

Unionized Mixed Oligopoly and Privatization with Excess Burden of Taxation

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Abstract By introducing the excess burden of taxation into unionized mixed and privatized oligopolies, we show that (i) regardless of the number of private firm, privatization of public firm is always desirable when the degree of the excess burden of taxation is small and (ii) when the degree of the excess burden of taxation is large, whether privatization or nationalization is desirable depends on the critical value of the excess burden of taxation. In contrast to the existing works on mixed oligopolies, we find that privatization can enhance social welfare regardless of the number of firms under mild conditions. Thus, the excess burden of taxation is relatively low for industrial countries since privatization is considered a powerful instrument for reducing distortionary taxation for policy reasons.

Keywords Union, Excess Burden of Taxation, Mixed Oligopoly, Privatization.

JEL Classification D43, H44, J51, L13, L33, M21.

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1. INTRODUCTION

The economic implications of mixed oligopoly markets have recently attracted research attention with respect to changes in competition for market structure efficiency and their effects on privatization. Pioneering works by De Fraja and Delbono (1989) and Beato and Mas-Colell (1984) on mixed oligopolies employed game-theoretic analyses of public and private firms. Other studies on mixed oligopoly have frequently assumed that the public firm (or government) maximizes social welfare while private firms maximize their own profits.

Although various theoretical studies have explained a mixed duopoly, Willner (2006), Kato and Tomaru (2007), Saha and Sensarma (2008) and Kato (2008) explicitly investigated different objective functions between the public firm and the government in a mixed duopoly¹. However, the government directly intervenes with some inefficiency in many mixed markets. As Meade (1944) first pointed out, in the absence of lump-sum transfers, the government must resort to distortionary taxes on income, capital, or consumption. In other words, in the absence of lump-sum tax instruments, if the government raises \$1, society pays $\$(1 + \lambda) > \1 . The parameter $\lambda > 0$ is usually called the shadow cost of public funds². In the literature on mixed oligopolies, Capuano and De Feo (2010), Wang and Chen (2011) and Matsumura and Tomaru (2013, 2015) have addressed this issue by examining the welfare effect of a change in a public firm's objective function when the government takes into account the shadow cost of public funds (or, henceforth, the excess burden of taxation). In the absence of lump-sum transfers, Capuano and De Feo (2010) apply this analysis to a public firm in a mixed duopoly and the effects of privatization since obtaining money to reduce public debt (or distortionary taxes) is a common goal of privatization.

None of the above works, however, discusses the problem of the unionized firms along with the excess of burden of taxation. Hence, the purpose of the present paper is to extend the literature to the case of how they interact with each other and give rise to implications. A motivating example for our analysis is as follows: From the viewpoint of the government, the present strength of either German or U.K. trade unions is major impediment to any privatization (Bos, 1991, pp. 3-6). For more real world examples related to the objective of the government and labor union, see Bos (1991), Haskel and Szymanski (1993) and

¹The result of different objective function between the government and the public firm in a mixed oligopoly is a new one since so far the literature same objective function between the government and the public firm in a mixed oligopoly has found various robust results. See also Choi (2011).

²This approach is frequently adopted in contract theory. See Laffont and Tirole (1993).

Vickers and Yarrow (1988, p. 159). Hence, we extend the analysis to a *unionized mixed oligopoly* and to the effects of privatization when the government takes into account the excess burden of taxation.

We present some rationale for discussing why it is necessary to consider unionized firms (including the presence of private firm), along with the excess burden of taxation. First, while previous works considered that the costs of firms to be exogenous, our study considers that by allowing *multiple private firms*, costs are determined via wage bargaining with the excess burden of taxation and the effects of privatization. To investigate the optimal privatization policy, we incorporate union behavior into the objectives of the government, with the excess burden of taxation to explain the government's incentive for privatization as a commitment device. Motivating empirical works for our analysis are Ballard *et al.* (1985) and Snower and Warren (1996). They reported that the excess burden of taxation is generally assessed to be around 0.3 in industrial countries, and this may have policy implications for the main results, which better reflects reality. As already stated above, we consider that the government puts more weight on the profit of the public firm with respect to non-zero profit in order to extract more consumer surplus without a lump-sum tax fashion—that is, we assume deadweight loss by taxation in social welfare function. In the utilitarian measure of welfare, the public firm's profit or deficit should be weighted as $(1 + \lambda)$ rather than as a private firm's profit or the consumer net surplus because they are not neutral transfers among agents in an economy³. Hence, as for the policy implications of the present paper, we consider how unionized firms interact with each other when there is the excess burden of taxation for the government.

On the other hand, the results for unions' wage setting correspond to the empirical findings that in Europe, Japan, and the United States, the government is heavily involved in the setting of public sector wages (Du Caju *et al.*, 2009). Moreover, Bordogna (2003, pp. 62-63) pointed out that “even where bargaining has been decentralized, governments have often maintained strong, centralized, financial controls in order to contain public expenditures and avoid inflationary consequences of the decentralization process.” Thus, the issue to be analyzed is whether the above results are robust to changes in the type of competition under a unionized mixed oligopoly with an excess burden of taxation⁴. As Bordogna

³Thus, our assumption of the welfare function has such a Ramsey problem. For more discussion of deadweight loss by taxation, see Capuano and De Feo (2010, pp. 175-182) and Matsumura and Tomaru (2013, pp. 531-532). We will mention this point later in the Model at Section 2.

⁴In addition, according to Lewin (1977, p. 140), “an additional shortcoming of the ‘union power’ thesis with respect to the governmental sector is its de-emphasis, even ignorance, of conditions that make for potentially diverse patterns of labor relations.” In fact, this growing importance

(2003) argued, the empirical facts reveal that union wage setting under a unionized mixed oligopoly is a better approach than competition in the product market. This is why, for simplicity, unions' strategic behavior under a unionized mixed oligopoly and a *decentralized* union are considered in this paper.

