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Evolution of preferences in one-shot prisoner's dilemma

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Abstract Can cooperation evolve in one-shot prisoner's dilemma? Utilizing an indirect evolutionary framework that separates objective payoffs and utilities, I show that preferences that are consistent with conditional cooperation in one-shot prisoner's dilemma can evolve along with strictly self-interested preferences. The ability to detect others' preference types is critical for the evolution of reciprocal preferences. In the evolutionary equilibria, both the strictly self-interested preferences and reciprocal preferences coexist and both cooperation and defection are observed. These results complement the folk theorem and the standard evolutionary models by providing an alternative evolutionary logic of cooperation.

Keywords Prisoner's Dilemma, Evolution, Cooperation, Reciprocity

JEL Classification C72, C79

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1 Introduction

Why do individuals, groups, and nations cooperate with others when there are incentives to defect, free-ride, and exploit? The folk theorem (Fudenberg and Maskin 1986, 1991; Rubinstein 1979) and the standard evolutionary models (Axelrod 1981, 1984; Axelrod and Hamilton 1981) address this question from different directions, but provide similar explanations; when interactions are repeated, cooperation is consistent with self-interest. Critical to both explanations is the assumption that the prisoner's dilemma is indefinitely repeated with either a high continuation probability or low discount rate or both. In this paper, I provide an evolutionary logic of cooperation without the assumption of indefinite repetition.

The one-shot prisoner's dilemma game is used to represent the class of dilemma situations in which rational, self-interested decision makers cannot achieve mutual cooperation. These situations include pure one-shot interactions, finitely repeated interactions with complete information, and indefinitely repeated interactions with a low continuation probability or a high discount rate.[1] Interactions between, for example, buyers and sellers of items on internet auction sites, customers and waiters in restaurants in tourist areas, and strayed travelers in a bad weather and local residents, are best characterized as one-shot, or if ever repeated, with very low continuation probabilities. Our ancestors lived in small groups in which interaction is repeated. But they often found themselves in hard times in which surviving the day easily overshadowed any considerations for the possible future gains. In those situations, what the ancestors faced was a repeated game, but with very high discount rates. Whether individuals and groups facing these one-shot like situations achieve mutual cooperation matters a great deal for themselves as well as for their communities.

In a one-shot prisoner's dilemma played by rational actors, cooperation is possible, if not guaranteed, when some players have preferences that are not entirely dictated by self-interest. The question is two fold. First, how do the conditionally cooperative preferences survive if there is any evolutionary mechanism by which less successful traits are selected against? Second, why is it the case that in most populations, neither conditional cooperators nor the pure egoists are dominant, but they coexist in an evolutionary equilibrium? The following summarizes the basic set up of the model and the logic of the evolution of cooperation via the evolution of preference in one-shot prisoner's dilemmas.

A population consists of three preference types: egoists, reciprocators, and altruists. Each type is defined by its unique ordering of the four outcomes of a prisoner's dilemma

[1] Kreps et al.'s (1982) seminal model is not one of the cases mentioned here, in that their model posits types that are not strictly motivated by self-interest. Their model, thus, presents the same question as the one raised in this paper: how can the types other than those motivated by self-interest evolve? Note in their model the strictly self-interested ones obtain at least as large and often larger payoff than either the "irrational" type (in Model 1) or the type that "enjoys" mutual cooperation (in Model 2).

in objective payoffs. (A precise definition is provided in a later section.) The players are randomly paired and play a one-shot sequential prisoner's dilemma game in each evolutionary stage. The interactions occur with limited information; sometimes the game is played with players knowing the preferences of paired others and other times without such knowledge. The preferences that are more successful in terms of obtaining objective payoffs proliferate while the proportions of less successful preferences shrink over time. These assumptions lead us to stable evolutionary equilibria, called attractors of the dynamic evolutionary system, in which egoists and reciprocators coexist without altruists, a result consistent with our daily experience and the findings from controlled laboratory experiments.[2]

2 What if the prisoner's dilemma is not indefinitely repeated?

Axelrod's (1981, 1984) studies suggest that strategies that are nice initially and retaliatory when encountered with nastiness are most likely to evolve in the indefinitely repeated prisoner's dilemma. The most famous of such strategies is Tit for Tat. Later studies by Boyd and Lorberbaum (1987), Foster and Young (1991), Lorberbaum (1994), and Bendor and Swistak (1997) show that Tit for Tat, or any other pure or mixed strategy, is not stable in the long run. That is, there are always possible combinations of strategies that can invade a population composed of a single strategy. However, the researchers also note that, in terms of relative stability, Tit for Tat style, conditionally cooperative strategies have the best chance of evolving.(Bendor and Swistak, 1995; 1997)

While infinite repetition characterizes an important subset of the real world collective action problems, there are also many collective action situations that are either one-shot or repeated for a finite number of times. Interactions other than those repeat indefinitely are more common among humans than among animals, and in advanced market economies than in small-scale subsistence communities. One-shot games are an approximation not only of single-play interactions in the real world, but also of the finitely repeated settings and the indefinitely repeated interactions with low continuation probability or high discount rates. In all of these settings, cooperation cannot be achieved or sustained among egoists. But contrary to the logical prediction based on the assump-

[2] If both players are strictly self-interested and it is common knowledge, the predicted outcome of the sequential prisoner's dilemma is mutual defection. A simple backward induction confirms this. The second mover's best response is defection whether the first mover defects or cooperates. The first mover also defects because the payoff from (D,D) outcome is larger than that from (C,D) outcome for the first mover. Thus the dilemma is preserved in the sequential version. The results of this paper does not depend on the assumption that the game is played sequentially. I use a sequentially played prisoner's dilemma to highlight the problem of trust and trustworthiness, but a set of similar results can be obtained if one assumed a one-shot simultaneous game. The basic logic of the extension to simultaneous game is laid out in Appendix A2. A full analysis of the evolution with simultaneous play can be found in a working paper of the author (Ahn, 2002).

tion of rational and self-interested actors, many, if not all, individuals do cooperate in such one-shot and one-shot like dilemma situations. Experiments on one-shot prisoner's dilemma games consistently report that between a third to two thirds of subjects participating in such experiments do cooperate in various treatment conditions. For example, one-shot prisoner's dilemma experiments conducted using the same protocol in different countries show that the rates of cooperation among the second movers who know that their counterparts have already cooperated range from 75% in Japan, 73% in Korea, and 61% in the U.S. But when the second movers know that first mover has defected, the rate goes down to 0% in Korea and the U.S. and 12% in Japan. The cooperation rates in simultaneous prisoner's dilemma and among the first movers of sequential games range between 36% and 83% in these experiments. (See Ahn, Ostrom, and Walker 2004, Cho and Choi 2000, Hayashi et al. 1999 for details of the experimental results and analyses.)

The experimental results as well as our daily experience tell us that many individuals cooperate and, thus, contribute to sustaining a beneficial social order. But also abundant are the incidents in which good intentions and cooperative initiations are betrayed. What characterizes social interactions in the modern societies, therefore, is not universal cooperation, but a mixture of cooperation and defection. The relative rates of cooperation and defection characterize an important aspect of culture in various societies. Therefore, what needs to be explained in evolutionary models of the prisoner's dilemma is the heterogeneity of behavior and the heterogeneity of intrinsic motivations.

