Journal of Economic Theory and Econometrics, Vol. 27, No. 1, Mar. 2016, 48–81

Optimal Monetary Policy and Limited Enforcement in a Small Open Economy

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Abstract This paper analyzes the optimal monetary policy problem on the basis of a small open New Keynesian dynamic stochastic general equilibrium (DSGE) model with enforcement constraints, where limited enforcement and limited spanning interact to create an endogenous debt limit. As a result, if there are fiscal policy measures to eliminate steady-state distortions, the inflation targeting regime is optimal even when there are enforcement constraints.

Keywords Limited enforcement; Optimal monetary policy; New Keynesian model; Small open economy.

JEL Classification D5, D52, E52, F34.

*This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2014S1A3A2044637). E-mail address: equilibrium@snu.ac.kr

Received December 05, 2015, Revised February 29, 2016, Accepted March 11, 2016

1. INTRODUCTION

This paper incorporates limited enforcement constraints into a New Keynesian small open DSGE model to analyze optimal monetary policy. In this paper, similar to the studies of Kehoe and Perri (2002), and Bai and Zhang (2010), enforcement constraints require that, at every point in time, the continuation utility be equal to or greater than the financial autarky. As such, imperfect risk sharing takes place endogenously because of the limited ability to enforce international credit arrangements between sovereign nations.

Recent literature studies enforcement constraints in open economies. For example, Kehoe and Perri (2002) include limited enforcement constraints into their two country and one-good model to explain that cross-country correlations for consumption are lower than those for output, and cross-country correlations of employment and investment are positive. Through interaction of limited enforcement and limited spanning to create an endogenous debt limit in a small open economy, Bai and Zhang (2010) solve the Feldstein-Horioka puzzle that shows a highly correlated relation of long-term average savings and investment rates across countries. The difference between the two papers in terms of enforcement constraints is that Kehoe and Perri (2002) has current value constraints, while Bai and Zhang (2010) uses continuation value constraints. To explain that the volatility of the real exchange rate is much higher than the volatility of consumption and the real exchange rate is negatively correlated with the ratio of domestic over foreign consumption (Backus-Smith puzzle), Bodenstein (2008) introduces enforcement constraints similar to Kehoe and Perri (2002), but unlike them, the goods market consists of two tradables and the non-tradable good.

As opposed to existing analyses based on a real business cycle model, this paper characterizes optimal monetary policy in a small open New Keynesian DSGE framework, in which prices are set following the Calvo-Yun price staggering model. When steady-state distortions can be eliminated through fiscal policy measures such as lump-sum subsidy or tax to households or firms, this paper analyzes enforcement effects on the central banks optimal policy decision. The results show that the inflation targeting regime is optimal even when there are enforcement constraints.

The remainder of the paper is organized as follows: section 2 introduces the canonical New Keynesian model in a small open economy and enforcement constraints; section 3 analyzes optimal monetary policy in a complete market and bond model with enforcement constraints; and section 4 concludes the study.

2. THE MODEL

This section first describes a small-open economy model modified from a two country model, following Gali and Monacelli (2005), De Paoli (2009), and Farhi and Werning (2013a, 2013b). Price adjustment of a firm producing a differentiated good under monopolistic competition is based on the Calvo-Yun model. Following Kehoe and Perri (2002), enforcement constraints are introduced to generate endogenous imperfect risk sharing.

2.1. HOUSEHOLDS

There are two countries, H (Home) and F (Foreign). In this scenario, a fraction of agents $[0,\ n)$ of unit mass belongs to country H and the other fraction $(n,\ \ 1]$ belongs to country F and a continuum of differentiated goods exists. Each type of these tradable goods is produced by either country H or country F, while each country produces a number of different brands with measure equal to population size.

The preferences of the representative household at period 0 in country H are represented by the following function:

$$E_0 \sum\nolimits_{t=0}^{\infty} \beta^t U(C_t, H_t) \equiv E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \nu \frac{H_t^{1+\chi}}{1+\chi} \right], \tag{1}$$

where C_t is the aggregate consumption index at period t, H_t is the number of hours worked during period t, β is the time discount factor, σ is the inverse of intertemporal elasticity of consumption, χ is the inverse of elasticity of labor supply and ν is the weight on leisure in the utility function. The aggregate consumption index in the utility functions of home and foreign residents is defined as

$$C_t = \left[a_H^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-a_H)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

and

$$C_t^* = \left[a_H^{*\,\frac{1}{\theta}} (C_{H,t}^*)^{\frac{\theta-1}{\theta}} + (1-a_H^*)^{\frac{1}{\theta}} (C_{F,t}^*)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where the parameter $\theta > 0$ is the intratemporal elasticity of substitution between $C_{H,t}(C_{H,t}^*)$ and $C_{F,t}(C_{F,t}^*)$, which are the subindices of home(foreign) consumption of domestic and foreign goods. As in Sutherland (2005) and De Paoli

(2009), the parameters $a_H = 1-(1-n)\lambda$ and $a_H^* = n\lambda$ are the share of home goods in the consumption index of home and foreign residents. Here, the parameter λ measures the degree of openness, and the small open economy can be derived by taking the limit for $n \to 0$. Hence, in the small open economy, it can be represented that $a_H = 1-\lambda$ and $a_H^* = 0$.

The subindices $C_{H,t}(C_{H,t}^*)$ and $C_{F,t}(C_{F,t}^*)$ are defined as

$$C_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n C_{H,t}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}_{,} \quad C_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 C_{F,t}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}_{,}$$

$$C_{H,t}^* = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n C_{H,t}^*(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}_{,} \quad C_{F,t}^* = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_n^1 C_{F,t}^*(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}_{,}$$

where $\varepsilon > 1$ is the elasticity of substitution across the differentiated products, $C_{H,t}(z)(C_{H,t}^*(z))$ is the home(foreign) consumption of the differentiated good produced by domestic firm z and $C_{F,t}(z)(C_{F,t}^*(z))$ is the home(foreign) consumption of differentiated good produced by foreign firm z.

The solutions to the following cost minimization problem yield the set of demand functions for domestic and foreign differentiated goods of home residents:

$$\min_{\{C_{H,t}(z)\}} \int_0^n P_{H,t}(z) C_{H,t}(z) dz \text{ s.t. } C_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n C_{H,t}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}_{,}$$

$$\min_{\{C_{F,t}(z)\}} \int_n^1 P_{F,t}(z) C_{F,t}(z) dz \text{ s.t. } C_{F,t}^* = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 C_{F,t}^*(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}_{.}$$

Foreign households also solve the same problem as above. The solutions are given by

$$C_{H,t}(z) = \frac{1}{n}\left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}, \quad C_{F,t}(z) = \frac{1}{1-n}\left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t},$$

$$C_{H,t}^*(z) = \frac{1}{n}\left(\frac{P_{H,t}^*(z)}{P_{H,t}^*}\right)^{-\varepsilon} C_{H,t}^*, \quad C_{F,t}^*(z) = \frac{1}{1-n}\left(\frac{P_{F,t}^*(z)}{P_{F,t}^*}\right)^{-\varepsilon} C_{F,t}^*.$$

Additionally, the set of demand functions for domestic and imported goods of both home and foreign residents can be derived by the following cost minimization problems:

$$\min_{\{C_{H,t},C_{F,t}\}} P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \;\text{ s.t. }\; C_t = \left[a_H^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-a_H)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

$$\min_{\{C^*_{H,t},C^*_{F,t}\}} P^*_{H,t}C^*_{H,t} + P^*_{F,t}C^*_{F,t} \;\text{ s.t. }\; C^*_t = \left[a_H^{*\frac{1}{\theta}} C_{H,t}^{*\frac{\theta-1}{\theta}} + (1-a^*_H)^{\frac{1}{\theta}} C_{F,t}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

Consequently, demand functions are given by

$$C_{H,t} = a_H \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, \;\; C_{F,t} = (1-a_H)\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_t,$$

$$C^*_{H,t} = a^*_H \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\theta} C^*_t, \;\; C^*_{F,t} = (1-a^*_H)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\theta} C^*_t.$$

Correspondingly, consumer price index (CPI) of home and foreign residents is given by

$$P_t = \left[a_H P_{H,t}^{1-\theta} + (1-a_H) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{2}$$

and

$$P^*_t = \left[a^*_H P_{H,t}^{*\,1-\theta} + (1-a^*_H) P_{F,t}^{*\,1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{3}$$

where $P_{H,t}(P^*_{H,t})$ and $P_{F,t}(P^*_{F,t})$ are the home(foreign) price subindices of domestic and foreign goods and are given by

$$P_{H,t} = \left[\frac{1}{n}\int_0^n P_{H,t}(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}}, \quad P_{F,t} = \left[\frac{1}{1-n}\int_n^1 P_{F,t}(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}},$$

$$P^*_{H,t} = \left[\frac{1}{n}\int_0^n P^*_{H,t}(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}}, \quad P^*_{F,t} = \left[\frac{1}{1-n}\int_n^1 P^*_{F,t}(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}}.$$

As the law of one price holds, the following equations reflect that:

$$P_{H,t}(h) = S_t P^*_{H,t}(h), \quad P_{F,t}(f) = S_t P^*_{F,t}(f),$$
$$P_{H,t} = S_t P^*_{H,t}, \qquad P_{F,t} = S_t P^*_{F,t},$$

where S_t is the nominal exchange rate. However, due to the home bias, $a_H \neq a^*_H$ generates deviations from the purchasing power parity in the CPI. Therefore, for $P_t \neq S_t P^*_t$, the real exchange rate is defined as

$$Q_t = \frac{S_t P^*_t}{P_t}. \tag{4}$$

The demand function of a firm h in the home country is composed of consumptions from domestic and foreign consumers and the government purchase, that is:

$$Y_t(h) = nC_{H,t}(h) + (1-n)C^*_{H,t}(h) + G_t(h),$$

where $G_t(h)$ is the government purchase for the goods of firm h in the home country. It is assumed that the government only purchases domestic-produced goods.[1] The resulting demand function of firm h is given by

$$Y_t(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\varepsilon} Y_t,$$

where Y_t denotes the aggregate demand at period t:

$$Y_t = P^{-\theta}_{h,t}\left\{a_H C_t + \frac{(1-n)a^*_H}{n} Q^\theta_t C^*_t\right\} + G_t, \tag{5}$$

where $P_{h,t} \equiv \dfrac{P_{H,t}}{P_t}$ and G_t is the government consumption of domestic goods.

2.2. FIRMS

Under monopolistic competition, a firm h in the home country produces a differentiated good and its production function is given by

$$Y_t(h) = A_t H_t(h),$$

[1]The demand function of government for firm h is $G_t(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\varepsilon} G_t$.

where $Y_t(h)$ is the level of output of firm h and $H_t(h)$ is the hours hired by the firm. Assume a prefectly competitive labor market and a completely flexible nominal wage.

