計量經濟學報, 제17권 제2호(2006. 6. 30.), 79-105

The Dynamics of Nominal Exchange Rates in a Fractional Cointegration Model*

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This paper introduces an efficient estimation for fractional cointegration models which capture long run economic equilibrium relationships while allowing for a wider range of mean reverting behavior than standard cointegration analysis. The Fully-Modified method which is suggested in Phillips and Hansen (1991) can be applicable to fractional cointegration models to reduce second order biases. The comovement of nominal exchange rates is analyzed in a fractional cointegration models. Exchange rates dynamics are estimated by the efficient estimation method. Empirical results based on data for the 1957~1997 period show that there exists a cointegration relationship in a group of exchange rates.

Keywords: Fractional Cointegration, Long Memory, Long Run Equilibrium, Cointegrating Vector, Exchange Rate Dynamics

JEL Classification : C22

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* I am grateful to a Co-Editor and two anonymous referees for their helpful comments, which have greatly improved an earlier version of this paper.

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투고일: 2006. 4. 5 심사일: 2006. 4. 6 최종심사완료일: 2006. 4. 25

Ⅰ. Introduction

The growing interest in fractional processes and long range dependence has been motivated in part by the extensive literature on unit root time series. Unit root nonstationarity gives rise to data with very different statistical properties with different economic interpretations from those associated with weakly stationary processes. Robinson (1994(a)) and others have pointed out that the gulf between weak stationarity and nonstationary unit root processes can be smoothly bridged by the class of fractionally integrated process.

Fractional cointegration is well defined as

$$z_t = x_t - \alpha y_t$$

where x_t, y_t are I(d) and z_t could be I($d-b$) while d, b do not have to be integers. Two series sharing the same stochastic trend are said to be cointegrated if a linear combination of the two series does not have that dominant property. In general, cointegration analysis enables us to establish long run relationships with little restriction on short run dynamics. From this point of view, the notion of fractional cointegration is of great economic importance since it implies the existence of a very general type of long run equilibrium. By avoiding the knife-edged I(1), I(0) distinction in the equilibrium error, fractionally cointegrated models allow a much wider class of mean reverting behavior in econometrics.

The analysis of fractional cointegration has been explored in recent years, for example, Robinson and Hualde (2003), Marinucci and Robinson (2003), Davidson and de Jong (2002), Kim and Phillips (2002) among others. It is known that ordinary least squares (OLS) is consistent in fractional cointegration models as shown in Robinson and Marinucci (2003). OLS, however, suffers from second order biases as shown in Kim and Phillips (2002) under specific model characterization.

When estimating the cointegrating vector in a linear regression model with I(1) variables, it is well known that OLS is consistent under general assumptions including endogeneity in the regressors and serial correlation in the errors. Phillips and Hansen (1990) therefore suggested a Fully-Modified (FM) OLS procedure that modifies OLS with corrections for these biases. Kim and Phillips (2002) has developed Fully modified estimation methods in the fractional cointegration model utilizing the original idea of FM-OLS, and they also showed that the second order bias of the OLS estimator in fractional cointegration adversely affects finite sample performance of OLS estimation. In host of empirical works, the biased reduced and more efficient estimation of cointegrating vectors seems important since the estimates itself can draw different conclusions on the same models, and it may come from finite sample distortion of estimation procedures or hypothesis testings.

We use the idea of efficient estimation of fractional cointegration models and apply the method into the analysis of nominal exchange rate dynamics in the present paper. The analysis of exchange rates dynamics provides an interesting empirical arena for the application of the concept of fractional cointegrations. The use of efficient estimation procedures that utilize potential links between exchange rates is important in these models, since cointegrating relationships between exchange rates tell us that there are restrictions on exchange rate dynamics and this will affect the future evolution of exchange rates from currently available information.

A series of papers, however, such as Baillie and Bolleslev (1989), Diebold, Gardeazabal, and Yilmaz (1994), and Baillie and Bolleslev (1994), provided somewhat conflicting evidence about the exchange rates dynamics based on the concept of long run economic equilibrium. Baillie and Bolleslev (1989) implement a test for the number of independent unit roots, or stochastic trends with seven nominal exchange rates. They can not reject the null hypothesis that six stochastic trends are present in each of the full seven-dimensional systems or only one cointegrating factor exists between a set of nominal exchange rates, which implies that the set of foreign exchange rates are tied together in one long-run relationship. Hakkio and Rush (1989) support this result and point out the lack of power on rejecting cointegration. However, Sephton and Larsen (1991) describe the cointegration relationship as being fragile since they found mixed conclusions about the cointegration relationship in the seven nominal exchange rates. They demonstrate that cointegration methods can not offer sound evidence on market efficiency against the empirical findings in Baillie and Bolleslev (1989) and Hakkio and Rush (1989).

Diebold *et al.* (1994) provide an evidence which concludes that there exists substantial uncertainty regarding the existence of cointegration relationships among nominal dollar exchange rates. Diebold *et al.* (1994) draws a different conclusion from Baillie and Bolleslev (1989) even though both implement the cointegration tests with the same set of data. Diebold *et al.* (1994) argue that these conflicts come from the fact that Baillie and Bolleslev (1989) does not allow a drift in the cointegration test. No cointegration are reinforced even in the longer time span (entire post-1973 floating exchange rate system) in Diebold *et al.* (1994). However, Baillie, and Bolleslev (1994) suggest some additional evidence on the existence of cointegration relationships which are associated with long memory processes. They note that the unit root tests have very low power against fractional alternative as in Diebold and Rudebusch (1991). Given this difficulty of distinguishing unit root processes and fractionally integrated processes, they consider the error correction term of seven nominal exchange rates to possibly be fractionally cointegrated.

