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Building an Identified Equilibrium Model of Aggregate Labor Market*

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This paper builds an econometrically identified equilibrium model of the labor market, by which one can jointly review labor supply and demand structure. A simple model economy is presented, where preferences are time-nonseparable, but technology is time-separable. The model allows recovery of the preference and technology parameters relevant to labor supply and demand behavior. The empirical model is verified to satisfy an econometric identification condition and estimated using a nonlinear instrumental method. The identified labor supply and demand parameters are consistent with theory. In conclusion, it appears that aggregate labor market fluctuations do not easily reject equilibrium interpretation.

Keyword: Identification of Aggregate labor demand and supply, Time-nonseparable preference, Intertemporal substitution in labor supply, Adjustment costs in labor demand, Interest rate and labor market, Econometric Identification and instrumental estimation of nonlinear system models.

JEL Classifications : E24, J40

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1. Introduction

Labor market data of employment (or hours worked) and real wages, usually show regular or irregular fluctuations. One conjecture is that such fluctuations are produced by labor demand and supply activities. This paper tests the conjecture by constructing an equilibrium model of the labor market. The test of equilibrium labor market theory has been attempted for a long time, but criticized because of the well-known identification problem.[1] The problem is very much classical in jointly identifying labor supply and demand schedules. The contribution of this paper is to build an econometrically identified equilibrium model where labor supply and demand structures are jointly estimated. The importance of such joint identification cannot be overemphasized and was pointed out by many economists.[2]

There have been many studies testing the aggregate labor market. However, the majority of studies on the labor market analyze only one side of the market by employing very restrictive assumptions for identification. For example, Rogerson and Rupert (1993) on labor supply, and Hammermesh (1993) on labor demand. These one-side analyses of the labor market suffer from the identification problem. When labor market data are plotted on two- dimensional space of employment and real wages, it can be seen why the studies on one side of the labor market fails to identify the correct labor market structure. The plotted data do not make the labor supply schedule nor the labor demand schedule alone, but make just a set of scattered employment and wages. Thus,

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1) Lucas (1969) has pointed out the imporance of understanding the structure of labor market, especially of aggregate labor supply as a cornerstone of both neoclassical growth theory and short-run Keynessian-type employment theory.

2) Among others, one bench mark and classical criticism has been raised by Geary and Kennan (1982). They conclude that it is difficult to reject the hypothesis specifying independence between real wages and employment from the data of 12 countries. They say that "Given dynamic supply and demand curves, each subject to random disturbances, there is no reason to believe that a competitive labor market would produce any particular relationship between employment and real wages". Also, see Deaton (1986), and similar empirical observation is in Dunlop (1938) and Tarshis (1939)

it is natural to presume that the observed data of employment and wage fluctuations are the outcome of the intersection of both schedules. This means that it is inappropriate to only look at the labor supply or labor demand schedule, while data have been generated by intersection of both schedules.[3] If labor supply and demand is affected by random exogenous variables, the resulting observations of wage and employment become a set of dispersed wage and employment combinations. They may look like "chaos", for which seemingly, economists can provide no deterministic structure. However, it might be "deterministic chaos" in the sense that one can identify the genuine structure generating the chaos. This paper is based on this idea and jointly identifies the structure of labor supply and demand by looking at seemingly uncorrelated data.

It means that we need to find a mapping from data to parameters of preference and technology (or labor supply and demand) which is theoretically just the inverse mapping of decision rules. This sounds simple, but has turned out to be difficult in practice. To find the inverse mapping, this paper specifies the dynamic structure of the labor market, where labor demand and supply are subject to random shocks. The model is not supposed to make observable ex post correlation between wage and employment. A similar methodology has been employed in Kennan (1988a) and Christiano and Eichenbaum (1992).[4] The former assumes the existence of serially correlated, independent and unobservable random shocks to both sides of the labor market, with no observable shocks, identifying the sets of parameters rather than each parameter, while the latter uses government spending as a variable disturbing labor markets, not identifying parameters.