Firms set their prices as in the Calvo-Yun price staggering model. A randomly chosen fraction, $(1-\alpha)$, of firms only resets their nominal prices each period, while the remaining fraction, α, of firms keeps their prices unchanged. The profit maximization problem of each firm that resets its price at period t is expressed by

$$\max_{P^*_{H,t}} \quad \sum_{k=0}^{\infty}(\alpha\beta)^k E_t\left[\Lambda_{t+k}\left(\frac{P^*_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{t+k}\left\{\frac{P^*_{H,t}}{P_{H,t+k}}-\frac{W_{t+k}}{A_{t+k}P_{H,t+k}}\right\}\right],$$

where $P^*_{H,t}$ is the optimal reset price at period t. Following Yun (2005), the first-order conditions for this profit maximization can be represented by a set of recursive equations and the detailed derivation is shown in appendix A.1:

$$F_t = C_t^{-\sigma}Y_t + \alpha\beta E_t[\Pi^{\varepsilon-1}_{H,t+1}F_{t+1}], \tag{6}$$

$$L_t = \frac{\nu\varepsilon}{\varepsilon-1}\frac{H_t^{1+\chi}}{\Delta_t P_{h,t}} + \alpha\beta E_t[\Pi^{\varepsilon}_{H,t+1}L_{t+1}], \tag{7}$$

$$\left(\frac{1-\alpha\Pi^{\varepsilon-1}_{H,t}}{1-\alpha}\right)^{\frac{1}{1-\varepsilon}} = \frac{L_t}{F_t}, \tag{8}$$

where F_t is the expected present-value of marginal revenue, L_t is the expected present-value of marginal cost and $\Pi_{H,t} \equiv \dfrac{P_{H,t}}{P_{H,t-1}}$ is the GDP deflator inflation. In addition, as in Yun (2005), the sub-price index of the home differentiated goods can be rewritten by the Calvo-Yun type staggering as follows and the detailed derivation as shown in appendix A.2:

$$P_{H,t} = \left[\alpha P^{1-\sigma}_{H,t-1} + (1-\alpha)P^{*\,1-\sigma}_{H,t}\right]^{\frac{1}{1-\sigma}}. \tag{9}$$

Dividing both sides of (9) by $P_{H,t}$ the following equation is obtained:

$$1 = (1-\alpha)\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{1-\varepsilon} + \alpha\Pi^{\varepsilon-1}_{H,t}, \tag{10}$$

which shows the relationship between the relative price of the optimal reset price and the GDP deflator inflation of domestic goods.

2.3. SOCIAL RESOURCE CONSTRAINTS

Following Yun (2005), the aggregate production function also can be written as

$$Y_t = \frac{A_t H_t}{\Delta_t}, \tag{11}$$

where the relative price distortion at period t, Δ_t is defined as

$$\Delta_t = \frac{1}{n}\int_0^n \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\varepsilon} dh. \tag{12}$$

To yield the relationship between the inflation and the relative price distortion under the Calvo-Yun type staggered price-setting as in Yun (2005), the measure of relative price distortion is represented as follows and the detailed derivation is shown in appendix A.3:

$$\Delta_t = (1-\alpha)\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{-\varepsilon} + \alpha \Pi^{\varepsilon}_{H,t}\Delta_{t-1}. \tag{13}$$

By substituting (10) into (13), one can obtain the evolution equation for the relative price distortion:

$$\Delta_t = (1-\alpha)\left(\frac{1-\alpha\Pi^{\varepsilon-1}_{H,t}}{1-\alpha}\right)^{\frac{1}{1-\varepsilon}} + \alpha \Pi^{\varepsilon}_{H,t}\Delta_{t-1}. \tag{14}$$

Taking the limit for $n \to 0$ in (5), the aggregate demand in the small open economy is

$$Y_t = P^{-\theta}_{h,t}\left\{(1-\lambda)C_t + \lambda Q^{\theta}_t C^*_t\right\} + G_t. \tag{15}$$

From (11) and (15), the social resource constraint at period t is given by

$$\frac{A_t H_t}{\Delta_t} = P^{-\theta}_{h,t}\left\{(1-\lambda)C_t + \lambda Q^{\theta}_t C^*_t\right\} + G_t. \tag{16}$$

The consumption price index of home(small open economy) and foreign economy(the rest of the world) from (2) and (3) is changed to

$$P_t = \left[(1-\lambda)P^{1-\theta}_{H,t} + \lambda P^{1-\theta}_{F,t}\right]^{\frac{1}{1-\theta}}, \tag{17}$$

$$P_t^* = P_{F,t}^*. \tag{18}$$

By dividing both sides of (17) by P_t and using (4) and (18), the ratio of GDP deflator to CPI, that is, terms-of-trade is derived:

$$P_{h,t}^{1-\theta} = \frac{1-\lambda Q_t^{1-\theta}}{1-\lambda}. \tag{19}$$

Moreover, the aggregate resource constraint at period t can be written as

$$P_{H,t}(Y_t - G_t) = P_t(C_t + \mathrm{NX}_t), \tag{20}$$

where NX_t are the real net exports expressed in the unit of consumption goods. The real net exports can be subsequently expressed as

$$\mathrm{NX}_t = -\lambda Q_t^{1-\theta}\left(C_t - \frac{1-\lambda Q_t^{1-\theta}}{1-\lambda} Q_t^{2\theta-1} C_t^*\right). \tag{21}$$

As shown in (21), the real net exports depend on the consumption ratio between home and foreign economy, and the real exchange rate. Specifically, under financial autarky where the trade balance is zero(i.e., $NX_t = 0$), (20) is changed by

$$Y_t = \frac{C_t}{P_{h,t}} + G_t. \tag{22}$$

2.4. LIMITED ENFORCEMENT CONSTRAINTS

Building on Kehoe and Perri (2002), and Bai and Zhang (2010), this paper considers the related problem of enforcement constraints, which is given by

$$\sum_{k=0}^{\infty} \beta^k E_t[U(C_{t+k}, H_{t+k})] \geq V^a(\Delta_{t-1}, A_t), \tag{23}$$

where $V^a(\Delta_{t-1}, A_t)$ denotes the value function at period t that would have been obtained if the economy were under financial autarky. The enforcement constraint means that the expected discounted sum of utilities from period t onward is equal to or greater than the value of autarky from period t onward. The value under financial autarky is given by

$$V^a(\Delta_{t-1}, A_t) = \max_{\{C_t, H_t, P_{h,t}, Q_t, F_t, L_t, \Delta_t, \Pi_{H,t}\}} U(C_t, H_t) + \beta E_t V^a(\Delta_t, A_{t+1}),$$

subject to the social resource constraint (16), firms' profit-maximization conditions (6), (7), and (8), the evolution equation for relative price distortion (14), the equilibrium relation between real exchange rate and terms-of-trade (19), and the restriction of zero trade balance (22).

To solve the optimal policy problem with the enforcement constraint, I follow the approach used in Marcet and Marimon (2011) and Kehoe and Perri (2002). First, let ϕ_{8t} be the Lagrange multiplier for the enforcement constraint, (23). Then Lagrangian takes the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t, H_t) + \phi_{8t} \left\{ \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, H_{t+k}) - V^a(\Delta_{t-1}, A_t) \right\} \right],$$

plus standard forms of the other constraints. As shown in appendix B, I can rewrite the Lagrangian in the following form:

$$\sum_{t=0}^{\infty} \beta^t E_0 [(1 + M_t) U(C_t, H_t) - \phi_{8t} V^a(\Delta_{t-1}, A_t)],$$

plus standard forms of the other constraints. In this equation, M_t is defined as a cumulative multiplier, and its evolution is written as

$$M_t = M_{t-1} + \phi_{8t},$$

where $M_{-1} = 0$. This cumulative multiplier M_t summarizes the impact of the central bank's commitment on consumption demand and labor supply decisions of households that should be made in the presence of the enforcement constraint. M_t helps keep track of impacts of the central bank's past commitments on the current one-period instantaneous utility function.

3. OPTIMAL MONETARY POLICY

This section analyzes the central bank's optimal monetary policy problem incorporating limited enforcement constraints in a complete market and a bond model. Specifically, the effect of fiscal policy on fixing the distortions associated with monopolistic competition in good market and the terms-of-trade on the optimal monetary policy problem is analyzed as in Yun (2005).

3.1. COMPLETE MARKET AND LIMITED ENFORCEMENT

First, I consider the optimization problem for households under the complete asset market. Households have access to a complete set of nominal state-contingent bonds denominated in foreign currency on this market. These bonds are traded in the international financial market for both foreign and domestic investors. The representative household maximizes (1) subject to a sequence of budget constraints of the form[2]

$$P_t C_t + S_t E_t[Q_{t,t+1} B_{F,t+1}] \leq +S_t B_{F,t} + W_t H_t - P_{H,t} T_t + \Phi_t,$$

where $Q_{t,t+1}$ is the stochastic discount factor to measure the nominal value at period t of one unit of foreign currency at period $t+1$, $B_{F,t}$ denotes foreign-currency denominated nominal bonds, W_t is the nominal wage, H_t is the hours worked at period t, T_t is the real tax in the unit of GDP, and Φ_t is the nominal profit at period t. The optimization condition for the domestic household's problem is given by

$$\nu C_t^{\sigma} H_t^{\chi} = \frac{W_t}{P_t}.$$

The optimization condition for foreign bond holdings of domestic household is as follows:

$$Q_{t,t+1} = \beta \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} \frac{S_{t+1}}{S_t}, \tag{24}$$

and the optimization conditions for bond holdings of foreign investors from the foreign household's problem can be written as

$$Q_{t,t+1} = \beta \frac{\Lambda_{t+1}^* P_t^*}{\Lambda_t^* P_{t+1}^*}, \tag{25}$$

where $\Lambda_t (\Lambda_t^*)$ represents the marginal utility of consumption at period t for domestic(foreign) residents. Following Gali and Monacelli (2005), and De Paoli (2009), and using (24), (25) and (4), it follows that

$$C_t^{\sigma} = Q_t (C_t^*)^{\sigma}. \tag{26}$$

The government's flow budget constraint at period t is given by

$$\frac{B_{H,t}}{1+i_{H,t}} = B_{H,t-1} - P_{H,t}(T_t - G_t).$$

[2] Domestic-currency-denominated nominal bonds issued by the government are traded only by home households, and their net supply is zero. Thus, the budget constraint does not explicitly include these bonds.

The home government issues nominal one-period riskless securities, where $B_{H,t}$ is the total outstanding issue of government debt at period t. T_t is the real amount of lump-sum taxes at period t, and G_t is the government's real expenditure at period t. In order to focus on the analysis of monetary policy, I assume that the Ricardian equivalence holds in this case.