The existing literature has produced a number of extensions of the excess burden of taxation in a mixed duopoly. Introducing the excess burden of taxation into an endogenous timing game in a mixed duopoly, and assuming the public firm to be less efficient than the private firm, Capuano and De Feo (2010) found that without a subsidy, private leadership emerges as more robust. Moreover, Matsumura and Tomaru (2013) investigated optimal subsidy schemes and the privatization neutrality. Assuming the public firm to be less inefficient than the private firm, Wang and Chen (2011) found that for the optimal subsidy, the level of welfare with privatization depends on the level of the cost efficiency gap and the excess burden of taxation⁵. Matsumura and Tomaru (2015) showed that in the mixed duopoly, the level of product differentiation is too low for social welfare. In this paper, following Capuano and De Feo (2010) and Matsumura and Tomaru (2013), we examine whether privatization is desirable with unions' strategic behavior when the excess burden of taxation is introduced into the government's objective function. To study the welfare effect of a change in the government's objective function in a mixed oligopoly, we allow for multiple private firms in the mixed oligopoly setting. Our main contribution of this paper is that regardless of the number of existing firms, the government's incentive to privatize the public firm always exists when the degree of the excess burden of taxation is relatively small, and does not exist when the degree of excess burden of taxation is relatively large.

More specifically, regardless of the number of private firms, the privatization of a public firm is always desirable from the welfare point of view when the degree of the excess burden of taxation falls within a small range, that is, $\lambda \in (0, 1]$. Because of the existence of unions, social welfare under either a mixed oligopoly or privatization consists of two factors in a reduced form: the representative consumer's utility and the profit of the public or privatized firm. Thus, on the one hand, when the excess burden of taxation is sufficiently small,

of the public sector in terms of union membership suggests that a more complete understanding of the role of unions in wage determination requires a better understanding of their role in the public sector (Bahrami *et al.*, 2009, p. 35). See, among others, De Fraja (1993a) and Haskel and Sanchis (1995) for theoretical view.

⁵Since welfare implications differ from Matsumura and Tomaru (2013, 2015) and Wang and Chen (2011), it is not meaningful to go beyond previous works with the excess burden of taxation to compare the results.

it forces the public firm's profit to be negative, while the privatized firm's profit is independent of the excess burden of taxation, which emerges as a positive welfare effect of privatization. On the other hand, (the representative) consumer's utility may be higher under a mixed oligopoly than under privatization when the excess burden of taxation is sufficiently small. The excess burden of taxation with public funding costs reduces the public firm's profit, and increases the representative consumer's utility as this burden increases under privatization given the number of firms and as privatization induces production substitution from the public firm to private firms. The latter two effects under privatization increase welfare, whereas the first effect reduces welfare under a mixed oligopoly. For this reason, the government uses the total wage as a commitment device to control unions' wage demands to maintain lower total wage levels under privatization. A decrease in wages improves welfare by increasing the total output under privatization.

When the degree of the excess burden of taxation falls within a large range—that is, $\lambda > 1$ —the desirable privatization and nationalization occur depending on the critical value of the excess burden of taxation. Given that $\lambda > 1$, privatization stimulates total output, and the consumer's utility under privatization starts to increase. However, after reaching the critical value of the excess burden of taxation, it is dominated by the excess burden effect, and the inverse is true when the degree of the excess burden of taxation becomes small, given the number of firms. In contrast to the first result, the government may or may not use the total wage as a commitment device to control unions' wage demands to maintain lower total wage levels since higher wages are still maintained under the mixed oligopoly when the degree of the excess burden of taxation is sufficiently great.

The main result of our paper is in contrast to the findings of De Fraja and Delbono (1989) that privatization can enhance social welfare when the number of existing private firms is relatively large. This finding holds when the effect of the excess burden of taxation does not exist. However, our result is crucial to the evaluation of the impact of welfare subsequent to the privatization of the public firm to maximize profits. With the excess burden of taxation, Capuano and De Feo (2010) found that under a mixed duopoly, privatization is not desirable when the public firm is assumed to be less efficient than private firms, which differs from our main result.

Hence, as for the policy implications of main results, the excess burden of taxation is relatively low for some countries since privatization is considered a powerful instrument for reducing distortionary taxation regardless of the number of firms. As stated above, Ballard *et al.* (1985) and Snower and Warren (1996)

reported that the excess burden of taxation is generally assessed to be around 0.3 in industrial countries⁶.

2. THE MODEL

Consider a mixed oligopoly situation for a homogeneous good that is supplied by a public firm (indexed by 0) and n private firms. Firm i ($i = 1, 2, \dots, n$) is a profit-maximizing private firm, and firm 0 is a public firm that maximizes social welfare. On the demand side of the market, we assume that the representative consumer's utility is a quadratic function given by

$$U = x_0 + \sum_{i=1}^n x_i - \frac{(x_0 + \sum_{i=1}^n x_i)^2}{2} + y; \quad i = 1, 2, \dots, n,$$

where x_0 is the level of output of the public firm, x_i is the level of output of the i^{th} private firm ($i = 1, 2, \dots, n$) and y is the amount of a numeraire good. Thus, the inverse demand is characterized by

$$p = 1 - x_0 - \sum_{i=1}^n x_i; \quad i = 1, 2, \dots, n,$$

where p is the market price.

To analyze the union's wage bargaining, we also assume that the public and private firms are unionized and that the wages, $w_j : j = 0, 1, \dots, n$, are determined as a consequence of bargaining between firms and their respective unions. Let \bar{w} and L_j denote the reservation wage and the number of workers who are employed by firm j , respectively. The firms are homogeneous with respect to productivity. Each firm adopts a constant returns-to-scale technology where one unit of labor is turned into one unit of the final good; thus, $x_j = L_j$. Taking \bar{w} as a given, the union's optimal wage-setting strategy regarding firm j , w_j , is defined as

$$\max_{w_j} u_j = (w_j - \bar{w})^\theta L_j; \quad j = 0, 1, \dots, n,$$

where θ is the weight that the union attaches to the wage level. Following Ishida and Matsushima (2009) in the literature on the unionized mixed duopoly, we

⁶Some readers may argue that the finding of this paper is quite technical depending on the parametric space of the degree of the excess burden, without much economic reasoning. However, with the crucial role by the union structure (i.e., the government's incentive for privatization as a commitment device when we incorporate union behavior into the objectives of the government), we may point out that for the policy implications, the relationship between privatization and mixed oligopoly is affected by the excess burden of taxation, which reflects more reality.

assume that $\theta = 1$ and $\bar{w} = 0$ to demonstrate our results simply. That is, the utility function of the union at the firm is its wage bill: $u_j(w_j; L_j) = w_j L_j = w_j x_j$. In our setting, no regulation is imposed on the public firm so that the public firm's union (union 0) is allowed to bargain collectively. Since the public firm is unionized just as the private firm, the union sets the wage w_0 to maximize $u_0 = w_0 L_0$. Thus, we consider the monopoly union model, which assumes that the unions set the wage while the firms choose the employment level once the wage is set by unions (see also Booth, 1995)⁷.

