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Belief Structure in Ultimatum Bargaining Game*

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This paper investigates the belief structure in the ultimatum bargaining game to test whether people care only about the pecuniary payoffs. In the normal-form quantal response equilibrium model, the responder's choice is made taking the proposer's intention into account while is independent in the agent normal-form game. Using Slonim and Roth's (1998, *Econometrica*) experimental data, it is shown that the experimental subjects who played the role of the responder cared not only the pecuniary payoffs but also the fairness of the amount of offers.

Keywords: Quantal-Response Equilibrium, Ultimatum Bargaining Game, Backward Induction, Fairness

JEL Classifications: C79, C92

I. Introduction

Ultimatum bargaining game could not be simpler. Two players are given a pie to divide. One player (the proposer) makes an offer which the other (the responder) may accept or reject. If the offer is accepted, the players receive their agreed payoffs. If not, both get nothing. In the ultimatum bargaining

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game, there is a continuum of Nash equilibria, but all but one of these can be thought of as involving an attempt by one of the bargainers to threaten a course of action which he would not wish to carry out if his bluff were called. The class of equilibria which does not involve such threats is called subgame perfect equilibria. When payoffs are discrete, there are two perfect equilibria; in one, the proposer offers 0 and the responder accepts anything; in the other, the proposer offers the smallest positive piece of the pie and the responder rejects 0 offer and accepts any positive offer.

Recent experimental studies of ultimatum bargaining games, however, have reported results markedly different from the subgame perfect equilibrium. The modal offer was a half of the pie and low but positive offers were often rejected (see Roth, 1995, Chapter 4, for a survey of the experimental literature.) The proposer's behavior of offering a significant share of the pie, although not consistent with perfection, is not very puzzling because the responder rejects bad offers, and making it is better not to demand too much. The problem is that the responders do reject as much as 40% of the pie. In the ultimatum bargaining game, since the responder makes a decision with an offer in hand, the problem the responder faces reduces to whether to take that offer or to choose 0 and thus self-interested behavior cannot be compatible with rejecting strictly positive offers.

Accordingly, attention shifted to fairness explanations assuming interdependent utility (Bolton, 1991; Rabin, 1993; Costa-Gomes and Zauner, 2001). However, a static notion of fairness does not seem to be able to explain the experimental results. If responders felt insulted for offers less than a certain "fair" amount, it might be better to reject unfair offers. According to backward induction argument, however, the distribution of offers should be concentrated around the "fair" offer and responders should made little rejections. Although it is conceivable to extend the model by allowing heterogeneity in fair offers across players, Binmore *et al.* (2002) and Johnson *et al.* (2002) reported it is not the case. Even controlling for social utility, experimental subjects do not always respect backward induction.

Along this line, Yi (2005) argued that parts of the experimental subjects' behavior can be explained by postulating that, as the game is played repeatedly, what the subjects learned is not only how to play the game but also the amount of fair offer among players who they play with.[1] Based on the empirical evidence that the positive correlation between the averages of initial offers and of subsequent offers as well as the average of rejected offers, he interpreted rejecting an offer as a way to signal that the offer is unfairly small comparing to what he experienced or expected. Although each individual player's idea of fairness has an influence on the "socially agreed" level of fair offer, it is limited and thus players need the average opinion on the fair offer in the group that he belonged to.[2]

That hypothesis provides a way to explain why responders rejected significant amounts of offers in bargaining experiments and how proposers' intention influences responders' rejecting decision. As an way to test the hypothesis, this paper compares two belief structures ; agent normal-form and normal-form representations of the ultimatum bargaining game.

In most ultimatum bargaining experiments, the game is represented in extensive form and naturally researchers developed models based on agent normal-form games. However, the experimental results suggests that agent normal-form might not be a correct way to model subjects' behavior.[3] To specify the belief structure, I apply the notion of "quantal response equilibrium" (henceforth, QRE) proposed by McKelvey and Palfrey (1995, 1998). QRE is motivated by human subjects' imperfectly optimizing behavior which is often observed in various experiments. A QRE is defined as an equilibrium in

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1) Another approach is based on learning models like the adaptive learning model (Prasnikar and Roth, 1992), the evolutionary model (Gale, Binmore, and Samuelson, 1995), and the reinforcement learning models (Roth and Erev, 1995). Without fairness argument, they were able to explain rejection of strictly positive offers. However, those approaches interpret rejection of positive offers as mistakes without any strategic consideration.

2) In most experiments, each player was informed only about his own game result. Thus he had to infer others' behavior based only on his own experience.

3) Binmore *et al*. (2002) argued that "game theory is typically used not as a literal description but as a model of a more complicated strategic interaction, and there is no reason to believe that the extensive form constructed by an analyst exactly captures the considerations used by players to analyze the interaction".

which players choose their strategies stochastically, with strategies that have higher expected payoffs chosen with higher probabilities.

While most notions of equilibrium produce an identical prediction in different representation of a game, the QRE discriminates game forms by allowing different belief structures. In particular, in the normal-form QRE of the ultimatum bargaining game, the responder could reject rarely expected offers with a high probability even without fairness consideration while all positive offers are rejected with probability less than 1/2 in its agent normal form. In this way, the structure of QRE provides a tractable way to test whether players care about the fairness of an offer by comparing two versions of the game.

