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On the Connection between the Expectations Hypothesis of the Term Structure of Interest Rates and Risk Neutrality*

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Using the non-expected recursive utility function of Epstein and Zin (1989), this paper re-examines the connection between the expectations hypothesis of the term structure theory and risk neutrality in three-date general equilibria. Major findings are : (i) To generate stochastic future interest rates in risk neutral economies, disentangling agents' two disparate preference components - intertemporal substitution and risk aversion - is critically important; (ii) As a result, term premia are non-zero and thus the expectations hypothesis does not hold under risk neutrality. These non-zero term premia are characterized as compensations for the investor being averse to intertemporal substitution.

Keywords: Term Structure, Expectations Hypothesis, Risk Neutrality, Non-expected Utility

JEL Classifications: E43, G12

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Ⅰ. Introduction

One traditional presumption in term structure literature is that the expectations hypothesis in the case of uncertainty will hold under risk neutrality. A more recent study by Cox, Ingersoll, and Ross [CIR, henceforth] (1981) have scrutinized this presumption in general equilibria. In a pure exchange economy, they show that all forms of the expectations hypothesis hold under risk neutrality, but their findings are valid only in the case where future interest rates become degenerate [see LeRoy (1982, 1983) also]. In a production economy, they obtain the following two results. When the marginal utility of consumption and the marginal utility of wealth are equal, the expectations hypothesis holds, but again in a degenerate case. When they are different, interest rates are stochastic and the expectations hypothesis does not hold. The second case is characterized by corner solutions. Overall, the validity of CIR's findings is greatly limited since the model fails to generate a stochastic environment for future interest rates except for a special case in the production economy.

To specify agents' preferences, the CIR model uses time-additive expected utility in which two disparate preference components — intertemporal substitution and risk aversion — are confounded. When the power function is chosen for one-period utility, the elasticity of intertemporal substitution is constrained to be the inverse of the risk aversion coefficient. For example, if agents are assumed to be risk neutral, it is necessary to assume infinite elasticity of intertemporal substitution. Then, one may raise the following questions : Is this built-in constraint likely to be transmitted into CIR's model and to put any restriction on their results? If so, how will these restricted results be altered if the two preference components are specified independently?

The purpose of this paper is to answer these questions. I present below three-date[1)] term structure models using the non-expected utility function developed by Epstein and Zin (1989) [EZ utility, henceforth], in which inter-

1) While it is possible to extend the model in this paper beyond three dates, basic insights are well illustrated in this simple context.

temporal substitution and risk aversion are disentangled. While Duffie and Epstein (1992) have employed a continuous-time version of EZ utility to study the term structure of interest rates, their focus is different from that of the model presented here. Assuming unit elasticity of substitution, they generalize the closed-form formula for interest rates presented in CIR's 1985 paper.

In brief, this paper finds : when examining the connection between the expectations hypothesis and risk neutrality in both exchange and production economies, disentangling agents' two disparate preference components — intertemporal substitution and risk aversion — plays a crucial role in generating a random future interest rate. In this stochastic environment, risk neutrality leads to non-zero term premia. Thus the conventional presumption that the expectations hypothesis holds under risk neutrality is legitimately rejected.

The remainder of this article is organized as follows : Section Ⅱ presents notation and definitions of terminologies. Section Ⅲ develops a term structure model in a pure exchange economy. Section Ⅳ explores the model in an economy with linear production technology. Section Ⅴ summarizes and concludes.

Ⅱ. Preliminary

Let ${}_t r_{t+n}$ denote the n-period annual real (gross) interest rate prevailing at time t. Suppose that at time 0, the investment horizon of an investor is one period. If the investor buys a one-period bond, he is guaranteed a sure return, ${}_0 r_1$. If he invests in a two-period zero-coupon bond and sell it at time 1, the return will be ${}_0 r_2^2 / {}_1\tilde{r}_2$, where ${}_1\tilde{r}_2$ denotes the future one-period interest rate and is uncertain at time 0. The expected excess return on the latter strategy over the former is $E({}_0 r_2^2 / {}_1\tilde{r}_2) - {}_0 r_1$.

