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Couponing When Preferences Change: To Whom and How Many?*

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Abstract Employing a differentiated product duopoly model with repeat purchase, this paper examines optimal targeted couponing when consumers have changing preferences and are forward-looking. To find optimal couponing, we consider the following sequential decision of couponing. Firms first decide how many of their own customers they will offer coupons to. Having determined the number of defensive coupons to be offered, firms decide how many of rivals' customers they will offer coupons to. Focusing on a symmetric equilibrium, we find that the firms can maximize profits by distributing coupons to all of their own customers and only them.

Keywords defensive coupon, offensive coupon, sequential decision of couponing, changing preference

JEL Classification D43, L13, M31

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1. INTRODUCTION

Couponing is a widely used promotional strategy in firms' competition. Depending on the method of distribution, coupons can be divided into *mass media coupons* and *targeted coupons*. Targeted coupons generally take two forms: *defensive coupons* and *offensive coupons*. Mass media coupons are randomly distributed to consumers. Defensive coupons are distributed to retain firms' own customers, while offensive coupons are offered to poach rival firms' customers.

Previous studies on couponing consider mass media coupons that lead to market segmentation through consumers' self-selection. Narasimhan (1984), among others, shows that coupons serve as a price discrimination vehicle, charging a lower price to coupon users (more price-elastic consumers).

Due to the development of more sophisticated methods for acquiring, storing, and analyzing consumer information, firms can now send targeted coupons to selected customers. Using a model of product differentiation à la Hotelling, Shaffer and Zhang (1995) examine the effects of targeted coupons on firm profits, prices, and coupon face values. Their main finding is that targeted coupons deteriorate firm profits due to increased competition for potential brand switchers. Bester and Petrakis (1996) study sales promotion via coupons in a duopoly. In their model, the role of (offensive) coupons is to reduce consumer switching costs. They show that, in equilibrium, couponing intensifies competition between firms, and hence lowers their profits.

In the context of price discrimination based on purchase history, Chen (1997) considers a two-period homogeneous product duopoly model. Here consumers incur costs when switching one firm to another, which enables firms to segment and price discriminate consumers. He shows that in equilibrium, each firm charges a lower price to the competitor's customers than to its own customers in the second period (paying customers to switch) and that such discriminatory pricing lowers profits. Fudenberg and Tirole (2000) analyze a two-period duopoly model in which consumers have different preferences for firms' products and each firm can set different prices in period 2, depending on whether or not consumers have bought its product in period 1. They find that each firm can poach the rival firm's customers by charging them a lower price (customer poaching) and that discrimination in prices charged to loyal and switching customers reduces firm profits.[1]

As reviewed above, the existing literature on targeted couponing has mainly focused on offensive coupons. There are two reasons for this: (i) consumers have to pay costs to switch between firms (Bester and Petrakis, 1996; Chen, 1997) and (ii) they prefer a specific firm's product (Fudenberg and Tirole, 2000). In either case, enticing brand switching through coupons (discounts) arises as an equilibrium outcome. The intuition for this is that consumers who bought from a competitor are revealed to have a lower relative preference for the firm's product (or a relatively high cost to switch to the firm), and so profit maximization requires the firm to charge a lower price to them. In addition, offensive couponing with exogenous switching costs and/or constant preferences leads to lower firm profits.[2]

Despite the extensive economic literature on targeted couponing, the case of considering both defensive and offensive couponing has received little attention. In Shaffer and Zhang (1995), firms can send defensive and offensive coupons together, but their study is based on a model with single purchase so that dynamic properties of consumer preferences are not considered, which may affect firms' couponing. Also in the literature, the question of what proportion of targeted customers firms should offer coupons to has not been definitively answered.

The purpose of the present study is to explore which and how many consumers firms should offer coupons to when consumers change their tastes across purchases and are forward-looking. To that end, we employ a two-period differentiated product duopoly model where firms first decide how many of their own customers they will offer coupons to and then how many of rivals' customers

[1] Taylor (2003) extends Chen (1997) to multiple periods and multiple firms, and Villas-Boas (1999) provides an analysis similar to Fudenberg and Tirole (2000) but in a duopoly with infinitely lived firms and overlapping generations of consumers.

[2] A well-established result in the price discrimination literature is that oligopolistic price discrimination intensifies competition and leads to lower profits (Thisse and Vives, 1988; Shaffer and Zhang, 1995; Bester and Petrakis, 1996; Liu and Serfes, 2004). One important environment for price discrimination to intensify competition and to reduce firm profits is best-response asymmetry, i.e., one firm's strong market is the other firm's weak market (see Corts, 1998).

they will offer coupons to.