In the experimental data in Slonim and Roth (1998), which I use for the empirical test of the Yi's (2005) hypothesis, the higher the average of initial offers was, invariantly, the larger the average offer in subsequent rounds were as well as the average of rejected offers. Such feature is consistent with the normal-form representation in which rarely expected offers are rejected more frequently. The estimation result also shows that normal-form representation describes the data better. This suggests that the responder's utility depends not only on the offer made but also on the proposer's intention to the extent that the responder would prefer rejecting "too" small offer relative to what he expected.

The rest of the paper is organized as follows. Section Ⅱ is a brief summary of QRE models of the ultimatum bargaining game. Section Ⅲ estimates the two versions of the game using experimental data in Slonim and Roth (1998) to test which model is better to describe human subjects' behavior in the experiments. The estimation result shows that the agent normal-form QRE model performs better to describe the responder's behavior while the normal- form logit equilibrium model does better for the proposer's behavior. In order to refine the empirical evidence, Section Ⅳ considers a QRE model incorporating fairness into players' preferences. With fairness consideration, the normal form outperforms in describing both parties' behavior and the estimation results suggest that the responders' did care about the proposers' intention to judge whether they got insulted by the offers in hands. Even with a small offer, if all the other responders were treated equally, a responder would not interpret the small offer as an insulting. Section Ⅴ concludes.

Ⅱ. Logit Equilibrium of Ultimatum Bargaining Games

This section compares the belief structures in agent normal form and normal form after a brief introduction of the game and the associated QRE model.[4)]

The ultimatum bargaining game is played by two players, the proposer and the responder who shall be denoted by a subscript $i \in \{p, r\}$. Let X_i denote the strategy set of player i with $x_i \in X_i$, which is discrete and bounded. In following, I will consider an ultimatum bargaining game where two players must split a pie of 1, which allows to interpret the strategy choice as the % share of the pie. In the present model, such normalization does not change any results. Then offers available to the proposer are $X_p = \left\{0, \frac{1}{n}, \cdots, 1\right\}$ with a given positive integer n and the responder can either accept the offer or reject, $X_r = \{Accept, Reject\}^{n+1}$ or $X_r = \{A, R\}^{n+1}$. Each player has a utility function of $u(x_p, x_r)$ that depends only on the pecuniary payoff and each player is assumed risk neutral. Let Σ_i be the set of probability distribution over X_i and an element $\sigma_i \in \Sigma_i$ is a mixed strategy and $\sigma_i(x_i)$ denote the probability of x_i being played. Given a strategy profile, player i's expected payoff is $\pi_i(x_i, \sigma_{-i}) = \int_{x_{-i} \in X_{-i}} \sigma_{-i}(y) u_i(x_i, y) dy$. When no confusion will arise, $\pi_i(x_i, \sigma_{-i})$ is denoted by $\pi_i(x_i)$.

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4) This section is a brief summary of Sections 2-4 in Yi (2005).

1. Agent Normal-form Ultimatum Bargaining Games

In agent-normal form, each information set is played by a different agent and each agent of the responder makes a decision based on the offer in his hand. Since the responder has the same expected payoff as the expected payoffs of each agent over terminal nodes, the expected payoffs are

$$\pi_p(x_p) = (1 - x_p)p_r(A \mid x_p),\ \pi_r(A \mid x_p) = x_p \text{ and } \pi_r(R \mid x_p) = 0\,. \tag{1}$$

2. Normal-form Ultimatum Bargaining Game

In the normal form, the responder's strategy is a complete contingent plan stating whether to accept or reject each possible offer and there are 2^{n+1} possible choices. In the analysis, the responder's strategy choice is identified with the minimum acceptable offers. In this way, one can reduce the payoff matrix of its normal-form representation of the ultimatum bargaining game from $(n+1) \times 2^{n+1}$ to $(n+1) \times (n+1)$. Note that this simplification rules out only the choices that the responder accepts small offers but reject larger offers.

The rule of the normal-form bargaining game is as follows. The proposer states an offer of a split of the pie, x_p, and the responder writes down the minimum acceptable offer, x_r, simultaneously. If $x_p \geq x_r$, then the proposer receives $1 - x_p$ and the responder receives x_p. Otherwise, both get nothing. The expected payoffs are given by :

$$\pi_p(x_p) = (1 - x_p)p_r(x_p) \text{ and } \pi_r(x_r) = \sum_{j = x_r}^{1} j_p p_p(j). \tag{2}$$

3. Logit Equilibrium Model

For an empirical test of QRE models, I use a specialized version of QRE where the choice probabilities are determined by an analogue of the standard multinomial logit distribution. The probability density, p_i, of player i's choosing x_i is a function of the expected payoff $p_i(x_i)$ and the density of each choice is an increasing function of the expected payoff for that choice.

$$p_i(x_i) = \frac{\exp(\lambda \pi_i(x_i))}{\sum_{X_i} \exp(\lambda \pi_i(x))} \tag{3}$$

where $0 \leq \lambda < \infty$ measures the amount of noise, or equivalently, the degree of rationality. A logit equilibrium for λ is defined by a fixed point of all players' choice densities with a given λ.