Suppose that the investment horizon is two periods. If the investor rolls over one-period bonds, the holding period return will be ${}_0 r_1 \cdot E({}_1\tilde{r}_2)$. If the investor holds a two-period bond until maturity, this will yield a sure return of ${}_0 r_2^2$, and thus the expected excess return in this case is ${}_0r_2^2 - {}_0r_1 \cdot E({}_1\tilde{r}_2)$.

These expected excess returns are referred to as the holding period premium [the HP, henceforth] and the rolling premium [the RP, henceforth], respectively [see Campbell (1986)] :

$$HP \equiv E({}_0r_2^2 / {}_1\tilde{r}_2) - {}_0r_1 \qquad (1)$$

$$RP \equiv {}_0r_2^2 - {}_0r_1 \cdot E({}_1\tilde{r}_2) \qquad (2)$$

The sign of these term premia will form a basis for examining the validity of the expectations hypothesis of the term structure theory. If the HP is zero, what CIR (1981) call the local expectations hypothesis holds. If the RP is zero, what they call the return-to-maturity expectations hypothesis is valid.2),3) In what follows, I examine the sign of these term premia in general equilibria where the interest rates are endogenously determined as functions of economic state variables. Two types of economy are analyzed below : a pure exchange economy where consumption goods are perishable, and a production economy where goods can be transformed into capital to produce future output.

Ⅲ. A Pure Exchange Economy

Consider a three-date (t=0,1,2) Lucas (1978) economy in which the amount of perishable consumption goods produced by capital is exogenous and stochastic. Suppose that the stochastic process for the amount of output over time is described by :

$$\tilde{y}_{t+1} = \tilde{g}_{t+1} \cdot y_t \quad \text{for } t=0,1 \qquad (3)$$

where y_t is the output at time t, and $\tilde{g}_{t+1}$ is the (gross) output growth rate from time t to t+1. $\tilde{g}_{t+1}$ is uncertain at time t but will be known at t+1.

2) In this particular three-date case, what CIR call the unbiased expectations hypothesis is equivalent to the return-to-maturity expectations hypothesis.

3) For details on the relationship between the HP and RP, see Woodward (1983).

The objective of a representative consumer in this economy is to maximize

$$U_0 = [c_0^{1-\rho} + \delta \cdot \hat{U}_1^{1-\rho}]^{1/(1-\rho)} \tag{4}$$

where

$$\hat{U}_1 \equiv [E_0 \tilde{U}_1^{1-\gamma}]^{1/(1-\gamma)}, \quad \tilde{U}_1 \equiv [\tilde{c}_1^{1-\rho} + \delta \cdot \hat{c}_2^{1-\rho}]^{1/(1-\rho)} \tag{5}$$

c_t and U_t are, respectively, consumption and utility at time t (t=0,1,2), E_t is the conditional expectation given all information up to time t, δ is the discount factor. $\gamma(0 \leq \gamma \neq 1)$ and $\rho(0 \leq \rho \neq 1)$ are coefficients of aversion to risk and intertemporal substitution, respectively [the elasticity of substitutions is $1/\rho$]. These two parameters represent the agent's attitudes toward riskiness and time variation of his consumption profiles, respectively. The objective function in equation (4) is the three-date version of EZ utility, which is a parametric representation of Kreps and Porteus (1978) non-expected recursive preferences. Taking the constant elasticity of substitution (CES) form, this utility function is an aggregator of current consumption and certainty equivalent of future utility, denoted by $\hat{U}_1$. Throughout the remainder of this paper, the hat($\hat{}$) notation is used to indicate the certainty equivalent of any random variable, and it is computed in the same fashion as shown in equation (5). Note that $\hat{c}_2$ is a random variable at time 0 since it is a function of a conditional expectation formed at time 1. By disentangling intertemporal substitution and risk aversion, EZ utility is characterized as a generalization of time-additive power expected utility in several periods [see Epstein and Zin (1989) for specific details].