Now, the optimal monetary policy problem with a limited enforcement in complete market is analyzed. The central bank at period 0 solves the following optimization problem:

$$\max_{\{C_t,H_t,P_{h,t},Q_t,F_t,L_t,\Delta_t,\Pi_{H,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t,H_t),$$

subject to the social resource constraint (16), firms' profit-maximization conditions (6), (7), and (8), the evolution equation for relative price distortion (14), the equilibrium relation between real exchange rate and terms-of-trade (19), the complete international risk sharing condition (26), and an enforcement constraint (23). The Lagrangian for this problem is given in appendix C.1.

The optimization conditions for optimal nominal prices of firms are given by

$$\phi_{2t} - \phi_{5t} \left(\frac{1-\alpha \Pi_{H,t}^{\varepsilon-1}}{1-\alpha} \right)^{\frac{1}{1-\varepsilon}} = \alpha \Pi_{H,t}^{\varepsilon-1} \phi_{2t-1}, \tag{27}$$

$$\phi_{3t} + \phi_{5t} = \alpha \Pi_{H,t}^{\varepsilon} \phi_{3t-1}. \tag{28}$$

The optimization condition for inflation is given by

$$\begin{aligned} &\phi_{4t} \left\{ \Delta_{t-1} \Pi_{H,t} - \left(\frac{1-\alpha \Pi_{H,t}^{\varepsilon-1}}{1-\alpha} \right)^{\frac{1}{\varepsilon-1}} \right\} \\ &= \frac{\phi_{5t} F_t}{(1-\alpha)\varepsilon} \left(\frac{1-\alpha \Pi_{H,t}^{\varepsilon-1}}{1-\alpha} \right)^{-\frac{\varepsilon}{\varepsilon-1}} + \frac{\varepsilon-1}{\varepsilon} \phi_{2t-1} F_t + \phi_{3t-1} \Pi_{H,t} L_t. \end{aligned} \tag{29}$$

The one for consumption is

$$(1+M_t)C_t^{-\sigma} - (1-\lambda)\phi_{1t} P_{h,t}^{-\theta} + \sigma \phi_{2t} \frac{A_t H_t}{\Delta_t C_t^{\sigma+1}} + \sigma \phi_{7t} C_t^{\sigma-1} = 0. \tag{30}$$

Moreover, the one for labor is given by

$$-\nu(1+M_t)H_t^{\chi} + \phi_{1t} \frac{A_t}{\Delta_t} - \phi_{2t} \frac{A_t}{\Delta_t C_t^{\sigma}} - \phi_{3t} \frac{\nu \varepsilon (1+\chi) H_t^{\chi}}{(\varepsilon-1)\Delta_t P_{h,t}} = 0. \tag{31}$$

For the terms-of-trade the condition is

$$\theta\phi_{1t}P_{h,t}^{1-\theta}\{(1-\lambda)C_t+\lambda Q_t^{\theta}C_t^*\}+\frac{\nu\varepsilon\phi_{3t}H_t^{1+\chi}}{(\varepsilon-1)\Delta_t}-\phi_{6t}(1-\lambda)(1-\theta)P_{h,t}^{2-\theta}=0. \quad (32)$$

The one for real exchange rate is

$$(1-\theta)\lambda\phi_{6t}Q_t^{-\theta}+\lambda\theta\phi_{1t}P_{h,t}^{-\theta}Q_t^{\theta-1}C_t^*+\phi_{7t}(C_t^*)^{\sigma}=0. \quad (33)$$

Additionally, the optimization condition for relative price distortion is

$$\begin{aligned}&\phi_{4t}+\beta E_t[\phi_{8t+1}V_1^a(\Delta_t,A_{t+1})]\\&=-\frac{\phi_{1t}A_tH_t}{\Delta_t^2}+\phi_{2t}\frac{A_tH_t}{C_t^{\sigma}\Delta_t^2}+\phi_{3t}\frac{\nu\varepsilon H_t^{1+\chi}}{(\varepsilon-1)P_{h,t}\Delta_t^2}+\alpha\beta E_t[\Pi_{H,t+1}^{\varepsilon}\phi_{4t+1}]. \quad (34)\end{aligned}$$

The evolution of a cumulative multiplier, M_t, is given by

$$M_t=M_{t-1}+\phi_{8t}, \quad (35)$$

where M_{-1} = 0. The complementary slackness condition associated with the enforcement constraint implies that the following condition holds:

$$\phi_{8t}(V^c(\phi_{t-1},\Delta_{t-1},A_t)-V^a(\Delta_{t-1},A_t))=0, \quad (36)$$

where $V^c(\phi_{t-1},\Delta_{t-1},A_t)$ denotes the value function under the optimal commitment plan and ϕ_t is a set of Lagrangian multipliers for the optimal policy problem of the central bank. Specifically, the value function under the optimal commitment plan has a recursive representation:

$$V^c(\phi_{t-1},\Delta_{t-1},A_t)=U(C_t,H_t)+\beta E_t[V^c(\phi_t,\Delta_t,A_{t+1})], \quad (37)$$

where ϕ_{-1} = 0. In addition, the evolution of M_t is affected by the relative size of the two value functions that can be obtained under the optimal state-contingent plan and financial autarky. Consequently, the complementary slackness condition associated with the enforcement constraint can be rewritten as

$$M_t-M_{t-1}=\begin{cases}0 & \text{if } V^c(\phi_{t-1},\Delta_{t-1},A_t)>V^a(\Delta_{t-1},A_t)\\ \phi_{8t}(>0) & \text{if } V^c(\phi_{t-1},\Delta_{t-1},A_t)=V^a(\Delta_{t-1},A_t).\end{cases}$$

Having described the central bank's optimization conditions, I now solve a set of 18 equations (8 equilibrium conditions: (6), (7), (8), (14), (16), (19), (26), and (37) and 10 optimization conditions, from (27) through (36)) to establish decision rules of 18 endogenous variables including the central bank's decision variables $\{C_t, H_t, \Pi_{H,t}, P_{h,t}, \Delta_t, Q_t, F_t, L_t, V_t^c\}$ and Lagrange multipliers $\{\phi_{1t}, \phi_{2t}, \phi_{3t}, \phi_{4t}, \phi_{5t}, \phi_{6t}, \phi_{7t}, \phi_{8t}, M_t\}$, given the value function under financial autarky (denoted by $V^a(\Delta_{t-1}, A_t)$), a Markov process for A_t, the initial values of Lagrange multipliers ϕ_{-1}, and the initial relative price distortion Δ_{-1}.

The effectiveness of enforcement constraint depends on the presence of fiscal policy measures to fix steady-state distortions. In order to show these results, I note that optimization conditions for consumption demand and labor supply result into the following condition at the steady-state

$$\nu C^{\sigma} H^{\chi} = \frac{(\varepsilon-1)P_h}{\varepsilon}. \tag{38}$$

As shown in Corsetti and Pesenti (2001), when the terms-of-trade, P_h, exists in (38), central bank has an incentive to use surprise inflations to raise output in models with inefficient steady-state deviations. Hence, through fiscal policy measures, such as lump-sum subsidy or tax to households or firms, it is possible to eliminate the two distortions associated with monopolistic competition in goods market and the terms-of-trade. In order to prove this result, following Yun (2005), I discuss a case for which the government imposes the labor supply tax on households. In this case, the optimization condition of labor supply is given by

$$\nu C_t^{\sigma} H_t^{\chi} = (1-\tau_t^H)\frac{W_t}{P_t},$$

where τ_t^H is the tax rate at period t for labor supply in the home. When $\Pi_H = 1 (\Delta = 1)$ at the steady-state, the optimization condition of firms is given by

$$MC_H = \frac{W}{P}\frac{1}{P_h A},$$

where MC_H is the nominal marginal cost of the home firm at the steady-state. From $L = F$ at the steady-state, $MC_H = \dfrac{\varepsilon-1}{\varepsilon}$. Thus, the equilibrium condition for labor market id given by

$$\nu C^{\sigma} H^{\chi} = (1-\tau^H)\frac{\varepsilon-1}{\varepsilon}P_h A.$$

The optimal labor tax to obtain the Pareto optimality is given by

$$1-\tau^H = \frac{\varepsilon}{(\varepsilon-1)P_h}.$$

Proposition 1. Let us suppose that there are fiscal policy measures to eliminate steady-state distortions. In the presence of the complete set of contingent claims home and abroad, the enforcement constraints have no role in the determination of the inflation of GDP deflator, real exchange rate, consumption, and the number of hours worked in each period.

If the assumption of fiscal policy eliminates steady-state distortions as in Woodford (2003) and Yun (2005), the Lagrangian multipliers for profit maximization conditions should be zero, that is, $\phi_{2t} = \phi_{3t} = \phi_{5t} = 0$. Firstly, the substitution of this solution into (29) leads to the following condition:

$$\Delta_{t-1}\Pi_{H,t} = \left(\frac{1-\alpha\Pi_{H,t}^{\varepsilon-1}}{1-\alpha}\right)^{\frac{1}{\varepsilon-1}}. \tag{39}$$

By substituting this condition into the evolution equation for relative price distortion (14) as in Yun (2005), the inflation of GDP deflator is given by

$$\Pi_{H,t} = \frac{\Delta_t}{\Delta_{t-1}}, \tag{40}$$

and, as shown in appendix C.2, the law of motion for relative price distortion is rewritten by

$$\Delta_t = \Delta_{t-1}\left\{\alpha + (1-\alpha)\Delta_{t-1}^{\varepsilon-1}\right\}^{-\frac{1}{\varepsilon-1}}. \tag{41}$$

In addition, the other optimal conditions can be solved to yield the following, the details of the derivation shown in appendix C.3:

$$\nu C_t^{\sigma} H_t^{\chi} = \frac{A_t P_{h,t}}{\Delta_t}\left[(1-\lambda)P_{h,t}^{1-\theta} + \sigma\theta\lambda(Q_t^{1-\theta} + P_{h,t}^{1-\theta}Q_t^{\theta-\frac{1}{\sigma}}) + \frac{\sigma\theta\lambda^2}{1-\lambda}Q_t^{1-\frac{1}{\sigma}}\right]^{-1}. \tag{42}$$

Moreover, the complete international risk sharing (26) and the social resource constraint (16) can be solved to show that consumption and labor are determined as follows:

$$C_t = Q_t^{\frac{1}{\sigma}} C_t^*, \tag{43}$$

$$H_t = \frac{\Delta_t}{A_t} P_{h,t}^{-\theta}\left\{(1-\lambda)Q_t^{\frac{1}{\sigma}} + \lambda Q_t^{\theta}\right\} C_t^*. \tag{44}$$

In summary, I have derived 9 equations (19), (35), (36), (37), and (40) - (44) for 9 variables, such as $\{C_t, H_t, \Pi_{H,t}, P_{h,t}, \Delta_t, Q_t, V_t^c, \phi_{8t}, M_t\}$, given the value function under financial autarky (denoted by $V^a(\Delta_{t-1}, A_t)$), a Markov process for A_t, the initial relative price distortion Δ_{-1}, and the initial asset holdings B_{-1}. An important implication of equations (42) - (44) is that the enforcement constraint has no impact on the determination of real exchange rate, consumption, and the number of hours worked in each period. Moreover, the evolution equation of relative price distortion and the inflation of GDP deflator are not affected by the Lagrange multipliers of the enforcement constraint as shown in equations (40) and (41). As a result, proposition 1 has been proved.