Proposition 1 (Yi, 2005). In the agent normal-form ultimatum bargaining game, only the perfect equilibrium can be the limit of logit equilibrium as $\lambda \rightarrow \infty$. In normal form, any offers between 0 and equal split, exclusively, can be the limit of logit equilibrium.

The existence of multiple QRE in normal form makes it possible that equilibrium selection is influenced by the history of the play, which is commonly observed in experimental data. Because of this, the estimated QRE in the following section can be viewed as a snap shot of the dynamic process of "converging" to an equilibrium.

Ⅲ. Estimation of the Logit Equilibrium Model

In this section, I estimate both the normal-form and the extensive-form logit equilibrium models assuming selfish players using an experimental data to test which model is better in describing human subjects' behavior. From the game theoretic point of view, there is no difference between those two versions of the ultimatum bargaining game because in the normal form the responder can make a conditional decision just as in the extensive form. However, the noisy best response assumed in QRE leads to different predictions in those two versions of the ultimatum bargaining game.

The QRE strategy is a function of expected payoffs and the key difference between those two versions is the way of calculating the responder's expected payoffs.[5] In the agent normal-form ultimatum bargaining game, the

5) Although the responder should make the decision before playing the game, the

noise in the proposer's strategy does not interact with the responder's decision because the responder's expected payoff from accepting an offer is the offer itself and his rejection leaves him 0. Thus the responder's situation is equivalent to a one-person decision making problem that does not involve any strategic consideration, and it is natural to expect that the QRE converges to the perfect equilibrium as the noise vanishes. In the normal-form logit equilibrium, by contrast, the proposer's QRE strategy mixture interacts with the responder's accepting decision and there can be a QRE in which the proposer makes a generous offer. That is, in a QRE where the noise is sufficiently small, since the proposer offers the "equilibrium" offer with a probability close to one, the expected cost of rejecting an offer less than the "equilibrium" offer to the responder is so small that the chance of the responder rejecting such offers is large enough to prevent the proposer from lowering the offer.

Note that main purpose of this paper is to compare belief structures implied by two different presentations of the game, not to make comparative statics predictions.[6] As Haile *et al.* (2003) argued, the QRE structure is flexible enough to match all the experimental data with different assumptions on noise structure and the ability to fit data is uninformative. In this regards, the present application of QRE framework should be considered as an effective way to discriminate the belief structure of the players. Although the main reason to reject significantly large offers would not be by mistake or payoff

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point is not the timing of move, but the noisy response and the way to calculate the expected payoffs. When there is noise in decision, under the best response assumption, since a strategy with a higher expected payoff in one presentation also has a higher expected payoff in the other, the outcome does not depends on the presentation.

6) QRE captures some important aspects of subjects' behavior but misses too much detail. For a prediction purpose, risk-aversion and inconsistent belief should be explicitly modeled. Since about 40% of the proposers made offers more than or equal to 45% of the pie in every session and round, it is not clear how this will affect responders' beliefs later on. Without taking care of such large offers by, for instance, allowing risk-aversion, it is likely that such offers push up the distribution of expected offers and the prediction on limiting offers should be upward biased.

perturbation, the QRE structure provides a tractable way to examine players' incentive structure by comparing agent normal-form and normal-form QRE of the ultimatum bargaining game.

1. Slonim and Roth's (1998) Experimental Design and Result[7)]

In the ultimatum bargaining game, subjects participated in a sequence of ten games against randomly matched anonymous opponents. During the ten game session a subject learned only the results of his or her own negotiations. Each subject was randomly assigned to be a proposer or responder, and a subject played the same role throughout the ten game session. In all games the pie was 1000 points and proposed divisions could be make in units of 5 points $(0, 5, 10, \cdots, 995, 1000)$. The redemption value of 1000 points was 60, 300, or 1500 Slovak Crowns (Sk), depending on the treatment.[8)]

Ten sessions consisted of 7 to 10 proposers were conducted, three at 60 Sk, four at 300 Sk, and three at 1500 Sk. Throughout this paper, offers made by subject 401 in treatment 300 Sk who offered 5 all the time and associated rejection decisions are excluded from analysis. Some summary statistics of the data are summarized in <Table 1>.

Slonim and Roth reported that responders in treatments with bigger pies rejected proportionally equivalent offers less often, although rejections still occurred even when substantial financial loss results. Moreover, in most sessions, with a few exceptional rounds, the proposers in a session with a higher average offer kept offering higher offers on average, and the same amount of offer were rejected more frequently in that session. On the other

7) See Slonim and Roth (1998) and Yi (2005) for more detailed experimental design and result.