How can those preferences evolve that cooperate often times in spite of material self-interest? Recently, many researchers utilize the indirect evolutionary approach, foretold by Frank (1987) and fully developed by Werner Güth and his colleagues (Güth and Kliemt, 1998; Güth, Kliemt, and Peleg, 2000; Güth and Yaari, 1992) to explain the existence and survivability of the preferences that are not strictly egoistic. These studies include Bohnet, Frey and Huck (2000), Friedman and Singh (2002), Samuelson (2001), and Sandholm (2001). The indirect evolutionary approach separates objective payoffs and utilities. Objective payoffs drive evolution and correspond to the concept of fitness in the standard evolutionary game models. Utilities represent preferences and, thus, guide choice of strategies in any given evolutionary stage.

In the evolutionary ecology studied in this paper, there are three different types of players – altruists, reciprocators, and egoists. A player's type is defined by his preference ordering over the four outcomes of a one-shot prisoner's dilemma. At any given evolutionary stage, each player plays a sequential version of the one-shot prisoner's dilemma with a randomly matched another player. Since types are defined not by strategies but by the underlying preferences, players do not carry fixed strategies. Instead, they play the stage game rationally, i.e., to maximize expected utility.

In this evolutionary environment, the level of information plays a key role in determining which type(s) of preferences and behavior will evolve in the population. Information about others' preference types in a single-shot interaction can be obtained from various biological and social signals and symbols (Ahn, Janssen, and Ostrom 2004). The reciprocators, who have preferences for conditional cooperation, are most dependent on the level and accuracy of the information about others' types. When a reciprocator knows the type of a paired other player, she can protect herself from an egoistic counterpart while harvesting the gains from mutual cooperation when encountered with another reciprocator or altruist. If the individual specific information is not available, a reciprocator plays the prisoner's dilemma based on his prior about the overall distribution of different preferences types. This lack of information often results in outcomes in which the reciprocator is getting exploited by an egoist or fails to cooperate with another reciprocator for fear of getting exploited. The altruists, whose preference implies unconditional cooperation, are not able to protect themselves when paired with egoists even when they know that they are interacting with egoists. In most general specification of an evolutionary dynamics that take into account mutations, the altruists will die out eventually, analogous to the ALL C strategy in the indefinitely repeated prisoner's dilemma setting. But the altruists' existence has significant ramifications; it affects the relative success of egoists versus reciprocators. The larger the proportion of altruists in a population, the better it is for the egoists, regardless of the level of information. Thus, egoists can invade a population with large proportions of altruists initially. What happens eventually depends on how often the players interact knowing others' preferences.

There are several ways in which one is able to guess with reasonable accuracy whether a random encounter is trustworthy. Biological signals such as facial expressions, body movements, and voice tones are spontaneous signals that most people are not capable of consciously controlling (Frank 1988; Schmidt and Cohn 2001). Certifications (Rao 1994), eBay reputation scores (Resnick and Zeckhauser 2001; Resnick et al. 2003), and sometimes mere appearances (Bacharach and Gambetta 2001) are examples of effective sociocultural symbols that are hard to fake and, thus, help people to judge unfamiliar others' trustworthiness. Experimental evidence abounds that when people can see and/or talk to each other the probability of accurately predicting others' future behavior significantly increases (Frank, Gilovich, and Regan 1993; Kikuchi, Watanabe, and Yamagishi 1997; Scharlemann et al. 2001). But these signals and symbols are neither always available nor always reliable. Thus, people often have to rely on their general belief about the trustworthiness of others when deciding whether or not to trust others.

Several other studies also explain cooperation in single-shot prisoner's dilemma games. Orbell and Dawes (1993) find that when subjects in one-shot prisoner's dilemmas have an option of not playing the game, the intended cooperators are more likely to choose to play the game and obtain the gains from mutual cooperation (see also Morikawa, Orbell, and Runde 1995; Orbell and Dawes 1991; Orbell, Schwartz-Shea, and Simmons 1984). Orbell et al. (2004), using computer simulations, show that cooperation can evolve along with

"Machiavellian intelligence," the capacity for manipulation and mindreading. While these studies rely on the exit option and use laboratory experiments or computer simulations, I use the standard one-shot game, treat the question as one of preference evolution, and provide analytical results.

The current study is most closely related to the study by Güth and Yaari (1992) who show that the trustworthy preference types do better than strictly self-interested ones in a trust game, and the study by Güth, Kliemt, and Peleg (2000) who treat investment in information as an individual-level trait. This paper complements Güth and his colleagues' studies by using the standard prisoner's dilemma game, positing three preference types, and studying a mixed information condition.

The remaining sections of this paper are organized as follows. In the next section, the stage game and the three preference types are defined. In the fourth section the evolutionary environment is introduced by defining the population state space, the replicator functions, and the stable and unstable steady states. In the fifth section, I analyze the equilibria of the stage games under complete and incomplete information conditions and calculate the average objective payoffs to the three preference types. In the sixth section, the evolutionary trajectories under complete and incomplete information conditions are analyzed. In the seventh section I study the evolution of preferences under mixed information condition and formalize the strongly stable population states, called attractors, with heterogenous preferences and behaviors. In the final section, I conclude the study and provide some of the implications for future studies of preference evolution.

3 Egoists, reciprocators, and altruists

The standard representation of prisoner's dilemma posits T (temptation), R (reward), P (punishment), and S (sucker's payoff) as the four payoffs and assumes $T > R > P > S$. In search of simpler parameters that characterize the cardinal incentive structure of a prisoner's dilemma, Rapoport and Chammah (1965) propose normalized greed ($G_n = \frac{T-R}{T-S}$) and normalized fear ($F_n = \frac{P-S}{T-S}$). Ahn et al. (2001) find that these normalized measures are strongly related to the rates of cooperation in one-shot prisoner's dilemmas. The payoff matrix we will be utilizing throughout this paper is a normalized version of the prisoner's dilemma with symmetric fear and greed, shown in the upper panel of Figure 1. The current representation of the prisoner's dilemma sets both F_n and G_n to be x.[3]

[3] The game also has an interpretation as a simple public-good provision game involving two individuals and two strategies of contributing and free-riding. In the linear public goods provision interpretation of the game, each of the two individuals, i and j, possesses an initial endowment of x ($0 < x < 0.5$, which ensures that the game is a prisoner's dilemma). Each must choose whether to contribute for the provision of a public good, or to free-ride. Contribution costs 1 to the contributor, but returns $1 - x$ to each of the two individuals. In other words, $1 - x$ is the marginal per capital return from a unit contribution. If both individuals choose to free-ride, the public good is not provided at all, and both are worse off than they can be if they both contributed.

Objective payoffs

		Individual j Contribute	Individual j Free-Ride
Individual i	Contribute	$1-x, 1-x$	0, 1
	Free-Ride	$1, 0$	x, x

*$0 < x < 0.5$

Utility Payoffs for Individual i

		Individual j Contribute	Individual j Free-Ride
Individual i	Contribute	$1-x$	$0+\beta_i$
	Free-Ride	$1-\alpha_i$	x

*$0 < x < 0.5$, $\quad 0 \leq \beta_i \leq \alpha_i < 1-x$

Figure 1: Objective and utility payoffs

To incorporate intrinsic motivations other than pure self-interest, this paper uses a utility payoff matrix shown in the lower panel of Figure 1. The utility matrix adds α_i and β_i to the objective payoffs. The parameter α_i can be interpreted as the magnitude of individual i's *guilt* when he defects while j cooperates. The parameter β_i can be interpreted as the strength of individual i's *altruism* in the sense that with a large enough β_i, individual i cooperates even when individual j defects. The restriction $\beta_i \leq \alpha_i$ reflects the experimental regularity that an individual is more likely to cooperate when the other also cooperate than when the other defects (Ahn, Ostrom, and Walker, 2003). The restriction $\alpha_i < 1-x$ implies that the outcome (D, C) is always preferred to the outcome (D, D), in which C denotes contribution and D denotes free-riding. Given the set of restrictions on α_i and β_i, we have three substantive preference-ordering types that are jointly determined by α_i, β_i, and x. For a given vector of type parameters (α_i, β_i) of individual i, his preference ordering depends also on the objectively given magnitude x of the gain and loss that results from different combinations of choices.