3.2. A BOND MODEL WITH LIMITED ENFORCEMENT

In this section, a bond model with limited enforcement is used, in which non-contingent nominal debt alone is available in international financial markets. The only difference in this section from the complete market of the previous section is the asset market structure. Hence, I highlight the optimization problem of the representative household, while other equilibrium conditions are identical to those of the previous section.

The optimization problem at period 0 of the representative household in country H can be written as follows. The representative household at period 0 chooses C_t, H_t, and $B_{F,t+1}$ to maximize (1) subject to a sequence of budget constraints of the form

$$\frac{S_t B_{F,t+1}}{1+i^*_{F,t}} = (1+\tau^b_t) S_t B_{F,t} + W_t H_t - P_t C_t - P_{H,t} T_t + \Phi_t, \tag{45}$$

for t = 0, 1, 2, $\cdots$, ∞ and where $i^*_{F,t}$ is the interest rate in country F and τ^b_t is a subsidy at period t for the household's holdings of foreign assets. The subsidy for holding foreign assets is financed by lump-sum taxes denoted by $P_{H,t}T_t$. Hence, the government imposes a tax on capital inflows (subsidy on capital outflows) in the home country, while proceeds of capital control taxes are redistributed to households in the home country, following the recent literature on capital controls such as Farhi and Werning (2013).

The optimization condition of domestic household are given by

$$\nu C_t^{\sigma} H_t^{\chi} = \frac{W_t}{P_t}.$$

The first-order condition for foreign bond holdings of domestic household is given by

$$1 = \beta(1 + i^*_{F,t})E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{S_{t+1}}{S_t\Pi_{t+1}}\right].$$

The first-order condition for bond holdings from the representative household's optimization problem in country F is given by

$$1 = \beta(1 + i^*_{F,t})E_t\left[\left(\frac{C^*_t}{C^*_{t+1}}\right)^{\sigma}(\Pi^*_{t+1})^{-1}\right].$$

By using the definition of the real exchange rate, it is possible to rewrite the Euler equation in country H for the accumulation of foreign assets as follows:

$$1 = \beta(1 + i^*_{F,t})E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Q_{t+1}}{Q_t\Pi^*_{t+1}}\right].$$

Abstracting from the issue of the home government's bonds in order to concentrate on the impact of foreign debt on the optimal design of monetary policy, I assume that the government follows the Ricardian equivalence like in the case of the complete market. The government's one-period flow budget constraint is thus given by

$$\tau^b_t S_t B_{F,t} + P_{H,t}G_t = P_{H,t}T_t.$$

Now, the optimal monetary policy problem in a bond model with limited enforcement is analyzed. Before proceeding, it is important to note that the foreign-currency denominated real bonds is the central bank's choice variable unlike a complete market. Thus, the one-period flow budget constraint (45) is transformed into the following social resource constraint:

$$\frac{A_tH_t}{\Delta_t} = \frac{C_t}{P_{h,t}} + G_t + \frac{Q_t}{P_{h,t}}\left(\frac{B_t}{1+i^*_{F,t}} - \frac{B_{t-1}}{\Pi^*_t}\right), \tag{46}$$

where $B_{t-1} = B_{F,t}/P^*_{t-1}$. This constraint means the net asset position that should hold for asset transactions in the incomplete market. The central bank at period 0 solves the following optimization problem:

$$\max_{\{C_t,H_t,B_t,P_{h,t},Q_t,F_t,L_t,\Delta_t,\Pi_{H,t}\}_{t=0}^{\infty}} E_0\sum\nolimits_{t=0}^{\infty}\beta^t U(C_t,H_t),$$

subject to subject to the social resource constraint (16), firms' profit-maximization conditions (6), (7), and (8), the evolution equation for relative price distortion (14), the equilibrium relation between real exchange rate and terms-of-trade (19), the net asset position in the incomplete market (46), and an enforcement constraint (23). The Lagrangian for this problem is given in appendix D.1.

The optimization conditions for the two variables associated with optimal nominal prices of firms can be written as follows:

$$\phi_{2t} - \phi_{5t}\left(\frac{1-\alpha\Pi_{H,t}^{\varepsilon-1}}{1-\alpha}\right)^{\frac{1}{1-\varepsilon}} = \alpha\Pi_{H,t}^{\varepsilon-1}\phi_{2t-1}, \tag{47}$$

$$\phi_{3t} + \phi_{5t} = \alpha\Pi_{H,t}^{\varepsilon}\phi_{3t-1}. \tag{48}$$

The optimization condition for inflation is given by

$$\begin{aligned}&\phi_{4t}\left\{\Delta_{t-1}\Pi_{H,t} - \left(\frac{1-\alpha\Pi_{H,t}^{\varepsilon-1}}{1-\alpha}\right)^{\frac{1}{\varepsilon-1}}\right\}\\ &= \frac{\phi_{5t}F_t}{(1-\alpha)\varepsilon}\left(\frac{1-\alpha\Pi_{H,t}^{\varepsilon-1}}{1-\alpha}\right)^{-\frac{\varepsilon}{\varepsilon-1}} + \frac{\varepsilon-1}{\varepsilon}\phi_{2t-1}F_t + \phi_{3t-1}\Pi_{H,t}L_t.\end{aligned} \tag{49}$$

At the same time, the optimality condition for consumption is

$$(1+M_t)C_t^{-\sigma} - \phi_{1t}P_{h,t}^{-\theta}(1-\lambda) + \sigma\phi_{2t}\frac{A_tH_t}{\Delta_tC_t^{\sigma+1}} - \frac{\phi_{7t}}{P_{h,t}} = 0. \tag{50}$$

The first-order condition for labor is

$$-v(1+M_t)H_t^{\chi} + \frac{A_t}{\Delta_t}(\phi_{1t}+\phi_{7t}) - \phi_{2t}\frac{A_t}{\Delta_tC_t^{\sigma}} - \phi_{3t}\frac{v\varepsilon(1+\chi)H_t^{\chi}}{(\varepsilon-1)\Delta_tP_{h,t}} = 0. \tag{51}$$

Additionally, the optimization condition for the terms-of-trade is

$$\begin{aligned}&\theta\phi_{1t}P_{h,t}^{1-\theta}\{(1-\lambda)C_t + \lambda Q_t^{\theta}C_t^*\} + \frac{v\varepsilon\phi_{3t}H_t^{1+\chi}}{(\varepsilon-1)\Delta_t} - \phi_{6t}(1-\lambda)(1-\theta)P_{h,t}^{2-\theta}\\ &\qquad + \phi_{7t}\left\{C_t + Q_t\left(\frac{B_t}{1+i_{F,t^*}} - \frac{B_{t-1}}{\Pi_t^*}\right)\right\} = 0.\end{aligned} \tag{52}$$

The one for real exchange rate is

$$(1-\theta)\phi_{6t} = -\theta\phi_{1t}P_{ht}^{-\theta}Q_t^{2\theta-1}C_t^* - \frac{\phi_{7t}Q_t^{\theta}}{\lambda P_{h,t}}\left(\frac{B_t}{1+i_{F,t^*}} - \frac{B_{t-1}}{\Pi_t^*}\right). \tag{53}$$

Moreover, the optimization condition for relative price distortion is

$$\phi_{4t} + \beta E_t[\phi_{8t+1} V_1^a(\Delta_t, A_{t+1})]$$
$$= \alpha\beta E_t[\Pi_{H,t+1}^{\varepsilon}\phi_{4t+1}] - \frac{(\phi_{1t}+\phi_{7t})A_t H_t}{\Delta_t^2} + \phi_{2t}\frac{A_t H_t}{C_t^{\sigma}\Delta_t^2} + \phi_{3t}\frac{\nu\varepsilon H_t^{1+\chi}}{(\varepsilon-1)P_{h,t}\Delta_t^2}. \quad (54)$$

The one for real bond holdings is given by

$$\frac{\phi_{7t}Q_t}{P_{h,t}}\frac{1}{1+i_{F,t^*}} = \beta E_t \frac{\phi_{7t+1}Q_{t+1}}{P_{h,t+1}\Pi_{t+1}^*}. \quad (55)$$

The evolution of a cumulative multiplier M_t can be written as

$$M_t = M_{t-1} + \phi_{8t}, \quad (56)$$

where $M_{-1} = 0$. The complementary slackness condition associated with the enforcement constraint implies that the following condition holds:

$$\phi_{8t}(V^c(B_{t-1},\phi_{t-1},\Delta_{t-1},A_t) - V^a(\Delta_{t-1},A_t)) = 0, \quad (57)$$

where $V^c(B_{t-1},\phi_{t-1},\Delta_{t-1},A_t)$ denotes the value function under the optimal commitment plan. Specifically, the value function under the optimal commitment plan has a recursive representation:

$$V^c(B_{t-1},\phi_{t-1},\Delta_{t-1},A_t) = U(C_t,H_t) + \beta E_t[V^c(B_t,\phi_t,\Delta_t,A_{t+1})], \quad (58)$$

where $\phi_{-1} = 0$. In the same way as in the previous section, it can be shown that the evolution of M_t is affected by the relative size of the two value functions that can be obtained under the optimal stat-contingent plan and financial autarky. In addition, the complementary slackness condition associated with the enforcement constraint implies that the following condition holds:

$$M_t - M_{t-1} = \begin{cases} 0 & \text{if } V^c(B_{t-1},\phi_{t-1},\Delta_{t-1},A_t) > V^a(\Delta_{t-1},A_t) \\ \phi_{8t}(>0) & \text{if } V^c(B_{t-1},\phi_{t-1},\Delta_{t-1},A_t) = V^a(\Delta_{t-1},A_t). \end{cases}$$

Having described the central bank's optimization conditions, I now solve a set of 19 equations (8 equilibrium conditions: (6), (7), (8), (14),(16), (19), (46), and (58), and 11 optimization conditions: from (47) through (57)) to determine the decision rules of 19 endogenous variables including the central bank's decision variables $\{B_t, C_t, H_t, \Pi_{H,t}, P_{h,t}, \Delta_t, Q_t, F_t, L_t, V_t^c\}$ and Lagrange multipliers $\{\phi_{1t}, \phi_{2t}, \phi_{3t}, \phi_{4t}, \phi_{5t}, \phi_{6t}, \phi_{7t}, \phi_{8t}, M_t\}$, given the value function under financial autarky (denoted by $V^a(\Delta_{t-1},A_t)$), a Markov process for A_t, the initial values of

Lagrange multipliers ϕ_{-1}, the initial relative price distortion Δ_{-1}, and the initial asset holdings B_{-1}.