8) Statistics were unavailable on student wages. The 20 to 30 Sk per hour average student wage rate came from personal observation. In terms of purchasing power, for example, a dormitory room cost 150 Sk per month, a monthly bus pass cost 80 Sk, a local phone call cost 2 Sk for 6 minutes, and a movie cost 24 Sk. The exchange rate was 31 Sk for $1; thus the stakes were $1.9, $9.7, and $48.4 for the 60, 300, and 1500 Sk treatments, respectively (Footnote 4 in Slonim and Roth, 1998).

hand, in all sessions in every treatment there is positive correlation between the average offers and the average of rejected offers. These features suggest that subjects' choices might not be independent as modeled in agent normal form.

〈Table 1〉 Summary of Ultimatum Bargaining Game Experiment

Offer ranges	60Sk, N=24		300Sk, N=33		1500Sk, N=25		# Offer
	% Offer	% Reject	% Offer	% Reject	% Offer	% Reject	
>500	6.7(16)	6.7(1)	6.7(22)	4.5(1)	7.2(18)	0.0(0)	56
=500	23.8(68)	0.0(0)	21.5(71)	1.4(1)	30.8(77)	2.6(2)	216
450-495	21.7(52)	9.6(5)	23.0(76)	5.3(4)	6.0(15)	0.0(0)	143
400-445	24.6(59)	23.7(14)	21.5(71)	12.5(9)	32.4(81)	4.9(4)	211
350-395	11.3(27)	40.7(11)	9.4(31)	9.7(3)	5.2(13)	0.0(0)	71
300-345	4.6(11)	45.5(5)	10.6(35)	20.0(7)	7.2(18)	11.1(2)	64
250-295	2.5(6)	66.7(4)	3.9(13)	30.8(4)	3.2(8)	37.5(3)	27
<250	0.4(1)	100.0(1)	3.3(11)	90.9(10)	8.0(20)	60.0(12)	32
All Offers	100(240)	17.1(41)	100(330)	12.1(40)	100(250)	9.2(23)	820
Offers Analyzed*	100(240)	17.1(41)	100(320)	9.7(31)	100(250)	9.2(23)	810
Average	445.0	375.7	435.7	377.2	427.5	275.2	810
S.E.	(69.3)	(71.8)	(77.0)	(97.54)	(113.0)	(113.3)	810

Source : Slonim and Roth (1998).
* The analysis excluded the offers made by subject 401 in session 1 of treatment 300Sk, who offered 5 in every round, and rejected 9 times.

The history dependence suggests that the analysis should be done period-by-period. However, since the purpose of the estimation is not to examine learning, I pool the data across rounds for each treatment. Since there is little differences in mean offer and the number of rejections between rounds 1-5 and 6-10, the pooling would not change the conclusion.[9] The results for the first half of rounds and the last half are provided for references. Moreover,

9) Since it is common that λ declines over time in repeated game experiments, some studies like Capra, Goeree, Gomez and Holt (1999) use the results of the last few periods. However, all the arguments below hold with estimation results of the last 5 periods.

since the players' intention is not explicitly modeled, the choice distribution is used to represent players' intention so that the data considered as the result of a game played by two players. Because of the limited number of observations, I also pool the data across sessions for each treatment.

2. Estimation of QRE

The logit model that underlies logit equilibrium makes it easy to test logit equilibrium model and it is straightforward to derive the following likelihood functions. From Eq. (1) with Eq. (2), for the agent normal-form logit equilibrium model, we have

$$\ln L(\lambda_p, \lambda_r) = \ln L_p(\lambda_p, \lambda_r) + \ln L_r(\lambda_r)$$

and, with Eq. (3), the likelihood function of the normal-form logit equilibrium model is

$$\ln L(\lambda_p, \lambda_r) = \ln L_p(\lambda_p, \lambda_r) + \ln L_r(\lambda_p, \lambda_r)$$

with

$$\ln L_p(\lambda_p, \lambda_r) = \sum_{i \in I_p} \ln p_p^i(x_p^i)$$

$$\ln L_r(\lambda_p, \lambda_r) = \sum_{i \in I_r} [1(Accept \;\; x_p^i) \ln P_r^i(x_r^i) + 1(reject \;\; x_p^i) \ln (1 - P_r^i(x_r^i))]$$

where $1(\cdot)$ is an indicator function, and I_p and I_r are sets of proposers and responders. P_i is the cumulative distribution function associated with p_i.

Since the size of the pie is normalized to 1, the estimates of λ's should be understood as 1000λ, and those for treatments 300 Sk and 1500 Sk should be multiplied by 5 and 25 to compare them across treatments. Because of the equilibrium condition on choices, the set of parameter values is found by a grid search.

The estimation results are reported in <Table 2> where 'reject' is conditional rejection rate of offers between 250 and 445 inclusively for which most rejection occurs and gives clear distinction between treatments, and 'offer' is average offer. The number in parentheses below the parameter estimate is standard error. The numbers in parentheses below $-\ln L$ is $-\ln L_r$ so that $\ln L_p = \ln L - \ln L_r$, and those under 'reject' and 'offer' are sample statistics. The number of offers made in each round is 24, 32, and 25 in treatments 60 Sk, 300 Sk, and 1500 Sk, respectively.