Focusing on a symmetric equilibrium, we obtain the following two results. (i) In the case of offering defensive coupons, it is optimal for the firms to distribute coupons to all of their own customers. (ii) Having distributed the defensive coupons, sending out offensive coupons to a rival's customers is detrimental to the firms' profits.

The rest of the paper is structured as follows. Section 2 sets up the model. In Section 3, two cases of couponing are analyzed and another couponing rule is discussed. Section 4 concludes.

2. THE MODEL

Consider the following two-period differentiated duopoly model à la Hotelling. Two firms (A and B) produce and sell competing goods for two periods (1 and 2) at a constant marginal cost, which we normalize to zero for simplicity. Firm A is located at point 0 and firm B at point 1 of the unit interval $[0,1]$. In each period, there is a continuum of consumers uniformly distributed on the interval $[0,1]$ with a unit mass. Each consumer is identified by her location on the interval, which corresponds to her ideal product. Consumers buy at most one unit of the good in each period and are willing to pay at most v. We assume that v is sufficiently high for non-purchase to be dismissed. A consumer located at $x \in [0,1]$ incurs a disutility of tx when purchasing from firm A, and of $t(1-x)$ when purchasing from firm B, where $t > 0$ measures the per-unit distaste's cost of buying away from her ideal product.

Each consumer's location in period 2 is allowed to vary randomly and independently of her first-period location.[3] In other words, all consumers change preferences from period 1 to period 2 and do not know their second-period preferences in period 1. For example, a consumer's preferences for different airlines may vary from one period to the next as travel plans change. A customer may change her preferred shopping venue depending on whether shopping trip starts

[3] A consumer's first-period choice contains no information about her second-period preference. Hence in equilibrium, there is no price discrimination by purchase history. Instead, the firms can use coupons or long-term contracts (commitment to future prices).

from home or work. All consumers are also forward-looking in the sense that in period 1, they buy from the firm that maximizes the sum of their first-period surpluses and expected second-period surpluses, anticipating second-period prices.

Coupons are distributed in period 1 and redeemed in period 2, which takes the form of price discounts. We assume that each consumer is equally likely to receive a coupon and that trading coupons is not possible.

To find the firms' optimal couponing, we consider the following sequential decision of couponing. The firms first decide how many of their own customers they will offer coupons to. Having determined the number of defensive coupons to be offered, the firms decide how many of their rival's customers they will offer coupons to.[4] In our model setup, there does not exist an equilibrium where only offensive coupons are distributed. This is because as the rival's customers of yesterday may prefer the firm's own product today, the firms have no incentives to poach them using offensive coupons. Thus, the other sequential decision of couponing (i.e., offensive-then-defensive couponing) reduces to defensive couponing, which is included in our analysis. The following formally defines the firms' couponing:

Definition 1. (1) Firm $i \in \{A, B\}$ is said to use *defensive couponing* $(\eta^i, 0)$ when it distributes coupons to a fraction $\eta^i \in (0, 1]$ of its own consumers and gives no coupons to its rival's consumers. η^i will be referred to as firm i's *defensive couponing intensity*. (2) Firm i is said to use *mixed couponing* (η^{i*}, θ^i) when it distributes coupons to a fraction $\theta^i \in [0, 1)$ of the rival firm's consumers with the optimal defensive couponing intensity η^{i*}. θ^i will be called firm i's *offensive couponing intensity*.

Each couponing is characterized by its intensity. In this regard, our model generalizes the one of Caminal and Matutes (1990) who analyze the case of $(\eta^i, 0) = (1, 0)$ only. Note also that in the definition of couponing, the cases of $\eta^i = 0$ and $\theta^i = 1$ are excluded because the former case means the absence of coupons and the latter case is similar to mass media couponing. Each case will be used as a benchmark in this study.

[4] A simultaneous decision of couponing is discussed in Section 3.

Prior to price competition, both firms announce an intensity of couponing (η^i or θ^i), which is a binding commitment. Hence, the effects of a couponing intensity on the equilibrium values take the form of a comparative statics analysis. It is also assumed that the discount factor for both firms and consumers is 1 and that there is no cost of distributing coupons.

Letting α_A denote firm A's market share in period 1, the timing of the couponing game is as follows:

- Period 1: Given a pre-announced couponing intensity (η^i or θ^i), firm $i \in \{A, B\}$ sets a first-period price (p_1^i) and a coupon face value (r^i), resulting in a portion α_A of consumers purchasing from firm A and the remaining portion $1 - \alpha_A$ purchasing from firm B.
- Period 2: Firm i chooses a second-period price (p_2^i). Each consumer decides whether or not to be loyal depending on her new location (preference) and the effective prices ($p_2^i - r^i$).

