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PROFIT TRANSFER WITHIN A VERTICAL RELATIONSHIP*

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Abstract

We consider a vertical relationship between a single upstream firm and a single downstream firm and examine the economic effects of the profit transfer program, where the downstream firm transfers a predetermined share of its profit to the upstream firm. We analyze the effects under two scenarios, according as how the price is determined in the upstream market. One is where the upstream firm sets the price of the intermediate good and the downstream firm takes the price as given. The other is where the downstream firm acts as a monopsonist and sets the price of the intermediate good. In the former scenario, the profit transfer alleviates the problem of 'double marginalization' and enhances economic efficiency. The downstream firm will hire more intermediate good and will produce more output. And the upstream firm will increase the effort level to reduce the production cost. The consumer surplus and the social welfare will rise. On the other hand, in the latter scenario, the profit transfer has opposite effects. It induces the downstream firm to hire less intermediate good. The upstream firm's effort level to reduce the production cost decreases. As a result, the output of the final good, the consumer surplus, and the social welfare decrease. We will also examine how the profit transfer affects the individual firms' profits.

Keywords Profit Transfer, Upstream Frim, Downstream Firm, Double Marginalization, Monopsony

JEL Classification D4, L1

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1. INTRODUCTION

Big corporates undoubtedly has led the Korean economy to its current status for several decades. It is true however that the gap between the big corporates and small- and medium-sized enterprises (SMEs) has widened in profitability, employee wages, and etc. during that time. Many experts point out the gap as one of main obstacles for the Korean economy to leap into another higher level. In 2011, amid such concerns on the widening gap and doubts on sustainable growth of the Korean economy, Unchan Chung, the first president of Korea Commission for Corporate Partnership and economist himself, proposed 'profit sharing' between the big corporates and their subcontractors, mostly SMEs. Since then, there has been large public controversy in Korea between the proponents and the opponents. It is expected to become even fiercer and more materialized as the current administration is known to take a positive view on 'profit sharing'. At present, however, the exact concept of 'profit sharing' and the details of the program and its implementations are still vague even in the academia as well as to the government authorities and the public. Nevertheless, it would be helpful to take a theoretical approach on this issue and to examine the effects of 'profit sharing' through a simple economic model.

It should be made clear that 'profit sharing' here is different from that under the same terminology widely used in economic literature. In economics, profit sharing commonly encompasses to any system which has a *direct* link between the profits of a company in a particular period and the compensation of employees in that period (Kruse, 1992). That is, profit sharing is the compensating plans to give employees a partial stake in their company's profits, aside from their regular salary and bonuses. There is a huge literature on the relationship of profit sharing plans to workers' productivity and firm's profitability, including Weitzman and Kruse (1990), Kruse (1992), Kim (1998), just to name a few. In contrast with the literature, 'profit sharing' is understood here as incentive plans within a vertical relationship, or between the firms who occupy different stages in the vertical chain of production or distribution of goods and services. In particular, in this paper it refers to profit sharing between the big manufacturing firms and the subcontractors who supply the inputs for them. Accordingly, we will refer to the former as the downstream firms and to the latter as the upstream firms. ‘Profit sharing’ is also different from revenue sharing, which has become prevalent practices in many industries.[1] In a revenue sharing contract, the downstream firm pays to the upstream firm a fixed share of the overall revenue it generates, in addition to the input price for each unit it purchases. On the other hand, ‘profit sharing’ in this paper is either a division of the joint profits of the downstream firm and the upstream firm, or a transfer of one firm’s profit to the other, as we will explain shortly.

It is well known in economics that independent decisions of the downstream firms and the upstream firms in vertical relationships lead to a loss of economic efficiency, called the problem of ‘double marginalization’, unless they are vertically integrated or there are some coordinating mechanisms between them. Since ‘profit sharing’ currently at issue in Korea addresses the vertical relations between the manufacturers and their suppliers, it clearly has some bearing on the problem of ‘double marginalization’ and thus will affect economic efficiency. Broadly we can think of two types of ‘profit sharing’, although the specific ones are not known at present, from a theoretically standpoint. One is the program under which the downstream firms and the upstream firms maximize their joint profits as a vertically integrated firm, and share the joint profits between them by a predetermined rule. This program will eliminate the problem of ‘double marginalization’ and achieve economic efficiency if there are no other complications such as asymmetric information between the downstream firms and the upstream firms. Then the only issue of the program is how to divide the realized joint profits and it purely becomes a problem of fairness, which is beyond the scope of this paper. The other type of ’profit sharing’ is a profit transfer program where the downstream firms and the upstream firms each maximize their own profits, and one party, say the former, transfers a predetermined share of its profit to the other, say the latter. In this paper, we only focus our attention to this profit transfer program. In particular, we consider a vertical relationship between

[1] Dana and Spier (2001) and Cachon and Lariviere (2005) provide examples of the prevalence of revenue sharing in the video retail industry

a single upstream firm and a single downstream firm and examine the economic effects of the profit transfer program, where the downstream firm transfers a predetermined share of its profit to the upstream firm. The direction of the profit transfer is mainly for the practical reason in Korea. The downstream firms in the main industries are big manufactures while their suppliers, the upstream firms, are small or medium- sized subcontractors who earn much smaller profits compared to their downstream counterparts.

Transactions in the intermediate good market between the upstream firm and the downstream firm occur in very complicated fashions, and differ in many respects from the final good market. While a simple uniform pricing is typical in the latter, more complex selling schemes, such as two-part tariffs, royalties, exclusive dealings, and etc., are widely used in the former market. As a matter of fact, there is an extensive literature on the contractual forms between the upstream firm and the downstream firm. Katz (1989) provides a comprehensive survey on this topic and Secrieru (2006) is another excellent survey paper. To sidestep the complications, we exclude nonlinear pricing in the intermediate good market and only consider a simple uniform pricing. In particular, we focus on two types of the intermediate good market, as regards how the price is determined. One is where the upstream firm sets the price of the intermediate good and the downstream firm takes the price as given. The other is where the downstream firm acts as a monopsonist and sets the price of the intermediate good. The former situation is more suitable if the upstream firm serves other downstream firms as well and provides essential inputs to a number of downstream firms. On the other hand, the latter fits better to situations where the downstream firm faces many suppliers who produce substitutable inputs and thus has much stronger bargaining power than the upstream firms. We believe that the latter situation is more similar to the reality in Korea.

We show that the economic effects of the profit transfer are completely different under the two scenarios. In the former scenario, the profit transfer will induce the upstream firm to charge a lower price for the intermediate good. As a result, the downstream firm will hire more intermediate good and will produce more output. And the upstream firm will increase the effort level to reduce the production cost. The consumer surplus and the social welfare will rise. Therefore, the profit transfer program enhances economic efficiency and alleviates the problem of 'double marginalization'. But in the latter scenario where the downstream firm is a monopsonist, the profit transfer hinders economic efficiency. It induces the downstream firm to charge a lower price for the intermediate good and to hire less intermediate good. The upstream firm's effort level to reduce the production cost decreases. As a result, the output of the final good, the consumer surplus, and the social welfare decrease. We also examine how the profit transfer affects the individual firm's profit. In the former scenario, the profit transfer will increase the upstream firm's profit, but it is ambiguous in general whether the downstream firm's profit increases or decreases. In the latter, the downstream firm's profit will decrease while the direction of the change in upstream firm's profit is ambiguous.

To our knowledge, there are not many economic studies to address the effect of 'profit sharing', let alone theoretical ones. Among a few policy-oriented articles, Kim (2012) identifies, through surveys and semi-structured interviews, a number of issues that can arise with introduction of 'profit sharing', and suggests several policy implications. There is however a research stream on the performance of revenue sharing, mostly in operation research and management science literature. As mentioned earlier, revenue sharing refers to a contract under which the downstream firm and the upstream firm share the final revenue that the former generates at an agreed ratio (Cachon, 2003). Dana and Spier (2001) analyze revenue sharing when there is uncertainty in the demand for the final good and the downstream market is perfectly competitive. They show that revenue sharing increases the joint profits of the industry as well as the upstream firm's profit by softening the competition in the downstream market and handling the inventory problems. Yao, Leung, and Lai (2008) investigate the performance of revenue sharing when one upstream firm faces two downstream firms and the demand is stochastic. Cachon and Lariviere (2005) study through a comprehensive model the environments under which revenue sharing performs well, and also identify several limitations of revenue sharing.

The paper is organized as follows. In section 2, we introduce the model of a vertical relationship between a single upstream firm and a single downstream firm. In section 3, we analyze the equilibrium under the former scenario where the upstream firm sets the price of the intermediate good, and examine the economic effects of the profit transfer. More precisely, we will look into how the profit transfer affects the price of the intermediate good, the output levels of the intermediate good and the final good, and the effort level of the upstream firm to reduce its production cost, as well as the individual firm's profit, the consumer surplus, and the social welfare. In section 4, we derive the equilibrium under the latter scenario where the downstream firm sets the price of the intermediate good, and investigate the economic effects of the profit transfer. Section 5 concludes. All the proofs of Propositions and Lemmas are relegated to the appendix.

2. THE MODEL

Suppose that there is a single upstream supplier and a single downstream firm. The upstream firm produces an intermediate good and sells it to the latter. We will describe shortly how the price of the intermediate good is determined. The downstream firm, who is a monopolist in the final good market, produces a final good using the intermediate good as a single input.[2] The inverse market demand function for the final good is given by $p = p(y)$, where p is the price of the final good and y is the output of the final good. $p(y)$ satisfies the regular conditions on the demand function: $p'(y) < 0$ and $MR'(y) = 2p'(y) + p''(y)y < 0$. We assume for simplicity that the production function of the downstream firm is $y = f(x) = x$, where x is the level of the intermediate good employed. We further assume that the downstream firm has no additional costs other than the expenditure cost on the intermediate goods. We can think of the downstream firm simply reselling the intermediate goods in the final market. Of course, the assumptions on the production function and the cost structure of the downstream firm is only for

[2] The setting that the downstream firm hires a single input is for simplicity and would not change the result of the paper. In section 5, we discuss the extension where the downstream firm hires another input.

simplicity. In particular, the production function $y = f(x) = x$ can be replaced by a standard one of $y = f(x)$, where $f'(x) > 0$ and the marginal revenue product $MRP(x) = p(f(x))f'(x) + p'(f(x))f'(x)f(x) = MR(f(x))f'(x)$ is decreasing in x.

The upstream firm's cost consists of two components. One is the production cost function and is given by $C_U(x,e)$, where x is the output level of the intermediate good and e is the upstream firm's effort level (or R&D investment level) to reduce the production cost. The other is the cost incurred by the effort, which we will denote by $F(e)$. We assume that $\frac{\partial C_U(x,e)}{\partial x} > 0$, $\frac{\partial^2 C_U(x,e)}{\partial x^2} \geq 0$, $\frac{\partial C_U(x,e)}{\partial e} \leq 0$, $\frac{\partial^2 C_U(x,e)}{\partial e^2} \geq 0$, and $\frac{\partial^2 C_U(x,e)}{\partial e \partial x} \leq 0$: the upstream firm's marginal production cost is positive and increasing in the output; increase in the effort level reduces the production costs, but in a decreasing rate; and the marginal production cost is lower when the effort level is higher. Because $\frac{\partial^2 C_U(x,e)}{\partial e \partial x} = \frac{\partial^2 C_U(x,e)}{\partial x \partial e}$, the last assumption also implies that $\frac{\partial C_U(x,e)}{\partial e}$, the marginal reduction in the production cost from an additional increase in the effort level, decreases in x. We also assume that $F'(e) > 0$ and $F''(e) \geq 0$. The cost incurred by the effort grows at an increasing rate as the effort level increases.

