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Core equivalence may fail without monotonicity*

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This paper looks at the role played by the assumption of monotonicity of preferences in core equivalence results. By means of two examples, we show that without strong monotonicity, the core equivalence may fail. We illuminate on the issue by introducing a stronger notion of core.

Keywords: Large Exchange Economy, Continuum Model, Perfect Competition

1. Introduction

Perfectly competitive market is arguably the cornerstone of economic theory. It is an ideal for economists because it satisfies the *fundamental theorems of welfare economics*, i.e. a competitive (or Walrasian) equilibrium is Pareto optimal and under suitable assumptions, any Pareto optimum allocation can be supported as a Walrasian equilibrium.

The core equivalence theorem is an extended justification of the fundamental theorems. (See Anderson (1992) for a survey.) The core is an extension of

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Pareto optimality because while Pareto optimality allows individuals to veto any change that is worse off individually, the core allows groups of individuals ("coalitions") to do so. The core equivalence result, meaning that the core allocations are identical to Walrasian equilibrium allocations in an ideal market, shows that perfect competition achieves even stronger level of optimality than Pareto.

Edgeworth first proposed a heuristic argument using the celebrated Edgeworth box diagram to show the core equivalence. Debreu and Scarf (1963) offered a rigorous treatment but it was Aumann (1964) who firmly established the exact equivalence in the limiting case of a continuum economy. Aumann′s works initiated the use of continuum models and measure theory in economic theory. One advantage of Aumann's (1964) continuum model is that it dispenses with convexity assumptions. Convexity is a meaningful but restrictive assumption on preferences. By dispensing with this assumption, Aumann (1964) established beautifully that a large economy has an inherent 'convexifying' mechanism, and we need not worry about whether convexity actually holds at the individual level.

However, another rather restrictive assumption used by Aumann (1964) that is not recognized duly in the literature is monotonicity of preferences. An economic agent with (strongly) monotonic preferences is a person who desires greater amounts in any commodity available for consumption. Textbook models usually assume monotonicity for convenience. But one can think of many real-world situations when monotonicity does not hold. We believe that if an important result is driven by the monotonicity in an otherwise very general model, one needs to be aware of it. The purpose of this paper is to illustrate this point.

We examine whether the core equivalence theorem of Aumann (1964) is robust with respect to the monotonicity of preferences. The theorem states that the core and the set of Walrasian allocations coincide in a continuum economy. We shall show that without monotonicity of preferences, this result can fail.

We are not the first to explicitly make this point in studies on core allocations. Manelli (1991) showed that monotonicity (which plays an "unimportant role" in Aumann's theorem) is crucial for core convergence theorems (i.e. results for large but finite economies, as in Debreu-Scarf (1963)). He presented an example of well-behaved sequences of finite-agent economies that do not exhibit core equivalence even approximately due to the lack of monotonicity. This led him to question the adequacy of the continuum model as an idealization of large finite economies. The force of his argument is clearer if one examines the limiting economy in his example: the exact core equivalence holds in the limiting case, so there is a discontinuity between large finite model and continuum model. In other words, in Manelli's (1991) example, the core equivalence holds in the continuum economy, but it does not hold in any finite version of it. So he concludes, so to speak, that core equivalence *should not* hold.

In contrast, we directly consider continuum economies with non-monotonic preferences. In other words, contrary to what Manelli's (1991) example might have suggested, even continuum economies may fail to show core equivalence if preferences are not monotonic. For such economies, Hildenbrand (1982, p.845) remarked that a core allocation is quasi-Walrasian if preferences are locally non-satiated, implicity suggesting that the exact core equivalence may not obtain in general. Although it may have been obvious, no explicit argument or example showing the failure of core equivalence was given (to the author′s knowledge) in the literature.

In Example 1 below, a continuum economy is presented where preferences are *weakly* monotonic. There is no Walrasian equilibria, but the core is non-empty, hence core equivalence fails. The nonexistence is related to the fact that some agents have their initial endowments on the *boundary* of consumption set.

The boundary endowments in Example 1 are entirely consistent with Aumann's assumptions (1964), but to show that the 'boundary-ness' is not the only driver of non-equivalence, we give another example. So in Example 2, another continuum economy is given where preferences are locally non-satiated (but not monotonic) and no agent has boundary endowments. Here a Walrasian equilibrium does exist, but there is a core allocation which is not

Walrasian. So equivalence fails again.[1)]

Of course, in both examples, Hildenbrand's (1982) remark applies–core allocations are quasi-Walrasian. Our method of argument is still worth noting: we introduce a stronger notion of core, which actually corresponds to that of core in a finite economy (Debreu-Scarf, 1963) and a natural generalization of Pareto optimality. In other words, Aumann's (1964) definition of core was somewhat weaker than standard definitions in finite-agent economies and Pareto optimality. But since he assumed monotonicity, it did not matter.