Each firm's profit is as following function

$$\pi_j = (p - w_j)x_j, \quad j = 0, 1, \dots, n, \quad (1)$$

where the price of labor (i.e., wage) that firm j has to pay is denoted by w_j , $j = 0, 1, \dots, n$.

We assume that the public firm maximizes a utilitarian measure of welfare taking into account the excess burden of taxation. That is, let $\lambda > 0$ denote the excess burden of taxation, which implies that distortionary taxation inflicts disutility $\$(1 + \lambda)$ on taxpayers in order to levy $\$1$ for the state. To compute the real cost incurred by a firm, the firm's cost and revenue are multiplied by $1 + \lambda$. This parameter is a measure of the dead-weight loss due to distortionary taxation. Then, in the presence of the excess burden of taxation, the maximization problem of the government is given by

$$SW(x_0, \sum_{i=1}^n x_i, \lambda) = U - p(x_0 + \sum_{i=1}^n x_i) + \sum_{i=1}^n (\pi_i + u_i) + u_0 + (1 + \lambda)\pi_0, \quad (2)$$

where λ represents the unit of excess burden⁸, $U - p(x_0 + \sum_{i=1}^n x_i)$ is consumer surplus, each firm π_0 and π_i is the profit of the public and private firms, and u_j

⁷As Ishida and Matsushima (2009) and Barcena-Ruiz and Garzon (2009) have suggested, this is because wage claims are decided by the elasticity of labor demand rather than the firm's profit. As a special case of the Nash bargaining solution, the monopoly union model is frequently adopted; see also Oswald and Turnbull (1985). On the other hand, adopting the asymmetric Nash bargaining, De Fraja (1993b) assumed that the public firm maximizes the weight of sum consumer surplus, profit and the union's utility. The weight attached to the union's utility assumed to be exogenously given. However, it is hard to obtain explicit solution due to the asymmetric Nash bargaining under the unionized mixed oligopoly.

⁸Usually, it is assumed that $\lambda > 0$. Since λ is a measure of the distortion by taxation, we may be comfortable assuming that $\lambda \in (0, 1]$ which reflects more reality. Ballard *et al.* (1985) and Snower and Warren (1996) reported that the shadow cost of public funds is generally assessed to be around 0.3 in industrial countries. However, Capuano and De Feo (2010) assume that λ has some higher-bound restriction.

is the union's utility of both the public and private firms. Moreover, the welfare can be expressed as a weighted average of welfare defined at the net surplus generated in the market and the public firm's profit,

$$\frac{SW(x_0, \sum_{i=1}^n x_i, \lambda)}{1 + \lambda} = \frac{1}{1 + \lambda} SW(x_0, \sum_{i=1}^n x_i, 0) + \frac{\lambda}{1 + \lambda} \pi_0,$$

where $SW(x_0, \sum_{i=1}^n x_i, 0) = U - p(x_0 + \sum_{i=1}^n x_i) + \sum_{i=1}^n (\pi_i + u_i) + u_0 + \pi_0$ and the former is weighted by $\frac{1}{1+\lambda}$, the latter by $\frac{\lambda}{1+\lambda}$. In this paper, the weight of the public firm's profit is exogenously correlated to the shadow costs of public funds (see also Matsumura, 1998; Matsumura and Tomaru, 2013).

Timing of the three-stage game is as follows. In the first stage, the government chooses whether or not to privatize the public firm. In the second stage, each firm's union negotiates over wages. In the third stage, each firm simultaneously chooses its quantity to maximize its respective objective knowing each union's choice of the wage level.

3. THE MARKET EQUILIBRIUM

Before comparing social welfare under the unionized mixed oligopoly with social welfare under the privatization, we first consider all firms' maximization problems. In this paper, since all unions are allowed to bargain collectively, we assume that all firms choose the same type of bargaining (see Ishida and Matsushima, 2009). Thus, the game is solved by backward induction, i.e., the solution concept used is the subgame perfect Nash equilibrium.

3.1. THE UNIONIZED MIXED OLIGOPOLY

In the third stage, given w_j for each firm, the public firm's maximization problem is as follows:

$$\max_{x_0} SW = U + \lambda \pi_0.$$

Given wage level w_j in the third stage, the best-reply functions of the public and private firms are derived, as usual, from the first order condition:

$$\frac{\partial SW}{\partial x_0} = 0 \Leftrightarrow x_0 = \frac{(1 + \lambda)(1 - nx_i) - \lambda w_0}{1 + 2\lambda}, \quad \frac{\partial \pi_i}{\partial x_i} = 0 \Leftrightarrow x_i = \frac{1 - w_i - x_0}{n + 1}. \quad (3)$$

Solving the first-order conditions (3), we obtain⁹,

$$x_0 = \frac{(1+\lambda)(1+nw_i) - \lambda(1+n)w_0}{1+\lambda(n+2)}, \quad x_i = \frac{\lambda(1+w_0) - (1+2\lambda)w_i}{1+\lambda(n+2)}. \quad (4)$$

In the second stage of this case, each wage is set to maximize its firm's union utility: $U_j = x_j w_j$. To do this, the two independent maximization problems should be considered simultaneously. Using (4), the problem for union $j = 0, 1, \dots, n$ is defined as

$$\begin{aligned} \max_{w_0} u_0 &= \frac{w_0[(1+\lambda)(1+nw_i) - \lambda(1+n)w_0]}{1+\lambda(n+2)}, \\ \max_{w_i} u_i &= \frac{w_i[\lambda(1+w_0) - (1+2\lambda)w_i]}{1+\lambda(n+2)}. \end{aligned}$$