Based on the approaches in the literature, we will provide the evidence for the existence of fractional cointegration among nominal exchange rates by using more efficient estimation of the cointegrating vector. In light of on the results of empirical study on exchange rates in this paper, it appears that there exists a fractional cointegration relationship between exchange rates, so that they move together through a cointegrating vector and tend not to drift apart without bound.

The plan of the paper is as follows. We first give underlying model and representation in the fractional cointegration model, and provide the FM estimation procedure in the context of a simple bivariate regression. Based on a fractional cointegration model and bias reduced estimation suggested, empirical results on exchange rates dynamics are provided.

Ⅱ. Estimation of a Fractional Cointegration Model

1. Model

(1) Assumptions

The time series $\{y_{1t}\}$, $\{y_{2t}\}$ which are assumed to be generated by

$$y_{1t} = \beta y_{2t} + u_{1t}$$
$$y_{2t} = y_{2t-1} + u_{2t}.$$

Assume the error processes in this model $\{u_{1t}\}$, $\{u_{2t}\}$ are fractionally integrated processes defined as follows.

$$(1-L)^{d_1}(u_{1t} - \mu_1) = e_{1t},\ (-\frac{1}{2} < d_1 < \frac{1}{2})$$
$$(1-L)^{d_2}(u_{2t} - \mu_2) = e_{2t},\ (-\frac{1}{2} < d_2 < \frac{1}{2})$$

For $0 < |d_i| < \frac{1}{2}$, $\{u_{1t}\}$, $\{u_{2t}\}$ are still stationary and ergodic. Especially, for $0 < d_i < \frac{1}{2}$, the process is long memory or long range dependent in a sense that j-lag autocovariance decreases very slowly, like the power law $j^{2d_i - 1}$ as $j \to \infty$ as is well known, see Baillie (1996). That is, it decays at a hyperbolic rate not a exponential rate. In practice, $-\frac{1}{2} < d_i < 0$ case is rarely encountered, mainly because this condition of zero spectral density value at the origin is very unstable (Beran, 1993). However, if we think of a nonstationary variable with $\frac{1}{2} < d_i < 0$ which is believed to characterize many economic variables, then innovations with $-\frac{1}{2} < d_i < 0$ are important since nonstationary variables with $\frac{1}{2} < d_i < 1$ can be analyzed by the partial sum of innovations with $-\frac{1}{2} < d_i < 0$. For $\frac{1}{2} < d_i < 1$, $\{u_{it}\}$ will not be covariance stationary any more so that the variance of $\{u_{it}\}$ does not exist. But, $d_i < 1$

assures the mean reverting property, namely, the impact of a unit innovation at time t on the process vanishes in the long run, which is different from $I(1)$ nonstationary process. Mean reversion property allows a fractionally integrated error in the cointegration model, since it implies the existence of long run equilibrium relationship. Therefore, mean reversion of fractionally integrated processes with $d_i < 1$ encompasses a broader model of economic relationship whereas only $0 < |d_i| < \frac{1}{2}$ case will be dealt with in this paper.

The sequences $\{e_{1t}\}$, $\{e_{2t}\}$ need not be assumed to be as independent, identically distributed (i.i.d.), they could be more general processes. Here, we assume that $\{e_{1t}\}$, $\{e_{2t}\}$ are autoregressive moving average of order (p, q) (ARMA(p, q)) processes, that is,

$$\phi_1(L)e_{1t} = \theta_1(L)\epsilon_{1t}$$
$$\phi_2(L)e_{2t} = \theta_2(L)\epsilon_{2t}$$

where we assume all the roots of $\phi_i(L)$ and $\theta_i(L)(i=1,2)$ lie outside the unit circle, there are no common roots and $\{\epsilon_{1t}\}$, $\{\epsilon_{2t}\}$ are i.i.d.. This model is not exactly same as autoregressive fractionally integrated moving average of order (p, d, q) (ARFIMA(p, d, q)) which was introduced in Granger and Joyeux (1980), and Hosking (1981). When $d > -\frac{1}{2}$, there exist infinite order moving average representation

$$u_{1t} = \sum_{k=0}^{\infty} \varphi_k e_{1t-k} = \varphi(L) e_{1t},$$

$$u_{2t} = \sum_{k=0}^{\infty} \varphi_k e_{2t-k} = \pi(L) e_{2t.}$$

where ϵ_{1t} and ϵ_{1t} are i.i.d. with variance

$$Var\begin{pmatrix}\epsilon_{1t}\\ \epsilon_{2t}\end{pmatrix} = \begin{pmatrix}\bar{\sigma}_{11} & \bar{\sigma}_{12}\\ \bar{\sigma}_{21} & \bar{\sigma}_{22}\end{pmatrix},$$

then the long run variance of $\begin{pmatrix}e_{1t}\\ e_{2t}\end{pmatrix}$ can be written as

$$\begin{pmatrix} \bar{\sigma}_{11}\phi_1(1)^{-2}\theta_1(1)^2 & \bar{\sigma}_{12}\phi_1(1)^{-1}\theta_1(1)\phi_2(1)^{-1}\theta_2(1) \\ \bar{\sigma}_{21}\phi_1(1)^{-1}\theta_1(1)\phi_2(1)^{-1}\theta_2(1) & \bar{\sigma}_{22}\phi_2(1)^{-2}\theta_2(1)^2 \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} .$$

Assumption : u_{1t} and u_{2t} *are Gaussian.*

In fact, the Gaussianity assumption is not necessary for the limit theory of partial sum of long memory processes. Invariance principle for fractionally integrated processes applies more generally without the Gaussianity assum-ption. That is, the limit theorem for partial sum of fractionally integrated processes generated by independent random variables and functionals induced by mixing processes is well established as in Davydov (1970). Moreover, the weak convergence of a linear combination of long memory processes without any distributional assumption is also established (Robinson, 1994(a)). However, the Gaussianity assumption is critical for the following spectral representation of the processes u_{1t} and u_{2t} and for the development of the weak convergence of sample covariances to integrals with respect to Brownian motion as in Chan and Terrin (1995).