In building a model, first of all, observable and measurable variables for disturbing labor supply and demand need to be specified. There are many variables presumably disturbing the labor market. For example, preference and technology shocks, the rental price of capital, government shock, net export

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3) Pointing out this problem, Kennan (1988) diagnoses that studies on the aggregate labor market have been suffered from "schizophrenic identification" problem.

4) To my knowledge, it looks that Kennan (1988) is unique in identifying the labor supply and demand jointly

shock, and so on. In this paper, the preference and technology shocks, and rental price, are first selected.[5)] Next, the empirical methods of identification and estimation are conducted as the following. In empirical work, Euler equations are estimated by exploiting the econometric implications of rational expectations, not the reduced forms of solutions. The set of Euler equations for empirical work creates a nonlinear model. After building the empirical model, it is proven that the model is econometrically identified, using Brown (1983)'s observational equivalence concept. The model is estimated by a nonlinear simultaneous instrumental variable procedure, specifically, the nonlinear three stage least square (N3LS).

Finally, the estimated parameters from Euler equations are used to recover the structure of labor supply and demand. The empirical results are interesting. The identified structure of labor supply and demand functions is consistent with the equilibrium labor market theory. It shows us that labor supply is a positive function of real wages and labor demand a negative function of real wages. Both schedules are found to be affected positively (but, very weakly) by real interest rate. The possibility of intertemporal substitution with respect to interest rate is observed empirically when the preference of leisure demand is time-nonseparable.[6)]

This paper is presented as follows. In section 2, a simplified model economy with one good and two inputs, is introduced, subject to random shocks and an exogenously given interest rate. Then, the features of the aggregate labor market composed of many identical agents are described in section 3, focusing on equilibrium as an optimal state of the economy. Section 4 explains the econometric methodology used for identification and estimation. Section 5 presents empirical results and interpretation. Section 6 provides concluding remarks.

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5) Empirically, it is often found that real interest rate has a remarkable relationship with labor markets, for which the sourc is not identified; see Alogoskoufis (1987), Seater (1985). In this paper, we presume that the interest rate transmits other shocks above mentioned to the labor market.

6) With time-separable specification and time lag in production, Hercowitz (1986) shows that hours can move negatively with interest rate, which is supported by data in his work.

2. Economic Environment and Equilibrium

(1) Preference and Its Optimum

Workers maximize the following utility function. While the optimization is smoothed by the presence of capital markets, and is subject to budget constraints and nontrivial adjustment costs in the form of time non-separable preferences for enjoying leisure.

$$U[C(t),\ N(t),\ N(t-1),\ x(t),\ e(t)] =$$
$$\alpha_1 C(t) - (\alpha_2 + e(t))\ N(t) - \alpha_3 N(t)\ N(t-1) - 0.5\alpha_4 N(t)^2 + x(t) \quad s.t \qquad (1)$$

$$A(t+1) = (1+R(t))[N(t)\ W(t) + A(t)\ C(t)],\ A(o)\ given. \qquad (2)$$

where C = consumption, N = hours worked, A = assets, W = real wage, R = real interest rate defined for one period, e = white noise random shock to marginal utility of leisure, and x = ex post shock to preference, which is not relevant for decision making, i.e., $x(t)$ is orthogonal to information set at time t. The random shocks are with well-defined density functions and finite support. α_1, α_2, α_3, α_4 are preference parameters.[7] Given this utility function, it is rational for a worker to optimize his (or her) lifetime objective function cross periods, subject to lifetime budget constraints. This is made possible by the capital market.

$$Max\ V = E(t)\Sigma_{i=0}^{\infty} q(t+i)\ U[C(t+i),\ N(t+i),\ N(t+i-1)]$$

where $E(t)$ = mathematical expectation on the information set at t, $q(t+i)$ = discounting factor defined as $1/[1+R(t)][1+R(t+1)]''\ [1+R(t+i-1)]$. This defi-

7) The utility function linear in consumption is a very strong assumption. As can be seen in next section, however, this assumption is useful in deriving the model for empirical work. And another reason that I used linear utility function is to compare the output with Kennan (1988)'s output which estimated labor supply and demand functions jointly. Anyway, with this specification of utility function, we are not allowed to use consumption data in the empirical work.

