計量經濟學報, 제17권 제2호(2006. 6. 30.), 1-20

An Efficient Double Auction*

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We study the double auction problem with interdependent values. We establish a payoff equivalence result for this environment. We also introduce the generalized Vickrey double auction which is an efficient and ex-post incentive compatible mechanism for the interdependent values environment. Using these tools, we find a necessary and sufficient condition for the existence of an efficient, Bayesian incentive compatible, interim individually rational, and ex-ante budget balancing mechanism. We show that positive interdependence makes the existence harder, while negative interdependence makes it easier.

Keywords: Double Auction, Interdependent Values, Efficiency
JEL Classifications: C72, D44, D82

I. Introduction

The research on the design of market mechanisms when sellers and buyers have private information is now well-established. This literature on double auctions has proposed various mechanisms and analyzed their efficiency properties.[1)] Most works in this literature, however, assume that values are

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* I thank Byoung Heon Jun, Benny Moldovanu, Gyuho Wang, Elmar Wolfstetter, and seminar participants at WZB as well as anonymous referees for many helpful comments and suggestions. This work was supported by Korea Research Foundation (KRF-2003-003-B00030).

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투고일: 2005. 10. 21 심사일: 2005. 10. 21 최종심사완료일: 2006. 4. 4

private. That is, traders' values for the good depend only on their own private information but not on others'.

In this paper, we study the double auction problem when traders' values are dependent on others' private information as well as their own, i.e., when values are interdependent. We establish a "payoff equivalence" result for the environment where (i) there are many traders in each side of a market, (ii) each trader may trade multiple units, and (iii) traders' values are interdependent (Theorem 1). This equivalence result is an extension of those in Williams (1999) and Krishna and Perry (2000) for the private values environment, which states that the payoff in any Bayesian incentive compatible mechanism is uniquely determined by the allocation rule alone up to a constant.

We also introduce the generalized Vickrey double auction which is an efficient and ex-post incentive compatible mechanism for the interdependent values environment. This mechanism corresponds to the generalized Vickrey auction in Ausubel (1999) and Perry and Reny (2002). By the equivalence result, we can safely restrict attention to the generalized Vickrey double auction when we search for an efficient and incentive compatible mechanism which also satisfies other desirable properties, e.g., individual rationality and budget balance. By analyzing the transfer rule of the generalized Vickrey double auction, we find a necessary and sufficient condition for the existence of an efficient, Bayesian incentive compatible, interim individually rational, and ex-ante budget balancing mechanism (Theorem 2). We can establish an explicit necessary and sufficient condition for the existence for the unitary case where each seller initially owns one unit and each buyer wants to buy at most one unit of the good (Theorem 3).[2] We show that positive interdependence makes the existence harder, while negative interdependence makes it easier.

Turning to the related literature, Gresik (1991) is the only work that we are aware of which studied the double auction problem with interdependent values. It analyzed the unitary case where there exist many sellers and many

1) The literature includes McAfee (1992), Rustichini, Satterthwaite, and Williams (1994), Yoon (2001), and Satterthwaite and Williams (2002).

2) Most papers on double auctions are concerned with the unitary case.

buyers but each seller initially owns one unit and each buyer wants to buy at most one unit of the good, and established an inefficiency result when a single seller and a single buyer trade. These results are straightforward extensions of those in Myerson and Satterthwaite (1983) to the interdependent values case. On the other hand, we allow each trader to sell or buy multiple units and provide a sufficient condition for efficiency. A recent paper by Fieseler *et al.* (2003), which studied the partnership dissolution with interdependent values, is also closely related to the present paper. In particular, we rely on their specification and insight when we analyze positive and negative interdependence in Section Ⅳ. Observe that, in comparison to the partnership dissolution in which the entire good is to be allocated to a single trader, the double action allows decentralized final allocations, i.e., the multiple units of the good are to be allocated to multiple traders in varying amounts. This makes the analysis and conclusion of the double auction problem quite different from those of the partnership dissolution problem, as Rustichini *et al.* (1994) for the double auction with private values differs from Cramton *et al.* (1987) for the partnership dissolution with private values. On the other hand, the case with one seller and one buyer can be easily analyzed with the apparatus of partnership dissolution problem since the single good the seller initially owns will be allocated to either one of the traders. Fieseler *et al.* (2003) and Kittsteiner (2003) do pursue this easy extension. Finally, the paper most closely related is Yoon (2001) for the double auction with private values, and the present paper is a natural generalization to the interdependent values case.

The rest of the paper is organized as follows. We set out the model in Section Ⅱ, and introduce the generalized Vickrey double auction in Section Ⅲ. Main results are contained in Section Ⅳ, and concluding comments appear in Section Ⅴ.

Ⅱ. The Model

There is a set $I=\{1,\cdots,m+n\}$ of traders, among whom traders $1,\cdots,m$ are sellers and traders $m+1,\cdots,m+n$ are buyers of a good. We assume $m\geq 1$

and $n \geq 1$, so that there are at least one seller and one buyer. We let $k(i)$ denote trader i's quantity restriction. If i is a seller then he initially owns $k(i)$ units that can be sold to buyers, while if i is a buyer then he does not initially own the good but may buy up to $k(i)$ units.[3] Hence, there are a total of

$$K \equiv \sum_{i=1}^{m} k(i)$$

units of the good available for trade. A *feasible allocation* is $q \equiv (q_1, \cdots, q_{m+n})$ such that $0 \leq q_i \leq k(i)$ for all $i \in I$ and $\sum_{i \in I} q_i = K$. Let Q denote the set of all feasible allocations. Given a feasible allocation, q_i is the quantity i buys if i is a buyer. On the other hand, $k(i) - q_i$ is the quantity i sells if i is a seller since i initially owns $k(i)$ units and finally owns q_i units. We will use

$$r_i \equiv k(i) - q_i$$

for $i = 1, \cdots, m$ to denote the quantity seller i sells.