Under the utility function in (4), the value function at time 0, V_0, may be written as :

$$V_0 \equiv \underset{\{c_0 \cdot c_1 \cdot c_2\}}{Max} U_0 = [c_0^{*1-\rho} + \delta \cdot \hat{V}_1^{1-\rho}]^{1/(1-\rho)} \tag{6}$$

where

$$\hat{V}_1 \equiv [E_0 \hat{V}_1^{1-\gamma}]^{1/(1-\gamma)}, \quad \tilde{V}_1 \equiv \underset{\{c_1 \cdot c_2\}}{Max} \tilde{U}_1 = [\tilde{c}_1^{*1-\rho} + \delta \cdot \hat{c}_2^{1-\rho}]^{1/(1-\rho)}, \tag{7}$$

and the asterisk (*) notation is used to indicate optimal solutions. Since the equilibrium of this economy is characterized by the condition

$$c_t^* = y_t \qquad \text{for } t=0,1,2, \tag{8}$$

substituting (8) into (6) and (7) will yield the value function at each time as follows : [4]

$$V_2 = y_2, \tag{9}$$

$$V_1 = \Theta_1 \cdot y_1, \quad \text{where } \Theta_1 \equiv [1 + \delta \cdot \hat{g}_2^{\,1-\rho}]^{1/(1-\rho)}, \tag{10}$$

$$V_0 = \Theta_0 \cdot y_0, \quad \Theta_0 \equiv [1 + \delta \cdot \hat{\phi}_1^{\,1-\rho}]^{1/(1-\rho)}, \quad \text{where } \phi_1 \equiv \Theta_1 \cdot g_1. \tag{11}$$

Due to the linear homogeneity of the utility function in (4), the value function is proportional to the current output in each period [see Epstein and Zin (1989)]. The proportionality factor, Θ_t, is interpreted as the marginal utility of output, and is a (time-varying) constant at time t but is stochastic before time t. That is, Θ_1 is a constant at time 1 but a random variable at time 0.

In this economy, the price at time t of a pure discount bond paying one unit of consumption good in n periods, denoted as ${}_td_{t+n}$, is determined as follows :

$${}_td_{t+n} = ({}_tr_{t+n})^{-n} = E_t(MRS_t^{t+n})^*, \tag{12}$$

where ${}_tr_{t+n}$ is as defined earlier, and MRS_t^{t+n} is the marginal rate of substitution of consumption between t+n and t. The asterisk (*) notation now indicates that all the values are evaluated in equilibrium. One can obtain the above formula by equating the utility loss from sacrificing ${}_td_{t+n}$ units of consumption at time t and the utility gain from recovering one unit of consumption at time t+n. Under the EZ utility function, it can be shown that

$$MRS_t^{t+n} = \delta^n \times \hat{U}_{t+1}^{\gamma-\rho} \times E_t(\widetilde{U}_{t+1}^{\rho-\gamma} \times \hat{U}_{t+2}^{\gamma-\rho} \times E_{t+1}(\widetilde{U}_{t+2}^{\rho-\gamma} \,..)..) \times \widetilde{U}_{t+n}^{\rho-\gamma} \times (\tilde{c}_{t+n}/c_t)^{-\rho}. \tag{13}$$

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4) All the derivations in this paper are available from the author upon request.

Evaluating the right-hand side of this equation in equilibrium and taking the expectation will yield the following current one- and two-period bond prices, respectively :

$$ {}_0d_1 = E_0(MRS_0^1)^* = \delta \times \hat{V}_1^{\gamma-\rho} \times E_0(\hat{V}_1^{\rho-\gamma} \cdot \tilde{y}_1^{-\rho} \cdot y_0^{\rho}), \tag{14} $$

$$ {}_0d_2 = E_0(MRS_0^2)^* = \delta^2 \times \hat{V}_1^{\gamma-\rho} \times E_0(\tilde{V}_1^{\rho-\gamma} \cdot \hat{V}_2^{\gamma-\rho}) \times E_0(\tilde{V}_2^{\rho-\gamma} \cdot \tilde{y}_2^{-\rho} \cdot y_0^{\rho}). \tag{15} $$

After calculations using the solutions in (9)-(11), equations (14) and (15) can be reduced to the following formulas :

$$ {}_0d_1 = \delta \times \hat{\phi}_1^{\gamma-\rho} \times E_0(\Theta_1^{\rho-\gamma} \cdot \tilde{g}_1^{-\gamma}), \quad \text{where } \phi_1 \equiv \Theta_1 \cdot g_1, \tag{16} $$

$$ {}_0d_2 = \delta^2 \times \hat{\phi}_1^{\gamma-\rho} \times E_0(\Theta_1^{\rho-\gamma} \cdot \hat{g}_2^{\gamma-\rho}) \times E_0(\tilde{g}_1^{-\gamma} \cdot \tilde{g}_2^{-\gamma}). \tag{17} $$