nition of $q(t+i)$ means that time preferences are equal to market interest rate. In solving this dynamic programming problem with respect to (w.r.t.) $N(t)$ *and* $C(t)$, and arranging (eliminating Lagrangian multiplier), the optimal condition for labor supply can be found.

$$\alpha_1 W(t) = \alpha_2 + e(t) + \alpha_3 N(t-1) + \alpha_4 N(t) + \alpha_3 q(t) E(t)[N(t+1)]. \tag{3}$$

The right side of (3) is the expected marginal utility ($=MU$) of leisure at time t. Based on this condition, one may discuss several properties of representative agent's preference structure.[8)]

Concerning the structure of labor supply, the slope of static labor supply with respect to wage are given by α_1/α_4, while the elasticity of labor supply with respect to interest rate (call ELSR) is $\Theta\alpha_3/\alpha_4(1+R)$, where $\Theta = N(t+1)/N(t)$.[9)] This ELSR can be positive or negative depending on whether current leisure is substitutable or complementary with respect to adjacent leisure. This distinction is characterized by the value of α_3: negative α_3 means the complementarity and positive α_3 means the substitutability.[10)] It is assumed to be positive in most studies of intertemporal substitution models.[11)] The elasticity of future work effort with respect to current work effort is defined as $-\alpha_4/\alpha_3\underline{\underline{\Theta}}$. With the linear marginal utility of consumption [$= MU(C)$], there is no income effect in labor supply because, as derived below, the marginal rate of substitution between consumption goods and hours worked depends on the latter and unpredicted shock, $e(t)$. Note, however, that observed $MU(C)$ is not

8) Note that unless preference is time non-separable, dynamic analysis is not possible with time preference equal to interest rate. With time separable preference, it is necessary to assume constant discounting factor, with changing interest rate, making misspecification.

9) Since $q(t) = 1/(1+R(t))$, the role of $R(t)$ is visible in affecting work efforts; movement of same direction as for $W(t)$ with positive α_2, and the inverse with negative α_2.

10) The role of interest rate in the labor market is mentioned longtime ago, even by Lucas and Rapping (1969). Mankiw et al.(1985) has estimated the elasticity of leisure demand with respect to interest rates.

11) Hal (1997) argues the possibility of atemporal substitution in leisure demand.

constant because of $e(t)$. It means that the resulting labor supply decision may look like it includes the income effect. With no predicted income effect, however, one can observe an intertemporal response for changing prices of wages and interest rate since preference is time non-separable.[12] Due to this property, every disturbance in relative price should be absorbed by changing leisure demand, and may generate fluctuations in working hours greater than with non-linear marginal utility of consumption.

In empirical work, the following version of (3) is employed.

$$\phi_2 + \phi_3 N(t-1) + \phi_3 q(t) N(t+1) + N(t) - \phi_1 W(t) + (1/\alpha_4)\, e(t) = 0 \tag{3)'}$$

which is calibrated by restrictions on parameters, simplifying the estimation procedure: $\phi_1 = \alpha_1/\alpha_4$, $\phi_2 = \alpha_2/\alpha_4$, $\phi_3 = \alpha_3/\alpha_4$. The restriction on preference parameter means that ϕ_1 is the slope of labor supply function.

(2) Optimum in Technology

The economy has access to following technology for producing consumption and capital goods.

$$F[N(t),\, K(t-1),\, z(t)] = \beta_1 N(t) - 0.5\beta_2 N(t)^2 + \beta_3 K(t-1) - 0.5\beta_4 K(t-1)^2 + \beta_5 N(t)\, K(t-1) + z(t) \tag{4}$$

where $K(t-1)$ is capital stocks available for production at time t, inherited from time t1.[13] $\beta_1, \beta_2, \beta_3, \beta_4$ and β_5 are technology parameters, satisfying concavity condition; $H = \beta_2 \beta_4 - \beta_5^2 > 0$. Technology shock $z(t)$ is unexpected and not reflected in Euler equations with a well-defined density function with finite support. The production function implies that the slope of the static labor demand curve w.r.t. real wage is $-1/\beta_2$.