Trader i's private information is represented by a signal θ_i. Let $\Theta_i = [\underline{\theta_i}, \overline{\theta_i}]$ be the set of i's possible signals. Note that signals are one-dimensional.[4] We use the usual notation such as $\theta = (\theta_1, \cdots, \theta_{m+n})$, $\theta_{-i} = (\theta_1, \cdots, \theta_{i-1}, \theta_{i+1}, \cdots, \theta_{m+n}), \Theta = \times_{i=1}^{m+n} \Theta_i$, and $\Theta_{-i} = \times_{j \neq i} \theta_j$. Traders' values for the good are dependent on the whole profile of signals $\theta = (\theta_1, \cdots, \theta_{m+n})$. Let $v_{ik}(\theta)$ denote trader i's marginal value for a k-th unit, given the signal vector θ.[5] We make the following assumptions on $v_{ik}(\theta)$'s throughout the paper.

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3) Note that $k(i)$ for the buyer may well be large enough, say K defined below.

4) Jehiel and Moldovanu (2001) have shown that efficiency is inconsistent with incentive compatibility when values are interdependent and signals are multi-dimensional.

5) We assume that the good is consumed in discrete units. The analysis is essentially the same when the good is perfectly divisible.

(1) $\forall i=1,\cdots,m;\ \forall k=1,\cdots,k(i)-1;\ \forall \theta\in\Theta: 0\leq v_{ik}(\theta)\leq v_{i,k+1}(\theta)\leq \bar{v}$, and

$\forall i=m+1,\cdots,m+n;\ \forall k=1,\cdots,k(i)-1;\ \forall \theta\in\Theta: 0\leq v_{i,k+1}(\theta)\leq v_{ik}(\theta)\leq \bar{v}$.

(2) $\forall i=1,\cdots,m+n;\ \forall k=1,\cdots,k(i);\ \forall \theta\in\Theta: \partial v_{ik}(\theta)/\partial\theta_i>0$.[6)]

(3) $\forall i,j=1,\cdots,m+n$ with $i\neq j; \forall k=1,\cdots,k(i);\ \forall l=1,\cdots,k(j);\ \forall \theta\in\Theta$:

$\partial v_{ik}(\theta)/\partial\theta_i > \partial v_{jl}(\theta)/\partial\theta_i$.

The first assumption says that marginal values for the sellers are weakly increasing in k, and that marginal values for the buyers are weakly decreasing in k.[7)] In other words, sellers' supply curves are weakly increasing, and buyers' demand curves are weakly decreasing. The first assumption also says that marginal values are non-negative and bounded above. The second assumption is a monotonicity condition, which implies that signals have a natural order. Finally, the third assumption is a single crossing condition. Observe that the single crossing property is necessary for efficient implementation in ex-post equilibrium.[8)]

Suppose trader i is allocated q_i units of the good. If i is a buyer then his payoff from the allocation is $\sum_{k=1}^{q_i} v_{ik}(\theta)$, while if i is a seller then his payoff from the allocation is $-\sum_{k=1}^{r_i} v_{ik}(\theta)$ where recall that $r_i=k(i)-q_i$. Define

$$w_i(\theta;q_i)\equiv\begin{cases} -\sum_{k=1}^{r_i} v_{ik}(\theta) & \text{if } i \text{ is a seller.}\\ \sum_{k=1}^{q_i} v_{ik}(\theta) & \text{if } i \text{ is a buyer.}\end{cases}$$

to denote traders' payoffs from the allocation. We assume quasi-linear preferences. Hence, when seller i sells r_i units of the good and receives a

6) In fact, we only need continuity of marginal values on θ for the results up to Section Ⅲ. We choose to assume differentiability mainly for notational simplicity.
7) It is natural to think of sellers as producers and their marginal values as marginal costs.
8) See, for example, Dasgupta and Maskin (2000) as well as Ausubel (1999) and Perry and Reny (2002).

sum t of money in return, his payoff is $t-\sum_{k=1}^{r_i} v_{ik}(\theta)\equiv w_i(\theta;q_i)+t$. When buyer i buys q_i units of the good and pays a sum t of money in return, his payoff is $\sum_{k=1}^{q_i} v_{ik}(\theta)-t\equiv w_i(\theta;q_i)-t$. The status-quo payoff level for every trader is normalized to zero.

Ⅲ. The generalized Vickrey double auction

Assume that the mapping $v_{ik}(\cdot)$ for all $i=1,\cdots,m+n$ and all $k=1,\cdots,k(i)$ is commonly known to the traders and the auctioneer. We now present a direct revelation mechanism that achieves efficiency. In this mechanism, which we call the generalized Vickrey double auction, traders report their signals to the auctioneer. Reported signals may be different from true signals, but traders will report truthfully in an ex-post equilibrium of our mechanism.

Suppose $\theta=(\theta_{1,}\cdots,\theta_{m+n})$ is the vector of reported signals. Then, the auctioneer can calculate the marginal values of all traders for all units, $v_{ik}(\theta)$'s for $i=1,\cdots,m+n$ and $k=1,\cdots,k(i)$. An *efficient allocation* given θ is a feasible allocation $q=(q_{1,}\cdots,q_{m+n})$ that maximizes

$$\sum_{i\in I} w_i(\theta;q_i)\equiv\sum_{i=m+1}^{m+n}\sum_{k=1}^{q_i} v_{ik}(\theta)-\sum_{i=1}^{m}\sum_{k=1}^{r_i} v_{ik}(\theta)$$

among all feasible allocations in Q. Hence, the available K units of the good are allocated to the highest K values in an efficient allocation. An *efficient allocation rule* is a mapping from signal vectors to feasible allocations such that it chooses an efficient allocation for every signal vector. We use $q^*\equiv(q_1^*\cdots,q_{m+n}^*):\Theta\to Q$ to denote an efficient allocation rule. We will only consider an efficient allocation rule such that $q_i^*(\theta)$ is weakly increasing in θ_i for all $i=1,\cdots,m+n$. Note that we can always choose such an allocation rule.