Solving first-order conditions of unions, the best reply functions can be written as follows:

$$w_i = \frac{\lambda}{2(1+2\lambda)} + \frac{\lambda}{2(1+2\lambda)} w_0, \quad w_0 = \frac{1+\lambda}{2\lambda(1+n)} + \frac{(1+\lambda)n}{2\lambda(1+n)} w_i.$$

Note that wages in the case of substitutes are strategic 'complements' with up-sloping best response functions from the terminology of Bulow *et al.* (1986)¹⁰. Moreover, comparing the slope of each best reply function yields

$$\text{"the slope" of } w_0 - \text{"the slope" of } w_i = -\lambda^2 + 3n\lambda + n.$$

Applying to a discriminant, we obtain one root, $\lambda^* \equiv \frac{3n+\sqrt{9n^2+4n}}{2} > 0$. Thus, if $\lambda^* > \lambda$, then the slope of w_i is smaller than the slope of w_0 . Otherwise, the slope of w_i is greater than the slope of w_0 if $\lambda^* < \lambda$. These relationships imply that the public firm's reaction function is flatter than the private firms' one if $\lambda^* < \lambda$. Otherwise, the public firm's reaction function is steeper than the private firms' one if $\lambda^* > \lambda$. This suggests that if $\lambda^* < \lambda$, in the public firm, the increase in the private firm's wage results in a greater increase in output than in private firms and vice versa if $\lambda^* > \lambda$.

Solving these problems and noting that superscript "*m*" denotes the optimal solution in a unionized mixed oligopoly, we have the following result.

⁹Some readers may argue that the unions have a common strange conjecture. The detailed computations are available from authors upon request.

¹⁰For more detailed-explanation, see De Fraja (1993a, pp. 463-464).

Lemma 1: Suppose that $\lambda > 0$. Then, the equilibrium wage, output, union's utility, the profit of private firms and social welfare are given by

$$\begin{aligned}
w_0^m &= \frac{(1+\lambda)[2+\lambda(4+n)]}{\lambda[4+3n+\lambda(8+7n)]}, & x_0^m &= \frac{(1+\lambda)(1+n)[2+\lambda(4+n)]}{[1+\lambda(2+n)][4+3n+\lambda(8+7n)]}; \\
w_i^m &= \frac{1+\lambda(3+2n)}{4+3n+\lambda(8+7n)}, & x_i^m &= \frac{(1+2\lambda)[1+\lambda(3+2n)]}{[1+\lambda(2+n)][4+3n+\lambda(8+7n)]}; \\
u_0^m &= \frac{(1+\lambda)^2(1+n)[2+\lambda(4+n)]^2}{\lambda[1+\lambda(2+n)][4+3n+\lambda(8+7n)]^2}; \\
u_i^m &= \frac{(1+2\lambda)[1+\lambda(3+2n)]^2}{[1+\lambda(2+n)][4+3n+\lambda(8+7n)]^2}; \\
\pi_0^m &= \frac{(1+\lambda)(1+n)[2+\lambda(4+n)]^2[\lambda^2(1+n)-\lambda(2+n)-1]}{\lambda[1+\lambda(2+n)]^2[4+3n+\lambda(8+7n)]^2}; \\
\pi_i^m &= \frac{(1+2\lambda)^2[1+\lambda(3+2n)]^2}{[1+\lambda(2+n)]^2[4+3n+\lambda(8+7n)]^2}; \\
SW^m &= \frac{(1+2\lambda)(4+16n+9n^2+A\lambda+B\lambda^2+C\lambda^3+D\lambda^4)}{2[1+\lambda(n+2)]^2[4+3n+\lambda(8+7n)]^2},
\end{aligned}$$

where $A = 24 + 108n + 92n^2 + 18n^3$, $B = 52 + 248n + 274n^2 + 98n^3 + 9n^4$;

$C = 48 + 212n + 268n^2 + 130n^3 + 22n^4$ and $D = 16 + 40n + 33n^2 + 10n^3 + n^4$.

3.2. THE UNIONIZED PRIVATIZED OLIGOPOLY

The previous subsection examined the impact of a unionized mixed oligopoly. This subsection compares the equilibrium of a unionized mixed oligopoly, which would be established in the case of a unionized privatized oligopoly. As discussed in the basic model, consider the situation of a privatized oligopoly for a homogeneous good that is supplied by firm $(k = 1, 2, \dots, n+1)$, which is a profit-maximizing private firm given that a new inverse demand is characterized by $p = 1 - \sum_{k=1}^{n+1} x_k : n \geq 1$. After privatization, as in Matsumura and Tomaru (2013) and Wang and Chen (2011), social welfare is given by

$$SW = U - px_0 - \sum_{k=1}^n px_k + (1+\lambda)R + u_0 + (\pi_0 - R) + \sum_{k=1}^n (\pi_k + u_k), \quad (5)$$

where R is the revenue from selling the stocks of public firm 0, u_0 is the union under selling the stocks of public firm 0 (i.e., privatized firm), and $U - px_0 - \sum_{k=1}^n px_k$ is consumer surplus. As Matsumura and Tomaru (2013) and Wang and

Chen (2011) assumed, we consider the case in where $R = \pi_0 = \pi_{n+1}$ since the financial market is complete¹¹. Thus, $SW = U + \lambda \pi_{n+1} = U + \lambda \pi_k$.

In the third stage, the firm k 's profit-maximization problem is to maximize $\pi_k = (p - w_k)x_k$. Hence, solving across the $n + 1$ first-order conditions, the $n + 1$ best reply functions can be rewritten as follows:

$$x_k = \frac{1 - (n+1)w_k + nw_l}{2+n}, \quad k \neq l; k, l = 1, 2, \dots, n+1. \quad (6)$$

In the second stage, given the output as a function of wage, each union at each firm sets the wage, w_k , that maximizes union rent, u_k . Thus, similar to the unionized mixed oligopoly in previous section, we have the following result as given, superscript “*” denotes the optimal solution under the unionized privatized oligopoly:

Lemma 2: *Suppose that $\lambda > 0$. Then, the equilibrium wage, output, union's utility, social welfare and the profit of private firms are given by*

$$\begin{aligned} w_k^* &= \frac{1}{2+n}, \quad x_k^* = \frac{1+n}{(2+n)^2}, \quad u_k^* = \frac{1+n}{(2+n)^3}, \\ SW^* &= \frac{(1+n)^2(7+6n+n^2+2\lambda)}{2(2+n)^4}, \quad \pi_k^* = \frac{(1+n)^2}{(2+n)^4}. \end{aligned}$$

4. COMPARISONS OF EQUILIBRIUM OUTCOMES

Having derived the market equilibrium for the fixed situation in the previous section, we will find the Nash equilibrium in the first stage for any given set of utilities of the unions and the profits of firms in both unionized mixed and privatized oligopolies.