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12) See Kennan (1988) for this points. If time-separable preference is linear in consumption, intertemporal substitution does not happen. See also Greenwood *et al.*(1988).

13) This simple form is useful for identifying parameters of estimated system of equations.

Marginal productivity(=MP) conditions for two inputs of capital and labor in terms of input prices are assumed to be satisfied.

$$\beta_1 - \beta_2 N(t) + \beta_5 K(t-1) = W(t) \tag{5}$$

$$\beta_3 - \beta_4 K(t-1) + \beta_5 E(t-1)[N(t)] = E(t-1)[R(t) + \delta] \tag{6}$$

where δ is a constant depreciation rate. Comparative statistics on these Euler equations shows the employment decision with respect to wage and interest rate: $dN/dW = -\beta_4/H$ and $dN/dR = -\beta_5/H$.[14] This result implies that the labor demand function is a negative function of real wages if technology is concave(strictly). However, its relationship with respect to interest rate depends on a cross derivative of the production function, for which no prior restriction is provided. If cross derivative is positive (negative), it becomes positive(negative). For the sake of simplicity, with no prior restriction on this derivative, discussions below proceed under the presumption of a positive derivative.

The above optimal conditions may be used to understand labor demand behavior explicitly as a function of real wages and interest rates. Substituting (6) into (5) and ignoring the expectation operator,

$$\pi_1 + \pi_2 N(t) - \pi_3 (R(t) + \delta) - W(t) = 0 \tag{7}$$

Here, $\pi_1 = (\beta_1 + \beta_3\beta_3/\beta_4)$, $\pi_2 = (\beta_5^2/\beta_4 - \beta_2)$, $\pi_3 = \beta_5/\beta_4$. The derivative conditions of labor demand are $dN/dW = 1/\pi_2$, $dN/dR = \pi_3/\pi_2$.[15] (7) is used for the empirical work.

3. Information and Equilibrium in the Aggregate Market

An important property of this economy is that real interest rate is not determined endogenously in a capital market, but given by outside mechanisms of the economy. That is, in each period, other authority (say, foreign markets

14) 'd' denotes a differential

15) They are the same as $-\beta_4/H$ and $-\beta_5/H$ respectively

or monetary authority) announces real interest rate for that period.

Then, this model economy should solve for an optimal level of $C(t)$, $K(t)$, $N(t)$ and prices of $W(t)$, with exogenously given shock $x(t-1)$, $z(t-1)$, and $R(t)$ in each period. The state of the economy is described by a vector $[K(t-1), A(t), R(t), x(t), z(t)]$ at the beginning of time t. Thus, the equilibrium quantity of labor $N(t)$ and its wage $W(t)$ are determined by conditions (3), (5) and (6), which means that real wages equalize labor demand and supply. The contemporaneous decision of $C(t)$ is not deterministic, but random depending on $e(t)$. However, optimal consumption $C(t)$ is intertemporally chosen by comparing $M[C(t)]/M[C(t+1)]$ and real interest rate. The possibility that consumption is not deterministic exists because of unpredictable structures for $e(t)$. In this case, consumption is chosen after capital stock, which is deterministic. The capital stock for next period $K(t)$ is determined by the condition that expected marginal productivity of capital is equal to expected interest rate $R(t+1)$. Now, the aggregate economy is closed by budget constraint: $C(t)+K(t)-(1-\delta)K(t-1)=Y(t)$.[16)]

In focusing on the movements of labor markets, serial correlations of employment and wage rate are generated by time non-separable preference and capital accumulation structures. Theoretically, as explained in the above sections, labor supply can respond positively or negatively, for exogenously changing interest rates, depending on the value of α_3, while labor demand responds in different directions, depending on the capital-labor relationship, i.e., on the sign of β_4, there is no restriction to these movements.[17)]

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16) It may be that this budget is not always satisfied because of utility linear in consumption, requiring inventory to be include explicitly. This point is beyond the scope of this paper. And the discussion about equilibrium for the case where firm makes two stage decisions is not discussed since the above arguments can be used with a little change.