The following example is helpful to better understand the couponing game:

Example 1. Suppose that the two firms employ $(\eta^{A*}, \theta^A) = (\eta^{B*}, \theta^B) = (1, 1/3)$. Consider a consumer who was located at x_1 and bought from firm A in period 1, i.e. $x_1 \in [0, \alpha_A]$. In period 2, if she buys from firm A again, she will pay $p_2^A - r^A$. On the other hand, if she buys from firm B, she will pay $p_2^B - r^B$ (resp. p_2^B) with probability 1/3 (resp. 2/3). Thus, when this consumer buys from firm A (resp. B) in period 2, she enjoys utilities $v - tx_1 - p_1^A$ and $v - tx_2 - (p_2^A - r^A)$ (resp. $v - t(1 - x_2) - (p_2^B - r^B/3)$) in periods 1 and 2, respectively, where x_2 is her new location (preference) in period 2.

3. ANALYSIS AND RESULTS

To find the firms' optimal couponing (intensity), the defensive couponing and mixed couponing games are analyzed in turn. For each couponing game, we use the subgame perfect Nash equilibrium as the solution concept and proceed by backward induction. Since the firms are identical ex ante, we focus on a (pure-strategy) symmetric equilibrium in which they use couponing with the same intensity.

3.1. DEFENSIVE COUPONING

Consider first the case where both firms use defensive couponing. We start by constructing the demand for firm A in the second period. In period 2, α_A and r^i are given. Recall that all consumers redraw their taste parameter at the beginning of period 2.

Consumers in period 2 can be divided into the following four groups, depending on which firm they bought from and whether they received coupons in period 1:

- (D1) Consumers in $[0, \alpha_A]$ with firm A's coupons; x_{10}; η^A
- (D2) Consumers in $[0, \alpha_A]$ without coupons; x_{00}; $1-\eta^A$
- (D3) Consumers in $(\alpha_A, 1]$ with firm B's coupons; x_{01}; η^B
- (D4) Consumers in $(\alpha_A, 1]$ without coupons; x_{00}; $1-\eta^B$.

where xs denote consumers who are indifferent between buying from A and buying from B in period 2. Specifically, x_{10} (resp. x_{01}) is the indifferent consumer of the group in which consumers receive coupons from firm A (resp. B) and x_{00} is the indifferent consumer of the group without coupons. These indifferent consumers are defined as follows:

$$
\begin{aligned}
&v - tx_{10} - (p_2^A - r^A) = v - t(1-x_{10}) - p_2^B \\
&v - tx_{00} - p_2^A = v - t(1-x_{00}) - p_2^B \\
&v - tx_{01} - p_2^A = v - t(1-x_{01}) - (p_2^B - r^B).
\end{aligned}
$$

For example, consumers belonging to group D3 bought from firm B and received coupons in period 1. In period 2, a consumer in this group prefers purchasing from firm A to purchasing from firm B (i.e., she switches to firm A) if her new taste parameter is smaller than x_{01}: $x \leq x_{01} = (p_2^B - p_2^A - r^B + t)/2t$.

Since a fraction η^i of firm i's consumers receive coupons, the second-period demand of firm A is given by

$$\begin{aligned} q_2^A(p_2^A, p_2^B) = & \underbrace{\alpha_A \eta^A x_{10}}_{\text{in D1}} + \underbrace{\alpha_A(1-\eta^A)x_{00}}_{\text{in D2}} + \underbrace{(1-\alpha_A)\eta^B x_{01}}_{\text{in D3}} \\ & + \underbrace{(1-\alpha_A)(1-\eta^B)x_{00}}_{\text{in D4}}. \end{aligned}$$

Then, in period 2, a fraction $\alpha_A \eta^A x_{10}$ of consumers buy from firm A at the discounted price $p_2^A - r^A$, while another fraction $\alpha_A(1-\eta^A)x_{00} + (1-\alpha_A)\eta^B x_{01} + (1-\alpha_A)(1-\eta^B)x_{00}$ buy from firm A at the full price p_2^A. Thus, firm A's second-period maximization problem can be written as

$$\begin{aligned} \max_{p_2^A} \pi_2^A = & (\alpha_A \eta^A x_{10})(p_2^A - r^A) + [\alpha_A(1-\eta^A)x_{00} + (1-\alpha_A)\eta^B x_{01} \\ & + (1-\alpha_A)(1-\eta^B)x_{00}]p_2^A. \end{aligned} \tag{1}$$

The first-order condition for the problem (1) gives firm A's best-response function. We can proceed in a similar way for firm B and then solve the system of the two best-response functions to find the second-period equilibrium prices. Substituting the second-period equilibrium prices into π_2^A yields the equilibrium profit for firm A in period 2, denoted by $\hat{\pi}_2^A(\alpha_A, r^A, r^B; \eta^A, \eta^B)$. Assuming that all consumers are forward-looking, we can now express firm A's profit maximization problem in period 1 as

$$\max_{p_1^A, r^A} \pi^A = \pi_1^A + \hat{\pi}_2^A, \tag{2}$$

where $\pi_1^A = p_1^A \alpha_A(p_1^A, p_1^B, r^A, r^B; \eta^A, \eta^B)$.