First, we observe $SW^m \rightarrow 0$ as $\lambda \rightarrow 0$ from Lemma 1. Note that if we assume $\lambda = 0$, there does not exist any subgame perfect Nash equilibrium since w_0 approaches infinity. Thus, we focus on the neighborhood of $\lambda = 0$. We summarize this result in Result 1.

Result 1: *Suppose that $\lambda \rightarrow 0$. Then, $SW^m < SW^*$.*

Result 1 indicates that regardless of the number of existing firms, social welfare under privatization is greater than under that the unionized mixed oligopoly.

¹¹ As Capuano and De Feo (2010) pointed out, we give full bargaining power to the government, i.e., it is able to extract the whole profit from the privatized firm.

This implies that if the public firm's union aims to maximize the wage level and does not face some budget constraint with a simple union function $u_j = w_j x_j$, the public firm's union can raise its wage without limit because the optimal output level of the public firm is independent of the wage (see Ishida and Matsushima, 2009).

We now present our main proposition as follows.

Proposition 1: *Suppose that $\lambda \in (0, 1]$. Then, $SW^m < SW^*$.*

Proof: See A-1 and Figure 1 in the appendix for more detailed calculations. Comparing the mixed oligopoly with privatization when $\lambda \in (0, 1]$, straightforward computations show that

$$SW^m - SW^* = \frac{-48 - 104n - 111n^2 - 84n^3 - 40n^4 - 8n^5 - E\lambda - F\lambda^2 - G\lambda^3 - H\lambda^4 - I\lambda^5}{2(2+n)^4[1+\lambda(2+n)]^2[4+3n+\lambda(8+7n)]^2} < 0. \quad (7)$$

where $E = 416 + 1032n + 1220n^2 + 970n^3 + 514n^4 + 148n^5 + 16n^6$;

$F = 1344 + 3744n + 4871n^2 + 4128n^3 + 2403n^4 + 864n^5 + 159n^6 + 10n^7$;

$G = 1920 + 5952n + 8494n^2 + 7682n^3 + 4818n^4 + 2016n^5 + 502n^6 + 60n^7 + 2n^8$;

$H = 1024 + 3584n + 5668n^2 + 5392n^3 + 3411n^4 + 1480n^5 + 427n^6 + 70n^7 + 4n^8$;

$I = 128n + 360n^2 + 248n^3 - 182n^4 - 340n^5 - 176n^6 - 36n^7 - 2n^8$. Q.E.D.

Proposition 1 suggests that regardless of the number of private firms, the privatization of a public firm is always desirable from a welfare point of view when $\lambda \in (0, 1]$. This proposition is different from the corresponding result of De Fraja and Delbono (1989), who found that the privatization of the public firm is desirable in terms of social welfare when the number of private firms is large, and is not desirable when the number of private firms is small.

The following is the intuition behind Proposition 1. When comparing welfare before and after the change in the public firm's objective function, the profits of the public firm and the privatized firm do matter—that is, when comparing the social welfare of the reduced form $SW^m = U^m + \lambda\pi_0^m$ under the mixed oligopoly with that of the reduced form, $SW^* = U^* + \lambda\pi_k^*$ under privatization. The government faces two effects when it privatizes its public firm. First, the excess burden of taxation forces the public firm's profit to be negative¹² when $\lambda \in (0, 1]$, while

¹²The calculation of the profit of the public firm is given by Lemma 1. However, when λ is

the privatized firm's profit is independent of the excess burden of taxation, which is a positive welfare effect of privatization. We call this effect the *excess burden effect*. Second, the representative consumer's utility may be higher under the mixed oligopoly than under privatization when $\lambda \in (0, 1]$. This is a positive welfare effect of nationalization¹³, which we call the *consumer effect*. The excess burden effect reduces the public firm's profit when $\lambda \in (0, 1]$, and increases the representative consumer's utility as λ increases under privatization given the number of firms. Privatization, in turn, induces production substitution from the public firm to the private firms. The latter two effects increase welfare whereas the first effect reduces welfare. This explanation implies that the consumer effect plus the excess burden effect under privatization dominates the effects obtained under the mixed oligopoly. That is, the excess burden effect is weaker because of the negative profit of the public firm, while the consumer effect is stronger when λ is larger, given the number of firms.

For this reason, the role of unionization can be understood as follows. The government uses the total wage as a commitment device to control the unions' wage demands to maintain lower total wage levels under privatization¹⁴. Thus, lower total wages under privatization work to improve welfare by increasing total output. This may lead to more output under privatization than under a unionized mixed oligopoly when $\lambda \in (0, 1]$, given that the number of existing firms increases.

However, we should not overemphasize the result that privatization increases welfare. Privatization can be harmful in alternative model formulations. For example, if the public firm faces some budget constraints (an issue ignored in this paper) with a simple union function when the union aims to maximize the wage level, the obtained welfare gain can dominate the welfare gain discussed above with the excess burden of taxation.

On the other hand, the welfare ranking may be reversed when λ is sufficiently large (i.e., when $\lambda > 1$). Thus, we have the following proposition.

sufficiently large (i.e., $\lambda > 1$), its profit may be either positive or negative depending on the critical value of λ . See A-2 in the appendix for more detailed calculations.

¹³See A-3 and A-4 in the appendix for comparisons of the representative consumer's utility and total output, respectively. Precisely speaking, there can exist a critical value of λ^U such that for all $\lambda > (\text{resp. } <) \lambda^U$ given $\lambda \in (0, 1]$, we obtain the difference as $U^m < (\text{resp. } >) U^*$ except for $n = 1$.