For a signal vector θ_{-i} of others and an efficient allocation rule $q^*(\cdot)$, define

$$\widehat{\theta}_{ik}(\theta_{-i}) \equiv \begin{cases} \sup \ \{\theta_i \mid q_i^*(\theta_i, \theta_{-i}) \leq k(i) - k\} & \text{if } i \text{ is a seller.} \\ \inf \ \{\theta_i \mid q_i^*(\theta_i, \theta_{-i}) \geq k\} & \text{if } i \text{ is a buyer.} \end{cases}$$

For the buyer, this is the minimum report he can make that still ensures him at least k units given θ_{-i} and $q^*(\cdot)$. For the seller, on the other hand, this is the maximum report he can make that entitles him no more than $k(i)-k$ units given θ_{-i} and $q^*(\cdot)$ In other words, this is the maximum report for the seller that let him trade at least k units given θ_{-i} and $q^*(\cdot)$.[9] Given the assumptions on v_{ik}'s, it is straightforward to see that $\widehat{\theta}_{ik}(\theta_{-i})$ is weakly decreasing in k for seller $i=1,\cdots,m$ and weakly increasing in k for buyer $i=m+1,\cdots,m+n$. It is also straightforward to see that

(i) $\forall i=1,\ldots,m$, $\forall \theta \in \Theta$: $\widehat{\theta}_{ik}(\theta_{-i}) \geq \theta_i$ when $k \leq r_i^*(\theta)$, and $\widehat{\theta}_{ik}(\theta_{-i}) \leq \theta_i$ when $k > r_i^*(\theta)$.

(ii) $\forall i=m+1,\ldots,m+n$, $\forall \theta \in \Theta$: $\widehat{\theta}_{ik}(\theta_{-i}) \leq \theta_i$ when $k \leq q_i^*(\theta)$, and $\widehat{\theta}_{ik}(\theta_{-i}) \geq \theta_i$ when $k > q_i^*(\theta)$.

Let us use the notation $v_{-i,l}(\theta)$ to denote, for a given θ, the marginal value of an l-th unit to bidders other than i when the units are allocated efficiently among them. That is, $v_{-i,l}(\theta)$ is the l-th highest among $I-\{i\}$'s marginal values. The *generalized Vickrey double auction* is a direct revelation mechanism in which (i) the allocation rule is an efficient allocation rule $q^*(\cdot)$, and (ii) the transfer rule is

$$t_i^*(\theta) \equiv \begin{cases} \displaystyle\sum_{k=1}^{k(i)-q_i^*(\theta)} v_{-i,K-k(i)+k}(\widehat{\theta}_{ik}(\theta_{-i}), \theta_{-i}) & \text{if } i \text{ is a seller,} \\ \displaystyle -\sum_{k=1}^{q_i^*(\theta)} v_{-i,K+1-k}(\widehat{\theta}_{ik}(\theta_{-i}), \theta_{-i}) & \text{if } i \text{ is a buyer,} \end{cases}$$

9) Observe that $\widehat{\theta}_{ik}(\theta_{-i})$ for seller i can be equivalently written as $\sup\{\theta_i \mid r_i^*(\theta_i, \theta_{-i}) \equiv k(i) - q_i^*(\theta_i, \theta_{-i}) \geq k\}$.

where $t_i^*(\theta)$ is the transfer of money from the auctioneer to trader i when the signal vector is θ.[10)]

We then have the following proposition. As is now well-known, ex-post equilibrium is an equilibrium concept for the interdependent values case which lies between Bayesian Nash equilibrium and dominant strategy equilibrium : A strategy profile is an ex-post equilibrium if truth-telling is optimal for any realization of others' private information. A formal definition is given below.

Proposition 1. *The generalized Vickrey double auction has truth-telling as an ex-post equilibrium.*

Proof : For seller $i=1,\cdots,m$, the payoff from reporting θ_i' when his true signal is θ_i and others' true and reported signals are θ_{-i} is

$$w_i(\theta; q_i^*(\theta_i', \theta_{-i})) + t_i^*(\theta_i', \theta_{-i})$$
$$= \sum_{k=1}^{r_i^*(\theta_i', \theta_{-i})} [v_{-i, K-k(i)+k}(\widehat{\theta_{ik}}(\theta_{-i}), \theta_{-i}) - v_{ik}(\theta_i, \theta_{-i})],$$

where recall that $r_i^*(\theta_i', \theta_{-i}) \equiv k(i) - q_i^*(\theta_i', \theta_{-i})$. We have, for $k=1,\cdots,r_i^*(\theta_i, \theta_{-i})$,

$$v_{-i, K-k(i)+k}(\widehat{\theta_{ik}}(\theta_{-i}), \theta_{-i}) - v_{ik}(\theta_i, \theta_{-i})$$
$$\geq v_{-i, K-k(i)+k}(\widehat{\theta_{ik}}(\theta_{-i}), \theta_{-i}) - v_{ik}(\widehat{\theta_{ik}}(\theta_{-i}), \theta_{-i}) = 0,$$

where the inequality follows from the fact that $\widehat{\theta_{ik}}(\theta_{-i}) \geq \theta_i$ for $k=1,\cdots,r_i^*(\theta_i, \theta_{-i})$ and the monotonicity condition, and the equality follows from the definition of $\widehat{\theta_{ik}}(\theta_{-i})$ and the continuity of value functions on θ. Likewise, for $k = r_i^*(\theta_i, \theta_{-i}) + 1, \ldots, k(i)$,

$$v_{-i, K-k(i)+k}(\widehat{\theta_{ik}}(\theta_{-i}), \theta_{-i}) - v_{ik}(\theta_i, \theta_{-i})$$
$$\leq v_{-i, K-k(i)+k}(\widehat{\theta_{ik}}(\theta_{-i}), \theta_{-i}) - v_{ik}(\widehat{\theta_{ik}}(\theta_{-i}), \theta_{-i}) = 0.$$

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10) The reader may want to consult Example 1 of next section.

