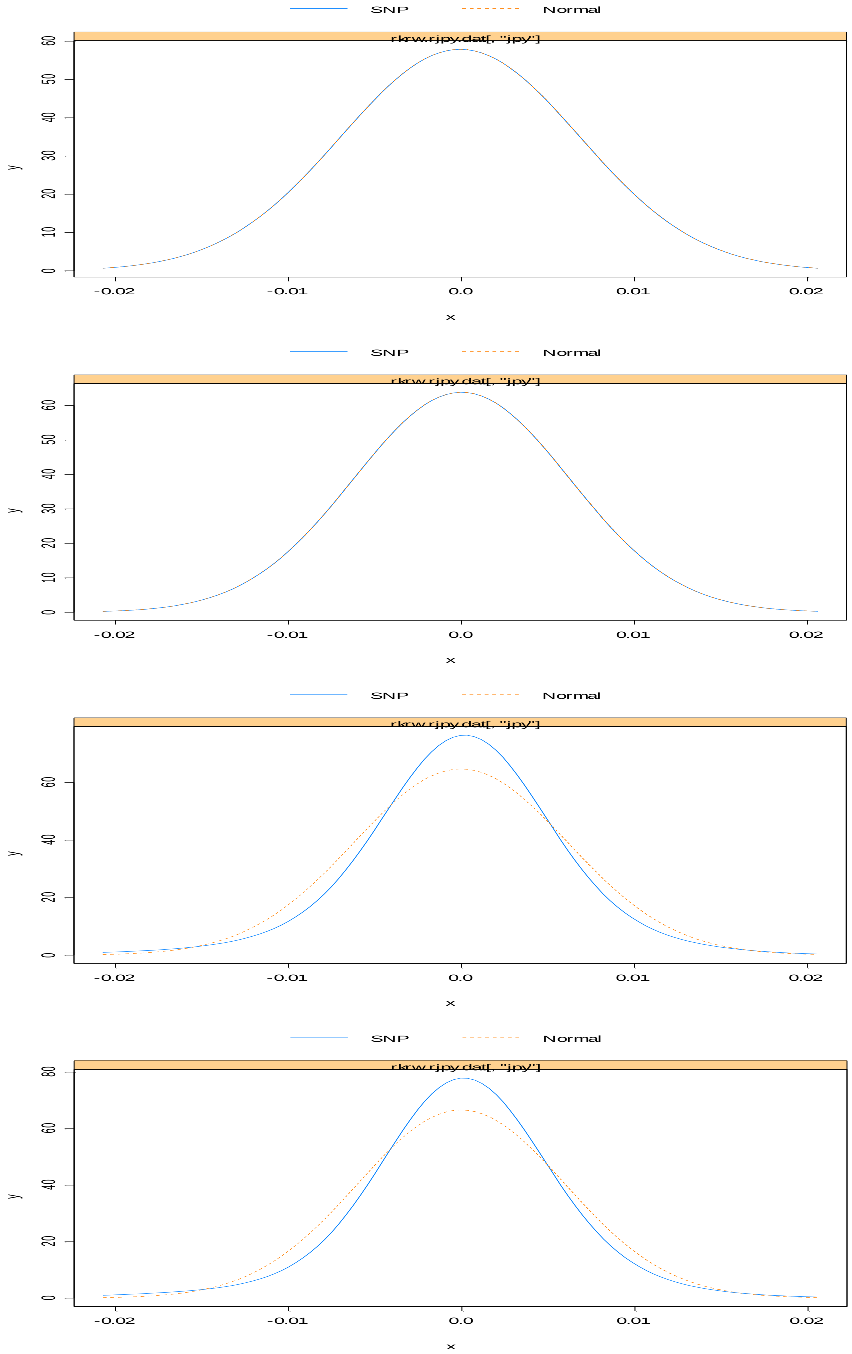
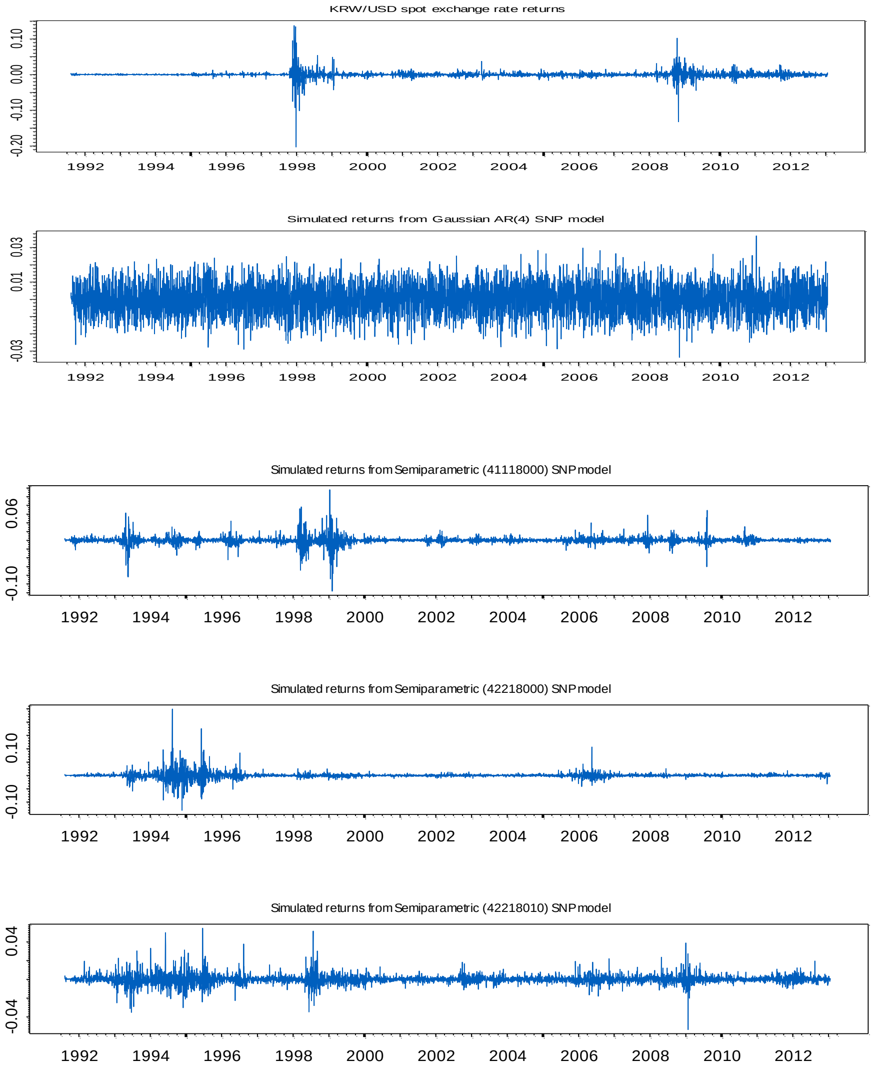