We will make additional assumptions, if necessary, to ensure the second order conditions of the upstream firm's and the downstream firm's profit maximization problems we will face in section 3 and section 4. In particular, we will assume throughout the paper that $\frac{\partial^2 C_U(x,e)}{\partial x^2}\left[\frac{\partial^2 C_U(x,e)}{\partial e^2} + F''(e)\right] - \left(\frac{\partial^2 C_U(x,e)}{\partial e \partial x}\right)^2 \geq 0$. This assumption will turn out to be slightly stronger than the second order conditions of the upstream firm's profit maximization problems in sections 3 and 4. But this assumption is innocuous in a sense that it is equivalent to the condition that the upstream firm's marginal production cost $\frac{\partial C_U(x,e)}{\partial x}$ is not decreasing when it chooses the optimal effort level for given x.[3] The main results of the paper would not change without the assumption.

Let us denote the price of the intermediate good by r. Without the profit transfer, the profits of the upstream firm and the downstream firm are then $\pi_U = rx - C_U(x,e) - F(e)$ and $\pi_D = p(x)x - rx$, respectively. With the profit transfer where $100 \times \alpha\%$ of the profit of the downstream firm is transferred to the up-

[3] See footnote 10.

stream firm, the profit of the upstream firm changes to $\pi_U + \alpha\pi_D = rx - C_U(x,e) - F(e) + \alpha(p(x)x - rx)$. The profit of the downstream firm becomes $(1-\alpha)\pi_D = (1-\alpha)(p(x)x - rx)$. Obviously $\alpha = 0$ refers to the case without the profit transfer.

We will consider two scenarios, as regards how the price of the intermediate good r is determined. One is where the upstream firm sets r first, before the downstream decides the level of the intermediate good to be employed. The other is the opposite case where the downstream firm sets the price of the intermediate good, and then the upstream firm decides the output level of the intermediate good for a given r. That is, the upstream firm is a monopolist in the intermediate good market in the former situation, while the downstream is a monopsonist in the intermediate good market in the latter.

Since the games we analyze under both scenarios are sequential, the equilibrium concept is backward induction equilibrium. To ensure existence of the equilibrium, we assume that $\lim_{x\to 0} MR(x) - \frac{\partial C_U(x,e)}{\partial x} > 0$ and $\lim_{x\to\infty} MR(x) - \frac{\partial C_U(x,e)}{\partial x} < 0$.[4] This is a trivial assumption that the marginal revenue (product) curve intersects the marginal production curve. We will have a unique equilibrium under this assumption along with the ones we already made on C_U and F.

3. WHEN THE UPSTREAM FRIRM IS A MONOPOLIST

3.1. THE DOWNSTREAM FIRM'S PROBLEM

We start with the former scenario in this section. The upstream firm simultaneously decides the price of the intermediate good r and the effort level (to reduce the production cost) e in the first period.[5] The downstream firm, after observing these, chooses the level of the intermediate good x in the second period. By

[4] In fact, the specific conditions for existence of the equilibrium in section 3 and section 4 are slightly different from this. But we will omit the specifications since both are similar and should be obvious from the main text.

[5] The change in the timing of the choice of the effort level e would not make the difference in the results.

backward induction, we first consider the downstream firm's profit maximization problem. Since the downstream firm's profit is $(1-\alpha)\pi_D = (1-\alpha)(p(x)x - rx)$, the first order condition is simply given by

$$\frac{d(1-\alpha)\pi_D}{dx} = (1-\alpha)\,[MR(x) - r] = 0 \quad or \quad MR(x) = r,$$

where $MR(x) = p(x) + p^{'}(x)x$ is the marginal revenue product of the intermediate good. The assumption of $MR^{'}(x) < 0$ ensures the second order condition $\frac{d^2(1-\alpha)\pi_D}{dx^2} < 0$. The first order condition requires the downstream firm to employ the intermediate good up to the level where $MR(x) = r$. Therefore $MR(x) = r$ becomes the downstream firm's demand curve for the intermediate good.

3.2. THE UPSTREAM FIRM'S PROBLEM AND THE EQUILIBRIUM

Inserting $MR(x) = r$ into $\pi_U + \alpha\pi_D$, the upstream firm's profit maximization problem can be regarded as choosing x and e, instead of r and e, and thus can be written as

$$Max_{x,e} \quad \pi_U + \alpha\pi_D = rx - C_U(x,e) - F(e) + \alpha(p(x)x - rx),$$

$$= (1-\alpha)MR(x)x + \alpha p(x)x - C_U(x,e) - F(e).$$

The first order conditions are

$$\frac{\partial(\pi_U + \alpha\pi_D)}{\partial x} = MR(x) + (1-\alpha)MR^{'}(x)x - \frac{\partial C_U(x,e)}{\partial x} = 0, \tag{3.1}$$

$$\frac{\partial(\pi_U + \alpha\pi_D)}{\partial e} = -\frac{\partial C_U(x,e)}{\partial e} - F^{'}(e) = 0. \tag{3.2}$$

We assume that $\pi_U + \alpha\pi_D$ is strictly concave in (x, e) so that the second order conditions for the upstream firm's profit maximization are satisfied.

To understand (3.1), notice first that the upstream firm's revenue changes to $(1-\alpha)rx + \alpha p(x)x = (1-\alpha)MR(x)x + \alpha p(x)x$ with the profit transfer, from $rx = MR(x)x$ in case of $\alpha = 0$. The cost $C_U(x, e) + F(e)$ remains the same. Therefore, (3.1) says that the upstream firm's marginal revenue after the profit transfer, $MR(x) + (1-\alpha)MR^{'}(x)x$, is equal to the marginal production cost. Because $MR^{'}(x) < 0$, the marginal revenue becomes higher compared to the case of $\alpha = 0$, and it gets higher and higher as α grows.

Interpretation of (3.2) is straightforward. The marginal benefit of the effort or the marginal reduction in the production cost, $-\frac{\partial C_U(x,e)}{\partial e}$, is equal to the marginal cost of the effort $F^{'}(e)$. It determines the upstream firm's optimal effort level as a function of x, which we will denote by $e = e(x)$. Before proceeding further, let us introduce a result on the upstream firm's optimal effort function $e(x)$.

Lemma 1: The upstream firm's optimal effort level increases in the output level of the intermediate good. More precisely,

$$\frac{\partial e(x)}{\partial x} = \frac{-\frac{\partial^2 C_U(x,e(x))}{\partial x \partial e}}{\frac{\partial^2 C_U(x,e(x))}{\partial e^2} + F''(e(x))} \geq 0.$$

Proof: See the appendix.

The intuition for Lemma 1 can be directly found from (3.2) and from the assumption of $\frac{\partial^2 C_U(x,e(x))}{\partial e \partial x} = \frac{\partial^2 C_U(x,e(x))}{\partial x \partial e} \leq 0$: The marginal benefit of the effort, or the marginal reduction in the production cost $-\frac{\partial C_U(x,e)}{\partial e}$, becomes higher as the output level increases while the marginal cost of the effort, $F^{'}(e)$, remains the same.

Now, putting $e = e(x)$ into (3.1), we have the equilibrium condition

$$MR(x) + (1-\alpha)MR'(x)x - \frac{\partial C_U(x, e(x))}{\partial x} = 0. \tag{3.3}$$

The difference between (3.1) and (3.3) is that the marginal production cost in (3.3), $\frac{\partial C_U(x,e(x))}{\partial x}$, is a reduced form in a sense that the effort level is evaluated at the optimum for given x. (3.3) determines the equilibrium level of the intermediate good as a function of α, $x^* = x^*(\alpha)$. The equilibrium effort level and the equilibrium price of the intermediate good are then determined by $e^*(\alpha) = e(x^*(\alpha))$ and $r^*(\alpha) = MR(x^*(\alpha))$. We provide the following results as to how the profit transfer affects the equilibrium.

Proposition 1: In equilibrium we have $\frac{dx^*}{d\alpha} > 0$, $\frac{de^*}{d\alpha} \geq 0$, and $\frac{dr^*}{d\alpha} < 0$. That is, the profit transfer increases both the equilibrium level of the intermediate good and the equilibrium effort level to reduce the production cost. Moreover, both increase in α, the ratio of the profit transfer from the downstream firm to the upstream firm. The equilibrium price of the intermediate good is lower with the profit transfer and decreases in α.

Proof: See the appendix.

The results of $\frac{dx^*}{d\alpha} > 0$ and $\frac{dr^*}{d\alpha} < 0$ can be easily verified from Figure 1. Figure 1 illustrates three levels of intermediate good. x^o and x^* denote the equilibrium levels in case of $\alpha = 0$ and in case of $0 < \alpha < 1$, respectively. x^J is the level that maximizes the joint profits of the upstream firm and the downstream firm, $\pi_U + \alpha\pi_D + (1-\alpha)\pi_D = \pi_U + \pi_D = p(x)x - C_U(x,e) - F(e)$. Notice that x^o, x^*, and x^J satisfy (3.3), the equilibrium condition $MR(x) + (1-\alpha)MR'(x)x = \frac{\partial C_U(x,e(x))}{\partial x}$, in cases of $\alpha = 0$, $0 < \alpha < 1$, and $\alpha = 1$, respectively. Clearly we have $x^o < x^* < x^J$, because the upstream firm's marginal revenue after the profit transfer $MR(x) + (1-\alpha)MR'(x)x$ gets higher as α grows, while the marginal production cost $\frac{\partial C_U(x,e(x))}{\partial x}$ remains the same. We also have $r^* < r^o$ since the equilibrium price of the intermediate good is determined by $r(x) = MR(x)$, and $MR(x)$ is downward-sloping. The result of $\frac{de^*}{d\alpha} = \frac{\partial e}{\partial x}\frac{dx^*}{d\alpha} \geq 0$ is obvious because $e(x)$ is an increasing function of x.