Therefore, trader i's payoff is maximized when $\theta_i' = \theta_i$, i.e., when i reports truthfully. The buyer case is similar. *Q.E.D.*

Ⅳ. Characterization results

We will use the generalized Vickrey double auction to find necessary and sufficient conditions for the existence of an efficient, Bayesian incentive compatible, interim individually rational, and ex-ante budget balancing mechanism. The key apparatus is an equivalence theorem which implies that any efficient and Bayesian incentive compatible mechanism is payoff-equivalent to some generalized Vickrey double auction from an interim perspective.[11] This technique was pioneered by Williams (1999) and Krishna and Perry (2000) for the private values environment, and applied by Fieseler *et al.* (2003) to the partnership dissolution with interdependent values.

We now make the following assumption on the distribution of signals.

Assumption 1. Traders' signals are independently distributed, and signal θ_i is drawn from a commonly known density function f_i which is continuous and positive almost everywhere on Θ_i.

1. The equivalence theorem

A direct mechanism is (q, t) such that $q: \Theta \rightarrow Q$ is an allocation rule, and $t: \Theta \rightarrow R^{m+n}$ is a transfer rule. Thus, if reported signals are $\theta' \in \Theta$, then $q(\theta')$ is the chosen allocation and $t_i(\theta')$ is the monetary transfer to trader i. A mechanism (q, t) is *Bayesian incentive compatible* if truth-telling is a Bayesian Nash equilibrium. That is, for all $i \in I$ and for all $\theta_i, \theta_i' \in \Theta_i$,

11) The term 'equivalence theorem' has been used in the literature to denote the fact that traders' interim payoffs depend only on the allocation rule, but not on the transfer rule.

$$E_{\theta_{-i}}[w_i(\theta; q_i(\theta)) + t_i(\theta)] \geq E_{\theta_{-i}}[w_i(\theta; q_i(\theta_i', \theta_{-i})) + t_i(\theta_i', \theta_{-i})].$$

Similarly, a mechanism (q, t) is *ex-post incentive compatible* if truth-telling is an ex-post equilibrium, i.e., for all $i \in I$ and for all $\theta_i, \theta_i' \in \Theta_i$, and for all $\theta_{-i} \in \Theta_{-i}$,

$$w_i(\theta; q_i(\theta)) + t_i(\theta) \geq w_i(\theta; q_i(\theta_i', \theta_{-i})) + t_i(\theta_i', \theta_{-i}),$$

while it is *dominant strategy incentive compatible* if truth-telling is a dominant strategy, i.e., for all $i \in I$ and for all $\theta_i, \theta_i' \in \Theta_i$, and for all $\theta_{-i}, \theta'_{-i} \in \Theta_{-i}$,

$$w_i(\theta; q_i(\theta_i, \theta'_{-i})) + t_i(\theta_i, \theta'_{-i}) \geq w_i(\theta; q_i(\theta_i', \theta'_{-i})) + t_i(\theta_i', \theta'_{-i}).$$

Note that ex-post incentive compatibility is stronger than Bayesian incentive compatibility but weaker than dominant strategy incentive compatibility. By the revelation principle, it is with no loss of generality to restrict our attention to direct mechanisms.

Given a mechanism (q, t), define

$$\begin{aligned} U_i(\theta_i, \theta_i') &\equiv E_{\theta_{-i}}[w_i(\theta_i, \theta_{-i}; q_i(\theta_i', \theta_{-i})) + t_i(\theta_i', \theta_{-i})] \\ &\equiv W_i(\theta_i, \theta_i') + E_{\theta_{-i}}[t_i(\theta_i', \theta_{-i})] \equiv W_i(\theta_i, \theta_i') + T_i(\theta_i'). \end{aligned}$$

$U_i(\theta_i, \theta_i')$ is trader i's interim expected payoff when his true signal is θ_i and reported signal is θ_i'. For notational simplicity, let $U_i(\theta_i) \equiv U_i(\theta_i, \theta_i)$. A mechanism (q, t) is interim individually rational (IR henceforth) if $U_i(\theta_i) \geq 0$ for all for all $i \in I$ and for all $\theta_i \in \Theta_i$, while it is ex-ante budget balancing (BB henceforth) if $E_\theta[\sum_{i \in I} t_i(\theta)] \leq 0$.

We have the following equivalence result.

Theorem 1. *(equivalence theorem) Assume that* $v_{ik}(\theta_i, \theta_{-i})$ *is continuously differentiable in* θ_i *and that, for all* $\theta_i \in \Theta_i$, $\lim_{\theta_i' \to \theta_i} q_i(\theta_i', \theta_{-i}) = q_i(\theta_i, \theta_{-i})$ *for almost every* $\theta_{-i} \in \Theta_{-i}$. *Then, for every Bayesian incentive compatible* *mechanism* (q, t), *trader* i*'s interim expected payoff can be written as*[12)]

$$U_i(\theta_i) = U_i(\underline{\theta_i}) + \int_{\underline{\theta_i}}^{\theta_i} D_1 W_i(x, x)\,dx.$$

Proof : Bayesian incentive compatibility implies, for all $\theta_i, \theta_i' \in \Theta_i$,

$$U_i(\theta_i, \theta_i) \geq U_i(\theta_i, \theta_i') \text{ and } U_i(\theta_i', \theta_i') \geq U_i(\theta_i', \theta_i).$$

