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Cournot duopoly and tacit collusion under fairness and reciprocal preferences*

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Abstract This paper studies the impact of fairness and reciprocity on collusion between firms competing in quantities in infinitely repeated games. A reciprocal firm responds to unkind behavior of rivals with unkind actions (destructive reciprocity), while at the same time, it responds to kind behavior of rivals with kind actions (constructive reciprocity). The paper shows that when firms are reciprocal, collusive quantity profiles are easier to sustain for reasonable perceptions of fair quantities of rivals. However, if only very low quantities deemed as fair, then sustaining collusion could be more difficult when the firms have fairness concerns.

Keywords Fairness; Reciprocity; Collusion; Cournot Duopoly; Repeated Games

JEL Classification D43, D63, L13, L21

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"*If they see me planting too much cocoa, they'll do things to my land and my family, and they won't bear fruit; really bad things; puripuri and other witchcraft.*"

1. INTRODUCTION

The quote is taken from a farmer in Papua New Guinea when he was explaining to Keir Martin, a social anthropologist, why he had only cultivated half of his three-hectare block and why, like him, none of his fellow villagers planted the whole of their blocks of land. According to Martin (2009): "Such an avoidance of profit maximization might appear economically irrational. But from the perspective of those villagers, putting that extra work just to make oneself target for jealousy of one's neighbors would be highly irrational behavior."

There is an ample evidence from psychology and from experimental economics that people do not only aim to maximize their material payoffs, despite its central role in economic analysis. Many observed departures from material payoff maximizing behavior arise through actions that favor fairness or reciprocity.[1]

Rabin (1993) argues that the parties of a transaction care about fairness in the sense that they "like to help those who are helping them, and hurt those who are hurting them" (pp. 1281). Fairness and reciprocity have been shown to explain behavior in bargaining games and in trust games. For example, in ultimatum games offers are usually much more generous than predicted by equilibrium and low offers are often rejected. These offers are consistent with an equilibrium in which players make offers knowing that other players may reject allocations that appear unfair.[2]

Fairness and reciprocity concern influence behavior in market experiments too. Huck, Muller, Normann (2001) shows that Stackelberg leader avoid exploiting the follower as the follower typically retaliate high quantities supplied by the leader by also supplying higher quantities than predicted by the subgame perfect

[1]Reciprocity is more traditionally referred as the result of optimizing actions of self-interested agents. In other-regarding preferences literature, reciprocity refers to a tendency to respond to perceived kindness with kindness and perceived unkindness with unkindness and to expect this behavior from others. Thus, it is a property of preferences. To differentiate the two, Sobel (2005) refers the initial one instrumental reciprocity and the latter intrinsic reciprocity. In this paper, I use the term reciprocity in the meaning of intrinsic reciprocity.

[2]Sobel (2005) argues that models of other-regarding preferences such as reciprocity can provide clearer and more intuitive explanations of interesting economic phenomena. İriş and Santos-Pinto (2014) employs another common other-regarding preference, inequity-aversion, and explains the some of the puzzling findings of Cournot experiments.

equilibrium. This behavior is consistent with the destructive reciprocity: the follower sacrifices some of their own material payoff in order to punish the unfair amount supplied by the leader. [3]

Motivated by this evidence, I ask: "can fairness and reciprocity facilitate cooperation between firms competing in quantities?" To answer this broad question I focus on infinitely repeated games. This important class of games tells how cooperative outcomes can be sustained when players interact repeatedly. By studying this question, I aim to contribute to the literature studying the factors that help or hinder collusion. For example, it is now well known that concentration, barriers to entry, cross-ownership, symmetry and multi-market contracts facilitate collusion–see Feuerstein (2005). Moreover, İriş and Santos-Pinto (2013) has studied the impact of fairness and reciprocity on collusion when firms' actions are strategic complements (e.g., price competition with products that are imperfect substitutes) and finds that fairness and reciprocity facilitate cooperation for fairly large and reasonable fair prices. Thus, this paper checks whether results are robust for the case in which firms' actions are strategic substitutes, e.g., quantity competition with products that are perfect substitutes.

To model reciprocal preferences I follow Segal and Sobel's (2007) and assume that players in a strategic environment have preferences not only over the outcomes but also the strategies.[4] A firm's utility is additively separable in monetary and fairness payoffs. Monetary payoffs are revenues minus costs and fairness payoffs are rival's weighted monetary payoffs in which the weight depends on how the rival's quantities are expected to differ from the fair ones. If a firm expects a rival to play a kind (mean) strategy, then it places a positive (negative) weight on rival's monetary payoff. If a firm expects a rival to play a fair strategy then it places zero weight on that rivals' monetary payoff.