Focusing on a symmetric equilibrium, we consider the case of $\eta^A = \eta^B = \eta \in (0,1]$. Taking the first-order conditions for the problem (2) and imposing symmetry, we can obtain the equilibrium values of defensive couponing as follows:

Lemma 1. *Suppose that firms A and B employ defensive couponing* $(\eta, 0)$. *In* *the equilibrium,*

(i) The first-period prices are

$$p_1^A = p_1^B = p_1^d = t + \frac{2t\eta(1+\eta)}{(2+\eta)^2}. \tag{3}$$

(ii) The values of the coupons are

$$r^A = r^B = r^d = \frac{2t}{2+\eta}. \tag{4}$$

(iii) The second-period prices are

$$p_2^A = p_2^B = p_2^d = t + \frac{t\eta}{2+\eta}. \tag{5}$$

(iv) The firms' profits are

$$\pi^A = \pi^B = \pi^d = t + t\left(\frac{\eta}{2+\eta}\right)^2. \tag{6}$$

Proof. See Appendix. □

From Lemma 1, we can see that in the equilibrium, as the firms distribute more coupons to their own customers, both the first- and second-period prices rise: $dp_1^d/d\eta > 0$ and $dp_2^d/d\eta > 0$. The intuition for the increase in the first-period price goes as follows. As more coupons are sent out, more consumers with the coupons accept to pay a higher price in period 1 since they anticipate their loyalty will be rewarded in period 2. On the other hand, the equilibrium value of a coupon decreases as η increases: $dr^d/d\eta < 0$. It is noteworthy that if coupons are not used, the game is similar, in each period, to the standard Hotelling model so that in period $j \in \{1,2\}$, $p_j^A = p_j^B = p^n = t$. Here we use the superscript n to denote the equilibrium values in the case of no coupons. Hence, the prices in both periods are higher than in the absence of coupons, although loyal consumers with coupons pay a lower price in period 2: $p_2^d - r^d = t + t(\eta - 2)/(2+\eta) < p^n = t, \forall \eta \in (0,1]$.

It can be also checked that as the firms offer more coupons to their customers, more of them are less tempted to switch to a rival firm. Formally, $\eta x_{10} + (1-\eta)x_{00}$ (resp. $\eta x_{01} + (1-\eta)x_{00}$) is the fraction of firm A's (resp. B's) consumers who decide to buy from firm A in period 2. Then we have $d[\eta x_{10} + (1-\eta)x_{00}]/d\eta > 0$ (resp. $d[\eta x_{01} + (1-\eta)x_{00}]/d\eta < 0$), which leads more consumers to benefit from coupons in period 2. In this sense, the second period becomes more competitive as more defensive coupons are sent out.

Finally, the equilibrium profit increases with the defensive couponing intensity: $d\pi^d/d\eta > 0$. Moreover, defensive couponing allows the firms to increase their profits compared with the case where no coupons are used: $\pi^d = t + t[\eta/(2+\eta)]^2 > \pi^n = t, \forall \eta \in (0,1]$. Unlike the general results in the oligopolistic price discrimination literature, price discrimination by defensive coupons considered here boosts firm profits.[5] This immediately gives the following result:

Proposition 1. *In the case of defensive couponing, it is optimal for the firms to distribute coupons to all of their own consumers.*

In terms of competitiveness of couponing, sending out defensive coupons softens competition. However, Caminal and Claici (2007) argue that loyalty-rewarding programs intensify competition unless the number of firms is sufficiently small and firms are restricted to use lump-sum coupons. Our result complements that of Caminal and Claici (2007) in the sense that, in a duopoly, couponing for rewarding loyalty becomes more anti-competitive as firms offer more coupons.[6]

3.2. MIXED COUPONING

We next turn to the case of mixed couponing. As before, we analyze the game by first deriving the demand for firm A in the second period.

[5] See footnote 2.

[6] In her empirical study on the airline industry, Lederman (2007) finds that frequent flyer programs (FFPs) increase demand for airlines, and interprets this finding as evidence that FFP reinforces firms' market power. Fong and Liu (2011) show that rewarding loyalty makes tacit collusion easier to sustain.