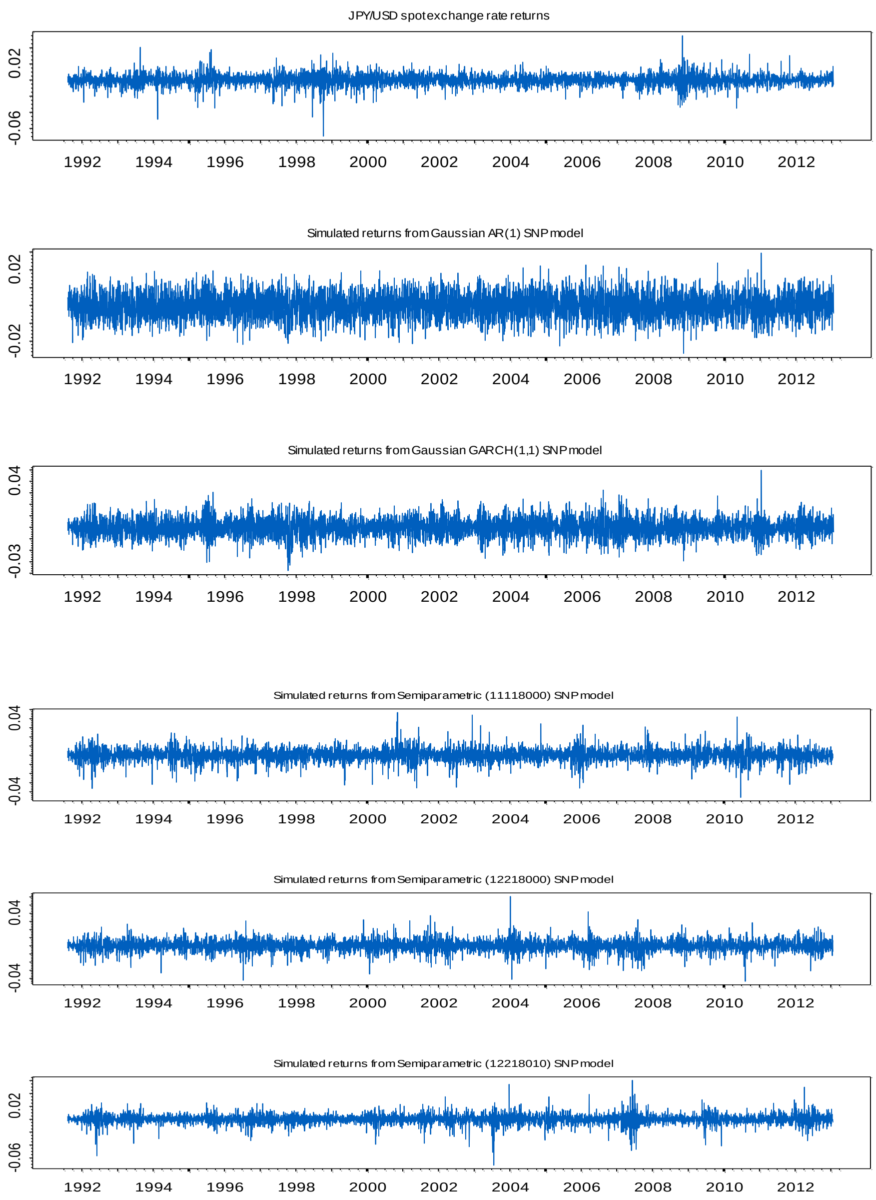