Now, consider fractional Brownian Motion which was defined as their d fractional derivative of regular Brownian motion by Mandelbrot and Van Ness (1968). The spectral representation of fractional Brownian motion is

$$\mathbf{B}_{kd_k}(r) = K_{d_k}\int_R \frac{e^{i\lambda r}-1}{i\lambda} \mid \lambda \mid^{-d_k} dB_k,\ k = 1, 2$$

where $K_{d_k} = \left(\int_R \left|\frac{e^{i\lambda}-1}{i\lambda}\right|^2 \mid \lambda \mid^{-2d_k}\right)^{\frac{1}{2}} = \left(\frac{(d_k+\frac{1}{2})\Gamma(1+2d_k)\sin\pi(d_k+\frac{1}{2})}{\pi}\right)^{\frac{1}{2}}$.

Denote $\mathbf{K}_{d_k} = K_{d_k}(2\pi)^{\frac{1}{2}}$ for a notational convenience. This spectral represen-tation of fractional Brownian motion is well explained in Samorodnitsky and Taqqu (1994). One merit of this representation is that it helps deliver the mean square convergence of fractionally integrated processes under Normality assumptions, as analyzed in Chan and Terrin (1995).

Now, define $S_{1T} = \sum_{t=1}^{T} u_{2t}$, and $S_{2T} = \sum_{t=1}^{T} u_{2t}$, then we have

$$\lim_{T\to\infty} T^{-(1+2d_1)}\mathbf{E}(S_{1T}^2) = w_{11},$$
$$\lim_{T\to\infty} T^{-(1+2d_2)}\mathbf{E}(S_{2T}^2) = w_{22},$$
$$\lim_{T\to\infty} T^{-(1+d_1+d_2)}\mathbf{E}(S_{1T}S_{2T}) = w_{12},$$

as shown in Kim and Phillips (2002).

Clearly, the long-run variance above is the same as the one in the Theorem 1 in Sowell (1990). Sowell (1990, 1992) used gamma and hypergeometric functions to derive long run variance and autocovariance functions of a fractionally integrated process, but by considering fractional Brownian motion in the limit, long-run variance of a fractionally integrated process can be well represented in the form of an integral with a deterministic function integrand. We also notice that consistent estimation of long-run variance can be implemented by consistent estimation of d and ARMA coefficients in the error term. Since various methods for the estimating d have been explored including full maximum likelihood (Sowell, 1992), consistent estimation of d is by now standard problem. Based on the limit theories in Chan and Terrin (1995) and Kim and Phillips (2002), it can be deduced that

Limit distribution of OLS estimator in a fractional cointegrated model.

$$T^{1+d_2-d_1}(\hat{\beta}-\beta) \xrightarrow{d} \left(\int_0^1 B_{2d_2}(t)\, B_{2d_2}(t)\, dt\right)^{-1}$$
$$\cdot\left(\iint_{R^2}\int_0^1 \frac{e^{it\mu}(e^{it\lambda}-1)}{i\lambda}\, dt\, |\lambda|^{-d_2}|\mu|^{-d_1}\, dB_2(\lambda)\, dB_1(\mu) + \int_0^1\int_R \frac{1-e^{-it\lambda}}{i\lambda}|\lambda|^{-d_1-d_2}d\lambda\, dt\cdot\sigma_{12}\right).$$

Here, $\iint_{R^2}$ indicates that integration on the diagonals $\mu = \pm\lambda$ is excluded, so that the multiple integral is defined over $\lambda \neq \pm\mu$, and has the form of a double Wiener-Ito integral just as in the definition of Chan and Terrin (1995). In a fractional cointegrated model, we have similar results as in the asymptotics of the standard I(1) variables with I(0) innovation. It is well known that

$T^{-1}\sum_{t=1}^{T} y_{2t}u_{1t}$ converges to $\int B_1 dB_2 + \lambda_{12}$ in the standard cointegration model where B_1, B_2 are Brownian motions and λ_{12} is a one side long-run variance. Similarly, the limit of OLS estimator contains random and non-random term which are corresponding to the $\int B_1 dB_2, \lambda_{12}$ respectively. Therefore, FM-estimation can be implemented here to eliminate the endogeneity bias and serial correlation bias term so that asymptotic efficiency can be obtained. In what follows, the efficient estimation procedure will be presented explicitly.

2. Estimation

The following estimation procedure can be found in Kim and Phillips (2002), and the same can be applied in the model here, and hence we briefly introduce the FM-OLS here.

(1) Notations

Some notations will be introduced here for simplicity of presentation.

$$\iint_{R^2}\int_0^1 \frac{e^{it\mu}(e^{it\lambda}-1)}{i\lambda}dt\,|\lambda|^{-d_2}|\mu|^{-d_1}dB_2(\lambda)dB_1(\mu)\frac{1}{2\pi} =: \int_0^1 B_{2d_2}(t)dB_{1d_1}(t) =: \Xi_{12}$$

which is defined when $\lambda \neq \pm\mu$. Here, writing $\int_0^1 B_{2d_2}(t)dB_{1d_1}(t)$ is not a stochastic integral in the conventional sense. Similarly, when $\lambda \neq \pm\mu$,

$$\iint_{R^2}\int_0^1 \frac{e^{it\mu}(e^{it\lambda}-1)}{i\lambda}dt\,|\lambda|^{-d_2}|\mu|^{-d_2}dB_2(\lambda)dB_2(\mu)\frac{1}{2\pi} =: \int_0^1 B_{2d_2}(t)dB_{2d_2}(t) =: \Xi_{22}$$

(2) Estimation of cointegrating vector using FM-OLS

From the previous results, fractional Brownian motion can be written as

$$\begin{pmatrix} B_{1d_1}(t) \\ B_{2d_2}(t) \end{pmatrix} \equiv FBM(\Omega) \text{ with long-run variance } \Omega = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix}.$$