Similarly, the one-period bond price at time 1 can be derived as :

$$ {}_1d_2 = MRS_1^{2*} = \delta \times \hat{g}_2^{\gamma-\rho} \times \tilde{g}_2^{-\gamma}. \tag{18} $$

Note that with ρ equal to γ, equations (16)-(18) will be reduced to the familiar bond price formulas under time-additive power expected utility [see equation (4) of Sun (1992)]. I use these three equations below to examine the connection between the expectations hypothesis and risk neutrality. Risk neutrality refers to the case where γ equals zero in the utility function in (4) and can be paired with three different cases depending on the degree of elasticity of intertemporal substitution : (i) infinite elasticity of substitution ($\rho = 0$) (ii) constant elasticity of substitution ($0 < \rho < \infty$) (iii) zero elasticity of substitution ($\rho = \infty$).

The case $\gamma = \rho = 0$ corresponds to the time-additive linear expected utility function, in which assuming risk neutrality necessarily constrains the elasticity of substitution to be infinite. In this case, one can easily show that the HP and the RP, as defined in (1) and (2) respectively, are zero and that the expectations hypothesis will hold. Yet this result has little significance since the model fails to generate a stochastic future interest rate. As the current and future consumptions are perfect substitutes, the future interest rate in equation (18) becomes degenerate (${}_1d_2 = \delta$). When the future interest rate is certain, the expectations hypothesis would hold by a simple arbitrage argument. CIR (1981) and others [e.g., LeRoy (1982, 1983) and Campbell (1986)] have analyzed this case when they examine the connection between the expectations hypothesis and risk neutrality in a pure exchange economy.

Let us consider the second case where the agent is risk neutral but averse to intertemporal substitution ($\gamma = 0$ and $0 < \rho < \infty$).[5] Using equations (16)-(18) and the inverse relationship between bond prices and interest rates yields the following expression for the HP :

$$HP = \delta^{-1} \times \bar{\phi}_1^{\rho} \times \left[\frac{E_0(\bar{g}_2^{-\rho})}{E_0(\Theta_1^{\rho} \cdot \bar{g}_2^{-\rho})} - \frac{1}{E_0(\Theta_1^{\rho})} \right]$$

$$= \delta^{-1} \times \bar{\phi}_1^{\rho} \times \left[\frac{1}{E_0(\Theta_1^{\rho}) + Cov(\Theta_1^{\rho}, \bar{g}_2^{-\rho})/E_0(\bar{g}_2^{-\rho})} - \frac{1}{E_0(\Theta_1^{\rho})} \right], \quad (19)$$

where the bar ($\bar{\ }$) notation indicates conditional expectations [e.g., $\bar{g}_2 \equiv E_1(\tilde{g}_2)$], and Θ_1 is now defined with $\bar{g}_2$ in place of $\hat{g}_2$ in equations (10) and (11). As it turns out, the sign of the HP depends on the sign of the covariance term in the denominator. Since the derivative of Θ_1 with respect to $\bar{g}_2$ [refer to equation (10)] is positive, the covariance is negative and the HP is positive. It thus follows that the local expectations hypothesis does not hold under risk neutrality.

This result can be explained as follows : At time 1, the marginal utility of output is low when the price of the two-period bond to be sold is high, and vice versa.[6] As a result, the expected utility gain from investing in the two-period bond is lower than from investing in the one-period bond. Hence the current value of the two-period bond must be low, or equivalently, it must offer a higher expected return. It is emphasized that while a higher expected

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5) This case represents the Risk Neutral Constant Elasticity (RINCE) preferences of Farmer (1990).

6) This is caused by the non-zero degree of intertemporal substitution. With infinite elasticity of substitution ($\rho = 0$), the marginal utility of output and the bond price are not correlated with each other.

return is required on the risky strategy, it should not be viewed as a usual risk premium since the risk neutral investor requires no reward for risk. It should rather be characterized as a compensation for the investor being averse to intertemporal substitution.