Therefore, we obtain

$$W_i(\theta_i', \theta_i') - W_i(\theta_i, \theta_i') \geq U_i(\theta_i', \theta_i') - U_i(\theta_i, \theta_i) \geq W_i(\theta_i', \theta_i) - W_i(\theta_i, \theta_i).$$

Dividing by $\theta_i' - \theta_i$, this expression when i is a buyer becomes

$$E_{\theta_{-i}}\left[\sum_{k=1}^{q_i(\theta_i', \theta_{-i})} \frac{v_{ik}(\theta_i', \theta_{-i}) - v_{ik}(\theta_i, \theta_{-i})}{\theta_i' - \theta_i}\right] \geq \frac{U_i(\theta_i', \theta_i') - U_i(\theta_i, \theta_i)}{\theta_i' - \theta_i}$$
$$\geq E_{\theta_{-i}}\left[\sum_{k=1}^{q_i(\theta_i, \theta_{-i})} \frac{v_{ik}(\theta_i', \theta_{-i}) - v_{ik}(\theta_i, \theta_{-i})}{\theta_i' - \theta_i}\right]$$

Since v_{ik}'s are (continuously) differentiable in θ_i and $\lim_{\theta_i' \to \theta_i} q_i(\theta_i', \theta_{-i}) = q_i(\theta_i, \theta_{-i})$ for almost every θ_{-i}, we obtain by the Dominated Convergence Theorem that

$$E_{\theta_{-i}}\left[\sum_{k=1}^{q_i(\theta_i, \theta_{-i})} \partial v_{ik}(\theta_i, \theta_{-i})/\partial \theta_i\right] \geq U_i'(\theta_i) \geq E_{\theta_{-i}}\left[\sum_{k=1}^{q_i(\theta_i, \theta_{-i})} \partial v_{ik}(\theta_i, \theta_{-i})/\partial \theta_i\right].$$

Therefore, $U_i(\theta_i)$ is differentiable with

$$U_i'(\theta_i) = E_{\theta_{-i}}\left[\sum_{k=1}^{q_i(\theta_i, \theta_{-i})} \partial v_{ik}(\theta_i, \theta_{-i})/\partial \theta_i\right] = D_1 W_i(\theta_i, \theta_i).$$

The desired formula now follows since $\partial v_{ik}(\theta_i, \theta_{-i})/\partial \theta_i$'s are continuous in θ_i. The seller case is similar. *Q.E.D.*

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12) $D_1 W_i(\theta_i, \theta_i')$ is the partial derivative of $W_i(\theta_i, \theta_i')$ with respect to the first argument, i.e., θ_i.

The theorem implies that each trader's interim expected payoff of any Bayesian incentive compatible mechanism is uniquely determined by the allocation rule alone up to a constant, i.e., its value at an arbitrarily chosen signal $\underline{\theta_i}$. Given a Bayesian incentive compatible mechanism (q, t), define trader i's worst interim expected payoff to be $\underline{U_i} = \inf\{U_i(\theta_i) | \theta_i \in \Theta_i\}$. We easily have:

Theorem 2. *Under the same hypothesis of Theorem 1, there exists a Bayesian incentive compatible, IR, and BB mechanism that implements an allocation rule* $q: \Theta \to Q$ *if and only if*

$$E_\theta[\sum_{i \in I} t_i(\theta)] \le \sum_{i \in I} \underline{U_i}.$$

The left-hand side of the inequality is the expected budget deficit of the mechanism, while the right-hand side can be regarded as the sum of maximal entry fees charged to traders. If we can cover the deficit by monetary payments collected from traders, then we can find an IR and BB mechanism.[13] This theorem is an extension of Williams' (1999) result for private values to interdependent values.

2. Unitary double auction

We now assume that $k(i) = 1$ for all $i \in I$. Hence, each seller $i = 1, \cdots, m$ has one indivisible unit to sell and each buyer $i = m+1, \cdots, m+n$ wants to buy at most one unit. This is the environment studied in most double auction literature.[14] This situation is also called multilateral bargaining problem since this is an extension of bilateral (i.e., two person) bargaining under incomplete information.[15]

13) *N.B.* : This theorem states that we can find a mechanism if and only if the condition of the theorem holds. It is obviously not true that any allocation rule can be made to satisfy Bayesian incentive compatibility, individual rationality, and budget balance.

14) See, for example, Satterthwaite and Williams (2002) for a recent overview of this literature.

15) The pioneering papers are Myerson and Satterthwaite (1983) and Chatterjee and

We strengthen Assumption 1 to impose some symmetry on sellers' (buyers', resp.) signals.

Assumption 2. Each seller's (buyer's, resp.) signal is drawn independently according to a commonly known distribution F (G, resp.) on the interval $[\underline{\theta}_s, \overline{\theta}_s]$ ($[\underline{\theta}_b, \overline{\theta}_b]$, resp.), with the density f (g, resp.) which is continuous and positive almost everywhere on respective intervals.

Since $k(i)=1$ for all $i \in I$, we now drop the subscript k from the marginal value $v_{ik}(\cdot)$ and simply denote trader i's marginal value as $v_i(\theta_1, \cdots, \theta_{m+n})$. We adopt Fieseler *et al.* (2003)'s specification and assume

$$v_i(\theta_1, \cdots, \theta_{m+n}) = \alpha(\theta_i) + \sum_{j \neq i} \beta(\theta_j) \text{ for all } i \in I.$$

We have $\alpha'(\cdot) > 0$ and $\alpha'(\cdot) > \beta'(\cdot)$ by the assumptions on marginal values. Hence, $v_i(\theta) = v_j(\theta)$ if and only if $\theta_i = \theta_j$ and $v_i(\theta) > v_j(\theta)$ if and only if $\theta_i > \theta_j$. Therefore, an efficient allocation rule $q^*(\cdot)$ dictates that the $K = m$ units of the good be allocated to those with m highest signals.