17) Concerning the movement of employment and real wages, if leisure demand is showing substitutability between two adjacent leisure cross periods, changing interest rates generate more volatile employment relative to real wages. This observation is easily verified from the arguments in the above sections, and hold true if preference and technology satisfy regular conditions required to have the equilibrium.

4. Identification and Estimation Strategy

(1) Identification

The model specified from equilibrium conditions includes equations (3) and (7). Therefore, it has two endogenous variables of employment and real wages. Identification problems ought to be diagnosed prior to estimating the model, since a simultaneous equation system is estimated. The model is linear in terms of parameters, but nonlinear in terms of variables, for which identification is generally easier than linear case.[18] There are a few studies about the identification conditions of a nonlinear model. To my knowledge, Brown (1983) is only one dealing with the issue, which is less restrictive and more visible than Fisher (1966).

Brown (1983)'s methodology is based on the concept of observational equivalence and may be summarized as follows. u is defined to be a $L \times 1$ vector of disturbances. Define Q as a $M \times 1$ vector of functions of endogenous and exogenous variables, denoted by Y_n and Y_x, respectively. Thus endogenous variables Y_n are a $L \times 1$ vector. Q is known to econometrician ex ante. B is defined to be a matrix of unknown coefficient; B is a $L \times M$ matrix. The systems of equation may be written as $u = BQ(Y_{n,} Y_x)$. A true structure *(B, f)* is identifiable if its structure can be determined from joint distribution of $f(Y_n, Y_x)$ and a priori restrictions. A similar condition is applied for each individual equation. While the model has the usual assumptions, one assumption is crucial to this argument. This assumption is called a homogeneous restriction requiring a matrix T_i as $B_i' \ T_i = 0$ for I = 1, 2, ... L.[19] Another matrix of $Q(J: Y_x)$ is required, denoting the Jacobian matrix as Q w.r.t. Y_x.

Condition for identification; The *ith* equation is identifiable if and only if the rank of $[E(Q) : Q(J : Y_x) : T_i] = M-1$, where $E(Q)$ = $E\{Q(Y_x = 0)\}$ as an

18) It turns out that the model in this paper is blessed by nonlinearity of the model so that it is identified. See Fisher (1966) for the argument that nonlinearity may be useful for the identification. And it is assumed that the model is linear in disturbance.

19) Here' denotes transpose.

expectation.[20]

In the proposed model, M = 6 is easily verified. Thus, equation (3) is (over) identified since its rank is 6. Equation (7) is (exactly) identified since its rank is 5.[21]

(2) Estimation Strategy

In the equation below, the empirical model is arranged for exposition, with expected values replaced by realized values under the assumption of rational expectation,[22]where error terms ($=u_i(t)$) are assumed to satisfy the properties given above.[23]

$$u_1(t) = \phi_2 + \phi_3 N(t-1) + \phi_3 q(t) N(t+1) + N(t) - \phi_1 W(t) \quad (3)'$$

$$u_2(t) = \pi_1 + \pi_2 N(t) - \pi_3 (R(t) + \delta) - W(t) \quad (7)'$$

The nonlinear three stage least square (N3LS) of Amemiya (1977) and Gallant (1977) is used to estimate the model.[24] This instrumental variable procedure is appropriate to this model, where some equations are identified and some are overidentified.[25] The more general version of the nonlinear instrumental variable procedure is General Method of Moment (GMM) estimation, suggested by Hansen (1982) and Hansen and Singleton (1982).[26] The

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20) It is global condition if the system has the solution in the neighborhood of true **B**, which is assumed.

21) A detailed procedure of checking the rank condition is available upon request

22) This may cause problems like bias because of measurement errors, predicting errors or random shocks. However, it is not unreasonable to assume that the Euler equations hold in expectation conditioning on some information sets.

23) For more delicate method in the case like these model where there are identified and overidentified equations in combination, see Zeller and Theil (1962).