Individual i is called an *egoist* if $\alpha_i < x$ (which also implies $\beta_i < x$) since his sense of guilt is not strong enough to overcome the temptation to defect when the other person co-operates. An egoist has preference ordering of $u(D,C) > u(C,C) > u(D,D) > u(C,D)$. Therefore, an egoist prefers the outcomes in which he free-rides no matter what the other does. An individual is called a *reciprocator* if $\beta_i < x < \alpha_i$. A reciprocator has a preference ordering of $u(C,C) > u(D,C) > u(D,D) > u(C,D)$ and, thus, prefers to cooperate if and only if the other player cooperates. Finally, individual i with $x < \beta_i$ is called an *altruist* because the person has an altruistic propensity strong enough to make him cooperate even when the other person defects. An altruist has a preference ordering of

$u(C,C) > u(D,C) > u(C,D) > u(D,D)$, which implies that cooperation is the dominant strategy.

The preference type of an individual is quite different from the way a type is defined in the standard evolutionary game theory. One's preference-ordering does not necessarily dictate her strategy. The actual strategy choice of a given preference-ordering type is based on the principle of utility-maximization. Therefore, a reciprocator playing as a second mover in the sequential version of the prisoner's dilemma cooperate when the first mover cooperates, but defects if the first-mover defects. An egoist, playing the sequential prisoner's dilemma as a first mover may cooperate if he assesses the probability of the second mover being a reciprocator at a high enough level.[4] More precise analyses of the equilibrium behavior are presented in the fourth section.

4 The evolutionary setting

For a given value of the temptation parameter x ($0 < x < 0.5$), a population can be represented by the relative proportions of altruists, reciprocators, and egoists. Let γ and δ be the proportions of altruists and reciprocators in a population whose size is normalized to 1; the proportion of egoists is $1 - \gamma - \delta$. A vector (γ, δ), then, characterizes the relative proportion of the three types. Figure 2 shows the triangular population state space: $\Delta = \{(\gamma, \delta) : (0,0), (1,0), (0,1)\}$. The horizontal axis of the triangle represents the proportion of altruists; the vertical axis represents the proportion of reciprocators. Each point in the triangle represents a composition of the three preference types. For example, the origin $(0,0)$ is a population state consisting entirely of egoists; the upper limit of the vertical axis $(0,1)$ is a population consisting entirely of reciprocators. The right-limit of the horizontal axis $(1,0)$ is a population state consisting only of altruists. Other points of the state space represent populations with two or more preference types.

In each evolutionary stage, every player is randomly matched with another player and plays a sequential version of the prisoner's dilemma game shown in Figure 1. A player has

[4] One might think that, because preferences ultimately translate into strategies in a given game context, modeling different types in terms of preferences is just another way of modeling them in terms of strategies. But this is not exactly true. A type defined in terms of preferences has much more flexibility, unless the type's preference implies a dominant strategy under all circumstances such as the altruistic preference in this paper. First, they can respond to the overall distribution of types in the population when information is incomplete. This implies that the stage game can be analyzed using the Bayesian Nash equilibrium concept and the equilibria will differ as the relative proportions of different types changes as a result of evolution. Even under complete information a type's strategy depends upon the type of the other player. A later section analyzing the stage game equilibria shows the details of this flexibility of strategies for egoists and reciprocators. Though some level of complexity is introduced by defining types in terms of preferences, it also is more realistic when applied to human interactions. After all, most humans in social dilemmas wish to obtain information about the types of specific individuals or the population in general and make conditional choices.

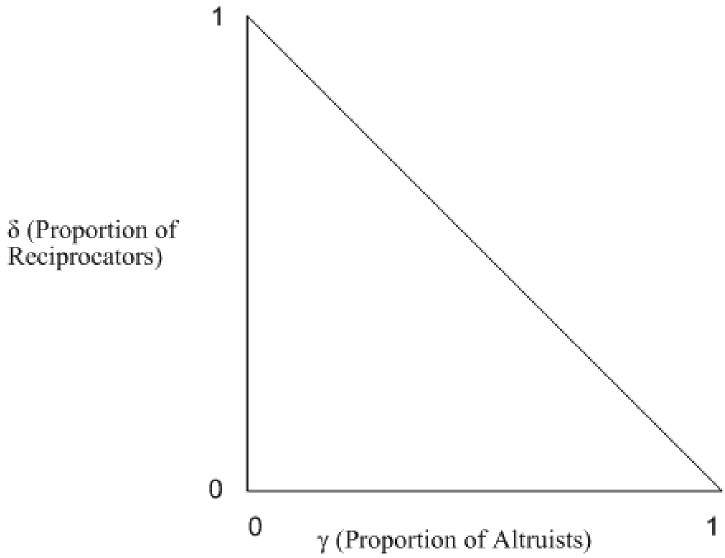

Figure 2: State space of a population defined by proportions of altruists, reciprocators, and egoists

an equal probability of playing the game as a first mover and as a second mover. With probability q the game is played with complete information, meaning that each player knows the preference type of the player with whom she is matched. With probability $1-q$, the one-shot game is played with incomplete information. When information is incomplete, a player's preference type is his private information, but players know the overall distribution of different preference types in the population.

Let $\pi_{e,t}$, $\pi_{a,t}$, and $\pi_{r,t}$ denote the average objective payoffs (fitness) to egoists, altruists, and reciprocators, respectively, at time t. The proportions of altruists and reciprocators at time $t+1$ are defined by the following standard, time-independent, replication functions: $\gamma_{t+1} = \gamma_t \times \pi_{a,t}/\bar{\pi}_t$ and $\delta_{t+1} = \delta_t \times \pi_{r,t}/\bar{\pi}_t$, where $\bar{\pi}_t$ denotes the average payoff of players in the entire population at time t. The replicator functions imply that the proportion of a type at time $t+1$ depends on its relative proportion at time t and the average objective payoff of the type at time t relative to the average payoff to all players in the population. The replication rule can be characterized as "linear proportional," and is used in Bendor and Swistak (1997, see equation (5) on page 295) who also point out that this is the replication rule used in Axelrod's tournaments.

The steady states of this evolutionary system can be studied by analyzing a vector field $\dot{\theta} = (\dot{\gamma}, \dot{\delta})'$, where $\dot{\gamma} = \gamma_{t+1} - \gamma_t$, and $\dot{\delta} = \delta_{t+1} - \delta_t$. A population state is *steady* if $\dot{\gamma} = \dot{\delta} = 0$. In other words, when a population is in a steady state, the relative proportions of the three types do not change over time: $\gamma_{t+1} = \gamma_t$ and $\delta_{t+1} = \delta_t$. However, steady states differ from one another in terms of the consequences that a small disturbance causes to the steady states. It is assumed that small mutation – change of type composition which is not caused by the deterministic replication rules – occurs whenever a population is in a steady state. A steady state of a population is *stable* if after a small disturbance caused by mutation, the population returns to the steady state before the mutation or its close vicinity. Otherwise, the steady state is *unstable*. A stable steady state is called an *attractor* if after repeated stochastic mutations at steady states, a large set of initial conditions reach the stable steady state.