〈Table 2〉 Estimation Results of Logit Equilibrium Models

	Extensive Form					Normal Form				
	λ_p	λ_r	$-\ln L$	reject	offer	λ_p	λ_r	$-\ln L$	reject	offer
60 Sk										
1-5	4.55	4.22	669.5	0.19	340.7	43.54	1.87	562.8	0.45	435.2
	(0.17)	(0.00)	(56.0)	(0.34)	(441.4)	(2.78)	(0.08)	(64.0)	(0.34)	(441.4)
6-10	4.30	4.75	658.7	0.17	344.4	60.66	1.62	541.8	0.45	438.3
	(0.76)	(0.00)	(44.5)	(0.32)	(448.7)	(4.97)	(0.06)	(63.6)	(0.32)	(448.7)
All	4.43	4.47	1328.5	0.18	342.4	50.74	1.75	1106.0	0.45	436.5
	(0.38)	(0.00)	(100.9)	(0.33)	(445.0)	(1.87)	(0.04)	(127.2)	(0.33)	(445.0)
300Sk										
1-5	4.09	5.67	856.8	0.13	345.0	40.61	2.13	740.2	0.43	428.8
	(0.56)	(0.00)	(38.4)	(0.16)	(445.8)	(3.14)	(0.07)	(76.8)	(0.16)	(445.8)
6-10	5.06	5.22	863.5	0.15	325.7	29.45	3.07	752.3	0.38	411.0
	(0.58)	(0.00)	(53.7)	(0.15)	(425.6)	(2.01)	(0.10)	(73.8)	(0.15)	(425.6)
All	4.56	5.42	1720.9	0.14	335.2	33.50	2.63	1495.0	0.40	419.2
	(0.29)	(0.00)	(92.5)	(0.15)	(435.7)	(1.38)	(0.05)	(148.7)	(0.15)	(435.7)
1500Sk										
1-5	3.81	6.14	668.2	0.11	349.9	11.53	5.35	621.6	0.29	404.3
	(0.46)	(0.00)	(27.3)	(0.09)	(434.5)	(0.79)	(0.40)	(36.3)	(0.09)	(434.5)
6-10	4.73	5.84	666.9	0.12	329.4	15.38	4.89	607.6	0.30	397.0
	(0.44)	(0.00)	(33.3)	(0.06)	(420.4)	(0.81)	(0.19)	(42.6)	(0.06)	(420.4)
All	4.25	5.97	1335.5	0.12	339.6	13.25	5.11	1230.5	.29	400.7
	(0.23)	(0.00)	(60.7)	(0.07)	(427.5)	(0.39)	(0.13)	(79.0)	(0.07)	(427.5)

In terms of overall $\ln L$, although the normal-form logit equilibrium model outperforms the extensive-form logit equilibrium model, $\ln L_r$ is smaller in the agent normal form while $\ln L_p$ is smaller in the normal form. Despite of such a big difference in two models' $\ln L$, the mixed results on the descriptive power of the behavior of each role calls for a closer look at the features of data and the implications of the logit equilibrium models to make a judgment.

In the experiment, the most challenging feature of the offering behavior is that more than 30% of proposers offered more than or equal to 500 in every treatment. On the other hand, the responders rejected offers less than 300 with high probabilities ; 71% (5/7), 36% (5/14), and 54% (15/28), in treatments 60 Sk, 300 Sk, and 1500 Sk, respectively. These features provide a way to evaluate the estimation results.[10)]

In the extensive-form logit equilibrium model, the responder cannot reject even the offer of 0 with probability more than 50% so that the proposer's expected payoff from offering 0 is 500 independent of λ's, and even offering 0 is not worse than offering 500. These properties would result in small predicted rejection rates and offers. The agent normal-form logit equilibrium model underestimates the conditional rejection rates in treatments 60 Sk and 300 Sk, and underestimates the average offers in all treatments by 100 on average. Although it is not compatible with high conditional rejection rates of small offers, the fact that 77% (626/810) of offers are bigger than or equal to 400 with a conditional rejection rate of 6% (40/626) makes $\ln L_r$ significantly better than the normal form. In contrast, in the normal-form logit equilibrium model, the high conditional rejection rate for offers less than 300 requires

10) In most ultimatum bargaining experiment, it has been very typical that offers average about 40% of the pie with a mode of 50%. Slonim and Roth's (1998) experiment is not an exception where offers of 500 were made 217 times out of 810 and only 2 of them were rejected, and offers of 400 were made 125 times and 18 of them were rejected. The expected payoffs from offering 500 and 400, which are 495 and 516, respectively, imply that there seems little hope for the self-interested individual assumption. However, this is only a half of the story. In treatment 1500 Sk, while most proposers seemed to try to find the level of minimum acceptable offer by engaging in experimentation, 6 proposers out of 25 kept offering more than or equal to 50% throughout the session. Their offers account for 63% (60/95) of such offers and they experienced virtually no rejections (only two offers of 50% rejected in rounds 7 and 10.) For the rest of proposers, self-interested behavior still has potential to explain such behavior. In rounds 1 and 2, 50 offers of 500 were made and none of them were rejected and 19 of offers of 400 were made and it is rejected 4 times. In the first two rounds the expected payoff from offering 400 is 473.7 and this could affect the offer behavior in the rest of rounds if learning occurs slowly.

small λ_r and, on the other hand, the small conditional rejection rate of offers bigger than 400 requires that $\sum_{x_r=400}^{1000} p_r(x_r)$ is close to 0, or a large λ_r. Such conditions make the normal-form logit equilibrium model incompatible to the responders' behavior, and the conditional rejection rates are overestimated to be several times larger than the actual rejection rates in all treatments.