The paper shows that when firms have fairness concerns, collusive outcomes are easier to sustain if firms perceive moderately low quantities as fair. The intuition is the following. Under this assumption on the concept of fairness (i) the incentive to deviate from the collusive scheme is less when firms are reciprocal and/or (ii) the possible punishment phases that can be sustained are harsher.

This result is consistent with findings in Rabin (1993, 1997) which show that in a fairness equilibrium it is possible to sustain cooperation in the one shot Prisoners' Dilemma and in every period of the finitely repeated Prisoners' Dilemma.

[3] One can be critical about these findings based on their external validity. Especially, one would expect such fairness concerns to have limited impact among large firms. Nevertheless, small firms in local markets would be more likely motivated by such fairness concerns.

[4] For a discussion on other approaches of modeling reciprocity and social preferences in general, see İriş and Santos-Pinto (2013) pages 3-4.

The result is also in line with Dufwenberg and Kirchsteiger (2004) which show that cooperation in the "Sequential Prisoners' Dilemma game" is a sequential reciprocity equilibrium.

On the other hand, if only very low quantities deemed as fair, then fairness concerns and reciprocity might impede sustaining collusion. The intuition is the following. Under this assumption on the concept of fairness (i) the incentive to deviate from the collusive scheme is higher when firms are reciprocal while (ii) the possible punishment phases that can be sustained are harsher.

The main policy implication of this paper is that fairness concerns among producers can have adverse welfare consequences for consumers. My work stands in contrast with findings in Rabin (1993) and Rotemberg (2011) which show that fairness concerns by the part of consumers can increase consumer welfare.[5] Thus, social preferences in imperfectly competitive markets might lead to different outcomes depending on who has such preferences (producers or consumers) and what is the comparison group.

The rest of the paper proceeds as follows. Section 2 sets-up the model. Section 3 studies the impact of fairness and reciprocity on a stage game. Section 4 studies the impact that fairness and reciprocity have on incentives for collusion when firms use grim trigger strategies. Section 5 concludes the paper. The Appendix contains the proofs of all results.

2. SET-UP

Segal and Sobel (2007) come up with an axiomatic foundation for interdependent preferences that can capture reciprocity, inequity aversion, altruism, and spitefulness. The advantage of their approach is that players in a strategic environment have both conventional preferences over outcomes and also preferences over strategy profiles. This allows players' preferences being affected by the others' behavior.

I apply Segal and Sobel (2007) approach to a dynamic quantity competition game where $N = 2$ firms play the same stage game infinitely many times.[6] More precisely, given that preferences are common knowledge and a quantity profile q describing how the game is expected to be played, each firm i, which expects

[5] For example, Rabin (1993) shows that a monopolist ought to set price lower than "the monopoly price" if consumers have concerns about fairness.

[6] The decision of focusing Cournot duopoly will be explained in the next section.

the rival to play q_j, decides how much to supply q_i. Its payoff in that stage is

$$u_i(q_i, q_j, q_j^f) = \pi_i(q_i, q_j) + \alpha w_i(q_j, q_j^f)\pi_j(q_i, q_j), \tag{1}$$

where $\pi_i(q_i, q_j)$ is the monetary payoff and $\alpha w_i(q_j, q_j^f)\pi_j(q_i, q_j)$ is the fairness payoff, with $\alpha > 0$. Firm i's monetary payoff, $\pi_i(q_i, q_j)$, is the difference between revenue and cost, that is,

$$\begin{aligned} \pi_i(q_i, q_j) = & \quad R_i(q_i, q_j) - C_i(q_i) \\ = & \quad P(Q)q_i - C_i(q_i), \end{aligned} \tag{2}$$

where $R_i(q_i, q_j)$ is revenue, $C_i(q_i)$ is the cost of production, and $P(Q)$ is the inverse market demand with $Q = \sum q_i$. I assume that $P(Q)$ is strictly positive on some bounded interval $(0, \bar{Q})$ with $P(Q) = 0$ for $Q \geq \bar{Q}$. I also assume that $P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ (in the interval for which $P(Q) > 0$). firms' costs of production are assumed to be twice continuously differentiable with $C_i'(q_i) \geq 0$. It is also assumed that the decreasing marginal revenue property holds, that is, $P'(Q) + P''(Q)q_i \leq 0$, and $P'(Q) - C_i''(q_i) < 0$.

Furthermore, I assume that the weight that firm i places on the rival's monetary payoff depends on firm i's perception of the fairness-neutral output of the rival, q_j^f, and on the actual output of the rival:

$$w_i(q_j, q_j^f) \begin{cases} > 0 \text{ if } q_j < q_j^f \\ = 0 \text{ if } q_j = q_j^f \\ < 0 \text{ otherwise} \end{cases}, \tag{3}$$

where $w_i(q_j, q_j^f)$ is assumed to be differentiable in both arguments with $\partial w_i / \partial q_j < 0$ and $\partial w_i / \partial q_j^f > 0$. The central behavioral feature of these preferences is the assumption that firms care about the intentions of the rivals. The first condition in (3) expresses constructive or positive reciprocity. If a firm expects its rival to produce less than her perception of fairness-neutral output, then it is willing to sacrifice some of its monetary payoff to increase the rival's monetary payoff. The third condition expresses destructive or negative reciprocity. If a firm expects its rival to produce more than its perception of fairness-neutral output, then it is willing to sacrifice some of its monetary payoff to reduce the rival's monetary payoff. Finally, the second condition states that if a firm expects its rival to produce exactly equal to its perception of fairness-neutral output, then u_i collapses to π_i.