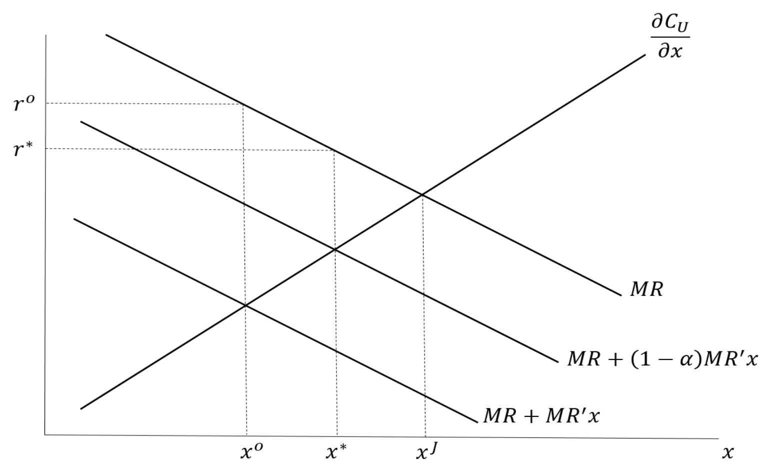

Figure 1: Equilibrium When The Upstream Firm Is A Monopolist

More clear intuition could be provided if we look at the upstream firm's incentive on pricing of the intermediate good. Compared to the case of $\alpha = 0$, the upstream firm has an extra revenue of $\alpha(p(x) - r)x$, the profit transfer from the downstream firm. This extra revenue will decrease if the price of the intermediate good r gets higher. Therefore, the upstream firm has stronger incentive to reduce the price, compared to the case of $\alpha = 0$, and this incentive gets stronger and stronger as α grows. In other words, the profit transfer alleviates the problem of 'double marginalization', by inducing the upstream firm to take the downstream firm's profit into account and to charge a lower price for the intermediate good in order to earn larger profit transfer. As a matter of fact, if $\alpha = 1$, the problem of double marginalization is perfectly internalized. The upstream firm's profit becomes to the joints profits of both firms and the equilibrium is determined at x^J, where the joint profits are maximized.

3.3. OTHER COMPARATIVE STATIC RESULTS

We will investigate how the profit transfer or increase in α affects other equilibrium variables, such as the profits of the upstream firm and the downstream firm, their joint profits, the consumer surplus, and the social welfare. First of all, notice that the joint equilibrium profits will increase compared to the case of $\alpha = 0$. This is because the joint profits $\pi_U + \pi_D$ increase in x until they are maximized at x^J, and decrease thereafter and we have $x^o < x^* < x^J$ as shown in Figure 1.[6] Increase in the equilibrium level of the intermediate good also results in increase in the equilibrium output of the final good, leading to reduction in the equilibrium price of the final good. Therefore the consumer surplus will increase and so will the social welfare along with the increase in the joint profits.

At an individual firm level, the equilibrium profit of the upstream firm increases compared to the case of $\alpha = 0$. To see this, notice that the upstream firm's profit increases by the amount of the profit transfer from the downstream firm, $\alpha\,[p(x) - r]\,x = \alpha[p(x) - MR(x)]x$, for all levels of x, compared to the case of $\alpha = 0$. And the equilibrium level x^* maximizes the upstream firm's profit $MR(x)x - C_U(x, e(x)) - F(e(x)) + \alpha\,[p(x) - MR(x)]\,x$. Hence the equilibrium profit of the upstream firm has to be higher in case of $\alpha > 0$ than that in case of $\alpha = 0$.[7] That is, if we denote the equilibrium profits of the upstream firm in cases of $\alpha = 0$ and $\alpha > 0$ by π_U^o and $\pi_U^* + \alpha\pi_D^*$, respectively, we have

$$\begin{aligned}
\pi_U^o &= MR(x^o)x^o - C_U(x^o, e(x^o)) - F(e(x^o)),\\
&< MR(x^o)x^o - C_U(x^o, e(x^o)) - F(e(x^o)) + \alpha\,[p(x^o) - MR(x^o)]\,x^o\\
&\leq MR(x^*)x^* - C_U(x^*, e(x^*)) - F(e(x^*) + \alpha\,[p(x^*) - MR(x^*)]\,x^*\\
&= \pi_U^* + \alpha\pi_D^*.
\end{aligned}$$

[6]Notice that the joint profits $\pi_U + \pi_D = p(x)x - C_U(x, e) - F(e)$ are strictly concave under our assumptions and thus are single peaked at $(x^J, e(x^J))$.

[7]In fact, this is the logic of the envelope theorem.

However, it is ambiguous whether the downstream firm's equilibrium profit will increase or decrease. The profit transfer changes the equilibrium profit from $\pi_D^o = [p(x^o) - r^o]x^o$ to $(1-\alpha)\pi_D^* = (1-\alpha)[p(x^*) - r^*]x^*$. It has two effects on the profit that work in the opposing directions each other. One is a direct effect. The profit transfer to the upstream firm decreases the downstream firm's profit, simply because it takes away $100 \times \alpha\%$ of the profit. On the other hand, it indirectly benefits the downstream firm by reducing the price of the intermediate good set by the upstream firm. That is, the downstream firm's equilibrium profit *before the profit transfer* will be higher when the upstream firm charges r^*, instead of r^o as in the case without the profit. The total effect depends on which effect dominates the other. Nevertheless, for two widely used demand functions, the linear demand function $p(x) = A - Bx$ and the constant elasticity demand function $p(x) = Ax^{-\frac{1}{\varepsilon}}$, we can show that the former effect dominates the latter, so that the downstream firm's profit decreases in equilibrium.[8]

Proposition 2 summarizes these results.

Proposition 2: (i) The profit transfer will increase the equilibrium joint profits of the upstream firm and the downstream. Moreover, they increase in α, the ratio of the profit transfer from the downstream firm to the upstream firm.

(ii) The profit transfer will increase the social welfare as well as the consumer surplus in equilibrium. Moreover, both increase in α.

(iii) The profit transfer will increase the upstream firm's equilibrium profit. It also increases in α.

(iv) It is ambiguous in general whether the profit transfer will increase or decrease the downstream firm's equilibrium profit. However, if the demand function for the final good is linear or of constant elasticity, it decreases the down-

[8] The profit transfer may increase the downstream firm's equilibrium profit, when the demand function for the final good is the form of $p(x) = A - 2Bx + Dx^2$, where $x < \frac{B}{D}$ or $x < \frac{B-\sqrt{B^2-A}}{D}$, and α is sufficiently close to 0. For instance, when $p(x) = 15 - 6x + x^2$, $C_U(x, e) = 3x$ and $F(e) = 0$, we can easily derive that the equilibrium level of the intermediate good is $x^* = \frac{2}{3-2\alpha}$ and the downstream firm's equilibrium profit is $(1-\alpha)\pi_D^* = \frac{8(1-\alpha)(7-6\alpha)}{(3-2\alpha)^3}$. We can show that $(1-\alpha)\pi_D^*$ is increasing in α if $\alpha < 0.22571$.

stream firm's equilibrium profit.

Proof: See the appendix.

The latter result of Proposition 2-(iv) depends on our assumption that the downstream firm's production function for the final good is $y = f(x) = x$. When the production function is replaced by a standard one of $y = f(x)$, we need additional conditions on f to establish the result of reduction in the downstream firm's equilibrium profit in case of the linear or the constant elasticity demand function for the final good. However, we will omit specifying the assumptions, because it will involve too much complication without much gain.[9] As mentioned earlier in section 2, the other results of Proposition 1 and Proposition 2 would not change as long as $f^{'}(x) > 0$ and the marginal revenue product $MRP(x) = \left[p(f(x)) + p^{'}(f(x))f(x)\right]f^{'}(x) = MR(y)f'(x)$, instead of $MR(x)$, is decreasing in x.

4. WHEN THE DOWNSTRAEM FIRM IS A MONOPSONIST

4.1. THE UPSTREAM FIRM'S PROBLEM

We now suppose that the downstream firm is a monopsonist in the intermediate good market. That is, the downstream firm sets the price of the intermediate good r in the first period. The upstream firm, after observing r, chooses the quantity of the intermediate good x and the effort level e in the second period.

Consider first the upstream firm's profit maximization problem given the price of the intermediate good r by backward induction. The first order conditions are given by

$$\frac{\partial(\pi_U + \alpha\pi_D)}{\partial x} = r - \frac{\partial C_U(x,e)}{\partial x} + \alpha(MR(x) - r)$$

[9] I appreciate a referee for pointing this out.

$$= (1-\alpha)r + \alpha MR(x) - \frac{\partial C_U(x,e)}{\partial x} = 0, \tag{4.1}$$

$$\frac{\partial(\pi_U + \alpha\pi_D)}{\partial e} = -\frac{\partial C_U(x,e)}{\partial e} - F^{'}(e) = 0. \tag{4.2}$$

As in the previous section, we will assume that the upstream firm's profit $\pi_U + \alpha\pi_D$ is strictly concave in (x,e) so that the second order conditions are satisfied.

When $\alpha = 0$, (4.1) simply states that the upstream firm produces the intermediate good up to the level where the price of the intermediate good is equal to the marginal production cost, *i.e.*, $r = \frac{\partial C_U(x,e)}{\partial x}$. When $\alpha > 0$, the upstream firm has two sources of revenue, one being the revenue from sales of the intermediate good and the other the profit transfer from the downstream firm. Therefore, it determines the output to the level where the sum of the price of the intermediate good(=the marginal revenue from sales of the intermediate good) and the marginal profit transfer from the downstream firm, are equal to the marginal production cost. This is exactly what (4.1) shows: $r + \alpha(MR(x) - r) = \frac{\partial C_U(x,e)}{\partial x}$. Rewriting it as $(1-\alpha)r + \alpha MR(x) = \frac{\partial C_U(x,e)}{\partial x}$ provides another interpretation. The left hand side rearranges the upstream firm's marginal revenue into another two components. One is $(1-\alpha)r$, which is the *net* price of the intermediate good. From r that the downstream firm charges for 1 unit of the intermediate good, the upstream firm only earns $(1-\alpha)r$ *after the profit transfer* because r is also counted as the cost of the downstream firm and thus αr is cancelled out of the profit transfer. The other component $\alpha MR(x)$ is the marginal revenue product transferred from the downstream firm. Therefore, we can interpret (4.1) as that the upstream firm's optimal output level is determined where the *net* price of the intermediate good, $(1-\alpha)r$, is equal to $\frac{\partial C_U(x,e)}{\partial x} - \alpha MR(x)$. Putting it differently, the minimum price of the intermediate good that the upstream firm is willing to accept for 1 unit increase *after the profit transfer*, becomes $\frac{\partial C_U(x,e)}{\partial x} - \alpha MR(x)$. Compared to the case of $\alpha = 0$, the upstream firm is willing to accept less amount by $\alpha MR(x)$, taking into account the marginal revenue product transferred from the downstream firm. By the same token, it is willing to accept less and less as α increases.

On the other hand, the first order condition on the effort level (4.2) is exactly the same as (3.2). Therefore the optimal effort function $e = e(x)$, determined by (4.2), is identical to that in the previous section. Notice that it only depends on x, but neither on r nor on α. Putting $e = e(x)$ into (4.1) gives the optimal output level of the intermediate good as a function of r, $x(r)$. This is the upstream firm's supply function for the intermediate good.