The problem posed in this paper is finding stable distribution of preference types as functions of x, a cardinal measure of the incentive to defect, and q, the degree of information. We will see that for a wide range of reasonable (x, q) combinations, there exist stable populations composed of egoists and reciprocators, but not altruists. In addition, the observed behaviors in such stable populations are also heterogenous, i.e., both cooperation and defection occur with substantial frequencies.

5 Stage game equilibria of one-shot, sequential prisoner's dilemma games and payoffs to different preference types

The first step towards finding the evolutionary consequences of the model is to analyze the equilibrium behavior of the three preference types in the stage game and each type's relative success in obtaining objective payoffs. Both, of course, are functions of the relative proportions of the three types at a given evolutionary stage, the temptation parameter x, and the information level q. In this section, we analyze the equilibria of the game under complete information and incomplete information and calculate the payoffs to the three preference types. Under the complete information condition, a player knows the exact type of the partner and knows that the other knows his own type, etc. That is, the types of players are common knowledge. Under the incomplete information condition, a player does not know the exact type of the other player. However, players are assumed to know the distribution of types within a population. In the following analyses, we utilize two simplifying assumptions that $\alpha_i = 0$ for all egoists, and $\alpha_i = \alpha$ $(x \leq \alpha \leq 1 - x)$ and $\beta_i = 0$ for all reciprocators.

5.1 Complete information condition

Because the second mover observes the first mover's choice before making a decision, the solution concept used to analyze the stage game played with complete information is a subgame perfect equilibrium. Table 1 shows the subgame perfect equilibrium outcomes for all of the nine possible combinations of types. The first panel of Table 3 reports the average objective payoffs to the three preference types under complete information condition as functions of the distribution of three preference types. For example, the egoists' average payoff under complete information condition can be calculated as follows.

		Second mover		
		Egoists	Reciprocators	Altruists
	Egoists	(D,D)	(C,C)	(D,C)
First mover	Reciprocators	(D,D)	(C,C)	(C,C)
	Altruists	(C,D)	(C,C)	(C,C)

Table 1: Equilibrium outcomes of complete information, sequential games

With probability γ, an egoist is matched with an altruist. Then, regardless whether he plays the game as a first mover or as a second mover, the egoist defects while the altruist cooperates. Therefore, the objective payoff to the egoist in this case is 1. With a probability δ, the egoist will be matched with a reciprocator. There is a half probability that the egoist will play the game as a first mover. Then the egoist cooperates and the reciprocator also cooperates. The objective payoff in this case is $1 - x$. There is also a half probability that he will play the game in the second mover's position. By backward induction, the reciprocator first mover defects and the egoist also defects. His payoff in this case, therefore, is x. Finally, there is a $1 - \gamma - \delta$ probability that the egoist will be matched with another egoist. In this case, regardless whether he plays the game as a first mover or as a second mover the outcome is mutual defection and the payoff is x. Therefore, the average objective payoff to an egoist, denoted $\pi_{e|c}$, under the complete information condition is $\gamma \times 1 + \delta \times (\frac{1}{2}(1-x) + \frac{1}{2}x) + (1-\gamma-\delta) \times x = (1-\gamma-\delta)x + \frac{1}{2}\delta + \gamma$. The average payoffs to the altruists and reciprocators under complete information condition, denoted $\pi_{a|c}$ and $\pi_{r|c}$ respectively, can be calculated in the same manner: $\pi_{a|c} = (1 - x)(\delta + \gamma)$, and $\pi_{r|c} = \frac{1}{2}(1 - \gamma - \delta) + (1 - x)(\delta + \gamma)$. Notice that when the proportion of altruists is larger than $1 - 2x$, egoists do better than reciprocators ($\pi_{e|c} > \pi_{r|c}$).

5.2 Incomplete Information Condition

When information is incomplete the stage game is played as a Bayesian game and the proper solution concept for the stage game is a Bayesian Nash equilibrium. Under incomplete information condition, a player does not know the exact type of the other player with whom he is randomly matched. However, the distribution of types within a population at a given evolutionary stage is common knowledge. The incomplete information condition features multiple equilibrium zones depending on the relative proportions of the three types. The optimal strategies for egoists and reciprocators change as functions of the distribution of types in the population.

There are three equilibrium zones under the incomplete information condition. The equilibrium strategy of a second mover is a direct function of the player's type in all

Equilibrium Zone		Equilibrium Outcome*
Zone I	$\frac{x}{1-x} \leq \delta$	$\text{Ego}^1 : C$, $\text{Ego}^2 : D$ always $\text{Rec}^1 : C$, Rec^2 : copy first mover's action
Zone II	$\frac{x-\alpha\gamma}{1-x} \leq \delta < \frac{x}{1-x}$	$\text{Ego}^1 : D$, $\text{Ego}^2 : D$ $\text{Rec}^1 : C$, Rec^2 : copy first mover's action
Zone III	$\delta < \frac{x-\alpha\gamma}{1-x}$	$\text{Ego}^1 : D$, $\text{Ego}^2 : D$ $\text{Rec}^1 : D$, Rec^2 : copy first mover's action

*Altruists always cooperate

**Ego: Egoist; Rec: Reciprocator; Alt: Altruist; Ego^1: Egoist first mover; Ego^2 : Egoists second mover; C : Cooperation; D : Defection

Table 2: Stage game equilibria of incomplete information game

Information Condition	Average Objective Payoff
Complete Information	
Altruists, $\pi_{a\|c}$	$(1-x)(\delta+\gamma)$
Reciprocators, $\pi_{r\|c}$	$\frac{1}{2}(1-\gamma-\delta)+(1-x)(\delta+\gamma)$
Egoists, $\pi_{e\|c}$	$(1-\gamma-\delta)x+\frac{1}{2}\delta+\gamma$
Incomplete Information Zone I	
Altruists, $\pi_{a\|1}$	$\frac{1}{2}(\delta+\gamma+1)(1-x)$
Reciprocators, $\pi_{r\|1}$	$\frac{1}{2}(\delta+\gamma+1)(1-x)$
Egoists, $\pi_{e\|1}$	$\frac{1}{2}((\delta+\gamma)(1-x)+1)$
Incomplete Information Zone II	
Altruists. $\pi_{a\|2}$	$\delta-\delta x+\gamma-x\gamma$
Reciprocators, $\pi_{r\|2}$	$\frac{1}{2}(2\delta-3\delta x+2\gamma-3x\gamma+x)$
Egoists, $\pi_{e\|2}$	$\frac{1}{2}(2x-\delta x-2x\gamma+2\gamma+\delta)$
Incomplete Information Zone III	
Altruists, $\pi_{a\|3}$	$\frac{1}{2}(\delta-\delta x+2\gamma-2x\gamma)$
Reciprocators, $\pi_{r\|3}$	$\frac{1}{2}(2x-3x\gamma+2\gamma)$
Egoists, $\pi_{e\|3}$	$x-x\gamma+\gamma$

Table 3: Average payoffs to three preference types

three zones: an egoist always defects, a reciprocator copies the first mover's choice, and an altruist always cooperates. The difference across the three equilibrium zones is the behavior of first movers. In the zone I, where the proportion of reciprocators is quite large ($\frac{x}{1-x} \leq \delta$), all three types of first movers cooperate. In the zone II, the proportion of reciprocators is not large enough to make it rational for an egoist first mover to cooperate, but it is still large enough to make a reciprocator first mover to cooperate. In the zone III, only altruistic first movers cooperate. Table 2 summarizes the equilibrium behavior of different preference types in the three equilibrium zones. Table 3 reports the average objective payoffs to the three preference types.