This observation suggests that the structure of the standard logit equilibrium model of the ultimatum bargaining game is limited to capture subjects' behavior in the experiment, and it is necessary to modify these models to test the differences in belief structure of the agent normal- and the normal-form logit equilibrium models.

Ⅳ. Estimation of the Logit Model of the Insulting Model

This section considers a model incorporating fairness into players' utilities to see how that affects the performances of models with different belief structure in describing experimental data and to deal with the problem with QRE model with self-interested players. For these purposes, I use an "insulting model" which assumes that the responder rejects offers too small, less than α, and accept offers big enough, bigger than β, with probability one. α and β represent players' notions of fairness. In this way, we can alleviate the problem with the sudden jumps in the distributions of offers and conditional rejection rates around offers of 300 and 500.

The insulting model is proposed by Ochs and Roth (1989), and puts a restriction on preferences in the way that the responder rejects offers less than a certain level by considering it as an insulting. Allowing possibly different levels of insulting offers across treatments, we can test the endogeneity of the level. Since such modification preserves the way of calculating the expected payoffs between two representations, the estimation of the insulting model would give better idea about which information structure describes the behavior better. In addition, to take care of some abnormal behavior in the data such as offering more than 800 and rejecting offers larger than 500, some noise is added independent of expected payoffs.

The agent normal-form insulting model is

$$\tilde{p}_p(x_p) = (1-\omega_p)p_p(x_p) + \omega_p \cdot \xi(x_p) \tag{4}$$

$$\begin{aligned} \tilde{p}_r(A|x_p) &= (\omega_r \zeta(x_r)) && \text{if } x_r < \alpha \\ &= (1-\omega_r)[\gamma + (1-\gamma)p_r(A|x_p)] + \omega_r \zeta(x_r) && \text{if } \alpha \le x_p \le \beta \\ &= (1-\omega_r) + \omega_r \cdot \zeta(x_r) && \text{if } x_p > \beta \end{aligned} \tag{5}$$

and the normal-form insulting model is

$$\tilde{p}_p(x_p) = (1-\omega_p)p_p(x_p) + \omega_p \cdot \xi(x_p) \tag{6}$$

$$\begin{aligned} \tilde{p}_r(x_r) &= (\omega_r \cdot \zeta(x_r)) && \text{if } x_r < \alpha \\ &= (1-\omega_r)[\gamma + (1-\gamma)p_r(x_r)] + \omega_r \cdot \zeta(x_r) && \text{if } x_r = \alpha \\ &= (1-\omega_r)p_r(x_r) + \omega_r \cdot \zeta(x_r) && \text{if } \alpha < x_p \le \beta \\ &= \omega_r(x_r) && \text{if } x_r > \beta \end{aligned} \tag{7}$$

where $\alpha, \beta, \gamma, \omega \in [0,1]$. ζ and ξ are cdf and pdf of uniform distribution on $\{0, 5, \cdots, 995, 1000\}$, respectively, so that $\tilde{p}$'s are convex combinations of p's, and ζ or ξ. Note that ζ and ξ are pure noise and the logit equilibrium strategies are calculated assuming γ and ω are zero.

In the insulting model, the responder plays logit equilibrium strategy as if he only could play strategies between α and β. Since $p_r(\alpha)$ could be unrealistically small in the normal form, γ is introduced to adjust the chance that the responder accepts the insulting offer α. Therefore, α and γ measure the contribution of the insulting model which assumes the responder reject an insultingly low offer (less than α) and accept any offer bigger than or equal to the threshold, α, with probability one. In the agent normal-form, since the probability of accepting any offer is equal to one half, γ would not be significant. The terms of $\omega \cdot \zeta$ and $\omega \cdot \xi$ alleviate the effect of outliers like offering 800 (in treatment 1500 Sk) or rejecting offer of 660 (in treatment 300 Sk).

With positive α and β, the responder could reject small offers and could accept large offers with a high probability independent of λ's. Although λ's have the same interpretations that λ's measure a player's ability to discern small differences in expected payoffs as in the logit equilibrium model, if α and β account for significant parts of the responders' behavior, the role of λ_r would become insignificant. In this case, α and β could undermine the implication of logit equilibrium on refinement. If α is big enough, the offers bigger than a half of the pie could be supported by the model and virtually there is no restrictions on the equilibrium offer.