3. STAGE GAME

In this section, I study the impact of fairness and reciprocity on the outcome of static quantity competition. There are four types of existence results which may apply to the Cournot model. The first type of result uses the standard existence theorem due to Nash and shows that every N-firm Cournot oligopoly has a Nash equilibrium if each firm's payoff is quasiconcave in q_i.[7]

The second type of result, due to Bamon and Frayssé (1985) and Novshek (1985), shows that every N-firm Cournot oligopoly has a Nash equilibrium if each firm's payoff depends on other firms' outputs only via their sum and marginal revenue is a decreasing function of the aggregate output of all other firms.

The third type of result deals with cases in which the Cournot game is a supermodular game. Here there are two different types of results, one for $N = 2$ and another one for $N \geq 2$. Milgrom and Roberts (1990) show that if the natural order on of one of the firms' action sets is reversed, then the Cournot duopoly is a supermodular game.[8] Amir (1996) provides conditions under which the N-firm Cournot oligopoly is a log-supermodular game. However, under these conditions, best replies are increasing which is not considered to be the "normal" case in Cournot games. Finally, Tarsky (1955) and Roberts and Sonnenschein (1976), show that every N-firm symmetric Cournot oligopoly has a Nash equilibrium if cost functions are convex.

The goal is not only to prove existence of equilibria for the Cournot game with reciprocal firms but also to state comparative static results. The assumptions required to state each of the four existence results imply different trade-offs between generality in existence versus generality in comparative static results. I decided to focus on the Cournot duopoly case and treat it as a supermodular game.

Let $\Gamma^s(\pi)$ and $\Gamma^r(u,w,q^f)$ denote the static game with self-interested and reciprocal firms, respectively, where $q^f = (q_j^f, q_i^f)$ is the vector of firms' perception of the fairness-neutral output of rivals. I am going to use the superscripts "*ns*" and "*nr*" to refer Nash equilibrium quantities of the games $\Gamma^s(\pi)$ and $\Gamma^r(u,w,q^f)$, respectively. The first result guarantees that the Cournot duopoly game with reciprocal managers is a supermodular game.

Lemma 1. *If u_i has decreasing differences in (q_i, q_j), then $\Gamma^r(u,w,q^f)$ is a supermodular game.*

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[7] This existence result is quite restrictive. See Ch. 4 in Vives (2001).

[8] This argument breaks down when there are three or more firms.

The assumption that the payoff function has decreasing differences in (q_i, q_j) means that the marginal returns to a manager from increasing output are lower if the rival produce a higher output. If u_i is differentiable, then this condition is equivalent to cross-partial derivatives to be non-positive, $\frac{\delta^2 u_i}{\delta q_i \delta q_j} \leq 0$. In other words, the choice variables are strategic substitutes.

Note that if firms only care about monetary payoffs, then the requirement that π_i has decreasing differences in (q_i, q_j) boils down to the assumption that the revenue of firm i has decreasing differences in (q_i, q_j). However, if firms have preferences for reciprocity, then the requirement that u_i has decreasing differences in (q_i, q_j) also implies that the weight that firm i places on the payoff from reciprocity can not be too large by comparison to the weight the firm places on monetary payoffs.

Thus, if preferences for reciprocity are very important relative to monetary payoffs, then quantities may be strategic complements over some output ranges and strategic substitutes over others. If that happens, then I can no longer use the theory of supermodular games to state general results that characterize the impact of reciprocity on Cournot competition. I rule out this possibility by assuming α to be small.

If $\Gamma^r(u, w, q^f)$ is a supermodular game, then it follows from Topkis (1979), that the equilibrium set is non-empty and has a smallest and a largest pure-strategy Cournot-Nash equilibrium.[9] The next result shows how firms' perceptions of the fairness-neutral quantities of their rivals change the outcome of Cournot competition.

Proposition 1. *If $\Gamma^r(u, w, q^f)$ is a supermodular game, and u_i has decreasing differences in $\left(q_i, q_j^f\right)$, then the smallest and the largest Cournot-Nash equilibria of $\Gamma^r(u, w, q^f)$ are nonincreasing functions of q^f.*

This result tells that if the weight that firms place on reciprocity is relatively small by comparison to the weight they place on monetary payoffs and the marginal returns from increasing output are decreasing with firms' perceptions of the fairness-neutral output of their rivals, then the higher are firms' perceptions of the fairness-neutral output of their rivals, the lower is the set of Cournot-Nash equilibria.[10]

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[9] This assumption that u_i has decreasing differences in (q_i, q_j) guarantees that best replies are decreasing and this implies existence of equilibrium.