We want to describe the transfer rule of the generalized Vickrey double auction for this particular environment. For a given $\theta \in \Theta$, order the marginal values as $v_{(1)}(\theta) \geq \cdots \geq v_{(m+n)}(\theta)$, or equivalently order the signals $\theta_{(1)} \geq \cdots \geq \theta_{(m+n)}$.[16] A seller whose signal (or marginal value) is one of the lowest n signals is successful (i.e., $q_i^*(\theta) = 0$), and the transfer $t_i^*(\theta)$ is equal to $v_{(m)}(\theta_{(m)}, \theta_{-i})$, which is his receipt of money in return for the good.[17] Likewise, a buyer whose signal (or marginal value) is one of the highest m signals is successful (i.e., $q_i^*(\theta) = 1$), and $t_i^*(\theta) = -v_{(m+1)}(\theta_{(m+1)}, \theta_{-i})$, which

Samuelson (1983).

16) Ties can be broken in any fashion.

17) The value $v_{(m)}(\theta_{(m)}, \theta_{-})$ is obtained by replacing θ_i with $\theta_{(m)}$ in $v_{(m)}(\theta)$.

is her payment of money for the good.[18] Other traders are unsuccessful, and they neither trade nor make monetary transfers. Note that $\widehat{\theta}_i(\theta_{-i})$ for sellers $i=1,\cdots,m$ is the same and equal to $\theta_{(m)}$, while $\widehat{\theta}_i(\theta_{-i})$ for buyers $i=m+1,\cdots,m+n$ is the same and equal to $\theta_{(m+1)}$. Nevertheless, monetary receipts may differ across successful sellers and, similarly, monetary payments may differ across successful buyers. This is in contrast with the private values case studied in Yoon (2001), where each successful seller receives the same amount of money and each successful buyer pays the same amount of money.

Example 1. There are two sellers (traders 1 and 2) and two buyers (traders 3 and 4). Let $k(i)=1$ and $v_i(\theta_1,\theta_2,\theta_3,\theta_4)=2\theta_i+\sum_{j\neq i}\theta_j$ for $i=1,2,3,4$. Suppose $\theta_1=1$, $\theta_2=2$, $\theta_3=3$, and $\theta_4=4$, so that $v_1(\theta)=11, v_2(\theta)=12$, $v_3(\theta)=13$, and $v_4(\theta)=14$. Then, we have $\widehat{\theta}_1=\widehat{\theta}_2=3$ and $\widehat{\theta}_3=\widehat{\theta}_4=2$. The transfers are given as $t_1^*(\theta)=15$, $t_2^*(\theta)=14$, $t_3^*(\theta)=-11$, and $t_4^*(\theta)=-10$.

For a given $\theta\in\Theta$, the budget deficit $\sum_{i\in I}t_i^*(\theta)$ of the generalized Vickrey double auction for the unitary case can be determined as follows. Let

$$S(\theta)=\{i=1,\cdots,m|\,\theta_i \text{ is one of } \theta_{(m+1)},\cdots,\theta_{(m+n)}\}$$

denote the set of successful sellers, and let

$$B(\theta)=\{i=m+1,\cdots,m+n\,|\,\theta_i \text{ is one of } \theta_{(1)},\cdots,\theta_{(m)}\}$$

denote the set of successful buyers. Note that the cardinality of $S(\theta)$ is the same as that of $B(\theta)$. Let us denote this number by $\hat{k}(\theta)$, which is the volume of trade. Then,

18) The value $v_{(m+1)}(\theta_{(m+1)},\theta_{-})$ is obtained by replacing θ_i with $\theta_{(m+1)}$ in $v_{(m+1)}(\theta)$.

$$\begin{aligned}\sum_{i\in I} t_i^*(\theta) &= \sum_{i\in S(\theta)} v_{(m)}(\theta_{(m)}, \theta_{-i}) - \sum_{i\in B(\theta)} v_{(m+1)}(\theta_{(m+1)}, \theta_{-i}) \\ &= \sum_{i\in S(\theta)} \left\{\alpha(\theta_{(m)}) + \sum_{j\neq i} \beta(\theta_j)\right\} - \sum_{i\in B(\theta)} \left\{\alpha(\theta_{(m+1)}) + \sum_{j\neq i} \beta(\theta_j)\right\} \\ &= \hat{k}(\theta)\{\alpha(\theta_{(m)}) - \alpha(\theta_{(m+1)})\} + \sum_{i\in B(\theta)} \beta(\theta_i) - \sum_{i\in S(\theta)} \beta(\theta_i).\end{aligned}$$

Example 2. For the previous example, $S(\theta)=\{1,2\}$, $B(\theta)=\{3,4\}$, and $\hat{k}(\theta)=2$. We have $\sum_{i\in I} t_i^*(\theta) = 2\times(6-4)+(3+4)-(1+2)=8$.