We write the linear combination of fractional Brownian motion

$$B^{1.2}_{d_1 \cdot d_2} = B_{1d_1} - w_{12}w_{22}^{-1}B_{2d_2}.$$

$B^{1.2}_{d_1 \cdot d_2}$ can be represented as

$$B^{1.2}_{d1 \cdot d_2} = \int_R \frac{e^{i\lambda r}-1}{i\lambda} \mid \lambda \mid^{-d_1} \frac{1}{\sqrt{2\pi}} dB_1(\lambda) - w_{12} w_{22}^{-1} \int_R \frac{e^{i\lambda r}-1}{i\lambda} \mid \lambda \mid^{-d_2} \frac{1}{\sqrt{2\pi}} dB_2(\lambda),$$

then, if $d_1 = d_2$,

$$B^{1.2}_{d_1 \cdot d_2} = \int_R \frac{e^{i\lambda r}-1}{i\lambda} \mid \lambda \mid^{-d_1} \frac{1}{\sqrt{2\pi}} (dB_1(\lambda) - w_{12} w_{22}^{-1} dB_2(\lambda)).$$

Since $B_1(\lambda) - w_{12} w_{22}^{-1} B_2(\lambda)$ is a Gaussian random measure with variance $\sigma_{11} - \sigma_{12}\sigma_{22}^{-1}\sigma_{21}$, $B^{1.2}_{d_1 \cdot d_2}$ is also fractional Brownian motion when $d_1 = d_2$. Therefore, in the $I(1)/I(0)(d_1 = d_2 = 0)$ cointegrating regression $B^{1.2}_{d_1 \cdot d_2}$ reduces to $B_1 - w_{12} w_{22}^{-1} B_2$. Now, the following holds obviously

$$Cov(B^{1.2}_{d_1 \cdot d_2}, B_{2d_2}) = Cov(B_{1d_1} - w_{12} w_{22}^{-1} B_{2d_2}, B_{2d_2}) = 0$$

which implies $B^{1.2}_{d_1 \cdot d_2}$, B_{2d_2} are independent, since they are Gaussian.

Consider the following augmented regression as

$$y_{1t} = \beta y_{2t} + w_{12} w_{22}^{-1} \triangle y_{2t} + u^{+}_{1t}, \quad \text{where } u^{+}_{1t} = u_{1t} - w_{12} w_{22}^{-1} u_{2t}.$$

Let $y^{+}_{1t} = y_{1t} - w_{12} w_{22}^{-1} \triangle y_{2t,}$ then the regression becomes

$$y^{+}_{1t} = \beta y_{2t} + u^{+}_{1t}.$$

Consistent estimation of long run variance of long memory processes will be explored later. Now consider FM-OLS estimator as

$$\hat{\beta}^{+} = \left(\sum_{t=1}^{T} y_{2t}^2 \right)^{-1} \left(\sum_{t=1}^{T} y_{2t} y^{+}_{1t} \right) - \hat{\lambda}^{+}_{12} T^{1+\hat{d}_1+\hat{d}_2} \left(\sum_{t=1}^{T} y_{2t}^2 \right)^{-1}$$

where $\hat{\lambda}^{+}_{12} = \hat{\lambda}_{12} - \hat{w}_{12} \hat{w}_{22}^{-1} \hat{\lambda}_{22}$.

This is almost the same form as that defined originally for $I(1)/I(0)$ cointegrating regression, so that the procedure is robust to the presence of long range dependence in the regressors and regression errors. Now, using the limit theories presented above, the asymptotic behavior of the FM- estimator can be developed in a fractional cointegration model. For $d_1 + d_2 > 0$, the limiting distribution of FM-OLS estimator can be obtained by the following result.

Theorem : Limit distribution of FM-OLS estimator in a fractional cointegrated model.

For $d_1 + d_2 > 0$,

$$T^{1+d_2-d_1}(\hat{\beta}^+ - \beta) \xrightarrow{d} (\Xi_{12} - w_{12}w_{22}^{-1}\Xi_{22})\left(\int_0^1 B_{2d_2}(t)B_{2d_2}(t)\right)^{-1}$$

$$=: \left(\int_0^1 dB_{d_1 \cdot d_2}^{1,2} B_{2d_2}\right)\left(\int_0^1 B_{2d2}(t)B_{2d_2}(t)\right)^{-1}.$$

The proof of the theorem is given in Kim and Phillips (2002). It can be easily deduced that the variance of limiting distribution contains constants such as $\mathbf{k}_{d_1}$, $\mathbf{k}_{d_2}$ the values of which are dependent upon d_1, d_2. The limiting distribution of FM-OLS in fractional cointegrated model is found to be free from biases. Another approach is to deal with the time series properties of u_t nonparametrically by the use of system spectral regression as in Phillips (1988). Robinson and Hidalgo (1997) show the asymptotics of GLS estimation following the Hannan's approach (e.g. Hannan, 1963) under long range dependence and Robinson and Marinucci (2003) show the consistency of narrow-band frequency domain least squares in the fractional cointegration model. Robinson and Hualde (2003) developed GLS type estimation in the time domain, and Kim (2004) suggested bias reduced frequency least squares. The convergence rate of FM-OLS estimator in a fractionally cointegrated model depends on the values of the fractional integrated parameters d_1, d_2 as expected.

The FM-OLS procedure involves unknown long run variances and covariances which need to be estimated consistently. This is complicated by the well known fact that fractionally integrated processes violate the strong mixing condition and, hence, the uniform mixing condition. Since the covariance function of long memory processes decays at a hyperbolic rate, which is slower than the geometric decay in the usual stationary or mixing sequences, results for the consistent estimation of the long run variance under weak dependence are not directly applicable to long range dependence series. Consistent estimation of long run variance and one-sided long run variance can be accomplished as following procedures.