[10] Note that this result does not imply that all Nash equilibria of $\Gamma^r(u, w, q^f)$ are nonincreasing functions of q^f. In fact a Nash equilibrium in the interior of the set of Nash equilibria of $\Gamma^r(u, w, \bar{q}^f)$ may be higher than the correspondent Nash equilibrium in the interior of the set of

The intuition behind this result is straightforward. The assumption that a firm's payoff function has decreasing differences in (q_i, q_j^f) means that the larger a reciprocal firm perceives the fairness-neutral output of its rival to be, the smaller are the marginal returns from increasing production. Thus, an increase in q_j^f shifts the best reply of a reciprocal firm i towards the origin. In other words, the more firm i perceives the fairness-neutral output of its rival to be high, the more it is willing to produce a smaller output level for any output level of the rival. If this happens for every firm, then the higher are firms' perceptions of the fairness-neutral quantities of their rivals the lower will be the set of Cournot-Nash equilibria.

Proposition 1 is a comparative statics result that characterizes the impact that firms' perceptions of the fairness-neutral output of their rivals have on equilibrium quantities of Cournot duopoly. I am also interested in comparing the outcome of Cournot duopoly among firms with reciprocal managers to that of Cournot duopoly among firms that only care about maximizing monetary payoffs. To do that I compare the equilibria of game $\Gamma^s(\pi)$, the standard supermodular Cournot game with self-interested firms, to the equilibria of $\Gamma^r(u,w,q^f)$, the supermodular Cournot game with reciprocal firms. I assume that these two games are identical in all respects (market demand, costs, and number of firms) with the exception of firms' preferences. However, allowing for multiple equilibria makes the comparison cumbersome. Thus, I assume that the game $\Gamma^s(\pi)$ has decreasing differences in (q_i, q_j), and that best replies have a slope greater than -1. It is a well known result that these two conditions guarantee that $\Gamma^s(\pi)$ has a unique equilibrium. Lemma 2 provides conditions under which the game $\Gamma^r(u,w,q^f)$ also has a unique equilibrium.

Lemma 2. *If $\Gamma^r(u,w,q^f)$ is a supermodular game, and firms' best replies have a slope greater than -1, then there exists an unique equilibrium of $\Gamma^r(u,w,q^f)$.*

This result guarantees that the supermodular Cournot game with reciprocal firms has a unique equilibrium. The condition that drives the result is the assumption that best replies have a slope strictly between $(-1,0)$. Before continuing with the next result, let me summarize the assumptions I made for the rest of the paper.

Assumption 1.

- *There are two firms, $N = 2$.*

Nash equilibria of $\Gamma^r(u,w,\hat{q}^f)$ with $\hat{q}^f$ higher than $\bar{q}^f$. Still, a decrease in equilibrium output can be justified by a coordination argument since the smallest Cournot-Nash equilibrium is the most preferred equilibrium for firms whereas the largest equilibrium is the less preferred one.

- $\Gamma^s(\pi)$ *is a supermodular game such that best replies have a slope greater than* -1,
- $\Gamma^r(u,w,q^f)$ *is a supermodular game such that*
  - α *is small,*
  - u_i *has decreasing differences in* (q_i, q_j^f),
  - *the firms' best replies have a slope greater than* -1.

I am now ready to state the first result that compares the outcome of Cournot duopoly with reciprocal firms to that of Cournot duopoly with self-interested firms.

Proposition 2. *If* $q_j^f = q_j^{ns}$ *for all* j, *then the Nash equilibrium of* $\Gamma^s(\pi)$ *coincides with that of* $\Gamma^s(u,w,q^f)$, *that is,* $q^{ns} = q^{nr}$.

Proposition 2 shows that if reciprocal firms compete à la Cournot and perceive the fairness-neutral quantities of their rivals to be equal to the quantity that the rivals would produce if they only cared about monetary payoffs, then they will produce the same quantities as the ones produced by self-interested firms. In this case preferences for reciprocity just pivot firms' best replies around the Cournot-Nash outcome of the game played self-interested firms and so the equilibrium is left unchanged. In this case market output, consumer welfare, and monetary payoffs are the same with reciprocal firms or self-interested firms.

Proposition 2 tells that a critical condition for the Cournot-Nash equilibrium of the game with reciprocal firms to differ from the Cournot-Nash equilibrium of the game with self-interested firms is that reciprocal firms' perceptions of the fairness-neutral output of their rivals are different from the equilibrium output of the rivals when firms are self-interested. The next result explores the implications of this possibility.