While the explicit form of $x(r)$ cannot be obtained, that of the inverse supply function for the intermediate good can be found from (4.1) and is written as

$$r(x) = \frac{1}{1-\alpha}\left[\frac{\partial C_U(x, e(x))}{\partial x} - \alpha MR(x)\right]. \tag{4.3}$$

Given the interpretation of $(1-\alpha)r = \frac{\partial C_U(x,e)}{\partial x} - \alpha MR(x)$ provided above, $r(x)$ represents the minimum price of the intermediate good that the upstream firm is willing to accept for 1 unit increase *before the profit transfer*, at a given level of x. We will simply call $r(x)$ and $(1-\alpha)r(x)$ the upstream firm's *gross* and *net* marginal willingness to accept, respectively.

Before proceeding to the downstream firm's problem, let us provide some results on $r(x)$ useful for further analysis. Let us denote by η the upstream firm's price elasticity of supply for the intermediate good. Then it can be written as

$$\eta = \frac{1}{r'(x)}\frac{r(x)}{x} = \frac{\frac{\partial C_U(x,e(x))}{\partial x} - \alpha MR(x)}{\left[\frac{\partial^2 C_U(x,e(x))}{\partial x^2} + \frac{\partial^2 C_U(x,e(x))}{\partial e \partial x}\frac{\partial e(x)}{\partial x} - \alpha MR'(x)\right]x},$$

where $\frac{\partial e(x)}{\partial x}$ is given in Lemma 1. The following Lemma characterizes useful properties of $r(x)$ and η.

Lemma 2: The upstream firm's inverse supply function (or *gross* marginal willingness to accept) $r(x)$ and its elasticity of supply $\eta(x,\alpha)$have the following properties: (i) $\frac{\partial r(x)}{\partial x} > 0$, (ii) $\frac{\partial r(x)}{\partial \alpha} < 0$ if and only if $MR(x) > \frac{\partial C_U(x,e(x))}{\partial x}$, and (iii) $\frac{\partial \eta}{\partial \alpha} < 0$.

Proof: See the appendix.

(i) is an obvious result because the upstream firm's optimal output level increases in the price of the intermediate good. It is equivalent to saying that the upstream firm's supply curve for the intermediate good is upward sloping. (ii) says that the upstream firm's *gross* (and *net*) marginal willingness to accept decreases in α if $MR(x) > \frac{\partial C_U(x,e(x))}{\partial x}$. As will be shown below, $MR(x) > \frac{\partial C_U(x,e(x))}{\partial x}$ holds in equilibrium. The result of $\frac{\partial r(x)}{\partial \alpha} < 0$ also appeals to the intuition because the marginal transfer from the downstream firm increases as α grows and therefore the upstream firm is willing to accept less and less. The third result that the upstream firm's elasticity of supply decreases in α could be explained in the same vein. As α grows, the relative importance of the revenue from sales of the intermediate good diminishes compared to the profit transfer from the downstream firm. Therefore the upstream will become less sensitive to the price of the intermediate good.

4.2. THE DOWNSTREAM FIRM'S PROBLEM AND THE EQUILIBRIUM

Let us now turn to the downstream firm's profit maximization problem. The downstream firm sets the price of the intermediate good r to maximize $(1-\alpha)\pi_D = (1-\alpha)[p(x(r))x(r) - rx(r)]$, where $x(r)$ is the upstream firm's supply function for the intermediate good. But it is more convenient for analysis to replace r by the upstream firm's inverse supply function $r(x)$ and to take x, the amount of purchase of the intermediate good, as the downstream firm's choice variable. From the perspective of the downstream firm, $r(x)$ becomes the price that it can charge per unit of the intermediate good *before the profit transfer*, while $(1-\alpha)r(x)$ being the price it can charge *after the profit transfer*. Hence the downstream firm's profit can be written as

$$(1-\alpha)\pi_D = (1-\alpha)\left[p(x) - r(x)\right]x = (1-\alpha)p(x)x - \left[\frac{\partial C_U(x,e(x))}{\partial x} - \alpha MR(x)\right]x.$$

Compared to the case of $\alpha = 0$, the revenue product of the intermediate good reduces to $(1-\alpha)p(x)x$ and the expenditure on the intermediate good reduces to $(1-\alpha)r(x)x = \left[\frac{\partial C_U(x,e(x))}{\partial x} - \alpha MR(x)\right]x$. Notice that the average expenditure reduces by $\alpha MR(x)$. This is the amount of marginal transfer that the upstream firm additionally earns besides the *net* price of the intermediate good. This also coincides with the amount of reduction in the upstream firms' *net* marginal willingness to accept.

The first order condition for the downstream firm's profit maximization problem is given by

$$\frac{d(1-\alpha)\pi_D}{dx} = (1-\alpha)MR(x) + \alpha MR(x) + \alpha MR^{'}(x)x - \frac{\partial C_U(x,e(x))}{\partial x}$$
$$-\left[\frac{\partial^2 C_U(x,e(x))}{\partial x^2} + \frac{\partial^2 C_U(x,e(x))}{\partial e \partial x}\frac{\partial e(x)}{\partial x}\right]x$$
$$= MR(x) + \alpha MR^{'}(x)x - \frac{\partial C_U(x,e(x))}{\partial x} - \left[\frac{\partial^2 C_U(x,e(x))}{\partial x^2} + \frac{\partial^2 C_U(x,e(x))}{\partial e \partial x}\frac{\partial e(x)}{\partial x}\right]x = 0.$$

Thus, the first order condition can be written as

$$MR(x) + \alpha MR^{'}(x)x = \frac{\partial C_U(x,e(x))}{\partial x} + \left[\frac{\partial^2 C_U(x,e(x))}{\partial x^2} + \frac{\partial^2 C_U(x,e(x))}{\partial e \partial x}\frac{\partial e(x)}{\partial x}\right]x. \tag{4.4}$$

We assume that the downstream firm's profit $(1-\alpha)\pi_D$ is strictly concave in x to ensure the second order condition $\frac{d^2(1-\alpha)\pi_D}{dx^2} < 0$.

When $\alpha = 0$, the left hand side of (4.4) is the marginal revenue product of the intermediate good while the right hand side is the marginal expenditure, $\frac{d}{dx}\left(\frac{\partial C_U(x,e(x))}{\partial x}x\right)$. The downstream firm, who is the monopolist in the final good market and the monopsony in the intermediate good market, hires the intermediate good to the level where the marginal revenue product is equal to the marginal expenditure. When $\alpha > 0$, interpretation of (4.4) is similar, but needs an explanation. As explained above, the downstream firm's revenue product reduces by $\alpha p(x)x$ and the expenditure reduces by $\alpha MR(x)x$. The reduction in the expenditure is due to the reduction in the upstream firms' *net* marginal willingness to accept for each unit of the intermediate good. We can take this reduction in the expenditure as the increase in the revenue product. Then the *new* revenue product becomes $(1-\alpha)p(x)x + \alpha MR(x)x$, and the *new* marginal revenue product is $MR(x) + \alpha MR^{'}(x)x$ while the expenditure and the marginal expenditure remains the same as in the case of $\alpha = 0$. Therefore (4.4) has the same interpretation as in the case of $\alpha = 0$. The only difference is that the marginal revenue product reduces by $-\alpha MR^{'}(x)x > 0$. As α grows, the marginal revenue product becomes lower and lower.

Given strict concavity of $(1-\alpha)\pi_D$, (4.4) uniquely determines the equilibrium level of the intermediate good $x^{**} = x^{**}(\alpha)$ as a function of α. The equilibrium effort level is given by $e^{**} = e(x^{**}(\alpha))$, where $e(x)$ is the upstream firm's optimal effort function. The equilibrium price of the intermediate good can also be obtained by inserting $x^{**}(\alpha)$ into (4.3). We will denote it by $r^{**} = r^{**}(\alpha) = r(x^{**}(\alpha))$.

Proposition 3 shows how the profit transfer affects the equilibrium.

Proposition 3: In equilibrium we have $\frac{dx^{**}}{d\alpha} < 0$ and $\frac{de^{**}}{d\alpha} \leq 0$. That is, the profit transfer will reduce both the equilibrium level of the intermediate good and the equilibrium effort level to reduce the production cost. Moreover, both decrease in α, the ratio of the profit transfer from the downstream firm to the upstream firm.

Proof: See the appendix.

The first result of $\frac{dx^{**}}{d\alpha} < 0$ can be verified in Figure 2. Figure 2 depicts the equilibrium levels of intermediate good in cases of $\alpha > 0$ and $\alpha = 0$, each of which is denoted by x^{**} and x^{oo}, respectively. In Figure 2, for notational simplicity, we denote the downstream firm's marginal expenditure by $ME(x) \equiv \frac{\partial C_U(x,e(x))}{\partial x} + \left[\frac{\partial^2 C_U(x,e(x))}{\partial x^2} + \frac{\partial^2 C_U(x,e(x))}{\partial e \partial x}\frac{\partial e(x)}{\partial x}\right]x$. Verify in Figure 2 that both x^{**} and x^{oo} satisfy the equilibrium condition $MR(x) + \alpha MR^{'}(x)x = ME(x)$ in cases of $\alpha > 0$ and $\alpha = 0$. We also compare these two equilibrium levels with x^J, which maximizes the joint profits of the upstream firm and the downstream firm. As in the previous section, x^J is determined at the level where $MR(x) = \frac{\partial C_U(x,e(x))}{\partial x}$. It is clear from Figure 2 that $x^{**} < x^{oo} < x^J$. Firstly $x^{**} < x^{oo}$ holds because the marginal revenue product is lower when $\alpha > 0$. And $x^{oo} < x^J$ holds because $ME(x) > \frac{\partial C_U(x,e(x))}{\partial x}$.[10] It is also easy to see that x^{**} decreases in α because the marginal revenue product $MR(x) + \alpha MR^{'}(x)x$ gets lower. The second result of $\frac{de^{**}}{d\alpha} = \frac{\partial e}{\partial x}\frac{dx^{**}}{d\alpha} \leq 0$ is obvious because the upstream firm's optimal effort $e(x)$ is an increasing function of x.

Figure 2 also shows that the downstream firm charges a lower price for the intermediate good in equilibrium, compared to the case of $\alpha = 0$. Let r^{**} and r^{oo} denote the equilibrium prices of the intermediate good, corresponding to the output level x^{**} and x^{oo}, respectively. Notice that $r^{**} = r(x^{**})$ and $r^{oo} = r(x^{oo}) = \frac{\partial C_U(x^{oo},e(x^{oo}))}{\partial x}$. Figure 2 shows clearly that $r^{**} < r^{oo}$ and that r^{**} decreases in α. The reason for the former is because the upstream firm's supply curve $r(x)$ is lower in case of $\alpha > 0$ than the supply curve $\frac{\partial C_U(x,e(x))}{\partial x}$ in case of $\alpha = 0$. The reason for the latter is because $r(x)$ is an increasing function of x and

[10] For expositional purpose, Figure 2 is drawn on the premises that (i) the marginal production cost curve $\frac{\partial C_U(x,e(x))}{\partial x}$ is nondecreasing, (ii)i.e., the marginal expenditure curve lies above the marginal production cost curve, and (iii) the marginal expenditure $ME(x)$ is nondecreasing. Both (i) and (ii) are identical to the condition $\frac{\partial^2 C_U(x,e)}{\partial x^2} + \frac{\partial^2 C_U \partial e}{\partial e \partial x}\frac{\partial e}{\partial x} \geq 0$. Replacing $\frac{\partial e}{\partial x}$ in the expression in Lemma 1, we can verify that this condition is equivalent to our assumption $\frac{\partial^2 C_U(x,e)}{\partial x^2}\left[\frac{\partial^2 C_U(x,e)}{\partial e^2} + F^{''}(e)\right] - \left(\frac{\partial^2 C_U(x,e)}{\partial e \partial x}\right)^2 \geq 0$, which we mentioned in section 2. However, (iii) requires a slightly stronger condition than the second order condition of the downstream firm's profit maximization. Of course the three premises are made only for the expositional purpose and the results of Proposition 3 do not depend on them.