¹⁴When comparing total wage, we obtain $(nw_i^m + w_0^m) - (n+1)w_k^* = 4 + 2n + \lambda(16 + 7n - n^2) + \lambda^2(16 + 3n - n^2 + 2n^3) - \lambda^3(6n - 2n^2 - 4n^3)$. On the other hand, $w_i^m - w_k^* = -(1+n) - \lambda(3 + 2n - n^2) - \lambda^2(2 - 2n^2)$. As stated already in the best reply functions of unions, if λ is sufficiently small, the public firm's reaction function is steeper than the private firms' reaction function, and vice versa if λ is sufficiently large.

Proposition 2: *Suppose that $\lambda > 1$. Then,*

- (i) *there can exist a critical value of λ^\dagger such that for all $\lambda \geq \lambda^\dagger$ and $n \geq 2$, we obtain the difference as $SW^m \geq SW^*$ and for all $\lambda < \lambda^\dagger$ and $n \geq 2$, as $SW^m < SW^*$.*
- (ii) *when $n = 1$, we obtain the difference as $SW^m < SW^*$ regardless of λ .*

Proof: See the appendix.

Q.E.D.

Proposition 2 suggests that depending on the range of the excess burden of taxation, with $\lambda > 1$ and $n > 1$, social welfare is determined under either the mixed oligopoly or privatization¹⁵.

Recalling the welfare of the reduced form— $SW^m = U^m + \lambda \pi_0^m$ under the mixed oligopoly and $SW^* = U^* + \lambda \pi_k^*$ under privatization—the welfare rankings in Proposition 2 become intuitive. When $\lambda > 1$, the improvement of social welfare under the mixed oligopoly is possible since the excess burden effect dominates the consumer effect under privatization when $\lambda \geq \lambda^\dagger$ and $n \geq 2$. This implies that if the degree of the excess burden of taxation is smaller than that of its critical value, the total output level under the mixed oligopoly may be smaller than that under privatization, and vice versa (see the appendix on the public firm's profit and the comparison of total output). In other words, given the number of firms, desirable privatization and nationalization both occur depending on the excess burden of taxation. If $\lambda < \lambda^\dagger$, privatization stimulates total output, and hence, the consumer effect under privatization starts to become stronger; however, with $\lambda > \lambda^\dagger$, it is dominated by an excess burden effect, and the inverse occurs if the degree of the excess burden of taxation becomes small with a small number of firms¹⁶. Consequently, we obtain Proposition 2 depending on λ^\dagger . Whether or not privatization improves welfare depends on which effect is stronger. As pointed out in Proposition 1, higher wages are still maintained under the mixed oligopoly when $\lambda > \lambda^\dagger$, whereas the excess burden effect starts to dominate the consumer effect. Maintaining a higher wage reduces welfare since wages are strategic complements between the unions, whereas the excess burden effect improves welfare even with a higher wages level under mixed oligopoly, and the inverse occurs if $\lambda < \lambda^\dagger$. This result means that when $\lambda > \lambda^\dagger$, the government may not use the total wage as a commitment device to control the unions' wage demands to maintain lower total wage levels under the mixed

¹⁵For simplicity, we exclude the case of $n = 1$ in order to analyze the effect of multiple private firms in oligopoly market structure.

¹⁶If we adopt the increasing marginal costs without union, it would similar results to proposition 2, except for the case, $n = 1, 2$. The detailed computations are available from author upon request.

oligopoly. This is because the excess burden effect under the mixed oligopoly dominates the consumer effect under privatization. However, when $\lambda < \lambda^\dagger$, it can use the total wage as a commitment device to set a lower wage level under privatization, which also works to improve welfare under privatization by increasing the total output.

In sum, Propositions 1 and 2 contrast sharply with the existing literature, which shows that the privatization of a public firm is desirable from a welfare point of view with a relatively large number of firms (De Fraja and Delbono, 1989). With both the excess burden of taxation and firms' endogenous cost via wage bargaining, the intuition of Propositions 1 and 2 relies on the different effects exerted by the representative consumer's utility and the profit of firms.

Comparing privatization with a mixed duopoly, Capuano and De Feo (2010) demonstrated that with no or with large efficiency gains, an inefficient public firm that maximizes welfare may still be preferred when there exists the excess burden of taxation in the government's objective function, which differs from our results. Their study does not consider the situation in which the public firm competes with multiple private firms and unionized firms; hence, the result that the mixed oligopoly reduces welfare does not hold even with the excess burden of taxation. Further, Matsumura and Tomaru (2015) showed that when the λ is high, privatizing the public firm improves welfare, while our results do hold when the λ is relatively small.

5. CONCLUDING REMARKS

By introducing the excess burden of taxation into a theoretical framework of unionized mixed and privatized oligopolies, this study provides new insights on a government's policy on privatization. When $\lambda \in (0, 1]$, privatization matters regardless of the number of firms. However, we show that whether privatization or nationalization is desirable depends on the critical value of the excess burden of taxation when $\lambda > 1$. In this paper, we suggest that under mild conditions, privatization can be used as a powerful instrument to reduce distortionary taxation. However, we show that an inefficient public firm may be preferred even when a large inefficiency exists.

We conclude by discussing the limitations of our paper. We have used a simple linear demand structure with the mixed oligopoly. It is worth to consider our results in the nonlinear demand structure. Further, we have used the simplifying assumption that private and public firms are symmetric because of a decentralized unionization structure in the monopoly union model. By making

this assumption, we do not take into account any cost difference that may arise from the mixed bargaining that occurs between private and public firms. Moreover, in this paper, it is assumed that the public firm is as efficient as the private firm with endogenous input costs (i.e., wages). If the cost between the public and private firms is characterized by increasing and decreasing returns to scale, privatization may reduce or improve welfare with different degrees of the excess burden of taxation. Finally, we have not extended the model to consider a situation in which the public firm competes with both domestic and foreign private firms. The extension of our model in these directions is left for future research.