Consumers in period 2 can be segmented into the following four groups, depending on their first-period choices and the firms' offensive couponing intensities:

- (M1) Consumers in $[0, \alpha_A]$ with both firms' coupons; x_{11}; θ^B
- (M2) Consumers in $[0, \alpha_A]$ with only firm A's coupons; x_{10}; $1-\theta^B$
- (M3) Consumers in $(\alpha_A, 1]$ with both firms' coupons; x_{11}; θ^A
- (M4) Consumers in $(\alpha_A, 1]$ with only firm B's coupons; x_{01}; $1-\theta^A$.

The indifferent consumers x_{10} and x_{01} are the same as in defensive couponing, while x_{11} is the indifferent consumer of the group in which consumers receive coupons from both firms and defined as

$$v - tx_{11} - (p_2^A - r^A) = v - t(1-x_{11}) - (p_2^B - r^B).$$

The second-period demand of firm A is then

$$\begin{aligned} q_2^A(p_2^A, p_2^B) = &\underbrace{\alpha_A \theta^B x_{11}}_{\text{in M1}} + \underbrace{\alpha_A(1-\theta^B)x_{10}}_{\text{in M2}} + \underbrace{(1-\alpha_A)\theta^A x_{11}}_{\text{in M3}} \\ &+ \underbrace{(1-\alpha_A)(1-\theta^A)x_{01}}_{\text{in M4}}. \end{aligned}$$

In period 2, a fraction $\alpha_A \theta^B x_{11} + \alpha_A(1-\theta^B)x_{10} + (1-\alpha_A)\theta^A x_{11}$ of consumers buy from firm A at the discounted price $p_2^A - r^A$, while another fraction $(1-\alpha_A)(1-\theta^A)x_{01}$ buy from firm A at the full price p_2^A. Hence, we can write the second-period maximization problem of firm A as

$$\begin{aligned} \max_{p_2^A} \pi_2^A = &\left[\alpha_A \theta^B x_{11} + \alpha_A(1-\theta^B)x_{10} + (1-\alpha_A)\theta^A x_{11}\right](p_2^A - r^A) \\ &+ [(1-\alpha_A)(1-\theta^A)x_{01}]p_2^A. \end{aligned} \tag{7}$$

The first-order condition for the problem (7) gives firm A's best-response function. Proceeding similarly for firm B and solving the system of the two best-response functions, we obtain the second-period equilibrium prices. Plugging those prices into π_2^A yields the equilibrium profit for firm A in period 2, denoted by $\tilde{\pi}_2^A(\alpha_A, r^A, r^B; \theta^A, \theta^B)$. With forward-looking consumers, firm A's profit maximization problem in period 1 is

$$\max_{p_1^A, r^A} \pi^A = \pi_1^A + \tilde{\pi}_2^A, \tag{8}$$

where $\pi_1^A = p_1^A \alpha_A(p_1^A, p_1^B, r^A, r^B; \theta^A, \theta^B)$.

As we focus on a symmetric equilibrium, the case of $\theta^A = \theta^B = \theta \in [0, 1)$ is considered. Taking the first-order conditions for the problem (8) and imposing symmetry yields the next lemma, which characterizes the equilibrium values of mixed couponing:

Lemma 2. *Suppose that firms A and B employ mixed couponing* $(1, \theta)$. *In the equilibrium,*

(i) The first-period prices are

$$p_1^A = p_1^B = p_1^m = t + \frac{2t(1-\theta)(2-3\theta)}{9(1+\theta)^2}. \tag{9}$$

(ii) The values of the coupons are

$$r^A = r^B = r^m = \frac{2t}{3(1+\theta)}. \tag{10}$$

(iii) The second-period prices are

$$p_2^A = p_2^B = p_2^m = \frac{4t}{3}. \tag{11}$$

(iv) The firms' profits are

$$\pi^A = \pi^B = \pi^m = t + \frac{t(1-\theta)(1-3\theta)}{9(1+\theta)^2}. \tag{12}$$

Proof. See Appendix. □

Lemma 2 provides different results from Lemma 1. In the equilibrium, the first-period price initially decreases and then increases in the offensive couponing intensity, approaching $p^n = t$. The reason is that coupons sent out offensively in mixed couponing intensify price competition in period 1 since each firm should lower a first-period price to prevent its own customers from being poached by the rival firm's coupons. The second-period price remains the same regardless of θ. The equilibrium value of a coupon, however, decreases with θ as in defensive couponing.