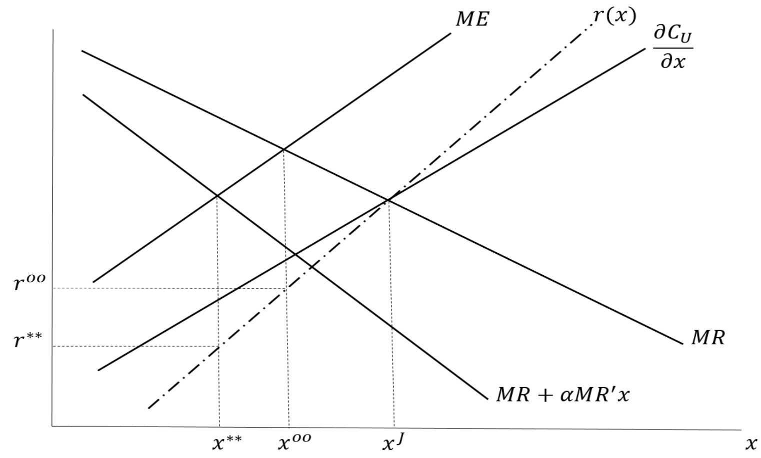

Figure 2: Equilibrium When The Downstream Firm Is A Monopsonist

$x^{**} < x^{oo}$.

Proposition 4: $\frac{dr^{**}}{d\alpha} < 0$. That is, the profit transfer will reduce the equilibrium price of the intermediate good. And it decreases in α. In particular, we have

$$\frac{dr^{**}}{d\alpha} = \frac{r^{**}}{\eta}\left[-\frac{1}{1-\alpha} + \frac{1}{x^{**}}\frac{dx^{**}}{d\alpha}\right] < 0,$$

where η is the upstream firm's price elasticity of supply for the intermediate good.

Proof: See the appendix.

Lemma 2-(iii) provides a clear intuition for Proposition 4. Recall that the upstream firm's price elasticity of supply for the intermediate good η decreases in α. Therefore, as α grows, the downstream firm has a stronger incentive to reduce the price of the intermediate good because the upstream firm becomes less sensitive to the price. This intuition can be verified from the equilibrium condition (4.4). A straightforward calculation shows that (4.4) can be rewritten as $MR(x) = r(x)\left(1+\frac{1}{\eta}\right)$. Since η decreases in α , $r(x)$ also has to decrease in α.[11] In the same line, the reduction in the equilibrium price of the intermediate good, $\frac{dr^{**}}{d\alpha} = \frac{r^{**}}{\eta}\left[-\frac{1}{1-\alpha} + \frac{1}{x^{**}}\frac{dx^{**}}{d\alpha}\right]$, gets larger when η is small.

4.3. OTHER COMPARATIVE STATIC RESULTS

Contrary to the previous section, the profit transfer reduces the equilibrium level of the intermediate good. Therefore, its effect on joint profits of the upstream firm and the downstream firm, consumer surplus, and social welfare will be the opposite. The equilibrium joint profits decreases. The profit transfer will decrease the equilibrium output of the final good and will raise the price of the final good. As a result, both the consumer surplus and the social welfare will decrease in equilibrium.

We can also show that the equilibrium profit of the downstream firm decreases, compared to the case of $\alpha = 0$. As we have explained above, the downstream firm's profit reduces by $\alpha[p(x) - MR(x)]x$ for all levels of x, with reduction in the revenue product and the expenditure by $\alpha p(x)x$ and $\alpha MR(x)x$, respectively. Therefore the equilibrium profit of the downstream firm in case of $\alpha > 0$ has to be lower than that in case of $\alpha = 0$. To see this, denote the equilibrium profits of the downstream firm in cases of $\alpha = 0$ and $\alpha > 0$ by π_D^{oo} and $(1-\alpha)\pi_D^{**}$, respectively. Then we have

$$
\begin{aligned}
&(1-\alpha)\pi_D^{**} = (1-\alpha)\left[p(x^{**}) - r(x^{**})\right]x^{**} \\
&= \left[p(x^{**}) - \frac{\partial C_U(x^{**}, e(x^{**}))}{\partial x}\right]x^{**} - \alpha[p(x^{**}) - MR(x^{**})]x^{**} \\
&\leq \left[p(x^{**}) - \frac{\partial C_U(x^{**}, e(x^{**}))}{\partial x}\right]x^{**} \leq \left[p(x^{oo}) - \frac{\partial C_U(x^{oo}, e(x^{oo}))}{\partial x}\right]x^{oo} = \pi_D^{oo}.
\end{aligned}
$$

[11]This argument, however, cannot provide the intuition for $\frac{dx^{**}}{d\alpha} < 0$, because the upstream firm's supply curve $r(x)$ in Figure 2 decreases in α for given x.

The second equality comes from the expression of $r(x)$ in (4.3). The last inequality holds because x^{oo} maximizes $\left[p(x) - \frac{\partial C_U(x,,e(x))}{\partial x}\right]x$, the downstream firm's profit in case of $\alpha = 0$.

However, it is ambiguous whether the equilibrium profit of the upstream firm will increase or decrease. This needs an explanation. To perform a comparative static analysis, denote the equilibrium profit of the upstream firm by $\pi_U^{**}(\alpha) + \alpha\pi_D^{**}(\alpha)$. It can be written as

$$\pi_U^{**}(\alpha) + \alpha\pi_D^{**}(\alpha) = r^{**}(\alpha)x^{**}(\alpha) - C_U(x^{**}(\alpha), e(x^{**}(\alpha))) - F(e(x^{**}(\alpha)))$$
$$+\alpha\left[p(x^{**}(\alpha)) - r^{**}(\alpha)\right]x^{**}(\alpha),$$

where $r^{**}(\alpha) = r(x^{**}(\alpha))$.

Increase in α by 1 unit directly increases the profit transfer from the downstream firm by $\pi_D^{**}(\alpha) = \left[p(x^{**}) - r^{**}\right]x^{**}$, raising the upstream firm's profit by the same amount. On the other hand, it also has the effect of reducing the upstream firm's profit by lowering the price of the intermediate good set by the downstream firm. The reduction in the upstream firm's profit due to the latter effect is expressed as $(1-\alpha)x^{**}\frac{dr^{**}}{d\alpha}$.[12] Hence the total effect of α on the equilibrium profit of the upstream firm is determined by these two countervailing effects. However, we know from Proposition 4 that $\frac{dr^{**}}{d\alpha} = \frac{r^{**}}{\eta}\left[-\frac{1}{1-\alpha} + \frac{1}{x^{**}}\frac{dx^{**}}{d\alpha}\right]$, where η is the upstream firm's price elasticity of supply for the intermediate good. The reduction in the equilibrium price of the intermediate good gets larger when η is small. So does the latter effect. Thus, which effect dominates the other depends on η. As a matter fact, we show in the proof of Proposition 5 that

$$\frac{d(\pi_U^{**}(\alpha) + \alpha\pi_D^{**}(\alpha))}{d\alpha} = \frac{p(x^{**})}{\varepsilon}x^{**} + \frac{(1-\alpha)r^{**}}{\eta}\frac{dx^{**}}{d\alpha},$$

[12]The remaining effect is the change in the upstream firm's profit caused by the change in x^{**}, fixing the price of the intermediate good. But this effect, expressed as $\left[(1-\alpha)r^{**} + \alpha MR - \frac{\partial C_U}{\partial x} - \left(\frac{\partial C_U}{\partial e} + F'\right)\frac{\partial e}{\partial x}\right]\frac{dx^{**}}{d\alpha}$, turns out to be 0 by the first order conditions for the upstream firm.

where ε is the price elasticity demand for the final good. Therefore, the profit transfer is more likely to decrease the equilibrium profit of the upstream firm as ε is larger and η is smaller.[13]

In sum, we have

Proposition 5: (i) The profit transfer will decrease the joint equilibrium profits of the upstream firm and the downstream firm. Moreover, they decrease in α.

(ii) The profit transfer will reduce the social welfare as well as the consumer surplus in equilibrium. Moreover, both decrease in α.

(iii) The profit transfer will decrease the downstream firm's equilibrium profit. It also decreases in α.

(iv) It is ambiguous in general whether the profit transfer will increase or decrease the upstream firm's equilibrium profit. It depends on the price elasticity demand for the final good ε and the upstream firm's price elasticity of supply for the intermediate good η. If the former is large and the latter is small, the upstream firm's equilibrium profit may decrease.

Proof: See the appendix.

As those of Proposition 1 and 2 in section 3, the results of Proposition 3, 4, and 5 would not change if the downstream firm's production function for the final good,$y = f(x) = x$, is replaced by $y = f(x)$, where $f^{'}(x) > 0$. We only need to replace $p(x)$ by $p(f(x))$ and $MR(x)$ by $MRP(x) = MR(f(x))f'(x)$. However, we need a minor modification on the formula of $\frac{d(\pi_U^{**}(\alpha)+\alpha\pi_D^{**}(\alpha))}{d\alpha}$ specified above. With $y = f(x)$, we can show that

$$\frac{d(\pi_U^{**}(\alpha)+\alpha\pi_D^{**}(\alpha))}{d\alpha} = p(f(x^{**}))f(x^{**})\left[1-(1-\frac{1}{\varepsilon})\frac{f^{'}(x^{**})x^{**}}{f(x^{**})}\right]+\frac{(1-\alpha)r^{**}}{\eta}\frac{dx^{**}}{d\alpha}.$$

[13]However, when the demand function is of a constant elasticity $p(x) = Ax^{-\frac{1}{\varepsilon}}$ and the upstream firm's production function is the form of $C_U(x,e) = C_U(x) = cx^n$, $\frac{d(\pi_U^{**}(\alpha)+\alpha\pi_D^{**}(\alpha))}{d\alpha} > 0$ for all $\varepsilon > 1$ and $n > 1$.

Here $\frac{f'(x^{**})x^{**}}{f(x^{**})}$ is the output elasticity. Therefore, whether the profit transfer increases or decreases the upstream firm's equilibrium profit still depends on the relative size of ε and η, with an additional effect of the output elasticity, which negatively affects $\frac{d(\pi_U^{**}(\alpha)+\alpha\pi_D^{**}(\alpha))}{d\alpha}$.