REFERENCES

- Bahrami, B., J. D. Bitzan, and J. A. Leitch, (2009) Union Worker Wage Effect in the Public Sector, *Journal of Labor Research* 30, 35-51.
- Ballard, C., J. B. Shoven and J. Whalley, (1985) General Equilibrium Computations of the Marginal Welfare Costs of Taxation in the United States, *American Economic Review* 75, 128-138.
- Barcena-Ruiz, J. C. and M. B. Garzon, (2009) Relocation and Public Ownership of Firms, *Journal of the Japanese and International Economies* 23, 71-85.
- Beato, P. and A. Mas-Colell, (1984) "Marginal Cost Pricing as a Regulation Mechanism in Mixed Markets," in M. Marchand, P. Pestieu and H. Tulkens, (eds.), *The Performance of Public Enterprises*, pp. 81-101, North-Holland, Amsterdam.
- Booth, A. (1995) *The Economics of the Trade Union*, Cambridge, Cambridge University Press.
- Bordogna, L. (2003) "The Reform of Public Firm Employment Relations in Industrialized Democracy," In Brock, J. and Lipsy, D. B. (eds), *Going Public: The Role of Labor-Management Relations in Delivering Quality Government Services*, Industrial Relations Research Association, pp. 23-67.
- Bos, D. (1991) *Privatization: A Theoretical Treatment*, Oxford University Press.
- Bulow, J. I., J. D. Geanakoplos and P. D. Klemperer, (1985) Multimarket Oligopoly: Strategic Substitutes and Complements, *Journal of Political Economy* 93, 488-511.
- Capuano, C. and G. De Feo, (2010) Privatization in Oligopoly: the Impact of the Shadow Cost of Public Funds, *Rivista Italiana Degli Economisti* 15, 175-208.
- Choi, K. (2011) Unions, Government's Preference, and Privatization, *Economic Modelling* 28, 2502-2508.
- De Fraja, G. (1993a) Unions and Wages in Public and Private Firms: A Game Theoretic Analysis, *Oxford Economic Papers* 45, 457-469.
- De Fraja, G. (1993b) Productive Efficiency in Public and Private Firms, *Journal of Public Economics* 50, 15-30.

- De Fraja, G. and F. Delbono, (1989) Alternative Strategies of a Public Enterprise in Oligopoly, *Oxford Economic Papers* 41, 302-311.
- Du Caju, P., E. Gautier, D. Momferatu and M. Ward-Warmedinger, (2009) Institutional Features of Wage Bargaining in 23 European Countries, the US and Japan, *Ekonomia* 12, 57-108.
- Haskel, J. and A. Sanchis, (1995) Privatization and X-Inefficiency: A Bargaining Approach, *Journal of Industrial Economics* 43, 301-321.
- Haskel, J. and S. Szymanski, (1993) Privatization, Liberalization, Wages and Employment: Theory and Evidence for the UK, *Economica* 60, 161-182.
- Ishida, J. and N. Matsushima, (2009) Should Civil Servants be Restricted in Wage Bargaining? A Mixed-Duopoly Approach, *Journal of Public Economics* 93, 634-646.
- Kato, H. (2008) Privatization and Government Preference, *Economics Bulletin* 12, 1-7.
- Kato, K. and Y. Tomaru, (2007) Mixed oligopoly, Privatization, Subsidization and the Order of Firms' Moves: Several Types of Objectives, *Economics Letters* 96, 287-292.
- Laffont, J. J. and J. Tirole, (1993) *A Theory of Incentives in Procurement and Regulation*, Cambridge, MA: MIT Press.
- Lewin, D. (1977) Public Sector Labor Relations, *Labor History* 18, 133-144.
- Matsumura, T. (1998) Partial Privatization in Mixed Duopoly, *Journal of Public Economics* 70, 473-483.
- Matsumura, T. and Y. Tomaru, (2015) Mixed Duopoly, Location Choice, and Shadow Cost of Public Funds, *Southern Economic Journal* 82, 416-429.
- Matsumura, T. and Y. Tomaru, (2013) Mixed Duopoly, Privatization and Subsidization with Excess Taxation Burden, *Canadian Journal of Economics* 46, 526-554.
- Meade, J. E. (1944) Price and Output Policy of State Enterprise, *Economic Journal* 54, 321-339.

- Oswald, A. J. and P. Turnbull, (1985) Pay and Employment Determination in Britain: What are Labour Contracts Really Like, *Oxford Review of Economic Policy* 1, 80-97.
- Saha, B, and R. Sensarma, (2008) Distributive Role of Managerial Incentives in a Mixed Duopoly, *Economics Bulletin* 12, 1-10.
- Snowder, A. and R. Warren, (1996) The Marginal Welfare Cost of Public Funds: Theory and Estimates, *Journal of Public Economics* 61, 289-305.
- Vickers, J. and G. Yarrow, (1988) *Privatization: An Economic Analysis*, MIT Press, Cambridge.
- Wang, L. F. S. and T. Chen, (2011) Privatization, Efficiency Gap, and Subsidization with Excess Taxation Burden, *Hitotsubashi Journal of Economics* 52, 55-68.
- Willner, J. (2006) A Mixed Oligopoly Where Private Firms Survive Welfare Maximization, *Journal of Industry, Competition and Trade* 6, 235-252.

APPENDIX

A-1. Proofs of Propositions 1 and 2

To draw real roots with quintic equation of $SW^m - SW^* \equiv \Delta SW$ from (7) in main text and escape the complicated calculations, using Mathematica (Wolfram, 1999) yields one positive root (say λ^\dagger), two different negative roots and no real two roots. The plots in Figure 1 are obtained using Mathematica (Wolfram, 1999) with one real positive root¹⁷. Thus, when $\lambda \in (0, 1] < \lambda^\dagger$, the comparison of $SW^m - SW^*$ over the parameter space $\{\lambda, n\}$ is drawn in Figure 1(a) and Figure 1(c). We can easily complete the proof of Proposition 1, $SW^m < SW^*$.