A closed-form solution to the optimization conditions shown above is desired, under the assumption that fiscal policy is used to eliminate steady-state distortions as discussed in Woodford (2003) and Yun (2005). In this case, the Lagrange multipliers for profit maximization conditions should be zero: $\phi_{2t} = \phi_{3t} = \phi_{5t} = 0$. First, applying these solutions into (49), the same result as in the complete market case can be obtained:

$$\Pi_{H,t} = \frac{\Delta_t}{\Delta_{t-1}}. \tag{59}$$

Therefore, the law of motion for relative price distortion is

$$\Delta_t = \Delta_{t-1}\{\alpha + (1-\alpha)\Delta_{t-1}^{\varepsilon-1}\}^{-\frac{1}{\varepsilon-1}}. \tag{60}$$

By combining (50) and (51), as shown in appendix D.2, the Lagrangian multiplier for the social resource constraint, ϕ_{7t}, is obtained. The substitution of ϕ_{7t} into the optimal condition for bond holdings (55) leads to the following condition:

$$1 = \beta(1+i_{F,t}^*)E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Q_{t+1}}{Q_t\Pi_{t+1}^*}\frac{Q_{t+1}^{\theta-1}(1+M_{t+1})(1-\nu(1-\lambda)\frac{\Delta_{t+1}}{A_{t+1}}C_{t+1}^{\sigma}H_{t+1}^{\chi}P_{h,t+1}^{-\theta})}{Q_t^{\theta-1}(1+M_t)(1-\nu(1-\lambda)\frac{\Delta_t}{A_t}C_t^{\sigma}H_t^{\chi}P_{h,t}^{-\theta})}\right]. \tag{61}$$

By combining (52) and (53), as shown in appendix D.3, the following condition arises:

$$\nu C_t^{\sigma}H_t^{\chi} = \frac{A_tP_{h,t}}{\Delta_t}\frac{1-\kappa_t}{1+(1-\lambda)\kappa_tP_{h,t}^{1-\theta}}, \tag{62}$$

where κ_t is defined as

$$\kappa_t \equiv \theta^{-1}\frac{(1-\lambda)P_{h,t}^{1-\theta}Q_t^{\theta}z_t + \lambda(Q_tz_t + C_t)}{\lambda P_{h,t}^{1-\theta}\{(1-\lambda)C_t + \lambda Q_t^{\theta}C_t^*\} + \lambda(1-\lambda)P_{h,t}^{2(1-\theta)}Q_t^{2\theta-1}C_t},$$

where z_t is defined as

$$B_t = (1+i_{F,t}^*)\left(\frac{B_{t-1}}{\Pi_t^*} + z_t\right). \tag{63}$$

Moreover, the combination of (16) and (46) can be solved to show that consumption and labor are determined as follows:

$$C_t = \lambda^{-1} Q_t^{\theta} \left(\lambda Q_t^{\theta-1} \frac{1-\lambda Q_t^{1-\theta}}{1-\lambda} C_t^* - z_t \right), \tag{64}$$

$$H_t = \frac{\Delta_t Q_t^{2\theta-1}}{A_t} \left(\frac{1-\lambda Q_t^{1-\theta}}{1-\lambda} \right)^{-\frac{\theta}{1-\theta}} \{C_t^* - \lambda^{-1}(1-\lambda) Q_t^{1-\theta} z_t\}. \tag{65}$$

In conclusion, I have derived 11 equations (19), (56), (57), (58), and (59) - (65) for 11 variables, such as $\{B_t, z_t, C_t, H_t, \Pi_{H,t}, P_{h,t}, \Delta_t, Q_t, V_t^c, \phi_{8t}, M_t\}$, given the value function under financial autarky (denoted by $V^a(\Delta_{t-1}, A_t)$), a Markov process for A_t, the initial relative price distortion Δ_{-1}, and the initial asset holdings B_{-1}.

Comparing this set of optimization conditions with the set of optimization conditions used to prove proposition 1, I can see that the asset market structure plays a substantial role in determining whether the enforcement constraints can have an impact on the determination of the inflation of GDP deflator, real exchange rate, consumption, and the number of hours worked in each period. In particular, equation (61) shows that the cumulative Lagrangian multiplier (denoted by M_t) can affect the relation between the level of consumption and the accumulation of foreign debt. The enforcement constraint in a bond model with limited enforcement can have an impact on the determination of the inflation of GDP deflator, real exchange rate, consumption, and the number of hours worked in each period, whereas it does not in the case of complete market. As a result, findings for the role of asset market structure on the impact of the enforcement constraint on the determination of optimal consumption and hours worked can be summarized as follows.

Proposition 2. Let us suppose that there are fiscal policy measures to eliminate steady-state distortions. The inflation targeting regime described by equations (59) and (60) is then optimal even in a bond model with limited enforcement. But the real exchange rate, consumption, and hours worked are affected by the presence of the enforcement constraints.

4. CONCLUSION

In this paper, I analyze the optimal monetary policy problem of a complete market and a bond model in a New Keynesian model of a small open economy with limited enforcement constraints. Enforcement constraints are shown to have different effects on the equilibrium allocation depending on the asset market structure. However, the inflation targeting is optimal even when there are enforcement constraints.

This paper presents only qualitative results on an impact of enforcement constraints without quantitative analysis. Thus, my future research plan is to develop the computation method to obtain a robust numerical solution to the optimal monetary policy problem in a small open New Keynesian DSGE with enforcement constraints.

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APPENDIX

A. THE OPTIMAL PRICE SETTING

Each firm that resets its price chooses it by maximizing the following expected present-value of profits:

$$\max_{\{P^*_{H,t}\}} \quad \sum_{k=0}^{\infty}(\alpha\beta)^k E_t\left[\Lambda_{t+k}\left(\frac{P^*_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{t+k}\left\{\frac{P^*_{H,t}}{P_{H,t+k}} - \frac{W_{t+k}}{A_{t+k}P_{H,t+k}}\right\}\right].$$

A.1. FIRST-ORDER CONDITIONS

$$\sum_{k=0}^{\infty}(\alpha\beta)^k E_t\left[\Lambda_{t+k}\left(\frac{P^*_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{t+k}\left\{\frac{P^*_{H,t}}{P_{H,t+k}} - \frac{\varepsilon}{\varepsilon-1}\frac{W_{t+k}}{A_{t+k}P_{H,t+k}}\right\}\right] = 0,$$

$$\sum_{k=0}^{\infty}(\alpha\beta)^k E_t\left[\Lambda_{t+k}\left(\frac{P^*_{H,t}}{P_{H,t+k}}\right)^{1-\varepsilon} Y_{t+k}\right] = \sum_{k=0}^{\infty}(\alpha\beta)^k E_t\left[\frac{\varepsilon}{\varepsilon-1}\Lambda_{t+k}\left(\frac{P^*_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}\frac{W_{t+k}Y_{t+k}}{A_{t+k}P_{H,t+k}}\right].$$

$$\begin{aligned}
LHS &= \Lambda_t\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{1-\varepsilon} Y_t + \alpha\beta E_t\Lambda_{t+1}\left(\frac{P^*_{H,t}}{P_{H,t}}\frac{P_{H,t}}{P_{H,t+1}}\right)^{1-\varepsilon} Y_{t+1} + (\alpha\beta)^2 E_t\Lambda_{t+2}\left(\frac{P^*_{H,t}}{P_{H,t}}\frac{P_{H,t}}{P_{H,t+2}}\right)^{1-\varepsilon} Y_{t+2} + \cdots \\
&= \left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{1-\varepsilon} E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[\Lambda_{t+k}\left(\frac{P_{H,t}}{P_{H,t+k}}\right)^{1-\varepsilon} Y_{t+k}\right] \\
&= \left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{1-\varepsilon} F_t,
\end{aligned}$$

where

$$\begin{aligned}
F_t &\equiv E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[\Lambda_{t+k}\left(\frac{P_{H,t}}{P_{H,t+k}}\right)^{1-\varepsilon} Y_{t+k}\right] \\
&= \Lambda_t Y_t + \alpha\beta\Lambda_{t+1}\left(\frac{P_{H,t}}{P_{H,t+1}}\right)^{1-\varepsilon} Y_{t+1} + (\alpha\beta)^2\Lambda_{t+2}\left(\frac{P_{H,t}}{P_{H,t+2}}\right)^{1-\varepsilon} Y_{t+2} + \cdots \\
&= \Lambda_t Y_t + \alpha\beta E_t\left[\left(\frac{P_{H,t}}{P_{H,t+1}}\right)^{1-\varepsilon}\sum_{k=0}^{\infty}(\alpha\beta)^k\Lambda_{t+1+k}\left(\frac{P_{H,t+1}}{P_{H,t+1+k}}\right)^{1-\varepsilon} Y_{t+1+k}\right] \\
&= \Lambda_t Y_t + \alpha\beta E_t[\Pi_{H,t}^{\varepsilon-1} F_{t+1}].
\end{aligned}$$

$$
\begin{aligned}
RHS &= \frac{\varepsilon}{\varepsilon-1}\Lambda_t\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{-\varepsilon}\frac{W_tY_t}{A_tP_{H,t}}+\alpha\beta\frac{\varepsilon}{\varepsilon-1}E_t\Lambda_{t+1}\left(\frac{P^*_{H,t}}{P_{H,t}}\frac{P_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon}\frac{W_{t+1}Y_{t+1}}{A_{t+1}P_{H,t+1}}\\
&\quad+(\alpha\beta)^2\frac{\varepsilon}{\varepsilon-1}E_t\Lambda_{t+2}\left(\frac{P^*_{H,t}}{P_{H,t}}\frac{P_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon}\frac{W_{t+2}Y_{t+2}}{A_{t+2}P_{H,t+2}}+\cdots\\
&=\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{-\varepsilon}E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[\frac{\varepsilon}{\varepsilon-1}\Lambda_{t+k}\left(\frac{P_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}\frac{W_{t+k}Y_{t+k}}{A_{t+k}P_{H,t+k}}\right]\\
&=\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{-\varepsilon}L_t,
\end{aligned}
$$

where

$$
\begin{aligned}
L_t &\equiv E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[\frac{\varepsilon}{\varepsilon-1}\Lambda_{t+k}\left(\frac{P_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}\frac{W_{t+k}Y_{t+k}}{A_{t+k}P_{H,t+k}}\right]\\
&=\frac{\varepsilon}{\varepsilon-1}\Lambda_t\frac{W_tY_t}{A_tP_{H,t}}+\alpha\beta\frac{\varepsilon}{\varepsilon-1}E_t\Lambda_{t+1}\left(\frac{P_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon}\frac{W_{t+1}Y_{t+1}}{A_{t+1}P_{H,t+1}}\\
&\quad+(\alpha\beta)^2\frac{\varepsilon}{\varepsilon-1}E_t\Lambda_{t+2}\left(\frac{P_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon}\frac{W_{t+2}Y_{t+2}}{A_{t+2}P_{H,t+2}}+\cdots\\
&=\frac{\varepsilon}{\varepsilon-1}\Lambda_t\frac{W_tY_t}{A_tP_{H,t}}+\alpha\beta E_t\left[\left(\frac{P_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon}\sum_{k=0}^{\infty}(\alpha\beta)^k\frac{\varepsilon}{\varepsilon-1}\Lambda_{t+1+k}\left(\frac{P_{H,t+1}}{P_{H,t+1+k}}\right)^{-\varepsilon}\frac{W_{t+1+k}Y_{t+1+k}}{A_{t+1+k}P_{H,t+1+k}}\right]\\
&=\frac{\varepsilon}{\varepsilon-1}\Lambda_t\frac{W_tY_t}{A_tP_{H,t}}+\alpha\beta E_t[\Pi^{\varepsilon}_{H,t+1}L_{t+1}].
\end{aligned}
$$