First, we can get the residuals of $\hat{u}_t = (\hat{u}_{1t}, \hat{u}_{2t})'$ from the OLS, the narrow band least squares (Robinson and Marinucci, 2003), or Kim (2004) of the model in the section 1, then using the following autoregressive representations of long memory processes, $e_t = (e_{1t}, e_{2t})'$ can be obtained. That is,

$$\sum_{k=0}^{t} \pi_k \hat{u}_{t-k} = e_t$$

where

$$\pi_k = \frac{\Gamma(k-\hat{d})}{\Gamma(k+1)\Gamma(-\hat{d})}.$$

Here, $\hat{d}$ denotes the consistent estimate of d which can be estimated by semi-parametric estimation procedures in the literature, for example, Geweke and Porter-Hudak (1983), Local Whittle Estimation (Robinson, 1995) among others. By the consistent estimation of d, we calculate the long run variance of e_t using the usual estimation methods. Once we obtain the long run variance of e_t, $\hat{\Sigma}_{ee}$, the only remaining thing to obtain the long run variances of u_t is to calculate the coefficient $\mathbf{K}_{d_i}$ which is defined before. The long run variance of u_t can be estimated by

$$\hat{\Omega} = \begin{pmatrix} \hat{w}_{11} & \hat{w}_{12} \\ \hat{w}_{21} & \hat{w}_{22} \end{pmatrix} = \begin{pmatrix} \hat{\boldsymbol{K}}_{d_1}^{-2} \hat{\sigma}_{11} & \hat{\boldsymbol{K}}_{d_1,d_2}^{-2} \hat{\sigma}_{12} \\ \hat{\boldsymbol{K}}_{d_1,d_2}^{-2} \hat{\sigma}_{21} & \hat{\mathbf{K}}_{d_2}^{-2} \hat{\sigma}_{22} \end{pmatrix}$$

where

$$\hat{\boldsymbol{K}}_{d_k} = \left(\frac{\left(\hat{d}_k + \frac{1}{2}\right)\Gamma(1+2\hat{d}_k)\sin\pi\left(\hat{d}_k + \frac{1}{2}\right)}{\pi} \right)^{\frac{1}{2}} (2\pi)^{\frac{1}{2}}, k = 1, 2$$

and

$$\hat{\boldsymbol{K}}_{d_1,d_2} = \left(\frac{\left(a + \frac{1}{2}\right)\Gamma(1+2a)\sin\pi\left(a + \frac{1}{2}\right)}{\pi} \right)^{\frac{1}{2}} (2\pi)^{\frac{1}{2}}, a = \frac{\hat{d}_1 + \hat{d}_2}{2}.$$

Similarly, one-sided long run variance of u_t can be obtained in a similar way. Therefore, once we get the consistent estimate of d, the long variance and one-sided long run in our model can be easily obtained by this procedure.

Ⅲ. Fractional Cointegration and Exchange rates Dynamics

As noted in the introduction, the analysis of exchange rates dynamics provides an interesting example for the application of methods developed in the previous sections. Moreover, exchange rates dynamics can be related to issues of efficiency in foreign exchange markets. One of the possible interpretations of cointegration relationships in foreign exchange markets is that if markets are efficient, spot exchange rates should contain all relevant information, and no future exchange rates should be predictable using current available information. So, if the series of exchange rates are not cointegrated, one might not reject the efficiency of foreign exchange market, whereas if we find a cointegrating relationship, then it would violate foreign exchange market efficiency. Another interpretation is that the linear combination of nominal exchange rates may be an instrument for the forward risk premium, so foreign exchange market efficiency can be tested by examining the time series properties of the forward premiums along with cointegration tests in nominal exchange rates as in Crowder (1994).

Baillie and Bolleslev (1989, 1994) used the data set which contains seven daily nominal exchange rates from 1980 to 1985. However, cointegration relationships are the statistical manifestation of long run equilibrium in economics, so that fairly long term span of data will be needed in analyzing cointegration. As noted in Hakkio and Rush (1991), the frequency of observation plays very little role in detecting a cointegration. That is, increasing observations by simply switching to more frequently observed data is not helpful in capturing cointegration relationships in variables, whereas adding to the total length of the sample yields better performance in detecting cointegration relationships.

We therefore study seven nominal quarterly spot exchange rates data from 1957 through 1997 which are much longer time spanning data than those examined before.[1] The quarterly data set includes ; the Canadian Dollar, French Franc, Deutsche Mark, Italian Lira, Japanese Yen, Swiss Franc, and British Pound, all relative to the U.S. Dollar. The data were taken from the Citibase and they run from 1957.1 through 1997.4 for a total of 164 observations.[2] The data set includes both fixed and flexible exchange rates periods, but the cointegration analysis for the flexible exchange rates period only (1973~1997) gives almost the same results as will be seen in the <table 4>.

1. Exchange Rates and Unit Roots

There appears to be a widespread agreement that nominal exchange rates are well characterized by I(1) processes. Most of previous results support the I(1) nonstationarity of nominal exchange rates including Corbae and Ouliaris (1988), Baillie and Bolleslev (1989). However, Cheung (1993) reports some evidence that exchange rates may be fractionally integrated. He implements the Geweke and Porter-Hudak (1983) test for five nominal spot exchange rates and draw a rejection to the unit root hypothesis in favor of the long memory alternative. Cheung (1993) only reports for possible deviation from I(1) behavior of the nominal exchange rates. But, it appears that the exact configuration of $I(0)/I(d)$ variates is not necessary for efficient estimation developed here, since our framework allows long range dependence in the regressors as well as in the regression errors as long as long memory coefficients d for all regressors are of the same as shown in the previous section. But, to confirm the exact specification of exchange rate data, unit root tests were implemented as follows.

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1) Some of recent nominal exchange rates data are not available since Euro currency system has started.

2) In fact, we found almost the same results in cointegration analysis with monthly data for the same time span, we do not report them in the present paper to save the space.