5. CONCLUSION

We consider a vertical relationship between a single upstream firm and a single downstream firm and examine the economic effects of the profit transfer program, where the downstream firm transfers a predetermined share of its profit to the upstream firm. We analyze the effects under two scenarios, according as how the price is determined in the upstream market. One is where the upstream firm sets the price of the intermediate good and the downstream firm takes the price as given. The other is where the downstream firm acts as a monopsonist and sets the price of the intermediate good. In the former scenario, the profit transfer will induce the upstream firm to charge a lower price for the intermediate good. As a result, the downstream firm will hire more intermediate good and will produce more output. And the upstream firm will increase the effort level to reduce the production cost. The consumer surplus and the social welfare will rise. Therefore, the profit transfer program enhances economic efficiency and alleviates the problem of 'double marginalization', which refers the efficiency loss occurring due to lack of coordination between the upstream firm and downstream firm. On the other hand, in the latter scenario, the profit transfer hinders economic efficiency. It induces the downstream firm to charge a lower price for the intermediate good and to hire less intermediate good. The upstream firm's effort level to reduce the production cost decreases. As a result, the output of the final good, the consumer surplus, and the social welfare decrease.

At an individual firm level, the profit transfer in the former scenario increases the upstream firm's profit, while it is ambiguous in general whether it increases or decreases the downstream firm's profit. However, if the demand for the final good is a linear function or a function of constant elasticity, it decreases the downstream firm's equilibrium profit. In the latter scenario, the profit transfer has a reverse effect. It decreases the downstream firm's profit, but how it affects the upstream firm's profit is not certain. The upstream firm's profit may decreases if the price elasticity demand for the final good is large and the upstream firm's price elasticity of supply for the intermediate good is small.

The policy implication of this paper should be clear. The market structure of the intermediate good or how the price of the intermediate good is determined is the most critical in evaluating the profit transfer program. When the upstream firm has enough power to set the price of the intermediate good, the profit transfer benefits the upstream firm, the consumers, and overall social efficiency, although it might decrease the downstream firm's profit. When there are several upstream firms and several downstream firms in the intermediate good market, we still expect a similar result as long as the downstream firms are price takers. In Salinger (1988), for instance, Cournot competition among the upstream firms in the first stage leads to the market clearing price of the intermediate good. The downstream firms take the price as given and decide their demands for the intermediate good. In this modification, the profit transfer will induce the upstream firms to internalize the problem of 'double marginalization' and will enhance economic efficiency, as in section 3.[14] When the downstream firms have more power in the vertical relationship, a natural extension of section 4 is an oligop-

[14]When there are multiple upstream firms and downstream firms, we implicitly assume the existence of a hypothetical auctioneer or a centralized institution in the intermediate good market that determines the market clearing price. We are also assuming that each upstream firm receives the profit transfers from all the downstream stream firms at a predetermined ratio α. For example, when there are two upstream firms, $U1$ and $U2$, and two downstream firms, $D1$ and $D2$, $U1$'s profit with the profit transfer becomes $\pi_{U1} = rx_{U1} - C_{U1}(x_{U1}) + \alpha\pi_{D1} + \alpha\pi_{D2}$. Thereby I disregard the possibility of bargaining process on the price and the transfer ratio between or among the upstream firms and downstream firms. As a referee pointed out, an alternative setting, which is probably more realistic in the relationship between big corporates and their subcontractors in Korea, would be decentralized bilateral trades and explicitly taking the bargaining process into account. Corominas-Bosch(2004) studied a bargaining problem between multiple buyers and sellers who are connected by a network, and showed that the numbers of buyers and sellers and the connectivity of the network affect the bargaining power and the division of surplus. From her work, with bargaining process into the model we could infer quite different results from the conjecture provided here

sony in the intermediate good market. In the first stage, the downstream firms simultaneously decide their quantity demanded for the intermediate good, generating a market clearing price of the intermediate good. Then the upstream firms take the price as given and decide their output levels for the intermediate good. In this extension, we believe that the profit transfer will harm the downstream firm, the consumers, and the overall efficiency, as in section 4. It is also dubious in general that the upstream firm gets benefits from the profit transfer.

Several assumptions have been made to simplify the analysis. In particular, we have assumed that the downstream firm only needs a single input produced by the upstream firm. We could extend our analysis to a model where the downstream firm has other substitutable inputs. For instance, it would be more realistic to replace the downstream firm's production function by $y = f(x,z)$, where y is the output of the final good and x,z are two inputs. The first input x is produced by the upstream firm to whom the downstream firm must transfer its profit. On the other hand, the second input z is internally provided by the downstream firm or is outsourced from another, possibly foreign, upstream firm, to which the downstream firm does not have to transfer its profit. That is, there is competition in the upstream market. The downstream firm can *partially* substitute the second input for the first input, where the first one is influenced by the profit transfer program, while the second is not so. Nevertheless, the main results of section 4(Propositions 3, 4 and 5) would not change with this modification. In section 4, where the downstream firm sets the price of the first, the profit transfer will induce the downstream firm to replace the first input by the second one in order to avoid the profit transfer. As a result, the downstream firm will reduce hiring of the first input even more than in Proposition 3. And it would not be difficult, if complicated, to show that the reduction in hiring of the first input dominates the increase in hiring of the second input under very mild conditions on $f(x,z)$. Therefore, the output of the final good as well as then consumer surplus and the social welfare, will decrease compared to the case without the profit transfer.

We also expect that the main results of section 3(Propositions 1 and 2) would remain valid. However, the result of Proposition 1 that the profit transfer increases x is less clear-cut with the modification than in our current model, since the downstream firm would replace the first input by the second one. Nonetheless, the downstream firm's equilibrium hiring of x would increase after all, compared to the case without the profit transfer, because the upstream firm anticipates the downstream firm's such response and will charge a lower price. Even if equilibrium hiring of x decreases, the output for the final good $y = f(x,z)$ would increase due to the increase in hiring of z. Hence the consumer surplus and the social welfare would increase, as in Proposition 2. However, these positive effects of the profit transfer will be limited in scale. By the same reason, so will the effect on the upstream firm's profit. We will leave the details for future research.

Another extension worthwhile to mention is to introduce competition in the downstream market. In particular, consider a situation where the downstream firm competes with a foreign firm in the downstream market, in the Cournot fashion, i.e. in quantities. Suppose also that the foreign rival has its own source for the intermediate good and it does not have to transfer the profit to its supplier. Then the results in sections 3 and 4 are robust to this extension. In section 3, where the upstream firm sets the price of the intermediate good, the profit transfer will reduce the domestic price of the intermediate good, and thus will benefit the domestic downstream firm. Both hiring of the intermediate good and output of the final good of the domestic firm will increase, while those of the foreign rival will decrease. In section 4, the profit transfer will have the opposite effects. The domestic downstream firm will reduce the hiring of the intermediate good and the output of the final good, while the foreign rival will increase those. As a result, the profit transfer will harm the domestic downstream firm, but will benefit the foreign rival.

A. APPENDIX

A.1. PROOF OF LEMMA 1

Putting $e(x)$ into (3.2) and differentiating both sides with respect to x gives

$$-\frac{\partial^2 C_U(x,e(x))}{\partial x \partial e} - \left[\frac{\partial^2 C_U(x,e(x))}{\partial e^2} + F^{''}(e(x))\right]\frac{\partial e(x)}{\partial x} = 0.$$

Thus we have $\frac{\partial e(x)}{\partial x} = \frac{-\frac{\partial^2 C_U(x,e(x))}{\partial x \partial e}}{\frac{\partial^2 C_U(x,e(x))}{\partial e^2} + F''(e(x))} \geq 0$. The sign of $\frac{\partial e(x)}{\partial x}$ is positive because the denominator is nothing but $-\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial e^2}$ and thus has a positive sign by the second order condition of the upstream firm's profit maximization, while the numerator is positive by the assumption of $\frac{\partial^2 C_U(x,e(x))}{\partial x \partial e} \leq 0$.

Q.E.D.

A.2. PROOF OF PROPOSITION 1

As a preliminary step, let us write down the second order conditions for the upstream firm's profit maximization.

$$\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial x^2} = (2-\alpha)MR^{'} + (1-\alpha)MR^{''}x - \frac{\partial^2 C_U}{\partial x^2} < 0,$$

$$\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial e^2} = -\left[\frac{\partial^2 C_U}{\partial e^2} + F^{''}\right] < 0,$$

$$H_U = \frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial x^2}\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial e^2} - \left(\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial e \partial x}\right)^2$$

$$= \left[\frac{\partial^2 C_U}{\partial x^2} - (2-\alpha)MR^{'} - (1-\alpha)MR^{''}\cdot x\right]\left[\frac{\partial^2 C_U}{\partial e^2} + F^{''}\right] - \left(\frac{\partial^2 C_U}{\partial e \partial x}\right)^2 > 0.$$

For notational simplicity, we denote $\frac{\partial^2(\pi_U+\alpha\pi_D)}{\partial x^2}\frac{\partial^2(\pi_U+\alpha\pi_D)}{\partial e^2} - (\frac{\partial^2(\pi_U+\alpha\pi_D)}{\partial e \partial x})^2$ by H_U

hereafter. Now putting $x^*(\alpha)$and $e(x^*(\alpha))$ into (3.3) and differentiating both sides with respect α gives

$$\left[(2-\alpha)MR' + (1-\alpha)MR'' \cdot x^* - \frac{\partial^2 C_U}{\partial x^2} - \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x}\right]\frac{dx^*}{d\alpha} = MR' \cdot x^*.$$

Replacing $\frac{\partial e}{\partial x}$ in $[\cdot]$ by $\frac{\partial e}{\partial x} = \frac{-\frac{\partial^2 C_U}{\partial x \partial e}}{\frac{\partial^2 C_U}{\partial e^2}+F''}$ from Lemma 1, it can be rewritten as

$$-\left[\frac{\partial^2 C_U}{\partial e^2} + F''\right]^{-1}\left\{\left[\frac{\partial^2 C_U}{\partial x^2} - (2-\alpha)MR' - (1-\alpha)MR'' \cdot x^*\right]\left[\frac{\partial^2 C_U}{\partial e^2} + F''\right] - \left(\frac{\partial^2 C_U}{\partial e \partial x}\right)^2\right\}\frac{dx^*}{d\alpha} = MR' \cdot x^*.$$

Observing that the expression in $\{\cdot\}$ in the left hand side is H_U, we have

$$\frac{dx^*}{d\alpha} = \frac{MR' x^*}{(2-\alpha)MR' + (1-\alpha)MR'' \cdot x^* - \frac{\partial^2 C_U}{\partial x^2} - \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x}} = \frac{-MR' x^*[\frac{\partial^2 C_U}{\partial e^2} + F'']}{H_U} > 0.$$

The sign of $\frac{dx^*}{d\alpha}$ is positive because both the numerator $-MR' x^*[\frac{\partial^2 C_U}{\partial e^2} + F'']$ and the denominator H_U have positive signs. Since $e^*(\alpha) = e(x^*(\alpha))$ and $r^*(\alpha) = MR(x^*(\alpha))$, it is obvious to see that $\frac{de^*}{d\alpha} = \frac{\partial e}{\partial x}\frac{dx^*}{d\alpha} \geq 0$ and $\frac{dr^*}{d\alpha} = MR' \frac{dx^*}{d\alpha} < 0$.