Proposition 3. *If* $q_j^f > (<) q_j^{ns}$ *for all* j, *then the Nash equilibrium of* $\Gamma^s(\pi)$ *is greater (smaller) than that of* $\Gamma^r(u,w,q^f)$, *that is,* $q^{ns} > (<) q^{nr}$.

Proposition 3 tells that if reciprocal firms perceive the fairness-neutral quantity of their rivals to be greater than the equilibrium quantity that the rivals would produce if all firms only cared about monetary payoffs, then reciprocal firms will produce less than self-interested firms. This is the constructive reciprocity equilibrium. On the other hand, if reciprocal firms perceive the fairness-neutral quantity of their rivals to be smaller than the equilibrium quantity that the rivals would produce if all firms only cared about monetary payoffs, then reciprocal firms will produce more than self-interested firms. This is the destructive reciprocity equilibrium.

In a constructive reciprocity equilibrium market output is smaller than the one of Cournot competition with self-interested firms. Thus, consumers are worse off if reciprocal firms' perceptions of fairness lead to a constructive reciprocity equilibrium than if firms only care about maximizing monetary payoffs. The opposite happens in a destructive reciprocity equilibrium: market output is larger than that in the equilibrium of the Cournot game with firms that only care about monetary payoffs and consumers are better off.

For the rest of the paper, the following assumption is now introduced:

Assumption 2. $q_j^f < q_j^{ns}$ *for all* j,.

This assumption means that too high output levels are considered unfair. Since I focus on the tacit collusion among firms, this is a reasonable assumption.

4. DYNAMIC GAME

In this section, I analyze the impact of fairness and reciprocity on collusion using a dynamic quantity competition set-up. To this end, the symmetric static quantity competition games $\Gamma^s(\pi)$ and $\Gamma^r(u,w,q^f)$ will be played over an infinite horizon.

The repeated game monetary payoff of firm i of choosing quantity $q_i = (q_i^1, q_i^2, \ldots)$ when the rival plays strategy q_j is given by

$$\Pi_i(q_i,q_j) = \sum_{t=1}^{\infty} \delta^{t-1}\pi_i(q_i^t,q_j^t), \tag{4}$$

where $\pi_i(q_i^t,q_j^t)$ represents firm i's monetary payoff at stage t, a function of firm i's quantity supplied at t, q_i^t, and the quantity supplied by the rival at t, q_j^t. Firms discount the future at rate $\delta \in (0,1)$.

To model reciprocity I assume that the weight firm i places on the rival's repeated game monetary payoff depends only on the rival's quantity and on firm i's perception of what is the fairness-neutral quantity for the rival to supply, q_j^f. I also assume throughout that firms' preferences as well as their exogenous perceptions of the fairness-neutral quantities of the rival are common knowledge. The repeated game payoff of reciprocal firm i of choosing strategy $q_i = (q_i^1, q_i^2, \ldots)$ when the rival play strategies q_j is given by

$$U_i(q_i,q_j,q_j^f) = \sum_{t=1}^{\infty} \delta^{t-1}\pi_i(q_i^t,q_j^t) + \alpha \sum_{t=1}^{\infty} \delta^{t-1} w_i(q_j^t,q_{ij}^f)\pi_j(q_i^t,q_j^t). \tag{5}$$

Denote the dynamic game with reciprocal firms by $\Gamma^r_\infty(u,q)$, where $u = (u_i,u_j)$ and $q = (q_i,q_j)$ and the dynamic game with self-interested firms by $\Gamma^s_\infty(\pi,q)$, where $\pi = (\pi_i,\pi_j)$. Firms are able to sustain a collusive outcome when the payoff from collusion is no less than the payoff from deviation. To understand how fairness and reciprocity influence collusion I will compare the incentive compatibility condition of self-interested firms in $\Gamma^s_\infty(\pi,q)$ to that of reciprocal firms in $\Gamma^r_\infty(u,q)$ assuming that these two games are identical in all respects (monetary payoffs and the number of firms) with the exception of firms' preferences.

The standard model used to study collusion in infinitely repeated games assumes that firms use grim trigger strategies to punish any deviation from collusion, that is, following a deviation firms switch to a Nash equilibrium of the stage game forever after. Thus, when self-interested firm uses grim trigger punishments in $\Gamma^s_\infty(\pi,q)$, each firm i will prefer to play its collusive quantity $q^c_i = (q^c_i, q^c_i, ...)$ if the payoff from collusion, $\pi_i(q^c)/(1-\delta)$, is no less than the payoff from defection which consists of the one period gain from deviating $\pi_i(BR^s_i(q^c_j),q^c_j)$ plus the discounted payoff of inducing Nash reversion forever $\delta\pi_i(q^{ns})/(1-\delta)$, that is,

$$\pi_i(BR^s_i(q^c_j),q^c_j) + \frac{\delta}{1-\delta}\pi_i(q^{ns}) \leq \frac{1}{1-\delta}\pi_i(q^c). \tag{6}$$