In the equilibrium zone I, it suffices to show that cooperation is rational for an egoist first mover. If she cooperates, the expected utility is $(1-\gamma-\delta)\times 0+\delta(1-x)+\gamma(1-x) = (\delta+\gamma)(1-x)$. If she defects, the expected utility is $(1-\gamma-\delta)\times x+\delta\times x+\gamma\times 1 = (1-\gamma)x+\gamma$. Therefore, an egoist prefers to cooperate if and only if $(\delta+\gamma)(1-x) \geq (1-\gamma)x+\gamma$, or $\frac{x}{1-x} \leq \delta$. Note that this equilibrium is only a function of the proportion of reciprocators.

In the equilibrium zone II, egoists defect and the reciprocators and altruists cooperate as first movers. Since $\delta < \frac{x}{1-x}$, there is no equilibrium in which first mover egoists cooperate. Therefore, it suffices to show that cooperation in the first mover's position is rational for a reciprocator. If a reciprocator first mover cooperates, his expected utility is $(1-\gamma-\delta)\times 0+\delta(1-x)+\gamma(1-x) = (\gamma+\delta)(1-x)$. If a reciprocator first mover defects, the expected utility is $(1-\gamma-\delta)\times x+\delta\times x+\gamma\times(1-\alpha) = x+\gamma-x\gamma-\alpha\gamma$. The expected utility of cooperation is greater than or equal to that of defection if $\frac{x-\alpha\gamma}{1-x} \leq \delta$.

Finally, if $\delta < \frac{x-\alpha\gamma}{1-x}$, only the altruists cooperate as first movers. The behavior of each type in the second mover's position is the same as that in zones I and II. Figure 3 shows two examples of the division of the population state space into the three equilibrium zones as a function of x and α using two numerical examples: ($x = 1/3$, $\alpha = 1/2$) and ($x = 1/4$, $\alpha = 1/2$).

The average objective payoff to each of the three types in each of the three equilibrium zones can be calculated based on the objective payoff matrix of Figure 1, the equilibrium outcome statements of Table 1, and the relative proportion of types. For example, the average objective payoff to altruists in the equilibrium zone I of the incomplete information condition, denoted $\pi_{a|1}$, can be calculated as follows. In the zone I, all three types of players cooperate in the first mover's position and only egoists defect in the second mover's position. An altruist, if he plays the game as a first mover, meets another altruist second mover with probability γ and obtains a payoff of $1-x$, meets a reciprocator second mover with a probability δ and obtains an objective payoff of $1-x$, and meets an egoist second mover and obtains an objective payoff of 0. If the altruist find himself in the second movers position his payoff is simply $1-x$ since all three types of first movers cooperate and the altruist second mover also cooperates. Adding up the objective payoffs with relevant probabilities, we have

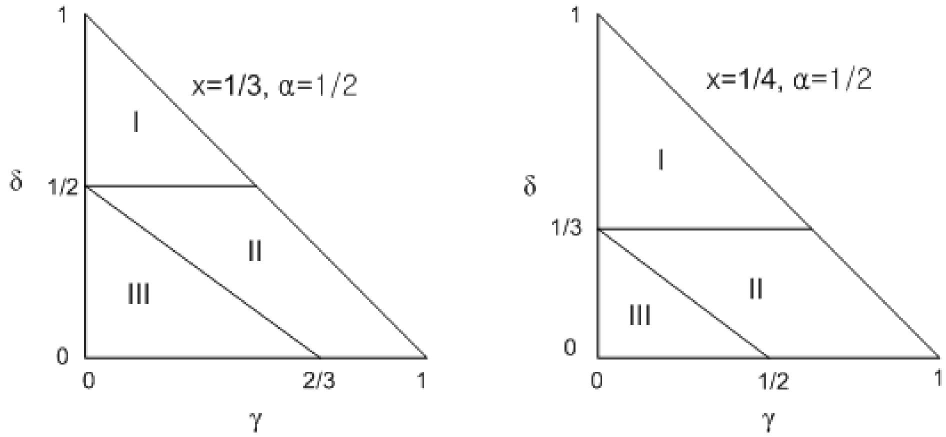

Figure 3: Division of population state space into three equilibrium zones of incomplete information game

$\pi_{a|1} = \frac{1}{2}(\gamma \times (1-x) + \delta \times (1-x) + (1-\gamma-\delta) \times 0) + \frac{1}{2}(1-x) = \frac{1}{2}(\delta+\gamma+1)(1-x)$. Other cases can be calculated in similar manners and the results are shown in Table 3.

5.3 Objective Payoffs Under Mixed Information Condition

Suppose that in each evolutionary stage, randomly matched players play the stage game under complete information with probability q, and under incomplete information with the complementary probability $1-q$. Call it a "mixed-information" condition. Under the mixed-information condition, the expected objective payoff to a type is a linear sum of the payoffs under complete and incomplete information conditions for the type. Thus, when the population composition belongs to the equilibrium zone k $(k = 1, 2, 3)$ of the incomplete information condition, the average payoff to type θ, denoted $\pi_{\theta|k}$, is $q\pi_{\theta|c} + (1-q)\pi_{\theta|k}$.

6 Evolution of preferences under complete and incomplete information

Using the stage game payoff results obtained in the previous section, it is now possible to study the evolutionary trajectories under varying information conditions. This section studies first how preferences evolve when information is complete, and then when information is incomplete. The study of these two extreme cases provide benchmarks for the evolution under mixed information conditions. Table 4 reports the vector derivatives $(\dot{\gamma}, \dot{\delta})$ and steady states in each equilibrium zone. The appendix at the end of the paper derives the vector derivatives. The vector derivatives, when evaluated at any population state (γ, δ), show the direction and the speed of the evolution at that population state. If the derivative for a type is positive the type's proportion increases; if the derivative for

Condition	$[\dot{\gamma}, \dot{\delta}]$	Steady States
Complete Info	$[\dot{\gamma} = \frac{-x\gamma(1-\gamma-\delta)}{x+\delta+\gamma(1-2x)}$, $\dot{\delta} = \frac{(1-2x)\delta(1-\gamma-\delta)}{2(x+\delta+\gamma(1-2x))}]$	$(\gamma = \delta = 0)^{*}$; $(\gamma + \delta = 1, \delta < 2x)^{*}$ $(\gamma + \delta = 1, 2x < \delta < 1)^{**}$; $(\delta = 1)^{***}$
Incomplete Info I	$[\dot{\gamma} = \frac{-x\gamma(1-\gamma-\delta)}{(1+\delta+\gamma)-2x(\delta+\gamma)}$, $\dot{\delta} = \frac{-x\delta(1-\gamma-\delta)}{(1+\delta+\gamma)-2x(\delta+\gamma)}]$	$(\gamma + \delta = 1)^{*}$
Incomplete Info II	$[\dot{\gamma} = \frac{-\gamma(1-\gamma-\delta)(2x+2x\delta-\delta)}{2x+(1-2x)\left(2\gamma+\delta+\delta\gamma+\delta^2\right)}$, $\dot{\delta} = \frac{-\delta(1-\gamma-\delta)(x+2x\delta-\delta)}{2x+(1-2x)\left(2\gamma+\delta+\delta\gamma+\delta^2\right)}]$	$(\gamma + \delta = 1)^{*}$
Incomplete Info III	$[\dot{\gamma} = \frac{\gamma(\delta-\delta x+2x\gamma-2x+2\delta x\gamma-\delta\gamma)}{2x+\gamma(1-2x)(2+\delta)}$, $\dot{\delta} = \frac{\gamma\delta(x-\delta(1-2x))}{2x+\gamma(1-2x)(2+\delta)}]$	$(\gamma = 0, 0 < \delta < \frac{x}{1-x})^{**}$ $(\gamma = \delta = 0)^{***}$

* unstable steady state, ** weakly stable state; *** attractor

Table 4: Vector Derivatives and Steady States

a type is negative the type's proportion decreases. When both derivatives are zero, the population is at a steady state. The remaining of this section discusses the steady states and their stability.