The budget deficit is dependent on how traders' marginal values are interdependent. Compared to the private values environment when $\beta'(\cdot)=0$, the budget deficit is larger (smaller, resp.) when $\beta'(\cdot)>0$ ($\beta'(\cdot)<0$, resp.). This is so since the signals of successful buyers are higher than the signals of successful sellers. Therefore, we observe that when marginal values are positively (negatively, resp.) interdependent, it is harder (easier, resp.) to achieve budget balance. We can even have budget surplus if the negative interdependence is strong enough.[19)]

We now turn to the worst interim expected payoff $\underline{U_i}$ to find an explicit necessary and sufficient condition for an efficient, Bayesian incentive compatible, IR, and BB mechanism. Since sellers are identical from an interim perspective, define $\underline{U_s}$ to be sellers' worst interim expected payoff. This is in fact equal to $U_i(\overline{\theta_s})$ for $i=1,\cdots,m$, that is, the value of interim expected payoff when a seller has the worst signal of $\overline{\theta_s}$. Likewise, buyers' worst interim expected payoff is $\underline{U_b}=U_i(\underline{\theta_b})$ for $i=m+1,\cdots,m+n$. Consider a seller and order *buyers*' signals in a decreasing order as $\theta^b_{(1)}\geq\cdots\geq\theta^b_{(n)}$. Then, since $t_i^*(\overline{\theta_s},\theta_{-i})-v_i(\overline{\theta_s},\theta_{-i})=\alpha(\theta^b_{(m)})+\beta(\theta^b_{(m)})+\sum_{j\neq i,(m)}\beta(\theta_j)-\alpha(\overline{\theta_s})-\sum_{j\neq i}\beta(\theta_j)=\alpha(\theta^b_{(m)})-\alpha(\overline{\theta_s})$ when $\overline{\theta_s}<\theta^b_{(m)}$, we have

$$\underline{U_s}=E_{\theta_{-i}}[\{\alpha(\theta^b_{(m)})-\alpha(\overline{\theta_s})\}1(\overline{\theta_s}<\theta^b_{(m)})].$$

19) This conclusion is obtained in Fieseler *et al.* (2003) for partnership dissolution and bilateral (i.e., two person) trading.

It is clear that $\underline{U_s}=0$ if $\overline{\theta_s}\geq\overline{\theta_b}$ or $m>n$. Observe that $\underline{U_s}$ for a seller does not depend on other sellers' signals since sellers' signals cannot be higher than $\overline{\theta_s}$ and thus $\theta^b_{(m)}$ is in fact the m-th highest signal out of $m+n-1$ other traders' signals when $\theta^b_{(m)}>\overline{\theta_s}$.[20] In a buyer case, we can order *sellers*' signals in an increasing order as $\theta^s_{(1)}\leq\cdots\leq\theta^s_{(m)}$, and

$$\underline{U_b}=E_{\theta_{-i}}[\{\alpha(\underline{\theta_b})-\alpha(\theta^s_{(n)})\}1(\underline{\theta_b}>\theta^s_{(n)})].$$

We have $\underline{U_b}=0$ if $\underline{\theta_s}\geq\underline{\theta_b}$ or $m<n$. It is also true that $\underline{U_b}$ does not depend on other buyers' signals. To summarize the discussion, we get by integration by parts

$$\underline{U_s}=\begin{cases}\sum_{r=m}^{n}\binom{n}{r}\int_{\overline{\theta_s}}^{\overline{\theta_b}}\alpha'(\theta_i)[G(\theta_i)]^{n-r}[1-G(\theta_i)]^r d\theta_i & \text{if } \overline{\theta_s}<\overline{\theta_b} \text{ and } m\leq n\\ 0 & \text{otherwise},\end{cases}$$

and

$$\underline{U_b}=\begin{cases}\sum_{r=n}^{m}\binom{m}{r}\int_{\underline{\theta_s}}^{\underline{\theta_b}}\alpha'(\theta_i)[F(\theta_i)]^{r}[1-F(\theta_i)]^{m-r} d\theta_i & \text{if } \underline{\theta_s}<\underline{\theta_b} \text{ and } m\geq n\\ 0 & \text{otherwise}.\end{cases}$$

It is noteworthy that neither $\underline{U_s}$ nor $\underline{U_b}$ depends on the function $\beta(\cdot)$.

Since an efficient allocation rule $q^*(\cdot)$ satisfies the hypothesis of Theorem 1, we can adapt Theorem 2 to the present environment to obtain the following necessary and sufficient condition.

Theorem 3. *There exists an efficient, Bayesian incentive compatible, IR, and BB mechanism if and only if*

$$E_\theta[\hat{k}(\theta)\{\alpha(\theta_{(m)})-\alpha(\theta_{(m+1)})\}+\sum_{i\in B(\theta)}\beta(\theta_i)-\sum_{i\in S(\theta)}\beta(\theta_i)]$$
$$\leq mE_{\theta_{-i}}[\{\alpha(\theta^b_{(m)})-\alpha(\overline{\theta_s})\}1(\overline{\theta_s}<\theta^b_{(m)})]+nE_{\theta_{-i}}[\{\alpha(\underline{\theta_b})-\alpha(\theta^s_{(n)})\}1(\underline{\theta_b}>\theta^s_{(n)})].$$

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20) More generally, $\theta^b_{(i)}=\theta_{(i)}$ for all $\theta^b_{(i)}>\overline{\theta_s}$.

This theorem is an extension of similar results in Williams (1999) and Yoon (2001) to the direction of interdependent values environment. We can determine how the existence depends on the numbers m and n of traders, the supports $[\underline{\theta_s}, \overline{\theta_s}]$ and $[\underline{\theta_b}, \overline{\theta_b}]$ and the distributions F and G of signals, and the nature of value interdependence $\alpha(\cdot)$ and $\beta(\cdot)$. Williams (1999, Theorem 4) has tabulated such analysis for the private values environment when $\beta'(\cdot)=0$. It is clear that positive interdependence (when $\beta'(\cdot)>0$) makes the existence harder, while negative interdependence (when $\beta'(\cdot)<0$ makes it easier. The following example shows that not only the sign but also the magnitude of value interdependence matters.