Therefore, using $\Lambda_t=C_t^{-\sigma}$, $Y_t=\dfrac{A_tH_t}{\Delta_t}$, and $\dfrac{W_t}{P_t}=C_t^{\sigma}H_t^{\chi}$:

$$F_t=C_t^{-\sigma}Y_t+\alpha\beta E_t[\Pi^{\varepsilon-1}_{H,t}F_{t+1}],$$

$$L_t=\frac{\varepsilon}{\varepsilon-1}\frac{H_t^{1+\chi}}{\Delta_tP_{h,t}}+\alpha\beta E_t[\Pi^{\varepsilon}_{H,t+1}L_{t+1}].$$

In addition, as LHS=RHS,

$$\frac{P^*_{H,t}}{P_{H,t}}=\frac{L_t}{F_t}$$

Applying (10),

$$\left(\frac{1-\alpha\Pi^{\varepsilon-1}_{H,t}}{1-\alpha}\right)^{\frac{1}{1-\varepsilon}}=\frac{L_t}{F_t}.$$

A.2. DERIVATION OF EQUATION (9)

Following the Calvo-type staggering of Yun (2005), firms' prices at period t and $t-1$ are distributed as follows

$$P_{H,t}^{1-\sigma} = (1-\alpha)P_{H,t}^{*\,1-\sigma} + (1-\alpha)\alpha P_{H,t-1}^{*\,1-\sigma} + (1-\alpha)\alpha^2 P_{H,t-2}^{*\,1-\sigma} + \cdots, \tag{A.1}$$

$$P_{H,t-1}^{1-\sigma} = (1-\alpha)P_{H,t-1}^{*\,1-\sigma} + (1-\alpha)\alpha P_{H,t-2}^{*\,1-\sigma} + (1-\alpha)\alpha^2 P_{H,t-3}^{*\,1-\sigma} + \cdots. \tag{A.2}$$

By subtracting (A.2) from (A.1), the sub-price index of the home differentiated goods can be rewritten:

$$P_{H,t} = \left(\alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)P_{H,t}^{*\,1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

A.3. DERIVATION OF EQUATION (13)

Following Yun(2005), under the Calvo-type staggered price-setting, one can rewrite the measure of relative price distortion:

$$\begin{aligned}
\Delta_t &= \frac{1}{n}\int_0^n \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\varepsilon} dh \\
&= \frac{1}{n}\left[(1-\alpha)n\left(\frac{P_{H,t}^*}{P_{H,t}}\right)^{-\varepsilon} + (1-\alpha)\alpha n\left(\frac{P_{H,t-1}^*}{P_{H,t}}\right)^{-\varepsilon} + (1-\alpha)\alpha^2 n\left(\frac{P_{H,t-2}^*}{P_{H,t}}\right)^{-\varepsilon} + \cdots\right] \\
&= (1-\alpha)\left(\frac{P_{H,t}^*}{P_{H,t}}\right)^{-\varepsilon} + \alpha\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\alpha)\left(\frac{P_{H,t-1}^*}{P_{H,t-1}}\right)^{-\varepsilon} + (1-\alpha)\alpha\left(\frac{P_{H,t-2}^*}{P_{H,t-1}}\right)^{-\varepsilon} + \cdots\right] \\
&= (1-\alpha)\left(\frac{P_{H,t}^*}{P_{H,t}}\right)^{-\varepsilon} + \alpha\Pi_{H,t}^{\varepsilon}\Delta_{t-1}.
\end{aligned}$$

B. LIMITED ENFORCEMENT

The Lagrangian with enforcement constraints can be written as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t, H_t) + \phi_{8,t} \left\{ \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, H_{t+k}) - V^a(\Delta_{t-1}, A_t) \right\} \right]$$

$$\begin{aligned}
&= U(C_0,H_0) + \phi_{8,0} E_0 \left\{ U(C_0,H_0) + \beta U(C_1,H_1) + \beta^2 U(C_1,H_1) + \cdots - V^a(\Delta_{-1},A_0) \right\} \\
&\quad + E_0 \left[\beta U(C_1,H_1) + \phi_{8,1} \left\{ \beta U(C_1,H_1) + \beta^2 U(C_2,H_2) + \beta^3 U(C_3,H_3) + \cdots - V^a(\Delta_0,A_1) \right\} \right] \\
&\quad + E_0 \left[\beta^2 U(C_2,H_2) + \phi_{8,2} \left\{ \beta^2 U(C_2,H_2) + \beta^3 U(C_3,H_3) + \beta^4 U(C_4,H_4) + \cdots - V^a(\Delta_1,A_2) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
&= (1+\phi_{8,0}) U(C_0,H_0) - \phi_{8,0} V^a(\Delta_{-1},A_0) + (1+\phi_{8,0}+\phi_{8,1}) U(C_1,H_1) - \phi_{8,1} V^a(\Delta_0,A_1) \\
&\quad + (1+\phi_{8,0}+\phi_{8,1}+\phi_{8,2}) U(C_2,H_2) - \phi_{8,2} V^a(\Delta_1,A_2) + \cdots \\
&\quad + (1+\phi_{8,0}+\phi_{8,1}+\cdots+\phi_{8,n}) U(C_n,H_n) - \phi_{8,n} V^a(\Delta_{n-1},A_n) + \cdots
\end{aligned}$$

$$\begin{aligned}
&= (1+M_0) U(C_0,H_0) - \phi_{8,0} V^a(\Delta_{-1},A_0) + (1+M_1) U(C_1,H_1) - \phi_{8,1} V^a(\Delta_0,A_1) \\
&\quad + (1+M_2) U(C_2,H_2) - \phi_{8,2} V^a(\Delta_1,A_2) + \cdots + (1+M_n) U(C_n,H_n) - \phi_{8,n} V^a(\Delta_{n-1},A_n) + \cdots
\end{aligned}$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left[(1+M_t) U(C_t,H_t) - \phi_{8,t} V^a(\Delta_{t-1},A_t) \right],$$

where the evolution of a cumulative multiplier M_t can be written as

$$M_t = M_{t-1} + \phi_{8t}$$

and $M_{-1} = 0$.

C. THE COMPLETE MARKET WITH LIMITED ENFORCEMENT

C.1. LAGRANGIAN

The Lagrangian for the optimal monetary policy problem with a limited enforcement in complete market is the following:

$$
\begin{aligned}
\max_{\substack{\{C_t, H_t, P_{h,t}, Q_t,\\ F_t, L_t, \Delta_t, \Pi_{H,t}\}_{t=0}^{\infty}}} \mathscr{L} = E_0 \textstyle\sum_{t=0}^{\infty} \beta^t \Big[& (1+M_t)U(C_t, H_t) - \phi_{8t} V^a(\Delta_{t-1}, s_t) \\
& + \phi_{1t} \left\{ \frac{A_t H_t}{\Delta_t} - P_{h,t}^{-\theta}((1-\lambda)C_t + \lambda Q_t^{\theta} C_t^*) \right\} \\
& - \phi_{2t} \left\{ \frac{A_t H_t}{\Delta_t C_t^{\sigma}} + \alpha\beta \Pi_{H,t+1}^{\varepsilon-1} F_{t+1} - F_t \right\} \\
& - \phi_{3t} \left\{ \frac{\nu\varepsilon}{\varepsilon-1} \frac{H_t^{1+\chi}}{\Delta_t P_{h,t}} + \alpha\beta \Pi_{H,t+1}^{\varepsilon} L_{t+1} - L_t \right\} \\
& + \phi_{4t} \left\{ (1-\alpha) \left(\frac{1-\alpha\Pi_{H,t}^{\varepsilon-1}}{1-\alpha} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \alpha \Pi_{H,t}^{\varepsilon} \Delta_{t-1} - \Delta_t \right\} \\
& - \phi_{5t} \left\{ \left(\frac{1-\alpha\Pi_{H,t}^{\varepsilon-1}}{1-\alpha} \right)^{\frac{1}{1-\varepsilon}} F_t - L_t \right\} \\
& + \phi_{6t} \left\{ 1 - \lambda Q_t^{1-\theta} - (1-\lambda) P_{h,t}^{1-\theta} \right\} \\
& + \phi_{7t} \left\{ C_t^{\sigma} - Q_t (C_t^*)^{\sigma} \right\} \Big].
\end{aligned}
$$

Applying $\phi_{2t} = \phi_{3t} = \phi_{5t} = 0$, first-order conditions are given by

$$
\phi_{4t} \left\{ \Delta_{t-1} \Pi_{H,t} - \left(\frac{1-\alpha\Pi_{H,t}^{\varepsilon-1}}{1-\alpha} \right)^{\frac{1}{\varepsilon-1}} \right\} = 0 \tag{C.1}
$$

$$
(1+M_t)C_t^{-\sigma} - (1-\lambda)\phi_{1t} P_{h,t}^{-\theta} + \sigma \phi_{7t} C_t^{\sigma-1} = 0 \tag{C.2}
$$

$$
-\nu(1+M_t)H_t^{\chi} + \phi_{1t} \frac{A_t}{\Delta_t} = 0 \tag{C.3}
$$

$$
\theta \phi_{1t} P_{h,t}^{1-\theta} \{(1-\lambda)C_t + \lambda Q_t^{\theta} C_t^*\} - \phi_{6t}(1-\lambda)(1-\theta) P_{h,t}^{2-\theta} = 0 \tag{C.4}
$$

$$(1-\theta)\lambda\phi_{6t}Q_t^{-\theta}+\lambda\theta\phi_{1t}P_{h,t}^{-\theta}Q_t^{\theta-1}C_t^*+\phi_{7t}(C_t^*)^{\sigma}=0 \tag{C.5}$$

$$\phi_{4t}+\beta E_t[\phi_{8t+1}V_1^a(\Delta_t,A_{t+1})]=-\frac{\phi_{1t}A_tH_t}{\Delta_t^2}+\alpha\beta E_t[\Pi_{Ht+1}^{\varepsilon}\phi_{4t+1}].$$