Solving for δ I obtain

$$\delta^s_{q^c} = \frac{\pi_i(BR^s_i(q^c_j),q^c_j) - \pi_i(q^c)}{\pi_i(BR^s_i(q^c_j),q^c_j) - \pi_i(q^{ns})} \leq \delta. \tag{7}$$

The collusion strategy profile q^c can be sustained by self-interested firms who are patient enough such that $\delta^s_{q^c} \leq \delta$ where $\delta^s_{q^c}$ is the critical discount factor above which q^c can be sustained by self-interested firms.

The same reasoning applies when firms have reciprocal preferences. A reciprocal firm i plays the collusive strategy q^c_i in $\Gamma^r_\infty(u,q)$ using a grim trigger strategy as long as the following condition holds

$$u_i(BR^r_i(q^c_j),q^c_j,q^f_j) + \frac{\delta}{1-\delta}u_i(q^{nr},q^f_j) \leq \frac{1}{1-\delta}u_i(q^c,q^f_j). \tag{8}$$

Solving for δ I obtain

$$\delta^r_{ac} = \frac{u_i(BR^r_i(q^c_j),q^c_j,q^f_j) - u_i(q^c,q^f_j)}{u_i(BR^r_i(q^c_j),q^c_j,q^f_j) - u_i(q^{nr},q^f_j)} \leq \delta. \tag{9}$$

When firms have reciprocal preferences it follows that the collusive strategy profile q^c can be sustained if firms are patient enough such that $\delta^r_{q^c} \leq \delta$ where $\delta^r_{q^c}$ is the critical discount factor above which q^c can be sustained by reciprocal firms.

I will use (7) and (9) to characterize the impact that fairness and reciprocity have on collusion when firms use grim trigger strategies. To perform this analysis I compare the critical discount factor above which the collusive strategy profile can be sustained when firms are self-interested to the critical discount factor when firms are reciprocal. I say that fairness and reciprocity facilitate collusion when the collusive strategy profile can be sustained at a lower critical discount factor when firms are reciprocal than when they are self-interested. If the opposite happens I say that fairness and reciprocity make collusion harder.

The main result shows that reciprocity facilitates collusion if firms think that the fairness-neutral output of their rivals is greater than or equal to the rivals' joint self-interested collusive output but smaller than or equal to the rivals' joint self-interested Nash output.

Proposition 4. *If $\Gamma^r(u,q)$ and $\Gamma^s(\pi,q)$ satisfy the conditions stated in Assumption 1, and $q^f_j \in [q^c_j, q^{ns}_j]$ for all j, then the critical (minimum) discount factor needed to sustain collusion at any q^c, which satisfies $\pi_i(q^c) > \pi_i(q^{ns})$ for all i, is lower in $\Gamma^r_\infty(u,q)$ than in $\Gamma^s_\infty(\pi,q)$, that is, $\delta^r_{q^c} < \delta^s_{q^c}$.*

Proposition 4 shows that fairness and reciprocity also facilitate collusion when firms' choices are strategic substitutes. It says that if firms think that the fairness-neutral quantity of the rivals is greater than or equal to the collusive quantity but less than or equal to the quantity of the rivals in the Nash equilibrium of the stage game with self-interested firms, then it is easier to sustain collusion when firms are reciprocal than when they are self-interested.

The intuition for this result is as follows. If reciprocal firms think that the fairness-neutral quantity of their rivals is greater than the joint self-interested collusive quantity of the rivals, then playing the collusive quantity is more attractive in the dynamic quantity-setting game with reciprocal firms than in the game with self-interested firms. This happens because the collusive monetary payoffs are the same as the ones obtained in the game with self-interested firms but in addition there are payoff gains from constructive reciprocity since reciprocal firms think that their rivals are being kind.

Additionally, if reciprocal firms perceive that the fairness-neutral quantity of their rivals is smaller than the quantity of the rivals in the Nash equilibrium of the stage game with self-interested firms, then the punishment imposed after cheating occurs becomes more severe in the dynamic game with reciprocal firms than in the dynamic game with self-interested firms. This happens because, the

Nash equilibrium of the stage game with reciprocal firms becomes a destructive reciprocity state. This is bad for firms since it reduces monetary payoffs (by comparison with the monetary payoffs of self-interested firms) and leads to payoff loses from destructive reciprocity since reciprocal firms think that the rivals are being mean.

In contrast, the single period deviation payoff in the game with reciprocal firms is larger than the single period deviation payoff in the game with self-interested firms. This happens because the unilateral single period deviation payoff of a reciprocal firm also includes the benefit that firm derives from being treated kindly by the rival (the rival is playing its collusive quantity). However, this effect is of second-order since monetary payoffs are larger by comparison with fairness payoffs.