Q.E.D.

A.3. PROOF OF PROPOSITION 2

Since (ii) is obvious given the result $\frac{dx^*}{d\alpha} > 0$, we will only provide the proofs for (i), (iii), and (iv). Let us denote the joint equilibrium profits, the upstream firm's equilibrium profit, and the downstream firm's equilibrium profit by $\pi_U^*(\alpha) + \pi_D^*(\alpha)$, $\pi_U^*(\alpha) + \alpha\pi_D^*(\alpha)$, and $(1-\alpha)\pi_D^*(\alpha)$, respectively. Then they can be written as

$$\pi_U^*(\alpha) + \pi_D^*(\alpha) = p(x^*(\alpha))x^*(\alpha) - C_U(x^*(\alpha), e(x^*(\alpha))) - F(e(x^*(\alpha))),$$

$$\pi_U^*(\alpha) + \alpha\pi_D^*(\alpha) = (1-\alpha)MR(x^*(\alpha))x^*(\alpha) + \alpha p(x^*(\alpha))x^*(\alpha)$$
$$-C_U(x^*(\alpha), e(x^*(\alpha))) - F(e(x^*(\alpha))),$$

$$(1-\alpha)\pi_D^*(\alpha) = (1-\alpha)\left[p(x^*(\alpha)) - r(x^*(\alpha))\right]x^*(\alpha) = (1-\alpha)\left[p(x^*(\alpha)) - MR(x^*(\alpha))\right]x^*(\alpha).$$

Firstly, total derivative of $\pi_U^*(\alpha) + \pi_D^*(\alpha)$ with α gives

$$\frac{d(\pi_U^*(\alpha) + \pi_D^*(\alpha))}{d\alpha} = \left[MR - \frac{\partial C_U}{\partial x}\right]\frac{dx^*}{d\alpha} - \left[\frac{\partial C_U}{\partial e} + F'\right]\frac{\partial e}{\partial x}\frac{dx^*}{d\alpha} = \left[MR - \frac{\partial C_U}{\partial x}\right]\frac{dx^*}{d\alpha}$$
$$= -(1-\alpha)MR' x^* \frac{dx^*}{d\alpha} > 0.$$

The second equality holds because $-\left[\frac{\partial C_U}{\partial e} + F'\right] = 0$ by (3.2), and the third equality comes from the equilibrium condition (3.3). The inequality holds because $MR' < 0$ and $\frac{dx^*}{d\alpha} > 0$. This proves (i).

Secondly, total derivative of $\pi_U^*(\alpha) + \alpha\pi_D^*(\alpha)$ with α is

$$\frac{d(\pi_U^*(\alpha) + \alpha\pi_D^*(\alpha))}{d\alpha} = [p - MR]x^* \left[(1-\alpha)MR' x^* + MR - \frac{\partial C_U}{\partial x}\right]\frac{dx^*}{d\alpha} - \left[\frac{\partial C_U}{\partial e} + F'\right]\frac{\partial e}{\partial x}\frac{dx^*}{d\alpha}$$
$$= [p - MR]x^* > 0.$$

Notice that the last two terms in the right hand side of the first equality are zero by (3.3) and (3.2). The inequality holds because $p - MR = -p'x^* > 0$.

Lastly, total derivative of $(1-\alpha)\pi_D^*(\alpha)$ with respect to α gives

$$\frac{d(1-\alpha)\pi_D^*(\alpha)}{d\alpha} = -[p - MR]x^* - (1-\alpha)MR' x^* \frac{dx^*}{d\alpha}$$
$$= -\left[p - (p + p'x^*)\right]x^* - (1-\alpha)MR' x^* \frac{dx^*}{d\alpha} = \left[p'x^* - (1-\alpha)MR' \frac{dx^*}{d\alpha}\right]x^*.$$

Hence $\frac{d(1-\alpha)\pi_D^*(\alpha)}{d\alpha} \leq 0$ holds if and only if $\frac{dx^*}{d\alpha} \leq \frac{p'x^*}{(1-\alpha)MR'}$. Since $\frac{dx^*}{d\alpha} = \frac{MR'x^*}{(2-\alpha)MR' + (1-\alpha)MR''x^* - \left(\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x}\right)}$ from the proof of Proposition 1, the condition $\frac{dx^*}{d\alpha} \leq \frac{p'x^*}{(1-\alpha)MR'}$ amounts to $MR''x^* - \frac{1}{1-\alpha}\left(\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x}\right) \leq \frac{MR'}{p'}\left[MR' - \frac{2-\alpha}{1-\alpha}p'\right] = \frac{MR'}{p'}[p''x^* - \frac{\alpha}{1-\alpha}p']$. Observe that $\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} = \frac{\frac{\partial^2 C_U}{\partial x^2}\left(\frac{\partial^2 C_U}{\partial e^2} + F''\right) - \left(\frac{\partial^2 C_U}{\partial e \partial x}\right)^2}{\frac{\partial^2 C_U}{\partial e^2} + F''}$ and its sign is positive under our assumptions. Therefore, a sufficient condition for $\frac{d(1-\alpha)\pi_D^*(\alpha)}{d\alpha} \leq 0$ is $MR^{''}x^* \leq \frac{MR^{'}}{p^{'}}[p^{''}x^* - \frac{\alpha}{1-\alpha}p']$. This inequality holds when the demand function is linear, $p(x) = A - Bx$, because $MR^{''} = p^{''} = 0$, $MR^{'} = -2B < 0$, and $p^{'} = -B < 0$. For a constant elasticity demand function $p(x) = Ax^{-\frac{1}{\varepsilon}}$, the left hand side of the inequality becomes $MR^{''}x^* = \frac{1}{\varepsilon}\left(1 - \frac{1}{\varepsilon}\right)\left(1 + \frac{1}{\varepsilon}\right)A(x^*)^{-\frac{1}{\varepsilon}-1}$, while the right hand side is $\frac{MR^{'}}{p^{'}}\left[p^{''}x^* - \frac{\alpha}{1-\alpha}p'\right] = \left(1 - \frac{1}{\varepsilon}\right)\left[\frac{1}{\varepsilon}\left(1 + \frac{1}{\varepsilon}\right) + \frac{\alpha}{1-\alpha}\frac{1}{\varepsilon}\right]A(x^*)^{-\frac{1}{\varepsilon}-1}$. The direction of the inequality is easily verified. Therefore $\frac{d(1-\alpha)\pi_D^*(\alpha)}{d\alpha} \leq 0$ also holds.

Q.E.D.

A.4. PROOF OF LEMMA 2

Let us start by writing the second order conditions for the upstream firm's profit maximization:

$$\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial x^2} = -\frac{\partial^2 C_U}{\partial x^2} + \alpha MR^{'} < 0,$$

$$\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial e^2} = -\frac{\partial^2 C_U}{\partial e^2} - F^{''} < 0,$$

$$\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial x^2}\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial e^2} - \left(\frac{\partial^2(\pi_U + \alpha\pi_D)}{\partial e\partial x}\right)^2$$

$$= \left[\frac{\partial^2 C_U}{\partial x^2} - \alpha MR^{'}\right]\left[\frac{\partial^2 C_U}{\partial e^2} + F^{''}\right] - \left(\frac{\partial^2 C_U}{\partial e\partial x}\right)^2 > 0.$$

From (4.3), we have $\frac{\partial r(x)}{\partial x} = \frac{1}{1-\alpha}\left[\frac{\partial^2 C_U(x,e(x))}{\partial x^2} + \frac{\partial^2 C_U(x,e(x))}{\partial e\partial x}\frac{\partial e(x)}{\partial x} - \alpha MR^{'}(x)\right]$. Replacing $\frac{\partial e(x)}{\partial x}$ by the expression given in Lemma 1 gives

$$\frac{\partial r(x)}{\partial x} = \frac{1}{1-\alpha}\frac{\left(\frac{\partial^2 C_U}{\partial x^2} - \alpha MR^{'}\right)\left(\frac{\partial^2 C_U}{\partial e^2} + F^{''}\right) - \left(\frac{\partial^2 C_U}{\partial e\partial x}\right)^2}{\frac{\partial^2 C_U}{\partial e^2} + F^{''}} > 0.$$

The sign of $\frac{\partial r(x)}{\partial x}$ is positive because both the numerator and the denominator have positive signs by the second order condition above.

An easy calculation shows that $\frac{\partial r(x)}{\partial \alpha} = -\frac{1}{(1-\alpha)^2}\left[MR(x) - \frac{\partial C_U(x,e(x))}{\partial x}\right]$. Thus $\frac{\partial r(x)}{\partial \alpha} < 0$ if and only if $MR(x) > \frac{\partial C_U(x,e(x))}{\partial x}$.

From the expression of η in the text, we have by a straightforward calculation

$$\frac{\partial \eta}{\partial \alpha} = \frac{-MR(\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x}) + MR'\frac{\partial C_U}{\partial x}}{x\left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]^2} < 0.$$

The sign of $\frac{\partial \eta}{\partial \alpha}$ is negative since $MR^{'} < 0$ and $\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} = \frac{\frac{\partial^2 C_U}{\partial x^2}\left(\frac{\partial^2 C_U}{\partial e^2} + F''\right) - \left(\frac{\partial^2 C_U}{\partial e \partial x}\right)^2}{\frac{\partial^2 C_U}{\partial e^2} + F''}$ ≥ 0 under our assumptions, and thus the numerator is negative.

Q.E.D.

A.5. PROOF OF PROPOSITION 3

First, let us write down the second order condition of the downstream firm's profit maximization.

$$\frac{d^2(1-\alpha)\pi_D}{dx^2} = (1+\alpha)MR' - 2(\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x})$$
$$-(\frac{\partial^3 C_U}{\partial x^3} + 2\frac{\partial^3 C_U}{\partial e \partial x^2}\frac{\partial e}{\partial x} + \frac{\partial^3 C_U}{\partial e^2 \partial x}(\frac{\partial e}{\partial x})^2 + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial^2 e}{\partial x^2} - \alpha MR^{''})x < 0.$$

Now putting $x^{**}(\alpha)$ and $e(x^{**}(\alpha))$ into the last line of the first order condition $\frac{d(1-\alpha)\pi_D}{dx} = 0$ and differentiating both sides with respect α gives

$$\left[(1+\alpha)MR^{'} - 2\left(\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x}\right) - \left(\frac{\partial^3 C_U}{\partial x^3} + 2\frac{\partial^3 C_U}{\partial e \partial x^2}\frac{\partial e}{\partial x} + \frac{\partial^3 C_U}{\partial e^2 \partial x}(\frac{\partial e}{\partial x})^2 + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial^2 e}{\partial x^2} - \alpha MR^{''}\right)x^{**}\right]$$
$$\times\frac{dx^{**}}{d\alpha} + MR^{'} \cdot x^{**} = 0.$$

The expression in $[\cdot]$ is exactly the same as $\frac{d^2(1-\alpha)\pi_D}{dx^2}$. Therefore we have $\frac{dx^{**}}{d\alpha} = -\frac{MR' \cdot x^{**}}{\frac{d^2(1-\alpha)\pi_D}{dx^2}} < 0$ because $\frac{d^2(1-\alpha)\pi_D}{dx^2} < 0$ by the second order condition and $MR' < 0$. The result of $\frac{de^{**}}{d\alpha} = \frac{\partial e}{\partial x}\frac{dx^{**}}{d\alpha} \leq 0$ is obvious because $\frac{\partial e}{\partial x} \geq 0$ from Lemma 1.