〈Table 1〉 Tests for unit root in the logarithm of spot exchange rates

Country	ADF 3 lags	ADF 5 lags	ADF 7 lags	z_{t_a} 3BW's	z_{t_a} 5 BW's	z_{t_a} 7 BW's
Canada	−2.73	−2.69	−2.91	−1.58	−1.77	−1.92
France	−2.50	−2.78	-3.05^c	−2.36	−2.51	−2.60
Germany	−2.87	−2.80	-3.16^c	−2.11	−2.29	−2.41
Italy	−2.49	−2.35	−2.49	−1.88	−2.01	−2.11
Japan	−2.52	−2.64	−2.62	−2.35	−2.47	−2.55
Switzerland	−2.72	−2.66	−2.84	−2.17	−2.32	−2.41
UK	−2.87	−2.63	-3.55^c	−2.45	−2.59	−2.67

(c denotes the rejection of null hypothesis of unit root at 95% level).

<Table 1> presents the results of unit roots test for seven nominal exchange rates. The Augmented Dickey-Fuller test (ADF) and the Phillips t-test (z_{t_a}) test were performed with linear time trend. The results were found to be similar for the different lag values and the different bandwidths (BW's). The critical value with 95% level is −2.89 for both tests. The testing results for seven nominal exchange rates suggest that the unit root null hypothesis can not be rejected for seven countries in all different lags and bandwidths with a few exceptions, which agrees with most of the previous results in the literature. Based on the unit root tests given above, we have $d_2 = 0$ and hence the fractional parameter d_2 needs not to be estimated in the following.

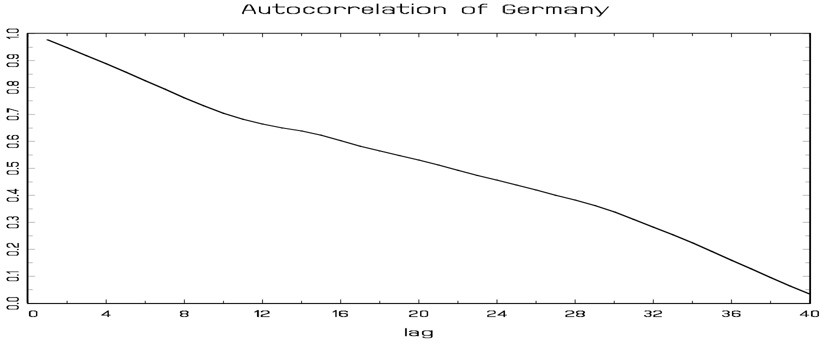
2. Efficient Estimation of the Fractional Cointegrating Vector

To further understand fractional cointegration in exchange rates system, the first 40 sample autocorrelations for exchange rate data of Germany which was regarded to be $I(1)$ nonstationary variables is plotted in the <figure 1> and autocorrelation function for the error correction term $\hat{a}'y_t$($\hat{a}$: estimate of the cointegration vector, y_t : seven nominal exchange rate vector) is plotted in the <figure 2>.[3] The error correction term and $\hat{a}$ were estimated by OLS which is consistent even though it entails the second order biases as mentioned earlier.

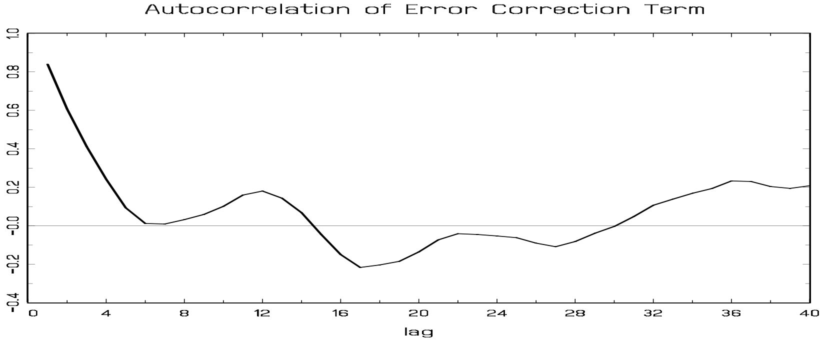
The graph represents that the autocorrelations for nominal exchange rate

3) The correlograms for other countries look similar with <figure 1>, and skipped here.

decline at very slow rate, which exhibits the typical property of an $I(1)$ process. But, the correlogram for the estimated error correction term exhibits long memory characteristic and the difference between autocorrelations of exchange rates and error correction term is somewhat clear from the figures below. That is, relatively rapid rate of decay of autocorrelation coefficient in error correction term can be noticed, which is contrast to autocorrelations of an $I(1)$ variable. Therefore, it can be deduced that the error correction term exhibits stationary characteristics rather than $I(1)$ nonstationary property.



〈Figure 1〉 Autocorrelation of German Mark



〈Figure 2〉 Autocorrelation of the Error Correction Term

The efficient estimation of cointegrating vector in exchange rates dynamics can be achieved from a regression of the logarithm of the US dollar rate for the German currency on a constant and the logarithm of other six nominal exchange rates for the currencies of United Kingdom, Japan, Canada, France, Italy, and Switzerland with 41 year quarterly data.

For the purpose of testing cointegration in exchange rates dynamics only, OLS can be used to obtain the estimates of the cointegrating vector and residuals from this regression although using the efficient estimation rather than OLS may increase the power of cointegration tests. That is, if $\alpha' y_t$ is stationary with fractional coefficient d, we can conclude that there exists one stochastic trend among seven nominal exchange rates. Since OLS estimator is still consistent in fractional cointegration system, we can use OLS estimator $\hat{\alpha}$ to test the stationarity of $\hat{\alpha}' y_t$. But, our aim in the analysis of exchange rates dynamics is not only to test the fractional cointegration but to estimate the cointegrating vector efficiently in a sense explained in the previous section.