However, as the following proposition shows, this is no longer necessarily true when the players are highly demanding from each other regarding their quantities.

Proposition 5. *If* $\Gamma^r(u,q)$ *and* $\Gamma^s(\pi,q)$ *satisfy the conditions stated in Assumption 1, and* $q_j^f < q_j^c$ *for all j, then the effect of fairness concerns on the critical (minimum) discount factor needed to sustain collusion at any* q^c*, which satisfies* $\pi_i(q^c) > \pi_i(q^{ns})$ *for all i, is ambiguous.*

The intuition for this result is as follows. If players think that the fairness-neutral quantity of the rivals are lower than the collusive output, then collusion becomes a negative reciprocity state. In this case players' monetary payoffs from collusion are the same as the ones obtained in the game with self-interested players but in addition there are fairness payoff losses since players think that their rivals are being unkind. This effect makes collusion less attractive when players are reciprocal than when they are self-interested.

On the other hand, the punishment imposed after cheating occurs is still more severe when players are reciprocal than when they are self-interested, makes collusion more attractive when players are reciprocal than when they are self-interested. These two effects offset each other and reciprocity might impede collusion when players are highly demanding from each other.

5. CONCLUSION

This paper studies the impact of fairness and reciprocity on firms collusive strategies. It shows that fairness and reciprocity can facilitate collusion in infinitely repeated games if firms have preference to punish rivals that are "unkind" and reward rivals that are "kind."

İriş and Santos-Pinto (2013) study similar problem when firms compete in prices with products that are imperfect substitutes or more generally when firms' choices are strategic complements, and find that fairness and reciprocity can facilitate collusion. This paper shows that this result is robust when firms' choices are strategic substitutes by studying quantity competition with products that are perfect substitutes. Furthermore, Proposition 5 shows that if only extremely low quantities are perceived fair, then the impact of fairness concerns on the sustainability of tacit collusion becomes ambiguous.

While this model provides results on the sustainability of tacit collusion for different levels of exogenously determined perception of fairness-neutral quantities, how these perception of fairness-neutral quantities determined remain to be an open question. The difficulty of endogeneizing the perception of fairness-neutral quantities in this strategic environment is, firms would set them in fully strategical manner and, thus, determine what is fair and unfair only to maximize monetary payoffs. Thus, they would not capture firms' fairness concerns. Nevertheless, such strategic use of "fairness concerns" can be an interesting future avenue of research.

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APPENDIX

Proof of Lemma 1. It is a well known result that a Cournot duopoly game with decreasing best replies, when one firm's strategy set is given the reverse order, is a supermodular game–see pp. 34 in Vives (2001). If u_i has decreasing differences in (q_i, q_j), then firms' best replies are decreasing and so $\Gamma^r(u, w, q^f)$ is a supermodular game. □

Proof of Proposition 1. This proposition follows immediately from Theorem 6 of Milgrom and Roberts (1990). □

Proof of Lemma 2. This lemma follows immediately from Theorem 2.8 in Vives (2001). □

Proof of Proposition 2. I know by Lemma 2 that $\Gamma^r(y, w, q^f)$ has a unique equilibrium. Let $q^{nr} = (q_1^{nr}, \ldots, q_n^{nr})$ denote the unique Nash equilibrium of $\Gamma^r(u, w, q^f)$. Let $q^{ns} = (q_1^{ns}, \ldots, q_n^{ns})$ denote the unique Nash equilibrium of $\Gamma^s(\pi)$. I would like to show that if $q_j^f = q_j^{ns}$, then $q_i^{nr} = q_i^{nr}$, for all i. To do that I only need show that a reciprocal firm i has no incentive to deviate from $q_i^{nr} = q_i^{ns}$ when its rival plays $q_j^{nr} = q_f^{ns} = q_j^f$. But, if $q_j^f = q_j^{ns} = q_j^{nr}$ then $w_i(q_j, q_j^f) = 0$. If that is the case, then the best reply of firm i to q_j^{nr} is indeed $q_i^{nr} = q_i^{ns}$. □

Proof of Proposition 3. I know from Proposition 2 that if $q^f = q^{ns}$, then $q^{nr} = q^{nr}$. If $q^f > (<) q^f = q^{ns}$, then Proposition 1 implies that the unique Cournot-Nash equilibrium of $\Gamma^r(u, w, q)^f)$ is smaller (greater) than the unique Cournot-Nash equilibrium of $\Gamma^r(u, w, q^f)$. □