6.1 Evolution When Information Is Complete

The complete information condition favors reciprocators. If we assume a constant mutation at the steady states the population will eventually evolve into one in which only reciprocators exist. There are stable steady states with a large proportion of reciprocators and some altruists in which invading egoists do worse than both reciprocators and altruists. But whenever egoists invade this type of steady states, the proportion of altruists slightly decreases in the recovered steady states. A long repetition of this process caused by a constant invasion of egoists moves the population into one in which reciprocators dominate. Figure 4 provides an illustration of the evolution under complete information.

Proposition 1 *When individuals' preference types are known to others, only the reciprocators exists in the attractor of the evolutionary system.*

The evolutionary dynamics under complete information can be studied by examining the vector derivatives and the steady states shown in the first row of Table 4. First, the proportion of altruists never increase: $\dot{\gamma} = \frac{-x\gamma(1-\gamma-\delta)}{x+(\delta+\gamma)(1-2x)} \leq 0$. In fact, whenever the population is not in a steady state, the proportion of altruists decreases. On the other hand, the proportion of reciprocators never decreases: $\dot{\delta} = \frac{(1-2x)\delta(1-\gamma-\delta)}{2(x+\delta+\gamma(1-2x))} \geq 0$. Thus, when mutations in steady states are taken into account, the altruists will keep decreasing and the reciprocators will keep increasing. This gives hints to analyzing the relative stability of different steady states under complete information condition.

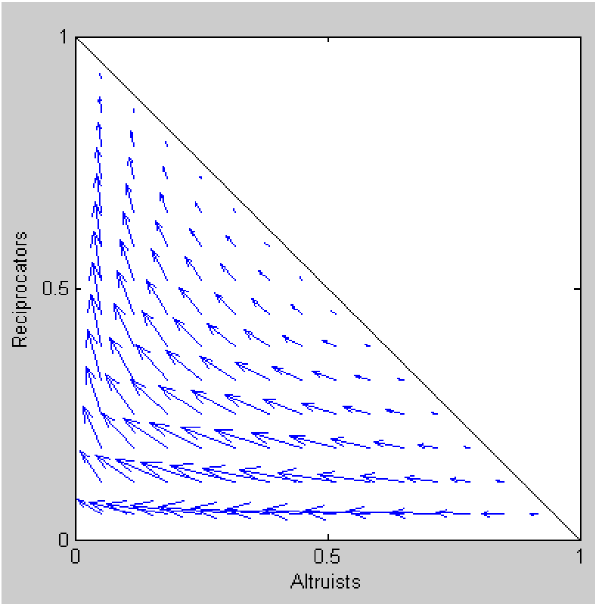

Figure 4: Evolution under complete information ($q = 1$ and $x = 0.3$)

The steady state ($\gamma = \delta = 0$), a population consists only of the egoists, is fundamentally unstable. Whenever a small number of reciprocators, by themselves or along with altruists, invade the population, the reciprocators do better than the egoists and, consequently, there proportion increases.

The other class of steady states consist of reciprocators and altruists, but not egoists: $(\delta+\gamma = 1)$. The stability of a steady state of this sort depends on the relative proportions of the reciprocators and altruists in the steady state. If the proportion of reciprocators is smaller than a threshold value $2x$ in the steady state, i.e., $(\gamma + \delta = 1 : \delta < 2x)$, the egoists do better than the reciprocators in the neighborhoods of the steady state.[5] On the other hand, in the neighborhoods of the steady states in which $\delta > 2x$, not only do

[5] This can be verified from Table 3 where the average expected payoffs are shown. For egoists do better than reciprocators, $\pi_{e|c} - \pi_{r|c} = x + \frac{1}{2}\gamma - \frac{1}{2} > 0$ or $\gamma > 1 - 2x$, which implies $\delta < 2x$.

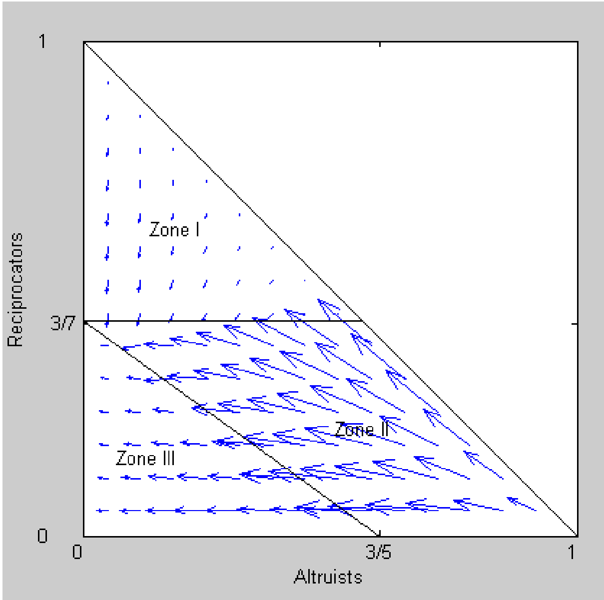

Figure 5: Evolution under incomplete information condition ($q = 0$ and $x = 0.3$)

the egoists do worse than the reciprocators, but they also do worse than the altruists. Thus, the ranking of objective payoffs among the three preferences around these steady states is in the order of reciprocators, altruists, and egoists. The invading egoists into these steady states will be driven out quickly and other steady states with reciprocators and egoists will be reached. However, since reciprocators do better than altruists in the neighborhoods of these steady states, the recovered steady states will always have slightly more reciprocators than those in the original steady states. Thus, though the steady states are stable, they will eventually attracted to the most stable steady state in which only the reciprocators exist ($\delta = 1$). To summarize, the only attractor of the population state space, when information is complete, is the one in which only reciprocators exist.

6.2 Evolution When Information Is Incomplete

The evolution under incomplete information condition can be summarized as follows. There are steady states with egoists and reciprocators: ($\gamma = 0$, $0 < \delta < \frac{x}{1-x}$). Altruists cannot invade the population. In the neighborhoods of these steady states, egoists grow faster than reciprocators. If there is a constant stream of random mutations, the population will eventually reach a point where only egoists exist. Figure 5 shows an example of the evolutionary dynamics under incomplete information.

Proposition 2 *When players know the distribution of preference types in the population but not the exact preference types of the current partners, only the egoists exist in the attractor of the evolutionary system.*

In zone I, all of the three types cooperate in the first mover's position. Reciprocators and altruists cooperate in return in the second mover's position, but egoists defect. The derivative for the altruists shows that the proportion of altruists is never increasing: $\dot{\gamma} = \frac{-x\gamma(1-\gamma-\delta)}{(1+\delta+\gamma)-2x(\delta+\gamma)} \leq 0$.[6] This is the same for the reciprocators whose proportion decreases except when the population is in a steady state: $\dot{\delta} = \frac{-x\delta(1-\gamma-\delta)}{(1+\delta+\gamma)-2x(\delta+\gamma)} \leq 0$. Therefore, the evolutionary trajectories in the equilibrium zone I are downward.