24) The maximum likelihood method may be asymptotically more efficient than N3LS if several assumptions are satisfied, particularly distributional assumption about error terms.

25) Among many studies estimating Euler equations by this method, some classical ones may be Mankiw et al.(1985), Hansen and Singlton (1982), Pindyck and Rotemberg (1983), Shapiro (1986), Singleton (1988). It is clear that this method of estimating orthogonality conditions ignores the transversality condition (See Hansen and Sargent (1982)).

essential difference between the GMM and the N3LS is in the assumption of the error terms. The GMM allows the error terms to be both serially correlated and conditionally heteroskedastic while N3LS assumes identically and independently distributed disturbances of the system for each observation.[27) Empirically, however, it is shown that the two methods generally do not make a significant difference in practice.[28)] Identically and independently distributed (i.i.d) error terms for each observation is an assumption necessary for identification. Therefore, i.i.d errors are assumed and N3LS is selected, simplifying the estimation procedure.

The optimal choice of instruments depends importantly on the structure of the error term. Here, a general rule is followed where the variables should be highly correlated with independent variables and be independent of error terms.[29)] It is also true that estimated values of parameters are severely affected by the choice of instrumental variables. In spite of this difficulty, one question in the search for correct instruments is the possibility of using the information at time *t* as instruments. The assumption necessary for identification makes it possible to use current variables as instruments, including exogenous variables of the model.

The overidentifying restriction caused by multiplying first order conditions with instrumental variables is tested by noticing that the number of observation times value of objective function is distributed as *X(Chi-square)* with a degrees of freedom.[30)]

26) Matyas (1999) summarizes some literatures on GMM.

27) The flexible assumption about the error terms is made possible by choosing different weighting matrix which is optimal in the sense that it makes asymptotic covariance matrix at least as small as any other element in the class. Furthermore, the flexible assumption about error terms is not free lunch, but should be paid by strong assumption of ergodic and stationary exogenous variables, which is often rejected by data.

28) See the above cited studies.

29) Gallant (1977) argues that the usage of exogenous variables as instruments might cause identification problem in some case of nonlinear system, unlike linear system. However, there is no evidence that his arguments apply to this model.

30) The degree of freedom is equal to the number of instruments times the number of equations minus the number of parameters, as suggested by Hansen (1982).

5. An Empirical Test of Model

(1) Data Decision and stylized facts

The data set is that of the Korean labor market. The total working hours are the product of number of employees and hours worked. The monthly earning of regular workers and the interest rate of loans are used to calculate real wage and interest rate. The consumer price index is employed for adjustment for inflation. All data are quarterly over the sample period 1984 : 1 to 2003 : 1. Since the result may depend on the trend estimation procedure, both deterministic and stochastic trends are assumed. The model is also estimated using seasonally adjusted and unadjusted data. Finally, instrumental variables used are the lagged total hours worked, the price index, output, lagged real wages and real interest rate.[31)]

Since this paper is estimating a labor market model based on the fluctuations of hours, employment and real wage, it may be useful to summarize their relative movements which has been called stylized facts in other studies. During the sample period, the standard deviation (s.d) of employment, hourd worked and real wages was 0.08, 0.03 and 0.07 respectively. So, employment was more volatile than other variables, especially hours. Unlike other countries, however, real wages are quite volatile in Korean labor market.[32)]

(2) Empirical Results

<Table 1> and <Table 2> report the estimation results of structural parameters, which are used to infer labor demand and supply behavior, reported in <table 3>. The primary interpretation is given to the result in <table 1>, while <table 2> reports additional results under the alternative trend estimation

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This test comes from the fact that number of orthogonality conditions used in estimation is greater than that of parameters.

31) This model shows a positive response for using them as instruments. Thus, like many other models, the hypothesis that current instruments are valid can't be rejected

32) For example, the well-known stylized fact in U.S.A has been "volatile employment relative to real wage" for which several models have been developed to explain.

procedure. The results in columns 1 and 2 of table 1 differ depending on the seasonal adjustment. Column 1 uses seasonally adjusted data and column 2 uses seasonally unadjusted data. For reference, the standard error for each parameter is given and is only valid asymptotically. Attention is not given to other summary statistics, the interpretation of which is not as clear as in linear models.