It should be noted that the estimates of α and β may not represent the levels of insulting offers because they do not impose any restriction on the proposer's preferences, but work indirectly. In the insulting model, if $\gamma = \omega = 0$, the expected payoff from offering lower than α is 0 and offering greater than β is always worse than offering β with probability one. For instance, when there were no offers less than 100, if $\lambda_r = 0$ and $\beta = 500$, there should be no difference between p_r with $\alpha = 0$ and p_r with $\alpha = 100$ because of γ which makes $\sum_{x_r=100}^{500} p_r(x_r)$ identical for $x_r \geq 100$ as it is clear in Eq. (5) and Eq. (7), but still the proposer's equilibrium strategies are different with respect to those p_r's. In estimation, therefore, if the play were far from the equilibrium, there could be a trade-off between $\ln L_p$ and $\ln L_r$ with different α and β, and we are not able to derive any inference about the insulting model from the estimates of α and β.

However, if play is close to an equilibrium, the estimates of α and β provide an useful information whether people share the same idea on the size of a fair offer or not. The different values of the estimates of α and β across sessions might be an evidence for the dependence of the rejection decision on the proposer's intention.

〈Table 3a〉 Estimation Results of Agent Normal-Form Insulting Model

	α	β	λ_p	λ_r	ω_p	ω_r	γ	$-\ln L$	reject	offer	SE
60 Sk											
1-5	330	495	29.79	2.97	0.10	0.12	0.00	551.7	0.30	450.1	122.5
	-	-	(2.84)	(0.42)	(0.03)	(0.08)	(0.11)	(50.3)	(0.34)	(441.4)	(74.6)
6-10	335	495	24.14	3.29	0.02	0.00	0.00	520.7	0.23	444.7	93.9
	-	-	(2.01)	(0.45)	(0.02)	(0.12)	(0.15)	(36.9)	(0.32)	(448.7)	(63.7)
All	335	495	27.27	3.14	0.07	0.08	0.08	1081.6	0.27	448.2	110.2
	-	-	(1.23)	(0.22)	(0.01)	(0.05)	(0.06)	(93.0)	(0.33)	(445.0)	(69.3)
300Sk											
1-5	290	445	19.65	2.98	0.00	0.03	0.00	683.9	0.27	438.4	86.7
	-	-	(1.93)	(0.05)	(0.04)	(0.03)	(0.13)	(38.6)	(0.16)	(445.8)	(72.7)
6-10	270	395	14.95	3.28	0.02	0.14	0.00	723.8	0.19	425.0	95.7
	-	-	(1.00)	(0.90)	(0.01)	(0.06)	(0.22)	(56.2)	(0.15)	(425.6)	(80.3)
All	270	445	17.64	3.11	0.01	0.05	0.00	1441.9	0.28	425.0	101.9
	-	-	(1.01)	(0.20)	(0.01)	(0.03)	(0.05)	(93.7)	(0.15)	(435.7)	(77.0)
1500Sk											
1-5	200	495	14.52	2.30	0.07	0.02	0.00	619.5	0.33	400.5	158.7
	-	-	(1.82)	(0.19)	(0.06)	(0.05)	(0.03)	(37.9)	(0.09)	(434.5)	(119.7)
6-10	200	495	15.15	2.67	0.02	0.04	0.00	608.3	0.31	381.1	142.0
	-	-	(2.92)	(0.42)	(0.03)	(0.39)	(0.03)	(42.0)	(0.06)	(420.4)	(105.9)
All	200	495	14.82	2.49	0.05	0.03	0.00	1229.2	0.32	390.8	150.8
	-	-	(1.30)	(0.11)	(0.03)	(0.05)	(0.02)	(80.1)	(0.07)	(427.5)	(113.0)

〈Table 3b〉 Estimation Results of Normal-Form Insulting Model

	α	β	λ_p	λ_r	ω_p	ω_r	γ	$-\ln L$	reject	offer	SE
60 Sk											
1-5	250	500	37.12	0.00	0.02	0.07	0.31	525.7	0.31	445.3	73.5
	-	-	(5.99)	(0.28)	(0.02)	(0.06)	(0.02)	(47.2)	(0.34)	(441.4)	(74.6)
6-10	250	500	36.93	0.00	0.00	0.00	0.30	501.2	0.29	448.6	60.8
	-	-	(4.10)	(0.34)	(0.03)	(0.12)	(0.01)	(34.3)	(0.32)	(448.7)	(63.7)
All	250	500	37.57	0.00	0.01	0.02	0.31	1029.0	0.30	446.7	68.0
	-	-	(3.30)	(0.18)	(0.01)	(0.04)	(0.01)	(82.8)	(0.33)	(445.0)	(69.3)
300Sk											
1-5	0	500	38.62	0.00	0.01	0.00	0.03	672.8	0.21	443.9	71.7
	-	-	(4.13)	(0.22)	(0.01)	(0.04)	(0.02)	(36.4)	(0.16)	(445.8)	(72.7)
6-10	0	500	45.39	0.00	0.03	0.05	0.08	707.5	0.23	426.3	81.3
	-	-	(11.06)	(0.38)	(0.02)	(0.05)	(0.03)	(55.6)	(0.15)	(425.6)	(80.3)
All	0	500	41.59	0.00	0.02	0.03	0.06	1384.9	0.22	435.0	77.6
	-	-	(3.46)	(0.14)	(0.01)	(0.02)	(0.01)	(93.4)	(0.15)	(435.7)	(77.0)
1500Sk											
1-5	0	500	77.96	0.00	0.28	0.00	0.05	582.8	0.19	461.4	160.2
	-	-	(62.29)	(0.89)	(0.03)	(0.17)	(0.09)	(26.0)	(0.09)	(434.5)	(119.7)
6-10	0	500	35.57	0.00	0.08	0.04	0.10	580.5	0.24	421.1	112.4
	-	-	(11.35)	(0.81)	(0.04)	(0.23)	(0.04)	(37.3)	(0.06)	(420.4)	(105.9)
All	0	500	58.23	0.00	0.22	0.02	0.07	1169.6	0.21	449.8	145.7
	-	-	(11.21)	(0.38)	(0.02)	(0.18)	(0.03)	(64.9)	(0.07)	(427.5)	(113.0)