The equilibrium profit first decreases and then increases as the firms distribute more coupons to their rival's customers, approaching $\pi^n = t$. In addition, if both firms send coupons to more than a fraction 1/3 of the rival firm's customers, then mixed couponing is less profitable than in the case of no coupons. From this, we can draw the next result:

Proposition 2. *In the case of mixed couponing, it is optimal for the firms not to distribute any coupons to their rival's consumers.*

Under mixed couponing, the coupons sent to each firm's customers defensively increase the cost of attracting the rival's customers because the discount required to entice an additional customer should cover this defensive coupon. Hence, the defensive coupons in mixed couponing reduce the firms' incentive to offer offensive coupons.

Combining Propositions 1 and 2 immediately yields the following result on optimal couponing:

Proposition 3. *Consider a two-period differentiated duopoly model where forward-looking consumers change their preferences over time. Then, the optimal couponing is such that the firms offer coupons to all of their own consumers and only them.*

The result of Proposition 3 has implications for marketing tactics such as frequent-flyer programs (by airline companies) and frequent-stay programs (by hotels).

3.3. SIMULTANEOUS DECISION OF COUPONING

Finally, we sketch optimal couponing when the firms simultaneously determine the defensive and offensive couponing intensities. From the above results, it seems that given an intensity of offensive couponing, the firms' profit increases as more defensive coupons are distributed. On the other hand, given an intensity of defensive couponing, the profit from offering the defensive coupons seems to dissipate as more offensive coupons are sent out. In these cases, optimal couponing in a simultaneous decision of couponing will be the same as in the sequential decision. In particular, when the firms distribute both types of coupons with the same intensity, the simultaneous decision of couponing is similar to mass media couponing in that consumers have the same possibility of receiving coupons from both firms, regardless of where they buy the products. Such couponing is not profitable compared with the case of no coupons since consumers only care about current prices in making purchasing decisions in period 1.[7]

4. CONCLUSION

Based on a two-period differentiated duopoly model with forward-looking consumers having changing preferences, we investigate targeted couponing to answer the question of which and how many consumers firms should offer coupons to. Considering the sequential decision of defensive and offensive couponing gives the following results.

First, defensive couponing allows the firms to increase their profits compared with the case of no coupons. The firms can then maximize profits by distributing coupons to all of their own customers. The intuition behind this result is that consumers with defensive coupons accept to pay a higher price in period 1 as they anticipate their loyalty will be rewarded in period 2. Second, having distributed coupons to all of the firms' own customers, sending out coupons to poach their rival's customers reduces profits. Moreover, it leads to lower profits than without

[7]Using the same procedure as in the previous two subsections, we can show that if the firms employ (η, θ) with $\eta = \theta$, then the first- and second-period equilibrium prices are t and the equilibrium value of a coupon is 0.

couponing when offering coupons to more than a fraction 1/3 of a competitor's customers. The reason is that each firm should lower its first-period price to prevent the rival's offensive coupons from luring away its customers, which results in intensified competition in period 1. These two results imply that, if consumers change their preferences over time and are forward-looking, the firms' optimal couponing is to distribute coupons to all of their own consumers and only them.

In future research, it would be interesting to study the optimality of a simultaneous decision of couponing when some consumers change their preferences across purchases and others do not.

APPENDIX

Proof of Lemma 1

The second-period equilibrium prices are

$$p_2^A = t + \alpha_A \eta^A r^A \text{ and } p_2^B = t + (1-\alpha_A)\eta^B r^B. \tag{13}$$

The equilibrium profit for firm A in period 2 is then calculated as

$$\hat{\pi}_2^A = \frac{t}{2} - \frac{1}{2t}\left[\alpha_A(1-\alpha_A)\eta^A\eta^B r^A r^B + \alpha_A(1-\alpha_A\eta^A)\eta^A(r^A)^2\right]. \tag{14}$$

Now we need to compute firm A's first-period market share α_A, which depends on p_1^i, r^i, and η^i. The indifferent consumer, $\hat{x}$, is such that the sum of the difference in her first-period surpluses from buying from firms A and B (denoted by $\Delta S_1 = S_1^A - S_1^B$) and the difference in her expected second-period surpluses (denoted by $\Delta S_2 = S_2^A - S_2^B$) is equal to zero. The first-period surplus difference is simply given by

$$\Delta S_1 = S_1^A - S_1^B = (v - t\hat{x} - p_1^A) - (v - t(1-\hat{x}) - p_1^B) = t - 2t\hat{x} + p_1^B - p_1^A.$$

Consumers do not know which will be their taste parameter in period 2. Note also that a fraction η^i of firm i's consumers receive coupons. Thus, the expected second-period surplus from buying from firm A in period 1 can be written as

$$\begin{aligned} S_2^A = \eta^A &\left[\int_0^{x_{10}} \left(v - tx - (p_2^A - r^A)\right) dx + \int_{x_{10}}^1 \left(v - t(1-x) - p_2^B\right) dx\right] \\ &+ (1-\eta^A)\left[\int_0^{x_{00}} \left(v - tx - p_2^A\right) dx + \int_{x_{00}}^1 \left(v - t(1-x) - p_2^B\right) dx\right]. \end{aligned}$$