This stronger notion of core is always contained in the standard core (in Aumann's sense) and also contains Walrasian allocations, so the equality between the two notions of core is a necessary condition for equivalence between standard core and Walrasian allocations. In our examples, the core strictly contains the "strong" core, hence failing core equivalence. Moreover, the strong core allocations are exactly Walrasian in Example 2, suggesting that in certain cases we can restore core equivalence just by using the stronger notion, which is better suited for non-monotonic preferences setting.

Our findings do not alter the accepted wisdom in the field in any fundamental way. But we stress that the usual practice (for example, as surveyed by Anderson (1992)) needs to be more careful regarding the monotonicity and the definition of core.

II. The Model

There are k commodities, with the commodity space $\mathbb{R}^k_+$. Vector inequalities $\geq, >, \gg$ have the usual meanings (see, e.g., Hildenbrand, 1974). Let P be the set of binary relations ("preferences") defined on $\mathbb{R}^k_+$. A preference relation $\succ \in P$ is said to be *weakly monotonic* if $x \gg y$ implies $x \succ y$ for

1) Experiences with similar examples suggest that weak monotonicity plus interior endowments probably guarantees equivalence but we need further verification to settle the issue.

$x, y \in \mathbb{R}^k_+$. It is said to be *strongly monotonic* if $x > y$ implies $x \succ y$.[2] On the other hand, the preference relation $\succ \in P$ is *continuous* if sets $\{y \in \mathbb{R}^k_+ \mid y \succ x\}$ and $\{y \in \mathbb{R}^k_+ \mid x \succ y\}$ are open.[3] We will assume continuity of preferences throughout the paper.

An *exchange economy* is a μ-measurable function $\mathcal{E}: A \to P \times \mathbb{R}^k_+$ where (A, M, μ) is an atomless measure space of agents. Let $\mathcal{E}(a) = (\succ_a, e(a))$ for $a \in A$ where $\succ_a$ is the preference and $e(a)$ the initial endowment of commodities for the agent a.

An *assignment* is a μ-integrable function $f: A \to \mathbb{R}^k_+$. Furthermore, if $\int_A f = \int_A e$ (feasibility) holds,[4] the assignment f is an *allocation*. A *coalition* is a μ-measurable subset of A with a positive measure (i.e. any subset $S \subset A$ such that $S \in M$ and $\mu(S) > 0$).

A pair of coalition and an assignment (S, g) is said to *block* another allocation f if (i) $g(a) \succ_a f(a)$ almost everywhere (abbreviated 'a.e.' hereafter) on S and (ii) $\int_S g = \int_S e$. In other words, if most members of the coalition $S \subset A$ prefer g to f and g is feasible with the endowments within the coalition, then (S, g) blocks f. The *core*, denoted $C(\mathcal{E})$, is the *set of all allocations that cannot be blocked*.

The definitions given so far are standard. Now we introduce a distinction suitable for our interest in this paper. First recollect that a weak preference relation $\succeq$ can be defined as: $x \succeq y$ if and only if $y \nsucc x$.

Definition 1. The pair (S, g) is said to *weakly block* another allocation f if

(i) $g(a) \succeq_a f(a)$ a.e. on S

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2) This is called 'monotonicity' by Aumann (1964) and is labeled (M4) by Anderson (1992), who also labels weak monotonicity as (M3). We add the adjective 'strong' for emphasis.

3) We put the standard Euclidean metric topology on the commodity space.

4) For simplicity, we write $\int_A f$ for the Lebesgue integral $\int_A f(a)\, d\mu$.

(ii) there exists a subcoalition $S_1 \subset S$, $\mu(S_1) > 0$ such that $g(a) \succ_a f(a)$ a.e. on S_1

(iii) $\int_{S_1} g = \int_{S_1} e$

Note that g need not satisfy $\int_{S_1} g = \int_{S_1} e$, i.e. the allocation need not be feasible for the subcoalition S_1. (It does have to be feasible for the coalition S.) Now we introduce a stronger notion of core using the weak blocking. It is called, naturally, a strong core.