C.2. DERIVATION OF EQUATION (41)

Equation (39) can be rewritten as follows

$$\Pi_{H,t}=\{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}\}^{\frac{1}{1-\varepsilon}}. \tag{C.6}$$

Applying this equation to (14)

$$\begin{aligned}
\Delta_t &= (1-\alpha)\left(\frac{1-\alpha\Pi_{H,t}^{\varepsilon-1}}{1-\alpha}\right)^{\frac{\varepsilon}{\varepsilon-1}}+\alpha\Pi_{H,t}^{\varepsilon}\Delta_{t-1}\\
&= (1-\alpha)\left(\frac{1-\alpha\{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}\}^{-1}}{1-\alpha}\right)^{\frac{\varepsilon}{\varepsilon-1}}+\alpha\{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}\}^{\frac{\varepsilon}{1-\varepsilon}}\Delta_{t-1}\\
&= (1-\alpha)\left(\frac{1}{1-\alpha}\,\frac{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}-\alpha}{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}}\right)^{\frac{\varepsilon}{\varepsilon-1}}+\alpha\left(\frac{1}{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\Delta_{t-1}\\
&= (1-\alpha)\left(\frac{\Delta_{t-1}^{\varepsilon-1}}{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}}\right)^{\frac{\varepsilon}{\varepsilon-1}}+\alpha\left(\frac{1}{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\Delta_{t-1}\\
&= \frac{\{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}\}\Delta_{t-1}}{\{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}\}^{\frac{\varepsilon}{\varepsilon-1}}}\\
&= \Delta_{t-1}\{\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1}\}^{\frac{1}{1-\varepsilon}}\\
&= \Delta_{t-1}\Pi_{H,t}.
\end{aligned}$$

Thus, the inflation becomes of the following form:

$$\Pi_{H,t}=\frac{\Delta_t}{\Delta_{t-1}}, \tag{C.7}$$

and the substitution of (C.7) into (C.6) yields the law of motion for relative price distortion (41).

C.3. DERIVATION OF EQUATION (42)

First, ϕ_{1t} derives from (C.3).

$$\phi_{1t} = \nu(1+M_t)\frac{H_t^{\chi}\Delta_t}{A_t}.$$

Using ϕ_{1t} and the social resource constraint (16), I can derive ϕ_{6t} from (C.4)

$$\phi_{6t} = \frac{\nu\theta}{(1-\lambda)(1-\theta)}\frac{(1+M_t)H_t^{1+\chi}}{P_{h,t}^{1-\theta}}.$$

And using ϕ_{1t}, I can also derive ϕ_{7t} from (C.2)

$$\phi_{7t} = \frac{(1+M_t)}{\sigma C_t^{\sigma-1}}\left[\nu(1-\lambda)\frac{H_t^{\chi}\Delta_t}{A_t P_{h,t}^{\theta}} - C_t^{-\sigma}\right].$$

Now, substituting ϕ_{1t}, ϕ_{6t}, and ϕ_{7t} into (C.5):

$$(1-\theta)\lambda\phi_{6t}Q_t^{-\theta} + \lambda\theta\phi_{1t}P_{h,t}^{-\theta}Q_t^{\theta-1}C_t Q_t^{-\frac{1}{\sigma}} + \phi_{7t}C_t^{\sigma}Q_t^{-1} = 0,$$

$$\frac{(1-\theta)\lambda\nu\theta}{(1-\lambda)(1-\theta)}\frac{(1+M_t)H_t^{1+\chi}}{P_{h,t}^{1-\theta}Q_t^{\theta}} + \lambda\nu\theta(1+M_t)\frac{H_t^{\chi}\Delta_t}{A_t}P_{h,t}^{-\theta}Q_t^{\theta-1-\frac{1}{\sigma}}C_t$$
$$+ C_t^{\sigma}Q_t^{-1}\frac{(1+M_t)}{\sigma C_t^{\sigma-1}}\left[\nu(1-\lambda)\frac{H_t^{\chi}\Delta_t}{A_t P_{h,t}^{\theta}} - C_t^{-\sigma}\right] = 0,$$

$$\frac{\lambda\nu\theta}{(1-\lambda)}\frac{H_t^{1+\chi}}{P_{h,t}^{1-\theta}Q_t^{\theta}} + \lambda\nu\theta\frac{H_t^{\chi}\Delta_t}{A_t}P_{h,t}^{-\theta}Q_t^{\theta-1-\frac{1}{\sigma}}C_t + \nu(1-\lambda)\frac{C_t}{\sigma Q_t}\frac{H_t^{\chi}\Delta_t}{A_t P_{h,t}^{\theta}} - \frac{C_t^{1-\sigma}}{\sigma Q_t} = 0.$$

Multiplying both sides of the above equation with $\sigma C_t^{\sigma-1}Q_t$, and using the social resource constraint (16) and the international risk sharing condition (26)(i.e., $\frac{A_t P_{h,t}}{\Delta_t} = P_{h,t}^{-\theta}(1-\lambda+\lambda Q_t^{\theta-\frac{1}{\sigma}})C_t$), I obtain (42):

$$\frac{\nu\sigma\theta\lambda}{(1-\lambda)}\frac{H_t^{1+\chi}C_t^{\sigma-1}}{P_{h,t}^{1-\theta}Q_t^{\theta-1}} + \nu\sigma\theta\lambda\frac{H_t^{\chi}\Delta_t}{A_t}P_{h,t}^{-\theta}Q_t^{\theta-\frac{1}{\sigma}}C_t^{\sigma} + \nu(1-\lambda)C_t^{\sigma}\frac{H_t^{\chi}\Delta_t}{A_t P_{h,t}^{\theta}} - 1 = 0,$$

$$1 = \nu C_t^{\sigma}H_t^{\chi}\frac{\Delta_t}{A_t P_{h,t}}\left[(1-\lambda)P_{h,t}^{1-\theta} + \sigma\theta\lambda P_{h,t}^{1-\theta}Q_t^{\theta-\frac{1}{\sigma}} + \frac{\sigma\theta\lambda}{1-\lambda}\frac{H_t A_t}{\Delta_t C_t P_{h,t}^{-\theta}Q_t^{\theta-1}}\right],$$

$$1 = \nu C_t^{\sigma} H_t^{\chi} \frac{\Delta_t}{A_t P_{h,t}} \left[(1-\lambda) P_{h,t}^{1-\theta} + \sigma\theta\lambda P_{h,t}^{1-\theta} Q_t^{\theta-\frac{1}{\sigma}} + \frac{\sigma\theta\lambda}{1-\lambda} \frac{P_{h,t}^{-\theta}}{C_t P_{h,t}^{-\theta} Q_t^{\theta-1}} (1-\lambda+\lambda Q_t^{\theta-\frac{1}{\sigma}}) C_t \right],$$

$$\nu C_t^{\sigma} H_t^{\chi} = \frac{A_t P_{h,t}}{\Delta_t} \left[(1-\lambda) P_{h,t}^{1-\theta} + \sigma\theta\lambda (Q_t^{1-\theta} + P_{h,t}^{1-\theta} Q_t^{\theta-\frac{1}{\sigma}}) + \frac{\sigma\theta\lambda^2}{1-\lambda} Q_t^{1-\frac{1}{\sigma}} \right]^{-1}.$$

D. A BOND MODEL WITH LIMITED ENFORCEMENT

D.1. LAGRANGIAN

The Lagrangian for the optimal monetary policy problem with a limited enforcement in the bond model is the following:

$$\begin{aligned}
\max_{\substack{\{C_t, H_t, B_t, P_{h,t}, Q_t, \\ F_t, L_t, \Delta_t, \Pi_{H,t}\}_{t=0}^{\infty}}} \mathscr{L} = E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \Bigg[& (1+M_t) U(C_t, H_t) - \phi_{8t} V^a(\Delta_{t-1}, s_t) \\
& + \phi_{1t} \left\{ \frac{A_t H_t}{\Delta_t} - P_{h,t}^{-\theta} ((1-\lambda) C_t + \lambda Q_t^{\theta} C_t^*) \right\} \\
& - \phi_{2t} \left\{ \frac{A_t H_t}{\Delta_t C_t^{\sigma}} + \alpha\beta \Pi_{H,t+1}^{\varepsilon-1} F_{t+1} - F_t \right\} \\
& - \phi_{3t} \left\{ \frac{\nu\varepsilon}{\varepsilon-1} \frac{H_t^{1+\chi}}{\Delta_t P_{h,t}} + \alpha\beta \Pi_{H,t+1}^{\varepsilon} L_{t+1} - L_t \right\} \\
& + \phi_{4t} \left\{ (1-\alpha) \left(\frac{1-\alpha \Pi_{H,t}^{\varepsilon-1}}{1-\alpha} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \alpha \Pi_{H,t}^{\varepsilon} \Delta_{t-1} - \Delta_t \right\} \\
& - \phi_{5t} \left\{ \left(\frac{1-\alpha \Pi_{H,t}^{\varepsilon-1}}{1-\alpha} \right)^{\frac{1}{1-\varepsilon}} F_t - L_t \right\} \\
& + \phi_{6t} \left\{ 1 - \lambda Q_t^{1-\theta} - (1-\lambda) P_{h,t}^{1-\theta} \right\} \\
& + \phi_{7t} \left\{ \frac{A_t H_t}{\Delta_t} - \frac{C_t}{P_{h,t}} - \frac{Q_t}{P_{h,t}} \left(\frac{B_t}{1+i_{F,t}^*} - \frac{B_{t-1}}{\Pi_t^*} \right) \right\} \Bigg].
\end{aligned}$$