Figure 7: Actual returns and simulated returns from a variety of SNP models: KRW/USD returns



Note: The figures show the original data along with the simulated data from a variety of estimated SNP conditional density functions. The first panel illustrates the actual returns on the KRW/USD exchange rate. From the second to fifth panel show the simulated data from the Gaussian AR(4), the semiparametric AR(4)-GARCH(1,1) with the tuning parameter $K_z = 8$, the semiparametric AR(4)-GARCH(2,2) with the tuning parameter $K_z = 8$ and the nonlinear nonparametric AR(4)-GARCH(2,2) with the tuning parameters $K_z = 8$ and $K_x = 1$ model, respectively.

Figure 8: Actual returns and simulated returns from a variety of SNP models: JPY/USD returns



Note: The figures show the original data along with the simulated data from a variety of estimated SNP conditional density functions. The first panel illustrates the actual returns on the JPY/USD exchange rate. From the second to sixth panel show the simulated data from the Gaussian AR(1), the Gaussian AR(1)-GARCH(1,1), the semiparametric AR(1)-GARCH(1,1) with the tuning parameter $K_z = 8$, the semiparametric AR(1)-GARCH(2,2) with the tuning parameter $K_z = 8$ and the nonlinear nonparametric AR(1)-GARCH(2,2) model with the tuning parameters $K_z = 8$ and $K_x = 1$ model, respectively.