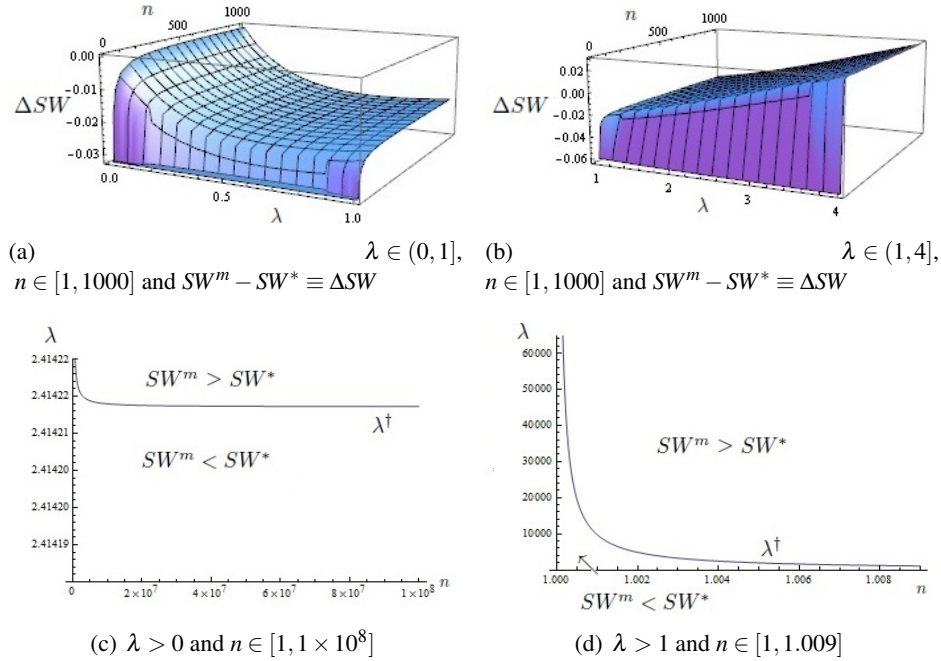


Figure 1: The Comparison of $SW^m - SW^*$

¹⁷We can provide only one locus λ^\dagger due to the positive value of root of λ^\dagger . That is, using the jargons of Mathematica, λ^\dagger is given by $\lambda^\dagger \equiv \text{Root}[-48 - 104n - 111n^2 - 84n^3 - 40n^4 - 8n^5 - (416 + 1032n + 1220n^2 + 970n^3 + 514n^4 + 148n^5 + 16n^6)\#1 - (1344 + 3744n + 4871n^2 + 4128n^3 + 2403n^4 + 864n^5 + 159n^6 + 10n^7)\#1^2 - (1920 + 5952n + 8494n^2 + 7682n^3 + 4818n^4 + 2016n^5 + 502n^6 + 60n^7 + 2n^8)\#1^3 - (1024 + 3584n + 5668n^2 + 5392n^3 + 3411n^4 + 1480n^5 + 427n^6 + 70n^7 + 4n^8)\#1^4 - (128n + 360n^2 - 248n^3 - 182n^4 - 340n^5 - 176n^6 - 36n^7 - 2n^8)\#1^5 \&, 3]$.

On the other hand, when $\lambda > 1$, there can exist a critical value of λ^\dagger such that for all $\lambda > \lambda^\dagger$ and $n \geq 2$, we obtain the difference as $SW^m > SW^*$ and for all $\lambda < \lambda^\dagger$ and $n \geq 2$, as $SW^m < SW^*$ (see Figure 1(c) that comes from Figure (b)). Moreover, when $\lambda > 1$ and n approximately approaches to 1, the comparison of $SW^m - SW^*$ over the parameter space $\{\lambda, n\}$ is drawn in Figure 1(d). Given the condition, $\lambda > 1$ and $n = 1$, we obtain $SW^m < SW^*$ when n approximately approaches to 1.

A-2. Comparison of Firm's Profit

$$\begin{aligned} \lambda \pi_0^m - \lambda \pi_k^* = & -64 - 192n - 224n^2 - 128n^3 - 36n^4 - 4n^5 \\ & - \lambda(720 + 2296n + 2921n^2 + 1892n^3 + 661n^4 + 116n^5 + 8n^6) \\ & - \lambda^2(3328 + 11208n + 15300n^2 + 10990n^3 + 4418n^4 + 986n^5 + 113n^6 + 5n^7) \\ & - \lambda^3(8064 + 28528n + 41481n^2 + 32090n^3 + 14246n^4 + 2978n^5 + 563n^6 + 40n^7 + n^8) \\ & - \lambda^4(10752 + 39808n + 60968n^2 + 49914n^3 + 23488n^4 + 6490n^5 + 1047n^6 + 97n^7 + 4n^8) \\ & - \lambda^5(7424 + 28800n + 46168n^2 + 39116n^3 + 18517n^4 + 4790n^5 + 664n^6 + 50n^7 + 3n^8) \\ & - \lambda^6(2048 + 8576n + 24528n^2 + 12248n^3 + 4828n^4 + 320n^5 + 340n^6 - 84n^7 - 4n^8) \\ & - \lambda^7(256n + 720n^2 + 496n^3 - 364n^4 - 680n^5 - 352n^6 - 72n^7 - 4n^8). \end{aligned}$$

As in A-1, $\lambda \pi_0^m - \lambda \pi_k^*$ has similar results when $\lambda > 0$.

A-3. Comparison of Total Output

$$\begin{aligned} X^m(\equiv nx_i^m + x_0^m) - X^*(\equiv (1+n)x_k^*) = & 4 + 9n + 4n^2 + \lambda(16 + 41n + 21n^2 + n^3) \\ & + \lambda^2(16 + 52n + 27n^2 - 3n^3 - 2n^4) + \lambda^3(12n + 2n^2 - 10n^3 - 4n^4). \end{aligned}$$

A-4. Comparison of Representative Consumer's Utility

$$\begin{aligned} U^m - U^* = & 80 + 280n + 337n^2 + 172n^3 + 32n^4 \\ & + \lambda(1372 + 3144n + 4718n^2 + 2490n^3 + 648n^4 + 52n^5) \\ & + \lambda^2(3456 + 14208n + 20465n^2 + 13636n^3 + 4217n^4 + 304n^5 + 3n^6) \\ & + \lambda^3(7168 + 32000n + 50104n^2 + 35964n^3 + 11686n^4 + 1036n^5 - 210n^6 - 30n^7) \\ & + \lambda^4(7424 + 37248n + 62808n^2 + 47196n^3 + 14781n^4 - 20n^5 - 977n^6 - 156n^7 - 4n^8) \\ & + \lambda^5(3072 + 19328n + 38327n^2 + 14628n^3 + 4320n^4 - 17096n^5 - 2252n^6 - 3296n^7 - 16n^8) \\ & + \lambda^6(2304n + 5016n^2 + 1936n^3 - 3260n^4 - 3920n^5 - 1684n^6 - 384n^7 - 12n^8). \end{aligned}$$