Definition 2. The *strong core*, denoted $C^*(\mathcal{E})$, is *the set of all allocations that cannot be weakly blocked.*[5]

If we restrict the coalition to be the whole set A, the definition reduces to the familiar Pareto optimality. One can also check that when Debreu and Scarf (1963) presented a "core convergence" theorem[6] their definition of finite-agent core corresponds to our notion of strong core. In other words, Aumann (1964) used a weaker notion for the first core equivalence result, perhaps for convenience, than Debreu and Scarf (1963).

The interpretation of the strong core is straightforward. A weak blocking is possible if a group S_1 of agents can persuade some other members ($S_0 = S \setminus S_1$) of the economy to join them in forming a coalition S and an allocation that will be make S_1 agents strictly better off and leave $S_0 = S \setminus S_1$ agents as before. In other words, allowing for the possibility of weak blocking presumes that agents will not be "spiteful" (i.e. willing to help others if one is not harmed). Therefore, if competitive allocations are shown to be equivalent to strong core, it establishes a very strong efficiency property for perfect competition.[7]

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5) Shitovitz (1973) used a similar notion in a somewhat different context and called it a *core ("star"-core).

6) Core convergence results show that cores defined for finite-agent economies approach Walrasian allocations as the number of agents grows to infinity.

7) I thank an anonymous referee for urging me to include a discussion on the notion

Given a price system p, the demand set is defined as $D(p, \succ, e) = \{x \in \mathbb{R}^k_+ : p \cdot x \leq p \cdot e,\ y \succ x$ implies $p \cdot y \geq p \cdot e\}$. An allocation f is said to be *Walrasian* if $f(a) \in D(p, \succ_a, e(a))$ for almost all $a \in A$. The set of all Walrasian allocations is denoted $W(\mathcal{E})$.

The quasi-demand set is defined as $D_q(p, \succ, e) = \{x \in \mathbb{R}^k_+ : p \cdot x \leq p \cdot e,\ y \succ x$ implies $p \cdot y > p \cdot e\}$. An allocation f is said to be *quasi-Walrasian* if $f(a) \in D_q(p, \succ_a, e(a))$ for almost all $a \in A$. The set of all quasi-Walrasian allocations is denoted $Q(\mathcal{E})$.[8] For quasi-Walrasian allocations, there can be better feasible allocations. So $Q(\mathcal{E})$ contains some non-equilibrium allocations.

Remark 1. The following are obvious from the definitions.

(a) $W(\mathcal{E}) \subset Q(\mathcal{E})$

(b) $C^*(\mathcal{E}) \subset C(\mathcal{E})$

(a) is obvious since quasi-Walrasian property is weaker than Walrasian property. (b) is obvious since blocking is always a weak blocking (simply take $S_1 = S$).

Theorem 1. $W(\mathcal{E}) \subset C^*(\mathcal{E})$

Proof. First suppose that $f \in W(\mathcal{E})$ and that (S, g) weakly blocks f, i.e. $f \notin C^*(\mathcal{E})$. Then we have $g(a) \succeq_a f(a)$ a.e. on S and $g(a) \succ \succ_a f(a)$ a.e. on S_1 for some $S_1 \subset S$. Let $S_0 = S \setminus S_1$.

Because $f \in Q(\mathcal{E})$, for almost all $a \in S$, we have $p \cdot g(a) \geq p \cdot e(a)$ for a suitable price p. Integrating both sides over S_0, we obtain $p\int_{S_0} g \geq p\int_{S_0} e$, therefore $\int_{S_0} g \geq \int_{S_0} e$. Also because $f \in W(\mathcal{E})$, for almost all $a \in S_1$, we have $p \cdot g(a) > p \cdot e(a)$. Integrating over S_1,

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of strong core.

8) We are following Anderson (1992) in terminology. Quasi-Walrasian equilibria in the sense of Debreu (1962) was shown to be equivalent to the current definition (Hildenbrand, 1968).

we get $\int_{S_1} g > \int_{S_1} e$.

Hence, $\int_S g = \int_{S_0} g + \int_{S_1} g > \int_S e$. This is a contradiction because a weak blocking has to be feasible (i.e. $\int_S g = \int_S e$). We conclude $f \in C^*(\mathcal{E})$. So, $W(\mathcal{E}) \subset C^*(\mathcal{E})$. ■

Remark 2. Hildenbrand (1982) shows $Q(\mathcal{E}) = C(\mathcal{E})$ if preferences are locally non-satiated. From Theorem 1, it is rather short of showing "core equivalence" which says $W(\mathcal{E}) = C(\mathcal{E})$. Obviously a necessary condition for core equivalence is $C^*(\mathcal{E}) = C(\mathcal{E})$. This obtains if preferences are strongly monotonic.