The RP under $\gamma = 0$ and $0 < \rho < \infty$ is shown to be :

$$RP = \delta^{-2} \times \bar{\phi}_1^{\rho} \times \left[\frac{1}{E_0(\Theta_1^{\rho} \cdot \bar{g}_2^{-\rho})} - \frac{1}{E_0(\Theta_1^{\rho})/E_0(\bar{g}_2^{\rho})} \right]. \tag{20}$$

By comparing the magnitudes of the two denominator terms in the bracket, I prove in the appendix that the RP is negative. Since the expected utility gain from investing in short-term bonds successively (the rollover strategy) is lower than from investing in the two-period bond, the cost of the rollover strategy must be low, or equivalently, it must offer a higher expected return. Hence the return-to-maturity expectations hypothesis does not hold under risk neutrality.

Contrasted with the case under time-additive linear expected utility ($\gamma = \rho = 0$), the above two results are notably characterized by the presence of a stochastic future interest rate. With $\gamma = 0$ and $0 < \rho < \infty$, the future bond price in equation (18) becomes $\delta \times \bar{g}_2^{-\rho}$ and thus contains a random component in $\bar{g}_2^{-\rho}$ [$\bar{g}_2^{-\rho}$ is a function of a conditional expectation, which is a random variable]. Hence, the classical association of the expectations hypothesis with risk neutrality is rejected in a non-singular case. For this conclusion, the non-zero degree of intertemporal substitution ($0 < \rho < \infty$) plays a crucial role. It is noted that using the portfolio approach of Stiglitz (1970), Kihlstrom (1992) and Cho (1998) obtain the above sign results in partial equilibrium, not in general equilibrium.

In the final case where $\gamma = 0$ and $\rho = \infty$, the current and future consumptions are perfect complements, so that bond prices are uniformly zero and interest rates are infinitely large as long as the economy is growing. Since this case is of little interest, I do not analyze further.

Ⅳ. A Production Economy

Let us consider a three-date one-good production economy in which the production technology is represented by a stochastic constant-returns-to-scale so that capital (or wealth) is accumulated by the following process :

$$\tilde{k}_{t+1} = (k_t - c_t) \cdot \tilde{s}_{t+1} \quad \text{for t=0,1,} \tag{21}$$

where k_t denotes the level of capital at time t (k_0 is given) and $\tilde{s}_{t+1}$ represents the random productivity of capital.

Suppose that the representative consumer maximizes the objective function given in (4) subject to the constraint in (21). Solving this simple dynamic programming problem by backward induction gives the following optimal consumption rules and value functions:

$$c_2^* = k_2 \quad \text{and} \quad V_2 = k_2, \tag{22}$$

$$c_1^* = \lambda_1 \cdot k_1 \quad \text{and} \quad V_1 = \theta_1 \cdot k_1,$$

where $\lambda_1 \equiv [1 + \delta^{1/\rho} \cdot \hat{s}_2^{(1-\rho)/\rho}]^{-1}$, $\hat{s}_2 \equiv \{E_1 \tilde{s}_2^{1-\gamma}\}^{1/(1-\gamma)}$, and $\theta_1 \equiv \lambda_1^{\rho/(\rho-1)}$, (23)

$$c_0^* = \lambda_0 \cdot k_0 \quad \text{and} \quad V_0 = \theta_0 \cdot k_0,$$

where $\lambda_0 \equiv [1 + \delta^{1/\rho} \cdot \hat{\Omega}_1^{(1-\rho)/\rho}]^{-1}$, $\Omega_1 \equiv \theta_1 \cdot s_1$, and $\theta_0 \equiv \lambda_0^{\rho/(\rho-1)}$, (24)

Again, due to the linear homogeneity of the utility function, the agent consumes a fraction of the capital stock at each date and invests the rest for future consumption. The value function is also linear in the initial capital stock each period, and the linear coefficient (θ_t) is the marginal utility of capital. Since the bond prices in the production economy are also described by the relationship in (12), substituting the solutions in (22)-(24) into (14) and (15), and simplifying them using the first-order conditions[7] will generate the following equations corresponding to (16) and (17), respectively :

7) The first-order conditions at time 1 and 0 are as follows, respectively :
$\lambda_1^{-\rho} = \delta \cdot (1-\lambda_1)^{-\rho} \cdot \hat{s}_2^{1-\rho}$; $\lambda_0^{-\rho} = \delta \cdot (1-\lambda_0)^{-\rho} \cdot \hat{\Omega}_2^{1-\rho}$.