Based on the OLS residual $\hat{u}_{1t}$, we estimate the fractional parameter d_1 using Geweke and Porter-Hudak (GPH) (1983).[4] The GPH estimator is a simple least squares in the frequency domain and hence has been one of the most popular estimation procedure for long memory parameters. We do not report the preliminary estimate results here to save space.

Using the estimate $\hat{d}_1$ based on OLS residual of cointegration model, we now can estimate the long run variances to have the augment FM-OLS regression model explained in the previous section. The estimation of long run variances is simple since they are only based on $\hat{d}_1$ and the variance of short memory components. The augmented FM-OLS regression model gives the following estimation results.

〈Table 2〉 Estimates of the Cointegrating Vector

	Constant	Canada	France	Italy	Japan	SW	UK
FM-OLS	-3.98 (0.37)	0.04 (0.07)	-0.08 (0.04)	0.45 (0.05)	0.19 (0.02)	0.79 (0.02)	-0.29 (0.04)

The numbers in parentheses are the standard errors of the corresponding parameter estimates. (SW(=Switzerland), UK(=United Kingdom)).

4) Recently, Andrews and Guggenberger (2003) developed a bias reduced Geweke and Porter-Hudak estimator, and the method gives the almost same value of fractional parameters in our case.

<Table 2> reports the estimates, which are obtained by FM-method, for full set of the seven exchange rates with the dependent variable (or normalized variable) in the cointegrating relationship taken to be Germany. Selecting a normalized variable to be another country (e.g. Canada, Italy) does give very similar results for the test of cointegrating relationship. All the estimated coefficients are significant except Canada.[5)]

3. Test of Fractional Cointegration and Estimation of ARFIMA Process

Given the estimates of cointegrating vector obtained by the efficient estimation method, we try to reestimate the fractional parameter d_1 to specify the degree of integrated order of regression errors in the exchange rates dynamics system. The non-zero estimate of d_1 indicates the degree of integration in the cointegration errors and indicates that the nominal exchange rates are fractionally cointegrated.

We assume an ARFIMA(p, d, q) model for the regression errors which is fairly general, then estimation of both the ARMA(p, q) coefficients and fractionally integrated parameter d should be performed. Semiparametric estimation of d in the frequency domain was suggested in Geweke and Porter-Hudak (1983), and Robinson (1994(c)) considered an estimation of d based on the average log-periodogram. Several semiparametric estimation methods in the time domain also has been suggested in the literature. (see E.g. Baillie, 1996; Beran, 1993).[6)]

Here we consider the Maximum Likelihood Estimation (MLE) in the joint estimation of the parameters in the ARFIMA(p, d, q) model. While Sowell's (1992) full MLE is theoretically appealing, it poses computational problems. In particular, the evaluation of inverse of covariance matrix in the likelihood function can be time-consuming and even unstable. (Beran, 1993). Therefore,

5) If Canada is taken out in the cointegration system, all the results are extremely similar and all the coefficient are significant.

6) The estimates of fractional parameter by different semiparametric estimation methods give almost the same result as in <table 3-4> and hence skipped.

maximizing approximation to the exact likelihood function may be a good alternative. Some simulation results in Chung and Baillie (1993) suggest that the resulting bias of the Conditional Sum of Square (CSS) method which is one of approximation method of the exact MLE can be sufficiently large to make inference extremely unreliable in small sample, e.g., T (number of sample observations) less than 150 when the mean of the processes are unknown. Actually, we do not know the mean of variables in most of empirical work in economics. Therefore, Using CSS method in our case where number of observations is 164 may cause a serious bias in estimating ARFIMA(p, d, q) parameters.

Another simulation results in Cheung and Diebold (1994) show that frequency domain MLE performs better than CSS especially when the mean of the process is unknown. Since it is well known that only the zero frequency ordinate periodogram depends on the mean, the frequency domain MLE defined below has its merit in that it is invariant to the unknown mean because the periodogram ordinate of frequency at zero will not be used. Moreover, in light of the computational burden associated with exact time domain MLE, frequency domain MLE will be a good approximation to the exact MLE.

Let X_t be a Gaussian process, then the log-likelihood of $X = (X_1, \ldots, X_T)$ is

$$\ell(X, \xi) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\log \mid \sum(\xi) \mid - \frac{1}{2}X'\sum^{-1}(\xi)X$$

where ξ denotes a parameter vector. If we let $f(\lambda)$ be the spectral density of process X and m^* be integer part of $\frac{T-1}{2}$, the approximation of Gaussian likelihood can be written as

$$L_W(\xi) = \sum_{j=1}^{m^*}\log g(\lambda_j, \xi) + \sum_{j=1}^{m^*}\frac{I(\lambda_j, \xi)}{f(\lambda_j, \xi)}$$

where

$$\lambda_j = \frac{2\pi j}{T},\ (j = 1, \ldots, m^*)$$

and $I(\lambda_j)$ is the periodogram of X at frequency λ_j given by

$$\frac{1}{T}\left|\sum_{j=1}^{T} X_t e^{-i\lambda_j t}\right|^2.$$

L_W stands for the Whittle likelihood, which was proposed by Whittle for the approximation in the context of short-memory time series. Following Fox and Taqqu (1986), we can use the fact that maximization of the Gaussian Likelihood is asymptotically equivalent to the minimization of

$$\sum_{j=1}^{m^*}\frac{I(\lambda_j)}{f(\lambda_j, \xi)}.$$

By minimizing this function with respect to parameters, all the parameters for ARFIMA(p, d, q) model can be simultaneously estimated. We can write the spectral density of an ARFIMA(p, d, q) model we have introduced in the previous section as

$$f(\lambda) = \left|\theta(e^{-i\lambda})\right|^2 \left|\phi(e^{-1\lambda})\right|^{-2} \left|1 - e^{-1\lambda}\right|^{-2d}$$

and $\xi = (\phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q, d)$.