In case the indifferent consumer receives (resp. does not receive) a coupon from firm A in period 1, she buys again from firm A if her new preference in period 2 is below x_{10} (resp. x_{00}); otherwise, she buys from firm B. In the same vein, the expected second-period surplus from buying from firm B in period 1 is

$$
\begin{aligned}
S_2^B = \eta^B &\left[\int_0^{x_{01}} \left(v - tx - p_2^A\right) dx + \int_{x_{01}}^1 \left(v - t(1-x) - (p_2^B - r^B)\right) dx\right] \\
&+ (1-\eta^B)\left[\int_0^{x_{00}} \left(v - tx - p_2^A\right) dx + \int_{x_{00}}^1 \left(v - t(1-x) - p_2^B\right) dx\right].
\end{aligned}
$$

After some algebra, we have

$$\Delta S_2 = S_2^A - S_2^B = t\eta^A(x_{10}^2 - x_{00}^2) - t\eta^B(x_{01}^2 - x_{00}^2) - \eta^B r^B.$$

Since $\hat{x}$ is defined by $\Delta S_1 + \Delta S_2 = 0$ and $\hat{x} = \alpha_A$, the first-period market share of firm A, α_A, is given implicitly by

$$t - 2t\alpha_A + p_1^B - p_1^A = t\eta^B x_{01}^2 - t\eta^A x_{10}^2 + t(\eta^A - \eta^B)x_{00}^2 + \eta^B r^B. \tag{15}$$

Note that the firms' couponing intensities are given as $\eta^A = \eta^B = \eta \in (0,1]$. Thus, the first-order conditions for the problem (2) are[8]

$$
\begin{cases}
\frac{\partial \pi^A}{\partial p_1^A} = \alpha_A + p_1^A \frac{\partial \alpha_A}{\partial p_1^A} - \frac{1}{2t}[(1-2\alpha_A)\eta^2 r^A r^B \\
\qquad + (1 - 2\alpha_A\eta)\eta(r^A)^2]\frac{\partial \alpha_A}{\partial p_1^A} = 0 \\
\frac{\partial \pi^A}{\partial r^A} = p_1^A \frac{\partial \alpha_A}{\partial r^A} - \frac{1}{2t}\left[\alpha_A(1-\alpha_A)\eta^2 r^B + 2\alpha_A(1-\alpha_A\eta)\eta r^A\right] \\
\qquad - \frac{1}{2t}\left[(1-2\alpha_A)\eta^2 r^A r^B + (1-2\alpha_A\eta)\eta(r^A)^2\right]\frac{\partial \alpha_A}{\partial r^A} = 0.
\end{cases}
\tag{16}
$$

Here the values of $\partial\alpha_A/\partial p_1^A$ and $\partial\alpha_A/\partial r^A$ are obtained by using the implicit function theorem for (15). We focus on a symmetric equilibrium so that $p_1^A = p_1^B = p_1$, $r^A = r^B = r$, and $\alpha_A = 1/2$. Then we have

$$\frac{\partial \alpha_A}{\partial p_1^A} = \frac{-t}{2(t^2 + \eta^2 r^2)} \quad \text{and} \quad \frac{\partial \alpha_A}{\partial r^A} = \frac{\eta[t + (1-\eta)r]}{4(t^2 + \eta^2 r^2)}. \tag{17}$$

Replacing $\partial\alpha_A/\partial p_1^A$ and $\partial\alpha_A/\partial r^A$ in (16) by (17) and solving the system gives (3) and (4). (5) is obtained by plugging (4) into (13).[9] With (3), (4), and

[8] It can be easily verified that the second-order conditions are satisfied.

[9] Switching occurs in period 2 if $0 < x_{01} \leq x_{00} \leq x_{10} < 1$ or $r^B - t < p_2^B - p_2^A < t - r^A$, which

(14) we get (6). $\square$

Proof of Lemma 2

The second-period equilibrium prices are as follows:

$$p_2^A = t + r^A\Theta \text{ and } p_2^B = t + r^B\Theta'. \tag{18}$$

where $\Theta = \alpha_A + (1-\alpha_A)\theta^A$ and $\Theta' = \alpha_A\theta^B + (1-\alpha_A)$.