In interpreting the results of <table 1>, the overall impression is that the sign of parameters are correct, but their significance is weak. The parameters relevant to the labor market support the theory. The positive value of ϕ_1 implies that the labor supply function is of positive slope with respect to real wage (with the values of 0.18 and 0.01),[33] even though the value is small. The small value is not surprising comparing to other studies. A positive value of ϕ_3 means the possibility of intertemporal substitution for changing interest rates. This (positive) movement of labor supply for the interest rate is related to the substitutability between two adjacent leisure consumptions: that is, $dN(t+1)/dN(t) = -1/\phi_3 < 0$, meaning that current hours worked may have a negative effect on hours worked in the future.[34] That is, if a worker worked longer hours last period, he (she) may want to work less this period.[35]

Looking at technology parameters, the slope of the labor demand with respect to real wage is negative (with values of -13.51 and -0.18), which shifts positively (but very weakly) with respect to the interest rate (the positive value of π_3/π_2).[36] This means indicates substitutability between capital and labor input in the aggregate economy. The production function can easily be verified as concave.

In combining both results, it can be concluded that the labor market is identi-

33) Comparing this result with other study, estimating supply function only, Choi (1995) reports that the values are insignificant or negative when total working hors are used.

34) Kennan (1988) and Eichenbaum et all. (1988) finds that leisure today and in subsequent period may be complement (See discussion in section 2).

35) However, this result is not consistent with other results. Remember that $\phi_3/\phi_2 = \alpha_3$. The estimate values of ϕ_2 and ϕ_3 mean a negative value of α_3, which implies a complementary relationship. This inconsistency may indicate that latent consumption data are not consistent with this model.

36) Kennan (1988) report that labor demand slope is in the range from -14.46 to -11.58.

fied by a positively slopped labor supply and negatively slopped labor demand curve. These curves appear to be positively affected by fluctuating interest rates. Plotting data of employment and real wages suggests that, other than interest rate, other variables affecting the decision of workers and labor demand exist.[37] Finally, two different pieces of evidence exist regarding model specification. The overidentifying restriction test result encourages model specification, and the (approximate) standard errors for parameters estimated are large. The latter problem may be improved by choice of more appropriate instrumental variables and a longer sample period.

As mentioned above, it is examined how the detrending procedure affects empirical results. The estimation results for other detrending procedures are in table 2. It can be seen that the empirical results in table 2 are similar to those in table 1, except that the adjacent leisure is complementary. The over-identifying restriction hypothesis is rejected when data is linearly decomposed into trends.

6. Conclusion and Remarks

This paper suggests that the structure of labor market is identified by estimating the preference and technology parameters. The labor supply and demand can be identified jointly by looking at "seemingly unrelated" data. The empirical results were found to be consistent with theory, while the size and significance of parameters estimated are not satisfactory. It is found that labor supply is a positive function of real wage and labor demand is a negative function of real wage. However, one weakness of the model used here may be that the concavity of the utility function is not supported by data. The absence of concavity in preference is based on the fact that estimated parameters imply inferiority of consumption. The overidentifying restriction test results, however, support model specification. While there are many directions for developing this kind of study, it may be desirable to apply different econometric methodologies to test theory, and to test the model using other data sets.

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37) One candidate may be nominal price of goods, the reason for which is obvious from Euler equations of preference and technology.

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Appendix

A : Empirical Results

Table 1. Parameters Estimated

	I	II
ϕ_1	0.176 (1.116)	0.008 (0.109)
ϕ_2	-0.007 (0.016)	-0.004 (0.001)
ϕ_3	0.174 (0.114)	0.019 (0.118)
π_1	-0.060 (0.040)	-0.037 (0.203)
π_2	-0.074 (0.421)	-5.659 (52.67)
π_3	-0.004 (0.002)	-0.004 (0.109)
Value of Objective Function	0.00	0.00
Critical values	5.99 (5%)	9.21 (1%)

( ) include approximate standard errors. I and II denote the total hours worked seasonally adjusted and unadjusted respectively. All series used are the first differences of logged values.