In the equilibrium zone II, reciprocators and altruists cooperate in the first mover's position, but egoists do not. In the second mover's position, the egoists always defect, the reciprocators copy first mover's action, and the altruists always cooperate The sign of the derivative for reciprocators ($\dot{\delta} = -\frac{\delta(1-\gamma-\delta)(x+2x\delta-\delta)}{2x+(1-2x)\left(2\gamma+\delta+\delta\gamma+\delta^2\right)}$) can be positive if $\delta + \gamma < 1$ and $\delta > \frac{x}{1-2x}$. The sign of the derivative for altruists ($\dot{\gamma} = -\frac{\gamma(1-\gamma-\delta)(2x+2x\delta-\delta)}{2x+(1-2x)\left(2\gamma+\delta+\delta\gamma+\delta^2\right)}$) can be positive if $\delta + \gamma < 1$ and $\delta > \frac{2x}{1-2x}$. But because $\delta < \frac{x}{1-x}$ in the zone II, neither condition can be satisfied. Therefore, proportions of both the reciprocators and altruists are decreasing unless $\delta + \gamma = 1$.

In the equilibrium zone III, only the altruists cooperate in the first mover's position. As second movers, the altruists always cooperate, the reciprocators copy first mover's action, and the egoists always defect. Notice that altruists never increase ($\dot{\gamma} = \frac{\gamma(\delta-\delta x+2x\gamma-2x+2\delta x\gamma-\delta\gamma)}{2x+\gamma(1-2x)(2+\delta)} \leq 0$) and the reciprocators never decrease ($\dot{\delta} = \frac{\gamma\delta(x-\delta(1-2x))}{2x+\gamma(1-2x)(2+\delta)} \geq 0$).[7]

In the zone III, there exists a continuum of steady states in which reciprocators and egoists coexist but no altruists ($\gamma = 0$). These steady states are relatively stable in the sense that after disturbance caused by invasion of altruists the population resettles in the neighborhoods of the original steady states. However, whenever the proportion of altruists becomes positive by their invasion, the average objective payoff to the egoists is larger than that to the reciprocators.[8] Therefore, in the recovered steady states, the proportions of reciprocators is slightly smaller than those in the original ones. After a long series of random mutations and recoveries, the population will be composed entirely of egoists.

[6] The denominator can be rewritten as $1+(\delta+\gamma)(1-2x)$, which is always positive because by definition $x < 0.5$. The numerator is never positive.

[7] The constraint for zone III is $\delta < \frac{x-\alpha\gamma}{1-x}$ which is smaller than $\frac{x-x\gamma}{1-x}$ because $\alpha > x$. For the proportion of altruists to increase, $\delta - \delta x + 2x\gamma - 2x + 2\delta x\gamma - \delta\gamma > 0$ or $\delta > \frac{2(x-x\gamma)}{1-x-\gamma(1-2x)}$. The numerator is never smaller than $x - x\gamma$ and the denominator is always smaller than $1 - x$. Thus, the condition cannot be met. For the reciprocators to decrease, $x - \delta(1 - 2x) < 0$ or $\delta > \frac{x}{1-2x}$. The numerator is greater than $x - \alpha\gamma$ and the denominator smaller than $1 - x$. Again, the condition cannot be met.

[8] This can be verified by comparing the payoffs to egoists and reciprocators in the zone III: $\pi_{e|3} - \pi_{r|3} = \frac{1}{2}x\gamma$ which is positive when $\gamma > 0$.

7 Stability of population with heterogeneous preferences

The analysis of evolutionary dynamics under complete and incomplete information shows that, even though there exist steady states with multiple types, the attractors contain only one preference type: reciprocators when information is complete and egoists when information is incomplete. Neither the assumption of a complete information nor that of incomplete information is a realistic representation of the real world. What is more reasonable to assume is a mixed information condition in which individuals can sometimes tell the preference types of their encounters, but other times they have to interact without knowing the preferences of their counterparts.

This section develops a general model of evolution in which the level of information is treated as a variable. With probability q players play the sequential prisoner's dilemma game under complete information condition; with probability $1-q$ they play the game under incomplete information condition. The parameter q in the model represents the extent to which the signals and symbols about others' types are available and reliable. The following analysis formally confirms our intuition that for a range of q, there must be strongly stable population states with mixed preference types.

We start with a proposition that describes the stable population states as functions of x, the degree of temptation to defect, and q, the degree of information.

Proposition 3 *When preferences of other players can be detected accurately with some positive probability, the attractor features either reciprocators only or a mixture of reciprocators and egoists. Specifically, if* $q > \frac{x}{1-x}$, $(\gamma = 0, \delta = 1)$ *is the only attractor of the evolutionary system. If* $q < \frac{x}{1-x}$, $(\gamma = 0, \delta = \frac{x}{1-x})$ *is the only attractor. If* $q = \frac{x}{1-x}$, *any population states with* $(\gamma = 0, \delta \in [\frac{x}{1-x}, 1])$ *are attractors.*

Figure 6 shows Proposition 3 graphically; all the possible combinations of initial conditions (x, q) are mapped into two difference kinds of attractors. Any (x, q) condition that is above the line $(\frac{x}{1-x})$ has an attractor at $(\delta = 1)$. Any (x, q) condition that is below the line had an attractor in which the proportion of reciprocators is the point on $\frac{x}{1-x}$ which is reached by the vertical extension of the point (x, q). Not shown in the figure is the cases in which q is exactly $\frac{x}{1-x}$. In those cases, any population with the proportion of reciprocators greater than of equal to $\frac{x}{1-x}$ and that of altruists zero, are attractors.

Figure 7 and Figure 8 provide examples of two kinds of attractors when $q \neq \frac{x}{1-x}$. In Figure 7, q is smaller than $\frac{x}{1-x}$ and the unique attractor contains both reciprocators and egoists. In Figure 8, q is greater than $\frac{x}{1-x}$ and, thus, the unique attractor inhabits only reciprocators: $(\gamma = 0, \delta = 1)$.

The evolutionary dynamics under zones II and III are rather obvious given that they are linear combinations of complete and incomplete information conditions we analyzed in the preceding subsection. In zone II, neither of the two extreme information condition

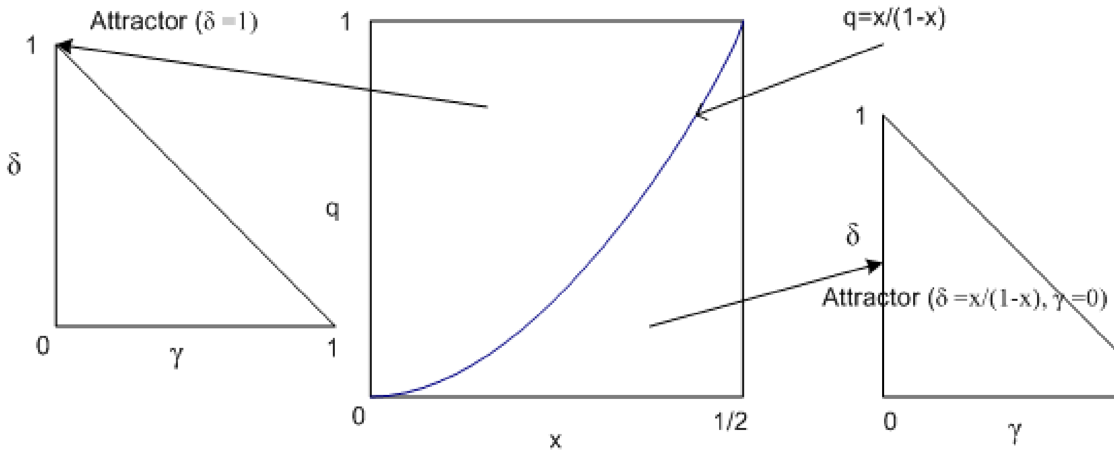

Figure 6: Attractors of Population State as Functions of normalized greed (fear) and the degree of information