Theorem 2. (Aumann, 1964) If the preferences are strongly monotonic, $C(\mathcal{E}) \subset W(\mathcal{E})$

Corollary. If the preferences are strongly monotonic,

(a) $C(\mathcal{E}) = W(\mathcal{E})$

(b) $C^*(\mathcal{E}) = C(\mathcal{E})$

Remark 3. (a) in Corollary is the core equivalence theorem. (b) shows equality between strong core and core. It is not straightforward to prove (b), but it follows from Theorems 1 and 2.

Now if the preferences are *not* strongly monotonic, $C(\mathcal{E})$ strictly contains $C^*(\mathcal{E})$ in general. In the next section, we will give examples where $C^*(\mathcal{E}) \neq C(\mathcal{E})$ with either weakly monotonic or locally non-satiated (but not strongly monotonic) preferences and the strong core is a strict subset of core for a continuum of agents.

Ⅲ. When preferences are not (strongly) monotonic

The fist example satisfies all of Aumann's (1964) assumptions except strong monotonicity. Both the core equivalence and the existence of Walrasian equilibria fail.

1. Example 1: Weakly monotonic preferences and boundary endowments for some agents

Consider an economy $\mathcal{E}_1$ with two commodities. As usual, the agent space is $[0,1]$ with Lebesgue measure. For $a \in [0, \frac{1}{2}]$ (**type 1**), preference is given as: $(x_1, y_1) \succ (x_2, y_2)$ if and only if $x_1 > x_2$ and $(x_1, y_1) \sim (x_2, y_2)$ for $x_1 = x_2$.[9] For $a \in (\frac{1}{2}, 1]$ (**type 2**), preference is given as: $(x_1, y_1) \succ (x_2, y_2)$ if and only if $y_1 > y_2$ and $(x_1, y_1) \sim (x_2, y_2)$ for $y_1 = y_2$. Hence, type 1 derives utility only from the first commodity, while type 2 derives utility only from the second commodity. The initial endowments are $(1, 0)$ for type 1 agents and $(1, 1)$ for type 2 agents.

Remark 4. The preferences are quite standard except that they are not monotonic. In other words, they are weakly monotonic, convex, continuous and measurable.

Result 1. $W(\mathcal{E}_1) = \emptyset$

Proof. Trying out a few simple diagrams can convince the reader easily. When p_i is nonpositive, type i agents' demand is unbounded, hence equilibrium is not feasible. So assume $(p_1, p_2) \gg 0$. Then we get a corner solution and the supply does not equal the demand. ■

Result 2. $C(\mathcal{E}_1) \neq \emptyset$

Proof. Let $f_x : A \to \mathbb{R}^2_+$ be given by $f_x(a) = (2 - x, 0)$ for type 1 agents and $(x, 1)$ for type 2 agents. Then $f_0 \in C^*(\mathcal{E}_1)$: any allocation which is

9) I thank an anonymous referee for pointing out the possible pitfall in not defining the indifference relation for the examples.

preferred to f_0 by at least one type is not feasible. Because $C^*(\mathcal{E}_1) \subset C(\mathcal{E}_1)$ by Theorem 1, the claim holds. ■

Remark 5. $C^*(\mathcal{E}_1) = C(\mathcal{E}_1)$ (i.e. the inclusion is strict)

Proof. Consider the initial endowment allocation e (which in fact equals f_1 using the notation introduced in the previous proof). Although e can be weakly blocked by f_0, it cannot be blocked by any coalition. In order to block e, the coalition needs to give type 2 agents an amount bigger than 1, which is not feasible. ■

With Hildenbrand's (1982) remark in mind, we check whether f_0 and e are quasi-Walrasian: they are, with the price $(0,1)$.

Remark 6. If there is only one agent in each type, this example reduces to a two-person exchange economy where the core contains the strong core strictly.

The nonexistence of Walrasian equilibria in this example is probably related to the fact that type 1 agents do not have the second commodity as their endowments (this is also the main reason the core strictly contains the strong core). One naturally wonders whether the result *solely* drives from the boundary endowments. Indeed, if we were to change the endowments of type 1 agents from $(1,0)$ to $(1,1)$, we restore the core equivalence. For this, we have the following useful lemma due to Shitovitz (1973).