3. CONCLUSION

The success of the GARCH model is attributed to the fact that GARCH model and its extensions can be fit to the leptokurtic data that have a higher peak near zero and fatter tails than a normal distribution even under the assumption of conditional normality. Although the assumption of conditional normality has been the norm in modeling the conditional distribution of asset returns, we have witnessed a voluminous literature where the GARCH model with conditionally normal, Students t, or the generalized error distribution cannot fully account for the observed characteristics in the exchange rate returns such as leptokurtosis, conditional heteroskedasticity and higher peak than the normal near zero. To resolve this misspecification, we investigate a SNP method in this paper. We expect the conditional density to successfully model not only conditional heteroskedasticity and nonlinear dynamics in the returns process but non-normality which proved to be difficult to capture with GARCH models.

We implement the mathematical procedure such as a series expansion methodology in order to apply an appropriate parametric procedure to estimate a conditional density function of the exchange rate changes. The aim of this paper is to show a seminonparametric (SNP) methodology in characterizing the conditional density of the process.

The model selection procedure based on the BIC is used by moving upward along an expansion path. We follow the recommendation by Gallant and Tauchen (2010) and estimate a variety of SNP models, starting from the Gaussian AR and the Gaussian ARCH/GARCH models to the semiparametric ARCH/GARCH and nonlinear nonparametric ARCH/GARCH models. We employ a random restart procedure in order not to be stuck with local minima and use randomly perturbed starting values in each step of the procedure.

We find that the BIC minimizing SNP model is the semiparametric AR(4)-GARCH(2,2) model with the tuning parameter $K_z = 8$ for the KRW/USD returns and the semiparametric AR(1)-GARCH(2,2) model with the tuning parameter $K_z = 8$ for the JPY/USD returns. The modeling strategy in this paper is particularly successful for the JPY/USD data. When we apply a nonparametric BDS test for i.i.d.-linearity to the standardized residuals from the estimated SNP models to test for the goodness of fit and omitted dynamics, we fail to reject the null hypothesis and conclude that the series is from i.i.d.-linear data generation process.

Simulations from a variety of estimated SNP models with the JPY/USD returns data show that the BIC preferred semiparametric AR(1)-GARCH(2,2) model with the tuning parameter of the Hermite polynomial $K_z = 8$ mimics the actual data well. The time dependent heterogeneity of the actual data is appropriately recognized by the simulations from the semiparametric AR-GARCH-type models and the nonlinear nonparametric AR-GARCH-type models.

Although a nonparametric BDS test for i.i.d.-linearity to the standardized residuals from the estimated SNP models is rejected for the KRW/USD returns, the LM test for ARCH effects and the modified Ljung-Box test for serial correlation show that the estimated residuals from the estimated SNP models seem to have no further heteroskedasticity. For the KRW/USD returns, the semiparametric AR(4)-GARCH(2,2) model with the tuning parameter $K_z = 8$ model and the nonlinear nonparametric model with the tuning parameter $K_z = 8$ and $K_x = 1$ model seem to fit the data relatively well. However, we have to note that conditional leptokurtosis of the observed data is not fully captured in the semiparametric GARCH or nonlinear nonparametric GARCH model. However, observing that the conditional coefficients of kurtosis are 29.30 and 29.14 from the aforementioned estimated SNP models, they are significantly lower than the conditional coefficient of kurtosis of 92.52 from the Gaussian AR(4) model. The SNP conditional densities from the (42218000) and (42218010) models have tails fatter than the normal and higher peak near zero, which reflects leptokurtosis and non-normality observed in the KRW/USD returns processes.

A summary of our findings from using SNP method suggest that the semiparametric and the nonlinear nonparametric AR-ARCH/GARCH-type models can explain conditional heterogeneity of the exchange rate changes and appropriately mimic the actual data in the simulation experiments.

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