Table 2. Parameters Estimated*

	III	IV
ϕ_1	0.168 (0.515)	0.481(0.883)
ϕ_2	0.004 (0.018)	0.006(0.017)
ϕ_3	-12.87 (19.65)	-10.99(14.62)
π_1	-0.217 (0.215)	-0.373(0.363)
π_2	-0.080 (0.762)	-0.229(1.018)
π_3	-0.015 (0.014)	-0.024(0.023)
Value of Objective Function	13.6	2.28
Critical values	5.99 (5%)	9.21 (1%)

* Total hours worked seasonally adjusted are used. ( ) includes approximate standard errors. III = logged and linear trend. IV = not logged and linear trend.

Table 3 Structure of Labor Supply and Demand

	I	II	III	IV
L_w^s	0.19	0.01	0.17	0.48
L_w^d	-13.51	-0.18	-5.56	-4.37

Note: Supscripts, s and d denote labor supply and demand respectively while subscript w does real wage. Number of columns are from <table 1> and <table 2>.

B : Assumptions and Rank Conditions for Identification

First, it is assumed to solve the system into endogenous variables.

(A-1) $y_n = g(u, y_x ; B)$

This does not require the possibility of solving system into reduced forms explicitly. Secondly, we need to assume the existence condition of derivatives satisfying

(A-2) $det(B(dQ/dy))$ is not zero.

(A-3) There exists a matrix T_i for each B_i' so that $B_i' T_i = 0$, where I = 1, 2,,, L.

(A-4) The distribution of u is independent of y_x with mean zero.

Now, consider the rank condition, which is in text without proof. The matrix B has following columns.

$$B_1 = [0 \ -1 \ \beta_5 \ 0 \ \pi_2 \ \pi_1]'$$

$$B_2 = [\phi_2 \ -\phi_1 \ 0 \ \phi_2 \ 1 \ \phi_3]'$$

And $Q = [N_{t+1} q_t \ W_t \ R_t \ N_{t-1} \ N_t \ 1]$. Then one can have

$$T_1 = [I_1 \ I_4], \qquad T_2 = [I_3]$$

Here I_i is M(= 16)dimensional unit vector with zero elements except ith term. And $y_x' = [N_{t-1}, \ R_t, \ N_t,]$. With these matrices, one can check the rank of each A_i Then, $A_i = [E(Q) : Q^* : T_i]$. $E(Q)$ makes a column. Q^* has columns equal to the number of exogenous variables. T_i has columns equal to the number of functions excluded in the ith equation (out of all functions in Q). Thus, A_t has rank of 1 + 3 + 2 = 6. A_2 has rank of 1 + 3 + 1 = 5.

〔Abstract〕

노동시장의 균형모형에 대한 연구 : 식별과 추정

최 창 곤

본 연구는 노동수요와 노동공급구조를 결합하여 동시에 분석하고자 하는 연구로서, 특히, 노동수요와 공급구조에 대한 계량경제학적인 식별이 가능함을 보인다. 분석을 위해, 시간-비분리적인 선호구조와 시간-분리적인 생산구조로 구성된 모형경제를 설정한다. 설정된 모형에서 유도되는 최적화조건의 추정을 통하여 선호와 생산구조의 모수들을 추정하고, 추정된 모수들을 이용하여 노동수요와 공급함수를 추정한다. 실증분석에서는 비선형연립방정식모형의 계량경제학적 식별을 확인하고 비선형도구변수추정방법을 사용하였다. 추정결과에 따르면 거시노동시장의 변동은 노동시장의 균형이론을 기각하지 않는 것으로 보인다.

핵심용어 : 거시노동시장에서 노동수요와 노동공급의 식별 및 추정, 이자율과 노동시장, 시간-비분리적인 선호, 노동공급의 기간별대체, 고용조정비용, 비선형연립방정식의 식별과 도구변수추정