$$ {}_0d_1 = \widehat{\Omega}_1^{\gamma-1} \times E_0(\theta_1^{1-\gamma} \cdot \tilde{s}_1^{-\gamma}), \tag{25} $$

$$ {}_0d_2 = \widehat{\Omega}_1^{\gamma-1} \times E_0(\theta_1^{1-\gamma} \cdot (1-\lambda_1)^{\gamma} \cdot \hat{s}_2^{\gamma-1}) \times E_0((1-\lambda_1)^{-\gamma} \cdot \tilde{s}_1^{-\gamma} \cdot \tilde{s}_2^{-\gamma}). \tag{26} $$

The future bond price corresponding to equation (18) is similarly derived as :

$$ {}_1d_2 = \hat{s}_2^{\gamma-1} \times \tilde{s}_2^{-\gamma}. \tag{27} $$

Using equations (25)-(27) produces the following formula for the HP under risk neutrality ($\gamma = 0$) and constant elasticity of substitution ($0 < \rho < \infty$) :

$$ \begin{aligned} HP &= \overline{\Omega}_1 \times \left[\frac{E_0(\bar{s}_2^{-1})}{E_0(\theta_1 \cdot \bar{s}_2^{-1})} - \frac{1}{E_0(\theta_1)} \right] \\ &= \overline{\Omega}_1 \times \left[\frac{1}{E_0(\theta_1) + Cov(\theta_1, \bar{s}_2^{-1})/E_0(\bar{s}_2^{-1})} - \frac{1}{E_0(\theta_1)} \right], \end{aligned} \tag{28} $$

where the bar($\bar{}$) notation is defined similarly as in the preceding section, and θ_1 is now defined with $\bar{s}_2$ in place of $\hat{s}_2$ in (23). Again, the sign of the HP depends on the sign of the covariance term. Taking the derivative of θ_1 with respect to $\bar{s}_2$ will give a positive sign. Hence, the covariance is negative and the HP is positive.

The RP under $\gamma = 0$ and $0 < \rho < \infty$ is computed as :

$$ RP = \overline{\Omega}_1 \times \left[\frac{1}{E_0(\theta_1 \cdot \bar{s}_2^{-1})} - \frac{1}{E_0(\theta_1)/E_0(\bar{s}_2)} \right]. \tag{29} $$

Following the same procedure as presented in the appendix, the RP will prove to be negative.

As before, risk neutrality combined with constant elasticity of substitution leads to non-zero term premia in a non-trivial case where the future bond price in (27) is stochastic [${}_1d_2 = \bar{s}_2^{-1}$]. These results can be explained in exactly the same manner as in the pure exchange economy. Under time-additive linear expected utility where ρ is constrained to be zero, one would not obtain unique interior solutions for optimal consumption as shown in (23)-(24), and equations (25)-(27) as well. By contrast, under risk neutrality with $\rho > 0$, optimal consumption rules (λ's) are uniquely determined, and these unique solutions make it possible to obtain a non-trivial result.

CIR (1981) used time-additive linear expected utility to examine the validity of the local expectations hypothesis in a risk neutral production economy and derived the following two results : When the marginal utility of consumption and the marginal utility of wealth are equal, the expectations hypothesis holds, but in a degenerate case. When they are different, interest rates are stochastic and the expectations hypothesis does not hold. The second case is characterized by corner solutions. The present model shows that even when the marginal utility of consumption and the marginal utility of wealth are equal, separating aversion to intertemporal substitution and risk aversion will permit the existence of a stochastic future interest rate, and that the expectations hypothesis does not hold.

V. Summary and Conclusion

Using the non-expected recursive utility function of Epstein and Zin (1989), this article presents three-date general equilibrium term structure models in both exchange and production economies. Each model explores an important implication of separating the two disparate preference components — intertemporal substitution and risk aversion — for examining the connection between the expectations hypothesis and risk neutrality.

Disentangling the two preference components proves to be critical in generating a stochastic future interest rate. When the agent's preferences exhibit risk neutrality and constant elasticity of intertemporal substitution, the future interest rate becomes uncertain and as a result, both the holding period premium and the rolling premium are non-zero. It is stressed that these non-zero term premia should be characterized as compensations for the investor being averse to intertemporal substitution, not being risk averse. Hence the classical assertion that the expectations hypothesis holds under risk neutrality is legitimately rejected. One cannot generally obtain this result under time-additive linear expected utility in general equilibrium.