Lemma. (Shitovitz) If preferences are continuous and weakly monotonic, and if the initial endowments are in the interior of consumption set for almost all agents, then

$C^*(\mathcal{E}) = C(\mathcal{E})$, where $\mathcal{E}$ is a continuum exchange economy.

For proof of this lemma, see Shitovitz (1973, Lemma 4 in Appendix). The argument can be given here roughly as follows: whenever we have a weak blocking, there are some agents who are indifferent between two assignments in question and some agents who strictly prefer the blocking. Due to continuity of preferences, we can take out a little bit of every commodities from the latter group so that they are still better off than the original assignment. Give this extra bundle to the former group, then they are strictly better off due to *weak monotonicity*. In doing this, we sometimes need to control the proportion between the two groups to meet feasibility-it is always possible because the space of agents is atomless. (For this reason, the lemma is false for a finite-agent economy.) We just created a blocking out of the weak blocking we began with. The lemma follows. Shitovitz (1973) did not assume that the endowments are in the interior, but our example here suggests it is necessary.

Therefore, if we insist avoiding boundary endowments, there is a force toward core equivalence. Then the next question arises as to whether the above lemma will extend to locally non-satiated preferences. The next example shows it does not.

2. Example 2: Locally non-satiated preferences and interior endowments

Here we present an economy where all agents have interior endowments and locally non-satiated (but not even weakly monotonic) preferences. The core strictly contains the strong core, which then equals the Walrasian allocations. In other words, core equivalence holds for strong core but not for Aumann's core.

Consider an economy $\mathcal{E}_2$ with three commodities and two types of agents. Both types have the identical initial endowments of $(1,1,1)$. As in Example 1, each type occupies the half of the population. For type 1 agents, the preferences are given as: $(x_1, y_1, z_1) \succ (x_2, y_2, z_2)$ whenever $x_1 > x_2$ *or* $z_1 < z_2$ and $(x_1, y_1, z_1) \sim (x_2, y_2, z_2)$ for $x_1 = x_2$ and $z_1 = z_2$. In other words, type 1 agents like commodity 1, are indifferent about commodity 2, and dislike commodity 3. For type 2 agents, the preferences are given as: $(x_1, y_1, z_1) \succ (x_2, y_2, z_2)$ whenever $x_1 < x_2$ *or* $y_1 > y_2$ and $(x_1, y_1, z_1) \sim (x_2, y_2, z_2)$ for $x_1 = x_2$ and $y_1 = y_2$. In other words, type 2 agents dislike commodity 1, like commodity 2, and are indifferent about commodity 3. These preferences may be incomplete (for example, suppose a type 1 agent gets more of commodity 1 and commodity 3 at the same time, she may be unable to determine whether she is better off or not).

Consider two allocations f and g defined as follows: (1) f gives $(2,0,1)$ to type 1 agents and $(0,2,1)$ to type 2 agents. (2) g gives $(2,0,0)$ to type 1 agents and $(0,2,2)$ to type 2 agents. It is obvious that $g \in C^*(\mathcal{E}_2)$ (hence the strong core is non-empty).

Result 3. $f \notin C^*(\mathcal{E}_2)$

Proof. f is weakly blocked by (A, g) (i.e. the whole coalition with g). ■

Result 4. $f \in C(\mathcal{E}_2)$

Proof. We need to show that there is no blocking for f. Obviously, there is no blocking by a coalition consisting of only one type of agents. Hence, consider a coalition $S_1 \cup S_2$ with measure μ_1 of type 1 agents (S_1) and measure μ_2 of type 2 agents (S_2).

(1) First, suppose the coalition $S_1 \cup S_2$ proposes an "equal treatment"[10] assignment h in order to block f.

1. h will give $(2+\varepsilon_1, \varepsilon_2, 1-\varepsilon_3)$ to type 1 agents where $\varepsilon_1 \geq 0$, $\varepsilon_2 \geq 0$, $0 \leq \varepsilon_3 \leq 1$ and not both ϵ_1 and ϵ_3 are zero.
2. h will give $(0, 2+\overline{\varepsilon_2}, 1+\overline{\varepsilon_3})$to type 2 agents where $\overline{\varepsilon_2} > 0$, $\overline{\varepsilon_3} \geq -1$.
3. Finally, $\mu_1(2+\varepsilon_1, \varepsilon_2, 1-\varepsilon_3)+\mu_2(0,2+\overline{\varepsilon_2}, 1+\overline{\varepsilon_3})=\mu_1(1,1,1)+\mu_2(1,1,1)$

The first two conditions require that the proposed assignment to be strictly preferred by all agents in the coalition. The last is the feasibility condition.