The equilibrium profit for firm A in period 2 is then

$$\tilde{\pi}_2^A = \frac{t}{2} - \frac{1}{2t}\left[r^A(1-\Theta)(r^A\Theta + r^B(1-\Theta'))\right]. \tag{19}$$

As in Lemma 1, we need to compute firm A's first-period market share α_A. Let $\tilde{x}$ be the indifferent consumer. Assuming forward-looking consumers, $\tilde{x}$ is such that $\Delta S_1 + \Delta S_2 = 0$. The first-period surplus difference is given by

$$\Delta S_1 = S_1^A - S_1^B = (v - t\tilde{x} - p_1^A) - (v - t(1-\tilde{x}) - p_1^B) = t - 2t\tilde{x} + p_1^B - p_1^A.$$

The expected second-period surpluses from buying from firms A and B in period 1 can be, respectively, expressed as

$$\begin{aligned}
S_2^A = \theta^B &\left[\int_0^{x_{11}} \left(v - tx - (p_2^A - r^A)\right) dx + \int_{x_{11}}^1 \left(v - t(1-x) - (p_2^B - r^B)\right) dx\right] \\
&+ (1-\theta^B)\left[\int_0^{x_{10}} \left(v - tx - (p_2^A - r^A)\right) dx + \int_{x_{10}}^1 \left(v - t(1-x) - p_2^B\right) dx\right] \\
S_2^B = \theta^A &\left[\int_0^{x_{11}} \left(v - tx - (p_2^A - r^A)\right) dx + \int_{x_{11}}^1 \left(v - t(1-x) - (p_2^B - r^B)\right) dx\right] \\
&+ (1-\theta^A)\left[\int_0^{x_{01}} \left(v - tx - p_2^A\right) dx + \int_{x_{01}}^1 \left(v - t(1-x) - (p_2^B - r^B)\right) dx\right].
\end{aligned}$$

A few lines of computations establish that

$$\Delta S_2 = S_2^A - S_2^B = t(x_{10}^2 - x_{01}^2) + t\theta^A(x_{01}^2 - x_{11}^2) + t\theta^B(x_{11}^2 - x_{10}^2) - (1-\theta^B)r^B.$$

is satisfied in the symmetric equilibrium.

Since $\tilde{x}$ is defined by $\Delta S_1 + \Delta S_2 = 0$ and $\tilde{x} = \alpha_A$, the first-period market share of firm A is given implicitly by

$$t - 2t\alpha_A + p_1^B - p_1^A = t(\theta^A - \theta^B)x_{11}^2 + t(1-\theta^A)x_{01}^2 - t(1-\theta^B)x_{10}^2 + (1-\theta^B)r^B. \tag{20}$$

Given the firms' couponing intensities as $\theta^A = \theta^B = \theta \in [0,1)$, the first-order conditions for the problem (8) are[10]

$$\begin{cases} \frac{\partial \pi^A}{\partial p_1^A} = \alpha_A + p_1^A \frac{\partial \alpha_A}{\partial p_1^A} - \frac{1}{2t}\big[(\theta-1)r^A(r^A\Theta + r^B(1-\Theta')) \\ \qquad + (1-\theta)r^A(r^A+r^B)(1-\Theta)\big]\frac{\partial \alpha_A}{\partial p_1^A} = 0 \\ \frac{\partial \pi^A}{\partial r^A} = p_1^A \frac{\partial \alpha_A}{\partial r^A} - \frac{1}{2t}\big[2r^A\Theta(1-\Theta) + r^B(1-\Theta)(1-\Theta')\big] \\ \qquad -\frac{1}{2t}\big[(\theta-1)r^A(r^A\Theta + r^B(1-\Theta')) + (1-\theta)r^A(r^A+r^B)(1-\Theta)\big]\frac{\partial \alpha_A}{\partial r^A} = 0. \end{cases} \tag{21}$$

Using the implicit function theorem for (20) and imposing symmetry (i.e., $p_1^A = p_1^B = p_1$, $r^A = r^B = r$, and $\alpha_A = 1/2$), we obtain

$$\frac{\partial \alpha_A}{\partial p_1^A} = \frac{-t}{2[t^2 + (1-\theta)^2 r^2]} \quad \text{and} \quad \frac{\partial \alpha_A}{\partial r^A} = \frac{(1-\theta)(t-\theta r)}{4[t^2+(1-\theta)^2 r^2]}. \tag{22}$$

Replacing $\partial \alpha_A / \partial p_1^A$ and $\partial \alpha_A / \partial r^A$ in (21) by (22) and solving the system, we get (9) and (10). Plugging (10) into (18) gives (11).[11] Using (9), (10), and (19), we obtain (12). □

[10]It can be easily verified that the second-order conditions are satisfied.

[11]Switching occurs in period 2 if $0 < x_{01} \leq x_{11} \leq x_{10} < 1$ or $r^B - t < p_2^B - p_2^A < t - r^A$, which is satisfied in the symmetric equilibrium.

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