The estimation results are summarized in <Table 3> where 'SE' is the standard error of offers. The insulting model improves the values of likelihood functions significantly in both presentations, Comparing to the previous result, a noteworthy feature of normal-from insulting model is that the estimates of λ_r are not significantly different from zero so that x_r is uniformly distributed over $[\alpha, \beta]$. This suggests that the responders are subject to large amount of uncertainty about the offer distribution as the sizes of SE show.

With the insulting model, the normal-form representation outperforms the agent normal-form in every treatment, and the differences in the likelihood function value are significant except in treatment 300 Sk. However, the agent normal-form of the insulting model performed poorer than that of the standard logit model. The main reason is that the insulting argument greatly improves the overall likelihood by $\ln L_p$ at the cost of $\ln L_r$. Since our main concern is the responder's behavior, the comparison is made between the agent normal-form of the logit equilibrium model and the normal-form of the insulting model.

In terms of the value of likelihood function, the offers were better described by the normal form of the insulting model in all treatments while the agent normal form without insulting argument does better for the rejection behavior except in treatment 60 Sk. The differences in $-\ln L_p$ $(-\ln L_r)$ between the normal-form insulting model and the agent normal-form model are 298.5 (18.1), 337.0 (-0.9), and 165.9 (-4.2) in treatments 60 Sk, 300 Sk, and 1500 Sk, respectively. However, only the difference of $-\ln L_r$ in treatment 60 Sk is significant (the value of $\chi^2_{(5)}$ at the significant level of 5% is 11.07.) On the other hand, in treatment 60 Sk, as SE showed, "convergence" of the offers allows a clearer interpretation of the result. The predicted rejection rate, mean offer, and standard error in most sessions are greatly improved. In particular, such an accurate descriptions of mean offer and conditional rejection rate in treatment 60 Sk, suggests that the normal-form insulting model provides a better way to describe the experimental result.

In the normal form presentation of the ultimatum bargaining game, the proposer's intention has a great influence on the responder's choice because the responder would prefer rejecting rarely expected offers. Together with the significant differences in mean offer and average of rejected offers across treatments, therefore, the estimation result suggests that the responder's choice is correlated with the proposer's intention and can be thought of an evidence in favor of the hypothesis that the consensus on the size of acceptable offer emerges through the repeated interactions.

Ⅴ. Conclusion

The present study uses the logit equilibrium model to compare the belief structures of the two different game representations of the ultimatum bargaining game to see whether intention matters in human subjects' decision making in the experimental bargaining game. Although the present analysis does not explain how intention affects the choice behavior, the result shows that the players' behavior in Slonim and Roth's (1998) experiment is more consistent with the normal-form logit equilibrium model even with interdependent preferences that considers only the relative sizes of payoffs. The result conforms to the previous findings of Binmore *et al.* (2002) and Johnson *et al.* (2002) that controlling for interdependent utility is limited to describe the subjects' behavior, and suggests that it is necessary to model the role of intention in an explicit way considering the dynamic features of the experimental result for a fuller explanation of the choice behavior.

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〔Abstract〕

최후통첩게임의 신념 구조

이 강 오

이 논문에선 최후통첩게임의 경기자들이 물질적인 보수만을 극대화하는지를 살펴보기 위해 양자적반응균형모형을 이용하여 두 가지 최후통첩게임 모형에서 경기자들의 신념 구조를 비교하였다. 전개형 개임에서 응답자의 선택은 오로지 자신에게 주어진 제안의 크기에 따라서 결정되는 반면 전략형 게임에서 응답자의 선택은 제안자의 의도가 어떠한 가에 영향을 받게 된다. 이러한 차이를 Slonim and Roth(1998, *Econometrica*)의 실험 자료를 이용해 살펴본 결과, 실제 게임은 전개형으로 진행된다하더라도 실제 반응자의 선택은 전략형 게임에서 유도된 신념체계와 더 부합하는 것으로 나타났다. 즉, 반응자의 최후 결정은 금전적인 보수 이외에도 제안자의 의도에 근거한 제안 자체의 공정성에 영향을 받는다는 것을 보여주고 있다.

핵심용어 : 양자적반응균형모형, 최후통첩게임, 후방귀납, 공정성