10) One important step in Debreu and Scarf's (1963) proof of core convergence theorem is the equal treatment property of the core, i.e. all agents in the same type (same preferences) receive the same bundle in a core allocation. This property holds if the preferences are strongly monotonic. (See Varian, 1992, p. 390) Because the preferences in this example are not strongly monotonic, we will also consider non-equal treatment case.

From the feasibility in commodity 1, we obtain $\mu_1(1+\varepsilon_1)=\mu_2$. Since $\varepsilon_1 \geq 0$, this implies $\mu_1 \leq \mu_2$ (i.e. the coalition should not include more type 1 agents than type 2 agents). From the feasibility in commodity 2, we obtain $\mu_1/\mu_2=(1+\overline{\varepsilon_2})/(1-\varepsilon_2)$. Since $\overline{\varepsilon_2} > 0$ and $\varepsilon_2 \geq 0$, it implies $\mu_1 > \mu_2$ (i.e. the coalition should include more type 1 agents. A contradiction!

(2) Next, suppose the same coalition $S_1 \cup S_2$ proposes a non-equal treatment assignment h'. The only significant difference from the equal treatment case (1) is that ε_i and $\overline{\varepsilon_i}$ are functions. In other words, we think of integrable functions $\varepsilon_i: S_1 \to \mathbb{R}$ and $\overline{\varepsilon_i}: S_2 \to \mathbb{R}$, with $\varepsilon_1(a)$ denoting the amount of commodity 1 that the agent a in subcoalition S_1 gets by h', etc. The value of the functions for each agent still need to satisfy the inequalities given in 1 and 2 above (although they may have different values for different agents). Hence, the same argument applies: The feasibility in commodity 1 requires $\mu_1+\int_{S_1}\varepsilon_1=\mu_2$. Since $\varepsilon_1(a) \geq 0$ for almost all $a \in S_1$, we have $\mu_1 \leq \mu_2$. The feasibility in commodity 2 requires $\mu_1-\int_{S_1}\varepsilon_2=\mu_2+\int_{S_2}\overline{\varepsilon_2}$. Again since $\overline{\varepsilon_2} > 0$ for almost all $a \in S_2$ and $\varepsilon_2(a) \geq 0$ for almost all $a \in S_1$, we obtain $\mu_1 > \mu_2$. A contradiction!

Hence, there is no blocking for f. ■

It is worth noting that the strong core allocation g is Walrasian with the price $(1/2, 1/2, 0)$. The intuition is clear: Since the commodity 3 is not desired by either type, it becomes a free good. Since decrease in commodity 3 is preferred by type 1 agents, all of the commodity 3 can be given to type 2 agents who do not mind (or particularly desire) it. However, such procedure is not a proper blocking. Also note that the core allocation f is quasi-Walrasian with the same price: g is preferred to f by the half of the agents but g costs as much as f since the commodity 3 is free.

Ⅳ. Concluding remarks

In this paper, we pointed out the important role played by monotonicity of preferences for core equivalence results. Our Example 2 suggests that the simple cure is to employ the strong core rather than the core. On the other hand, Example 1 shows even the strong core may fail equivalence if preferences are not strongly monotonic. Research on core equivalence is still progressing on many fronts. For example, an Econlit search on "core equivalence" as keywords yielded more than 50 items since mid-1990s. When applying the insights from these works, one has to be careful about monotonicity assumptions.

There have been a few more recent contributions regarding the monotonicity assumption. One notable work is Hara (2005a) in which the author compares core, anonymous core (Hara (2002)), Walras allocations, and bargaining set (an extension of core concepts that allows counter-blockings, see Mas-Colell (1989)) without monotonicity assumption. (Also see Hara (2005b)) It is of interest to see how our examples in this paper compares with these recent results. I leave this for future work.

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[Abstract]

선호의 단조성을 가정하지 않을 경우 코어 동등성 정리의 실패에 대해

김 성 현

본 연구는 코어 동등성(core equivalence) 정리에서 선호의 단조성(monotonicity) 가정이 어떤 역할을 하는지를 살펴본다. 대형 교환경제에 대한 2개의 예를 통해 강(strong)단조성이 가정되지 않으면 코어 동등성이 성립하지 않음을 보인다. 기존보다 강화된 코어의 개념을 소개하고 이를 통해 관련된 이슈를 설명한다.

핵심용어 : 대형 교환경제, 연속체 모형, 완전 경쟁