Applying $\phi_{2t} = \phi_{3t} = \phi_{5t} = 0$, first-order conditions are given by

$$\phi_{4t} \left\{ \Delta_{t-1} \Pi_{H,t} - \left(\frac{1-\alpha \Pi_{H,t}^{\varepsilon-1}}{1-\alpha} \right)^{\frac{1}{\varepsilon-1}} \right\} = 0 \qquad \text{(D.1)}$$

$$(1+M_t)C_t^{-\sigma} - \phi_{1t}P_{h,t}^{-\theta}(1-\lambda) - \frac{\phi_{7t}}{P_{h,t}} = 0 \tag{D.2}$$

$$-v(1+M_t)H_t^{\chi} + \frac{A_t}{\Delta_t}(\phi_{1t}+\phi_{7t}) = 0 \tag{D.3}$$

$$\begin{aligned}\theta\phi_{1t}P_{h,t}^{1-\theta}\{(1-\lambda)C_t + \lambda Q_t^{\theta}C_t^*\} - \phi_{6t}(1-\lambda)(1-\theta)P_{h,t}^{2-\theta}& \\ +\phi_{7t}\left\{C_t + Q_t\left(\frac{B_t}{1+i_{F,t^*}} - \frac{B_{t-1}}{\Pi_t^*}\right)\right\} = 0&\end{aligned} \tag{D.4}$$

$$(1-\theta)\phi_{6t} = -\theta\phi_{1t}P_{h,t}^{-\theta}Q_t^{2\theta-1}C_t^* - \frac{\phi_{7t}Q_t^{\theta}}{\lambda P_{h,t}}\left(\frac{B_t}{1+i_{F,t^*}} - \frac{B_{t-1}}{\Pi_t^*}\right) \tag{D.5}$$

$$\phi_{4t} + \beta E_t[\phi_{8t+1}V_1^a(\Delta_t, A_{t+1})] = \alpha\beta E_t[\Pi_{H,t+1}^{\varepsilon}\phi_{4t+1}] - \frac{(\phi_{1t}+\phi_{7t})A_tH_t}{\Delta_t^2} \tag{D.6}$$

$$\frac{\phi_{7t}Q_t}{P_{h,t}}\frac{1}{1+i_{F,t^*}} = \beta E_t\frac{\phi_{7t+1}Q_{t+1}}{P_{h,t+1}\Pi_{t+1}^*}. \tag{D.7}$$

D.2. DERIVATION OF EQUATION (61)

Combining (D.2) and (D.3):

$$\phi_{1t} = \frac{(1+M_t)C_t^{-\sigma} - \dfrac{\phi_{7t}}{P_{h,t}}}{(1-\lambda)P_{h,t}^{-\theta}}$$

$$\nu(1+M_t)H_t^{\chi}\frac{\Delta_t}{A_t} = \frac{(1+M_t)C_t^{-\sigma} - \dfrac{\phi_{7t}}{P_{h,t}}}{(1-\lambda)P_{h,t}^{-\theta}} + \phi_{7t}$$

$$\nu(1+M_t)H_t^{\chi}\frac{\Delta_t}{A_t}(1-\lambda)P_{h,t}^{-\theta} = (1+M_t)C_t^{-\sigma} - \frac{\phi_{7t}}{P_{h,t}} + (1-\lambda)P_{h,t}^{1-\theta}\frac{\phi_{7t}}{P_{h,t}}$$

$$\frac{\phi_{7t}}{P_{h,t}}(1-(1-\lambda)P_{h,t}^{1-\theta}) = (1+M_t)C_t^{-\sigma} - \nu(1+M_t)H_t^{\chi}\frac{\Delta_t}{A_t}(1-\lambda)P_{h,t}^{-\theta},$$

and then applying (19), ϕ_{1t} and ϕ_{7t} are solved:

$$\frac{\phi_{7t}}{P_{h,t}} = \frac{(1+M_t)C_t^{-\sigma}}{\lambda Q_t^{1-\theta}}\left(1-\nu(1-\lambda)\frac{\Delta_t}{A_t}C_t^{\sigma}H_t^{\chi}P_{h,t}^{-\theta}\right), \tag{D.8}$$

$$\phi_{1t} = \frac{(1+M_t)C_t^{-\sigma}}{(1-\lambda)P_{h,t}^{-\theta}}\left[1-\frac{1}{\lambda Q_t^{1-\theta}}\left(1-\nu(1-\lambda)\frac{\Delta_t}{A_t}C_t^{\sigma}H_t^{\chi}P_{h,t}^{-\theta}\right)\right]. \tag{D.9}$$

Applying ϕ_{7t} into (D.7), (61) is derived:

$$1 = \beta(1+i_{F,t}^{*})E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Q_{t+1}}{Q_t\Pi_{t+1}^{*}}\frac{Q_{t+1}^{\theta-1}(1+M_{t+1})(1-\nu(1-\lambda)\frac{\Delta_{t+1}}{A_{t+1}}C_{t+1}^{\sigma}H_{t+1}^{\chi}P_{h,t+1}^{-\theta})}{Q_t^{\theta-1}(1+M_t)(1-\nu(1-\lambda)\frac{\Delta_t}{A_t}C_t^{\sigma}H_t^{\chi}P_{h,t}^{-\theta})}\right].$$

D.3. DERIVATION OF EQUATION (62)

Let $z_t \equiv \frac{B_t}{1+i_{F,t^*}} - \frac{B_{t-1}}{\Pi_t^*}$. Combining (D.4) with (D.5) gives

$$\begin{aligned}\phi_{6t} &= \frac{1}{(1-\lambda)P_{h,t}^{2-\theta}}\left[\theta\phi_{1t}P_{h,t}^{1-\theta}\{(1-\lambda)C_t+\lambda Q_t^{\theta}C_t^{*}\}+\phi_{7t}(C_t+Q_tz_t)\right]\\ &= -\theta\phi_{1t}P_{h,t}^{-\theta}Q_t^{2\theta-1}C_t^{*}-\frac{\phi_{7t}Q_t^{\theta}}{\lambda P_{h,t}}z_t.\end{aligned}$$

From the equation in the second equality:

$$\left[\frac{\theta}{(1-\lambda)}P_{h,t}^{-1}((1-\lambda)C_t+\lambda Q_t^{\theta}C_t^{*})+\theta P_{h,t}^{-\theta}Q_t^{2\theta-1}C_t^{*}\right]\phi_{1t} = \left[\frac{C_t+Q_tz_t}{(1-\lambda)P_{h,t}^{2-\theta}}+\frac{Q_t^{\theta}}{\lambda P_{h,t}}z_t\right]\phi_{7t},$$

$$\begin{aligned}&\frac{\theta}{(1-\lambda)}\left[P_{h,t}^{-1}((1-\lambda)C_t+\lambda Q_t^{\theta}C_t^{*})+(1-\lambda)P_{h,t}^{-\theta}Q_t^{2\theta-1}C_t^{*}\right]\phi_{1t}\\ &\quad= \frac{1}{\lambda(1-\lambda)P_{h,t}^{2-\theta}}\left[\lambda(C_t+Q_tz_t)+(1-\lambda)P_{h,t}^{1-\theta}Q_t^{\theta}z_t\right]\phi_{7t},\end{aligned}$$

$$\begin{aligned}&\theta\left[\lambda P_{h,t}^{1-\theta}((1-\lambda)C_t+\lambda Q_t^{\theta}C_t^{*})+\lambda(1-\lambda)P_{h,t}^{2(1-\theta)}Q_t^{2\theta-1}C_t^{*}\right]\phi_{1t}\\ &\qquad= \left[\lambda(C_t+Q_tz_t)+(1-\lambda)P_{h,t}^{1-\theta}Q_t^{\theta}z_t\right]\phi_{7t}.\end{aligned}$$

Subsequently,

$$\frac{\phi_{1t}}{\phi_{7t}} = \kappa_t, \quad \text{where} \quad \kappa_t \equiv \theta^{-1} \frac{\lambda(C_t + Q_t z_t) + (1-\lambda)P_{h,t}^{1-\theta} Q_t^{\theta} z_t}{\lambda P_{h,t}^{1-\theta}((1-\lambda)C_t + \lambda Q_t^{\theta} C_t^*) + \lambda(1-\lambda)P_{h,t}^{2(1-\theta)} Q_t^{2\theta-1} C_t^*}.$$

Applying (D.8) and (D.9) to ϕ_{7t} and ϕ_{1t}:

$$\begin{aligned}
\frac{\phi_{1t}}{\phi_{7t}} &= \frac{\frac{(1+M_t)C_t^{-\sigma}}{(1-\lambda)P_{h,t}^{-\theta}} \left[1 - \frac{1}{\lambda Q_t^{1-\theta}} \left\{1 - \nu(1-\lambda)\frac{\Delta_t}{A_t} C_t^{\sigma} H_t^{\chi} P_{h,t}^{-\theta}\right\}\right]}{\frac{(1+M_t)C_t^{-\sigma} P_{h,t}}{\lambda Q_t^{1-\theta}} \left\{1 - \nu(1-\lambda)\frac{\Delta_t}{A_t} C_t^{\sigma} H_t^{\chi} P_{h,t}^{-\theta}\right\}} \\
&= \frac{\lambda Q_t^{1-\theta}}{(1-\lambda)P_{h,t}^{1-\theta}} \frac{\frac{1}{\lambda Q_t^{1-\theta}}(\lambda Q_t^{1-\theta} - \omega_t)}{\omega_t} \\
&= \frac{1}{(1-\lambda)P_{h,t}^{1-\theta}} \frac{\lambda Q_t^{1-\theta} - \omega_t}{\omega_t} \\
&= \frac{1}{(1-\lambda)P_{h,t}^{1-\theta}} \left(\frac{\lambda Q_t^{1-\theta}}{\omega_t} - 1\right),
\end{aligned}$$

where $\omega_t \equiv 1 - \nu(1-\lambda)\frac{\Delta_t}{A_t} C_t^{\sigma} H_t^{\chi} P_{h,t}^{-\theta}$.

Additionally,

$$\begin{aligned}
\omega_t &= \frac{\lambda Q_t^{1-\theta}}{1 + (1-\lambda)P_{h,t}^{1-\theta}\kappa_t} \\
1 - \nu(1-\lambda)\frac{\Delta_t}{A_t} C_t^{\sigma} H_t^{\chi} P_{h,t}^{-\theta} &= \frac{\lambda Q_t^{1-\theta}}{1 + (1-\lambda)P_{h,t}^{1-\theta}\kappa_t} \\
\nu C_t^{\sigma} H_t^{\chi} \frac{\Delta_t}{A_t}(1-\lambda)P_{h,t}^{-\theta} &= 1 - \frac{\lambda Q_t^{1-\theta}}{1 + (1-\lambda)P_{h,t}^{1-\theta}\kappa_t} \\
\nu C_t^{\sigma} H_t^{\chi} &= \frac{A_t P_{h,t}^{\theta}}{(1-\lambda)\Delta_t} \frac{1 + (1-\lambda)P_{h,t}^{1-\theta}\kappa_t - \lambda Q_t^{1-\theta}}{1 + (1-\lambda)P_{h,t}^{1-\theta}\kappa_t}.
\end{aligned}$$

Thus, applying $(1-\lambda)P_{h,t}^{1-\theta} = 1 - \lambda Q_t^{1-\theta}$:

$$\nu C_t^{\sigma} H_t^{\chi} = \frac{A_t P_{h,t}}{\Delta_t} \frac{1 + \kappa_t}{1 + (1-\lambda)P_{h,t}^{1-\theta}\kappa_t}.$$