Example 3. Let $[\underline{\theta_s}, \overline{\theta_s}]=[0, 1]$, and $[\underline{\theta_b}, \overline{\theta_b}]=[0.5, 1.5]$, and F and G are uniform. Let $\alpha'(\cdot)=1$ and $\beta'(\cdot)=c$ with $0<c<1$. Now consider the case when $m>n$. We have $\underline{U_s}=0$. As we increase m while fixing n, we observe that $\underline{U_b}\to \underline{\theta_b}-\underline{\theta_s}=0.5$, $\hat{k}(\theta)\to n$, $\alpha(\theta_{(m)})-\alpha(\theta_{(m+1)})\to 0$, and $E_\theta[\sum_{i\in B(\theta)}\beta(\theta_i)-\sum_{i\in S(\theta)}\beta(\theta_i)]\to \sum_{r=1}^{n} c\,(\underline{\theta_b}+\frac{r}{n+1}-\underline{\theta_s})=cn$. Hence, the inequality in Theorem 3 is satisfied if and only if $c\le 0.5$.

The next example shows that, for a canonical symmetric case, negative interdependence (however small) makes the existence asymptotically possible.

Example 4. Let $[\underline{\theta_s}, \overline{\theta_s}]=[\underline{\theta_b}, \overline{\theta_b}]=[0, 1]$, and F and G are uniform. Let $\alpha'(\cdot)=1$ and $\beta'(\cdot)=-c<0$. We have $\underline{U_s}=\underline{U_b}=0$. Fix an arbitrary n and increase m, and we will get $\hat{k}(\theta)\to n$, $\alpha(\theta_{(m)})-\alpha(\theta_{(m+1)})\to 0$, and

$E_\theta[\sum_{i\in B(\theta)}\beta(\theta_i)-\sum_{i\in S(\theta)}\beta(\theta_i)]\to -cn/2.$[21)]

Therefore, the existence is asymptotically possible. We stress that the

21) A similar conclusion holds when we increase n while fixing m.

existence is impossible with private values, while it is asymptotically possible even with a vanishingly small negative interdependence.

V. Conclusion

We have found a necessary and sufficient condition for the existence of an efficient, Bayesian incentive compatible, IR, and BB mechanism. This is achieved by the payoff equivalence theorem and the generalized Vickrey double auction. These results are extensions of similar results for the private values case to the interdependent values case. We have shown that positive interdependence makes the mechanism design problem harder, while negative interdependence makes it easier.

In the generalized Vickrey double auction, traders submit their signals directly to the auctioneer. Then, an efficient allocation and transfers are determined by the auctioneer who knows the marginal value functions $v_{ik}(\cdot)$'s of all traders. We can relax this informational requirement so that $v_{ik}(\cdot)$'s are commonly known to the traders but *not* to the auctioneer. Following Perry and Reny (2002), we can construct a double auction mechanism where traders submit *offers and bids,* rather than signals, in the first and second round of bidding. In this mechanism, traders infer others' signals in the first round, and then implement an efficient allocation without the auctioneer's intervention in the second round.[22)]

〔References〕

Ausubel, L.(1999), "A generalized Vickrey auction," manuscript, University of Maryland.

Chatterjee, K. and W. Samuelson(1983), "Bargaining under incomplete in-

22) Details may be provided upon request.

formation," *Operations Research*, 31, 835-851.

Cramton, P., R. Gibbons, and P. Klemperer(1987), "Dissolving a partnership efficiently," *Econometrica*, 55, 615-632.

Dasgupta, P. and E. Maskin(2000), "Efficient auctions," *Quarterly Journal of Economics*, 115, 341-388.

Fieseler, K., T. Kittsteiner, and B. Moldovanu(2003), "Partnerships, lemons and efficient trade," *Journal of Economic Theory*, 113, 223-234.

Gresik, T.(1991), "Ex ante incentive efficient trading mechanisms without the private valuation restriction," *Journal of Economic Theory*, 55, 41-63.

Jehiel, P. and B. Moldovanu(2001), "Efficient design with interdependent valuations," *Econometrica*, 69, 1237-1259.

Kittsteiner, T.(2003), "Partnership and double auctions with interdependent valuations," *Games and Economic Behavior*, 44, 54-76.

Krishna, V. and E. Maenner(2001), "Convex potentials with an application to mechanism design," *Econometrica*, 69, 1113-1119.

Krishna, V. and M. Perry(2000), "Efficient mechanism design," manuscript, Penn State University.

McAfee, P.(1992), "A dominant strategy double auction," *Journal of Economic Theory*, 56, 434-450.

Myerson, R. and M. Satterthwaite(1983), "Efficient mechanisms for bilateral trading," *Journal of Economic Theory*, 29, 265-281.

Perry, M. and P. Reny(2002), "An efficient auction, *Econometrica*, 70, 1199-1212.

Rustichini, A., M. Satterthwaite, and S. Williams(1994), "Convergence to efficiency in a simple market with incomplete information," *Econometrica*, 62, 1041-1063.

Satterthwaite, M. and S. Williams(2002), "The optimality of a simple market mechanism," *Econometrica*, 70, 1841-1863.

Williams, S.(1999), "A characterization of efficient, bayesian incentive compatible mechanisms," *Economic Theory*, 14, 155-180.

Yoon, K.(2001), "The modified Vickrey double auction," *Journal of Economic Theory*, 101, 572-584.

〔Abstract〕

효율적 경매매

윤 기 호

이 논문은 상호의존적 가치모형에서 경매매 문제를 다루고 있다. 논문은 상호의존 가치환경에서 보수 동등성 결과를 도출하고 있으며, 또한 효율적이고 사후적 유인합치 메카니즘인 ‘일반화된 비크리 경매매 메카니즘’을 소개한다. 이러한 도구들을 이용해 논문은 효율적, 베이지안 유인합치적, 사중적 개인합리적, 사전적 예산균형적 메카니즘의 존재에 관한 필요충분조건을 제시하고 있다. 논문은 양의 상호의존성은 이러한 메카니즘의 존재를 더욱 어렵게 하는 반면, 음의 상호의존성은 더욱 쉽게 한다는 점도 보이고 있다.

핵심용어 : 경매매, 상호의존적 가치, 효율성