Proof of Proposition 4. I need to show that $q_j^f \in [q_j^c, q_j^{ns}]$ for all j, implies $\delta_{q^c}^r < \delta_{q^c}^s$, where $\delta_{q^c}^r$ is the critical discount factor above which q^c can be sustained in $\Gamma_\infty^r(u, q)$ and $\delta_{q^c}^s$ is the critical discount factor above which q^c can be sustained in $\Gamma_\infty^s(\pi, q)$. From (7) and (9) sufficient conditions are that

$$u_i(BR_i^r(q_j^c), q_j^c) - u_i(q^c) \leq \pi_i(BR_i^s(q_j^c), q_j^c) - \pi_i(q^c) \tag{10}$$

and

$$u_i(BR_i^r(q_j^c), q_j^{cs}) - u_i(q^{nr}) \geq \pi_i(BR_i^s(q_j^c), q_j^{cs}) - \pi_i(q^{ns}). \tag{11}$$

(i) I start by showing that $q_j^f \in [q_j^c, q_j^{ns}]$ implies (10) is satisfied as a strict inequality. I have that

$$\begin{aligned} & u_i(BR_i^r(q_j^c), q_j^c) - u_i(q^c) = \pi_i(BR_i^r(q_j^c), q_j^c) - \pi_i(q^c) \\ & + \alpha w_i(q_j^c, q_j^f)\left[P(BR_i^r(q_j^c) + q_j^c) - P(q^c)\right] q_j^c \\ & \qquad \leq \pi_i(BR_i^r(q_j^c), q_j^c) - \pi_i(q^c) < \pi_i(BR_i^s(q_j^c), q_j^c) - \pi_i(q^c) \end{aligned}$$

The strict inequality follows from the fact that $BR_i^s(q_j^c)$ is the best reply to q_j^c for self-interested firms. If $q_j^c \leq q_j^f$ then $w_i(q_j^c, q_j^f) \geq 0$. Furthermore, $q_j^f \leq q_j^{ns}$ implies $BR_i^r(q_j^c) > q_i^c$ which in turn implies $P(BR_i^r(q_j^c) + q_j^c) < P(q^c)$, since $P'(\cdot) < 0$.

(ii) I now show that $q_j^f \in [q_j^c, q_j^{ns}]$ implies that (11) is satisfied. Rewrite (11) as

$$[u_i(BR_i^r(q_j^c), q_j^c) - \pi_i(BR_i^s(q_j^c), q_j^c)] + [\pi_i(q^{ns}) - u_i(q^{nr})] \geq 0.$$

I have that

$$u_i(q^{nr}) = \pi_i(q^{nr}) + \alpha w_i(q_j^{nr}, q_j^f)\pi_j(q^{nr}) \leq \pi_i(q^{ns}).$$

The inequality follows from $w_i(q_j^{nr}, q_j^f) \leq 0$ and Proposition 3, which shows that $q_j^f \leq q_j^{ns}$ for all j implies $q_i^{ns} \leq q_i^{nr}$ and $\pi_i(q^{nr}) \leq \pi_i(q^{ns})$, for all i. Taking a first-order Taylor series expansion of $u_i(BR_i^r(q_j^c), q_j^{cs})$ around $\alpha = 0$ I have that

$$\begin{aligned} u_i(BR_i^r(q_j^c), q_j^c) \quad \approx \quad & \pi_i(BR_i^s(q_j^c), q_j^c) \\ & + \alpha w_i(q_j^c, q_j^f)\pi_j(BR_i^s(q_j^c), q_j^c). \end{aligned}$$

which is equivalent to

$$\begin{aligned} & u_i(BR_i^r(q_j^c), q_j^c) - \pi_i(BR_i^s(q_j^c), q_j^c) \approx \\ & \qquad \qquad \alpha w_i(q_j^c, q_j^f)\pi_j(BR_i^s(q_j^c), q_j^c) \geq 0 \end{aligned}$$

since $q_j^c \leq q_j^f$ implies that $w_i(q_j^c, q_j^f) \geq 0$. Thus, $q_j^f \in [q_j^c, q_j^{ns}]$ for all i, implies $\delta_{q^c}^r < \delta_{q^c}^s$. □

Proof of Proposition 5. If $q_j^f < Q_j^c$ for all j, then the following term

$$\alpha w_i(q_j^c, q_j^f)\left[P(BR_i^r(q_j^c) + q_j^c) - P(q^c)\right] q_j^c \tag{12}$$

I used to show (10) in the proof of proposition 4 might become negative, and might lead the reverse condition to hold,

$$u_i(BR_i^r(q_j^c), q_j^c) - u_i(q^c) > \pi_i(BR_i^s(q_j^c), q_j^c) - \pi_i(q^c) \tag{13}$$

Therefore, the effect of fairness and reciprocity on the critical (minimum) discount factor needed to sustain collusion at q^c is ambiguous, i.e., either $\delta_{q^c}^r < \delta_{q^c}^s$ or $\delta_{q^c}^r \geq \delta_{q^c}^s$. □