Q.E.D.

A.6. PROOF OF PROPOSITION 4

Observe in (4.3) that $r(x)$ depends not only on x, but also on α. For expositional purpose we will rewrite $r(x)$ as $r(x,\alpha)$. Plugging $x^{**}(\alpha)$ into (4.3) gives $r^{**}(\alpha) = r(x^{**}(\alpha),\alpha)$. Then total derivative of $r^{**}(\alpha)$ with respect to α is

$$\begin{aligned}
\frac{dr^{**}}{d\alpha} &= \frac{\partial r}{\partial \alpha} + \frac{\partial r}{\partial x}\frac{dx^{**}}{d\alpha} = -\frac{1}{(1-\alpha)^2}\left(MR - \frac{\partial C_U}{\partial x}\right) + \frac{1}{1-\alpha}\left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]\frac{dx^{**}}{d\alpha} \\
&= -\frac{1}{(1-\alpha)^2}\left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]x^{**} + \frac{1}{1-\alpha}\left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]\frac{dx^{**}}{d\alpha} \\
&= \frac{r(x^{**})}{\eta}\left[-\frac{1}{1-\alpha} + \frac{1}{x^{**}}\frac{dx^{**}}{d\alpha}\right] < 0.
\end{aligned}$$

The second equality comes from the expression of $r(x)$ in (4.3) or from the proof of Lemma 2-(i) and 1-(ii). The third equality holds because we have from (4.4) $MR - \frac{\partial C_U}{\partial x} = \left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]x^{**}$. The last equality comes from the definition of η, $\eta = \frac{\frac{\partial C_U}{\partial x} - \alpha MR}{\left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]x} = \frac{(1-\alpha)r(x)}{\left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]x}$. Lastly, to see that the sign of $\frac{dr^{**}}{d\alpha}$ is negative, notice that $\left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]$ in the right hand side of the third equality has a positive sign. It is because replacing $\frac{\partial e}{\partial x}$ by the expression given in Lemma 1 gives $\left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right] = \frac{\left(\frac{\partial^2 C_U}{\partial x^2} - \alpha MR'\right)\left(\frac{\partial^2 C_U}{\partial e^2} + F''\right) - \left(\frac{\partial^2 C_U}{\partial e \partial x}\right)^2}{\frac{\partial^2 C_U}{\partial e^2} + F''}$. Its sign is positive by the second order condition for the upstream firm's profit maximization. Therefore the sign of $\frac{dr^{**}}{d\alpha}$ is negative because $\frac{dx^{**}}{d\alpha} < 0$.

Q.E.D.

A.7. PROOF OF PROPOSITION 5

Given the result $\frac{dx^{*}}{d\alpha} < 0$, (ii) is obvious. We will provide the proofs for (i), (iii), and (iv). The joint equilibrium profits, the downstream firm's equilibrium profit, and the upstream firm's equilibrium profit can be written as

$$\pi_U^{**}(\alpha)+\pi_D^{**}(\alpha) = p(x^{**}(\alpha))x^{**}(\alpha) - C_U(x^{**}(\alpha), e(x^{**}(\alpha))) - F(e(x^{**}(\alpha))),$$

$$(1-\alpha)\pi_D^{**}(\alpha) = (1-\alpha)\left[p(x^{**}(\alpha)) - r^{**}(\alpha)\right]x^{**}(\alpha)$$

$$= (1-\alpha)p(x^{**}(\alpha))x^{**}(\alpha) - \left[\frac{\partial C_U(x^{**}(\alpha), e(x^{**}(\alpha)))}{\partial x} - \alpha MR(x^{**}(\alpha))\right]x^{**}(\alpha),$$

$$\pi_U^{**}(\alpha)+\alpha\pi_D^{**}(\alpha) = (1-\alpha)r^{**}(\alpha)x^{**}(\alpha) - C_U(x^{**}(\alpha), e(x^{**}(\alpha))) - F(e(x^{**}(\alpha)))$$

$$+\alpha p(x^{**}(\alpha))x^{**}(\alpha) = \left[\frac{\partial C_U(x^{**}(\alpha), e(x^{**}(\alpha)))}{\partial x} - \alpha MR(x^{**}(\alpha))\right]x^{**}(\alpha)$$

$$-C_U(x^{**}(\alpha), e(x^{**}(\alpha))) - F(e(x^{**}(\alpha))) + \alpha p(x^{**}(\alpha))x^{**}(\alpha).$$

Firstly, total derivative of $\pi_U^{**}(\alpha)+\pi_D^{**}(\alpha)$ with α gives

$$\frac{d(\pi_U^{**}(\alpha)+\pi_D^{**}(\alpha))}{d\alpha} = \left[MR - \frac{\partial C_U}{\partial x}\right]\frac{dx^{**}}{d\alpha} - \left[\frac{\partial C_U}{\partial e} + F'\right]\frac{\partial e}{\partial x}\frac{dx^{**}}{d\alpha} = \left[MR - \frac{\partial C_U}{\partial x}\right]\frac{dx^{**}}{d\alpha}$$

$$= \left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e\partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]x^{**}\frac{dx^{**}}{d\alpha}$$

$$= \frac{\left(\frac{\partial^2 C_U}{\partial x^2} - \alpha MR'\right)\left(\frac{\partial^2 C_U}{\partial e^2} + F''\right) - \left(\frac{\partial^2 C_U}{\partial e\partial x}\right)^2}{\frac{\partial^2 C_U}{\partial e^2} + F''}x^{**}\frac{dx^{**}}{d\alpha} < 0.$$

The second equality holds because $-\left[\frac{\partial C_U}{\partial e} + F'\right] = 0$ by (4.2), and the third equality comes from the equilibrium condition (4.4). The last equality obtains by replacing $\frac{\partial e}{\partial x}$ by the expression in Lemma 1. The inequality holds because the sign of $\frac{\left(\frac{\partial^2 C_U}{\partial x^2}-\alpha MR'\right)\left(\frac{\partial^2 C_U}{\partial e^2}+F''\right)-\left(\frac{\partial^2 C_U}{\partial e \partial x}\right)^2}{\frac{\partial^2 C_U}{\partial e^2}+F''}$ is positive by the second order conditions of the upstream firm's profit maximization and from $\frac{dx^{**}}{d\alpha} < 0$. This proves (i).

Secondly, total derivative of $(1-\alpha)\pi_D^{**}(\alpha)$ with α is

$$\frac{d(1-\alpha)\pi_D^{**}(\alpha)}{d\alpha} = -[p-MR]x^{**} + \left[(1-\alpha)MR - \frac{\partial C_U}{\partial x} + \alpha MR - \left(\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right)x^{**}\right]\frac{dx^*}{d\alpha}$$
$$= -[p-MR]x^* < 0.$$

Observe that the second term in the right hand side of the first equality is zero because of the equilibrium condition (4.4).

Lastly, total derivative of $\pi_U^{**}(\alpha) + \alpha\pi_D^{**}(\alpha)$ with respect to α gives

$$\frac{d(\pi_U^{**}(\alpha)+\alpha\pi_D^{**}(\alpha))}{d\alpha} = [p-MR]x^{**} + \left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]x^{**}\frac{dx^{**}}{d\alpha}$$
$$-\left[\frac{\partial C_U}{\partial e} + F'\right]\frac{\partial e}{\partial x}\frac{dx^{**}}{d\alpha} = [p-MR]x^{**} + \left[\frac{\partial^2 C_U}{\partial x^2} + \frac{\partial^2 C_U}{\partial e \partial x}\frac{\partial e}{\partial x} - \alpha MR'\right]x^{**}\frac{dx^{**}}{d\alpha}$$
$$= [p-MR]x^{**} + \frac{\frac{\partial C_U}{\partial x} - \alpha MR}{\eta}\frac{dx^{**}}{d\alpha} = \frac{p}{\varepsilon}x^{**} + \frac{(1-\alpha)r^{**}}{\eta}\frac{dx^{**}}{d\alpha}.$$

The second equality holds because the last term in the right hand side of the first equality $\left[\frac{\partial C_U}{\partial e} + F'\right]$ is zero by the upstream firm's first order condition (4.2). The third equality comes from the definition of η and the last one obtains because $MR = p(1-\frac{1}{\varepsilon})$ and $(1-\alpha)r^{**} = \frac{\partial C_U}{\partial x} - \alpha MR$.

Q.E.D.

REFERENCES

Cachon, G. (2003), " Supply Chain Coordination with Contracts," in S. Graves and T. de Kok, eds., *Handbooks in Operation Research and Management Science*, 11, Elsevier, Boston, 229-340.

Cachon, G., and M. Lariviere (2005), " Supply Chain Coordination with Revenue-sharing Contracts: Strengths and Limitations," *Management Science*, 51(3.1), 30-44.

Corominas-Bosch, M. (2004), " Bargaining in a Network of Buyers and Sellers," *Journal of Economic Theory*, 115, 35-77.

Dana, J.D., and K.E. Spier (2001), " Revenue Sharing and Vertical Control in the Video Rental Industry," *Journal of Industrial Economics*, 49(3.3), 223-245.

Katz, M. (1989), " Vertical Contractual Relations," in Richard Schmalensee and Robert D. Willig, eds., *The Handbook of Industrial Organization*, Amsterdam: North-Holland, 655-721.

Kim, G. (2012), " In Search of the Best Profit-Sharing Practices in a Korean Society," *Korea Business Review*, 16(3.1), 51-79, *Published in Korean*.

Kim, S. (1998), " Does Profit Sharing Increase Firms' Profits?," *Journal of Labor Research*, 19 (3.2), 351-370.

Kruse, D. (1992), " Profit Sharing and Productivity: Microeconomic Evidence from the United States," *Economic Journal*, 102(3.1), 24-36.

Salinger, M. (1988), " Vertical Mergers and Market Foreclosure," *Quarterly Journal of Economics*, 103(May), 345-356.

Secrieru, O. (2006), " The Economic Theory of Vertical Restraint," *Journal of Economic Surveys*, 20(4.2), 797-822

Weitzman, M., and D. Kruse (1990), " Profit Sharing and Productivity," in Alan S. Binder, eds., *Paying for Productivity: A Look at the Evidence*, Washington, D.C.: Brookings Institution, 95-140.

Yao, Z., S.C.H. Leung, and K.K. Lai (2008), " Manufacturer's Revenue-sharing Contract and Retail Competition," *European Journal of Operational Research*, 186, 637-651.