<Table 1> summarizes the results in this paper by providing comparison between the time-additive utility case and the non-expected recursive utility case.

〈Table 1〉 The Validity of the Expectations Hypothesis of the Term Structure of Interest Rates under Risk Neutrality

	Time-additive Expected Utility with RN (⇔ IES)	Epstein-Zin Utility with RN & CES
Pure Exchange Economy	• The future interest rate is non-stochastic. • The EH holds.	• The future interest rate is stochastic • The EH does not hold.
Production Economy	MUC = MUW, • The future interest rate is non-stochastic. • The EH holds. MUC ≠ MUW, • The future interest rate is stochastic. • The EH does not hold.	MUC = MUW, • The future interest rate is stochastic. • The EH does not hold. MUC ≠ MUW, Not Available

Notes) RN : risk neutrality; IES : infinite elasticity of substitution; CES : constant elasticity of substitution; EH : expectations hypothesis; MUC : marginal utility of consumption; MUW : marginal utility of wealth.

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Appendix

By comparing the denominator terms in the bracket of equation (20) in the text, I determine the sign of the rolling premium below :

$$
\begin{aligned}
& E_0(\Theta_1^{\rho} \cdot \bar{g}_2^{-\rho}) - E_0(\Theta_1^{\rho})/E_0(\bar{g}_2^{\rho}) \\
&= [1/E_0(\bar{g}_2^{\rho})] \cdot [E_0(\bar{g}_2^{\rho}) \cdot E_0(\Theta_1^{\rho} \cdot \bar{g}_2^{-\rho}) - E_0(\Theta_1^{\rho})] \\
&= [1/E_0(\bar{g}_2^{\rho})] \cdot [E_0(\bar{g}_2^{\rho} \cdot \Theta_1^{\rho} \cdot \bar{g}_2^{-\rho}) - Cov(\bar{g}_2^{\rho} \cdot \Theta_1^{\rho} \cdot \bar{g}_2^{-\rho}) - E_0(\Theta_1^{\rho})] \\
&= - Cov(\bar{g}_2^{\rho}, \Theta_1^{\rho} \cdot \bar{g}_2^{-\rho})/E_0(\bar{g}_2^{\rho}).
\end{aligned}
$$

Let $\psi \equiv \Theta_1^{\rho} \cdot \bar{g}_2^{-\rho}$. Using the definition of Θ_1 in (10), ψ is rewritten as :

$$
\begin{aligned}
\psi \equiv \Theta_1^{\rho} \cdot \bar{g}_2^{\rho} &= [1 + \delta \cdot \bar{g}_2^{1-\rho}]^{\rho/(1-\rho)} \times \bar{g}_2^{\rho(\rho-1)/(1-\rho)} \\
&= [\delta + \bar{g}_2^{\rho-1}]^{\rho/(1-\rho)}
\end{aligned}
$$

Taking the derivative of ψ with respect to $\bar{g}_2$ will give a negative sign for the covariance term above. Therefore, the first denominator term in (20) is greater than the second one and so the RP is negative.

Following this procedure, it is not difficult to show that the rolling premium in the risk neutral production economy [see equation (29)] is also negative.

〔Abstract〕

이자율의 기간구조에 관한 기대가설과 위험중립의 관계에 대하여

조 재 호

이 논문은 Epstein and Zin(1989)이 개발한 비기대 효용함수를 이용하여, 이자율의 기간구조에 관한 기대가설이 위험중립 하에서 성립한다는 기존의 이론을 재분석하였다. 교환경제와 생산경제의 두 가지 경우를 분석한 결과, 다음과 같은 사실을 발견하였다 : (i) 미래이자율이 불확실한 상황을 만들기 위해서 시점간 자원배분에 대한 선호와 위험에 대한 선호를 구별하는 일은 매우 중요하다 ; (ii) 이자율이 불확실한 상황에서 투자자들이 위험에 대해 중립이더라도 기간 프리미엄(term premium)은 영이 아니다. 따라서 위험중립 하에서 기대가설이 성립한다는 이론은 기각되어야 한다.

핵심용어 : 이자율의 기간구조, 기대가설, 위험중립, 비기대 효용함수