The estimates of ARFIMA(p, d, q) model by frequency domain MLE in this exchange rates dynamics system are reported in the <table 3>. (The <table 4> shows the same estimates with the quarterly data in the flexible exchange rate system period (1973~1997). Similar results are obtained for those two periods).

〈Table 3〉 Estimates of ARFIMA(p, d, q) model (1957~1997)

p	q	d	ϕ_1	θ_1	θ_2	AIC
0	1	0.328 (0.218)	0.650 (0.211)			0.01
0	1	0.545 (0.070)		0.328 (0.218)		0.06
1	1	−0.222 (0.227)	0.822 (0.147)	0.518 (0.094)		3.17
0	2	0.445 (0.206)		0.698 (0.118)	0.118 (0.108)	1.32

〈Table 4〉 Estimates of ARFIMA(p, d, q) model in the flexible exchange rate system period (1973~1997)

p	q	d	ϕ_1	θ_1	θ_2	AIC
0	1	0.267 (0.280)	0.635 (0.276)			0.45
0	1	0.442 (0.091)		0.609 (0.092)		0.75
1	1	−0.216 (0.320)	0.714 (0.268)	0.519 (0.109)		3.44
0	2	0.214 (0.113)		0.827 (0.118)	0.241 (0.128)	2.32

The numbers in parentheses are the standard errors of the corresponding parameter estimates.

Even though some of standard errors of estimates for the fractional integrated parameter d are rather big, the fractional cointegration relationship seems to hold in the exchange rates dynamics system.

Akaike information criterion values are also calculated for the model, however, as noted in Hosking (1984), the assessment of goodness of fit of an ARFIMA(p, d, q) by AIC criterion is not straightforward. Conventional AIC method concentrates on the short term forecasting ability of the fitted model and may not give sufficient criterion for the fractionally integrated model. Bearing that AIC may not give the best criterion for applications involving fractionally integrated model in mind, we cautiously select the ARFIMA($1, d, 0$) model for the best fit model for the error correction term in the exchange rate dynamics system.

The results above support the fractional cointegration relationship in Baillie and Bolleslev (1994). That is, the deviation from the cointegration relationship in the exchange rate dynamics reveals the property of long memory and mean reversion. Baillie and Bolleslev (1994) used OLS without any asymptotic theories with very limited time span, we give more rigorous approach to find the fractional cointegration relationship in the nominal exchange rate dynamics. Our results in <table 3 and table 4> provide an possible explanation for the previous mixed conclusion about the existence of standard cointegration. The fractionally cointegrated model may lead the different testing results on the existence of standard cointegration. Our empirical findings confirm the long run economic equilibrium on the nominal exchange rate system and the mean reverting behavior of the disequilibrium error based on the general fractional cointegration model.

Moreover, the foreign market efficiency hypothesis can be rejected on the basis of the existence of a cointegration relationship. Since cointegration relationships capture a long run equilibrium of economic variables, the results above are meaningful in that they are based on much longer time span of data than other previous results related to the cointegration in exchange rates dynamics.

We used the FM-OLS method in estimating the cointegrating vector in the exchange rates dynamics without testing Gaussianity which is assumed to derive the limit theory of sample covariances of fractionally integrated processes in the previous section. Using quarterly data can avoid the heavy tail feature, but the Gaussianity assumption is probably far from being correct. Likewise, the presence of conditional heterogeneity has never questioned in the analysis of exchange rate dynamics. Further research for relaxation of the distributional assumption and consideration of fractional stable processes in the limit is necessary to analyze more general econometrics modeling.

Ⅳ. Conclusion

This paper introduces an statistical inference in a fractionally cointegrated regression model. The technique is based on the Fully Modified regression method involving fractional Brownian motion in a fractional cointegration model. Unlike the limit theory for OLS regression, the asymptotic distribution of the FM regression estimator is free from endogeneity bias and second order bias from serial correlation. Significantly, the FM procedure is robust to the presence of long range dependence in the regression errors.

Since cointegrating relationships are the statistical manifestation of many interesting long-run equilibrium relationships in economics, efficient and robust estimation of these relationships is important in a host of different empirical applications. Some immediate practical applications include models in finance, international finance, and linkages between financial variables and macroeconomic fundamentals. Based on an efficient estimation method of a fractional cointegration vector, a group of exchange rates are found to be fractionally cointegrated, implying that the error correction term in the cointegration model is mean reverting and that the exchange rates are tied together through a fractionally integrated process.

One limitation of the results in this paper, is that the results are based on some strong assumptions such as Normality of errors and the restriction $d_1 + d_2 > 0$, although the latter may not be too restrictive in practice. The development of an asymptotic theory under more general assumptions will be a significant extension of the preceding results. One interesting direction for future research will be the systematic analysis of cointegrating relationships when the regression error is I(d) with $d > \frac{1}{2}$. In this case, the fractionally integrated process is nonstationary but still mean reverting and a fractional cointegrating relationship exists in the sense explained in the introduction.

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〔Abstract〕

분수 공적분 모형에서의 명목 환율의 동태적 분석

김 창 식

이 논문은 표준 공적분 모형보다 더 넓은 범위의 평균 회귀적 성향을 포함하면서 장기적 경제 변수의 균형관계를 모델화 할 수 있는 분수 공적분 모형에서의 효율적인 추정방법을 소개한다. Phillips and Hansen(1991)의 Fully-Modified 방법이 분수 공적분 모형에서도 이차 편의를 줄일 수 있는 추정 방법이 될 수 있다는 사실에 의거하여 명목환율의 동태적 움직임을 분수 공적분 모형하에서 분석한다. 이 논문의 실증 분석에서는 1957~1997 사이의 명목환율에서 장기적 균형관계, 즉 공적분 관계가 있음을 보인다.

핵심용어 : 분수 공적분, 장기 기억, 장기 균형, 공적분 벡터, 장기 분산 추정