– complete and incomplete – has stable steady states. Therefore, the population states inside zone II evolve to either zone I or III. In zone III, reciprocators increase in both complete and incomplete information conditions, thus, increase in mixed information condition as well. The altruists, on the other hand, always decrease under both complete and incomplete information conditions, thus, also decrease under mixed information condition. Therefore, the only question worth formally analyzing is what happens in zone I, in which while altruists (after taking disturbances into account) always decrease, but the reciprocators may or may not. Intuitively, we can tell that there is a level of information q, given x, such that the evolution in zone I favors either reciprocators or egoists. In the former case we will have an attractor composed only of reciprocators. In the latter case the attractor will consist of both egoists and reciprocators.

In the equilibrium zone I,

$$\begin{aligned}\dot{\gamma} &= \frac{\gamma\pi_{a|m1}}{\gamma\pi_{a|m1}+\delta\pi_{a|m1}+(1-\gamma-\delta)\pi_{e|m1}}-\gamma \\ &= \frac{-\gamma x(1+q)(1-\gamma-\delta)}{1+(1-2x)(\gamma+\delta)-q(1-2x)(1-\gamma-\delta)}, \text{ and} \qquad (6\text{-}1)\\ \dot{\delta} &= \frac{\delta\pi_{r|m1}}{\gamma\pi_{a|m1}+\delta\pi_{a|m1}+(1-\gamma-\delta)\pi_{e|m1}}-\delta \\ &= \frac{\delta(q(1-x)-x)(1-\gamma-\delta)}{1+(1-2x)(\gamma+\delta)-q(1-2x)(1-\gamma-\delta)} \qquad (6\text{-}2)\end{aligned}$$

By examining the derivatives, we can tell how the types fare in the zone. Note that the

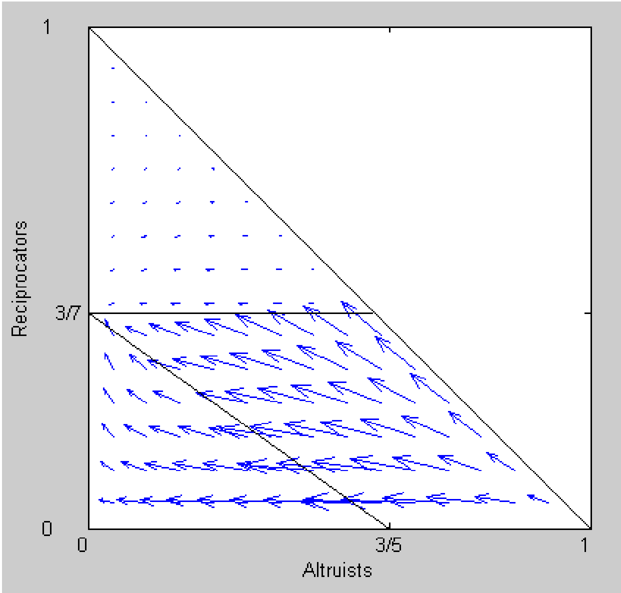

Figure 7: Evolution of preferences with a medium level of information ($q = 0.4$ and $x = 0.3$)

denominator of the vector derivatives is positive.[9] Setting (1)= 0 and (4)= 0, and solving them simultaneously subject to $\delta \geq \frac{x}{1-x}$ we can find the steady states of the population in zone I. First $(\gamma + \delta = 1)$ is a steady state. However, since the sign of $\dot{\gamma}$ is always negative, unless $\gamma + \delta = 1$, the steady state is unstable. Another family of steady states is $(\gamma = 0, \delta = \delta \mid q = \frac{x}{1-x})$. The steady states imply that any points on the vertical axis in zone I is a steady state if it happens that $q = \frac{x}{1-x}$. These are all stable in the sense that the population recovers equilibrium, after small disturbance, at the original steady state or it close neighborhood. If $q > \frac{x}{1-x}$, $(\gamma = 1,\ \delta = 0)$ is the unique stable state and attractor since all the evolutionary trajectories are upward. On the other hand, if $q < \frac{x}{1-x}$, the evolutionary trajectories in Zone I are downward and, thus, the mixed population with $(\gamma = 0, \delta = \frac{x}{1-x})$ is the unique attractor of the population.

Notice that the internal attractor $(\gamma = 0, \delta = \frac{x}{1-x})$ when exists consists only of two types – reciprocators and egoists. Since the internal attractor is the bordering point of all the three equilibrium zone, we can assume that the observed behavior at and around the

[9] The denominator is positive if

$$\begin{aligned} 1 + (1-2x)(\gamma+\delta) - q(1-2x)(1-\gamma-\delta) &> 0 \\ 1 + \frac{2((\gamma+\delta)(1-2x)+x)}{(1-\gamma-\delta)(1-2x)} &> q. \end{aligned}$$

The second term in the left-hand side is never negative and q, by definition is smaller than 1. Thus the inequality always holds.

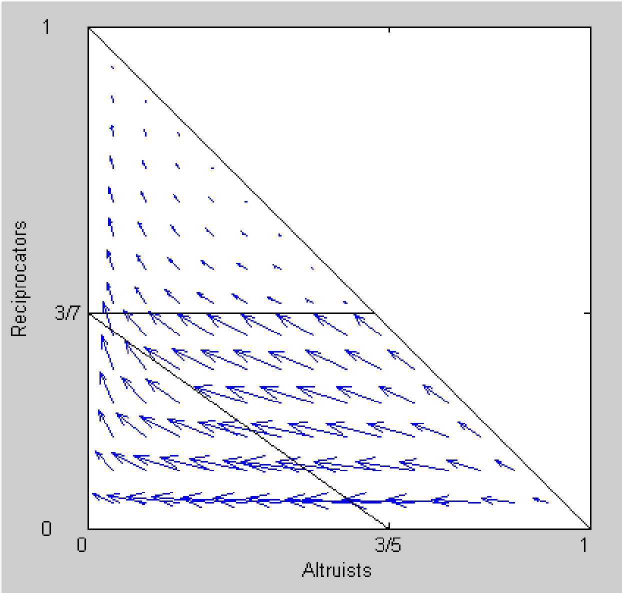

Figure 8: Evolution with a high level of information ($q = 0.7$ and $x = 0.3$)

attractor would be a mixture of behaviors in all three zones. That is, both reciprocators and egoists may or may not cooperate in the first mover's position, though the probability of cooperation is higher for the reciprocators than that for the egoists.

8 Discussion

This paper has shown that reciprocal preferences can evolve in the one-shot prisoner's dilemma, along with purely self-interested ones, when players are able to tell others' preferences with some positive probability. Thus, repetition of interactions, though important both theoretically and practically, is not a necessary condition for the evolution of cooperation. With ever increasing mobility and the expansion of the scope of interactions, people frequently face situations that are closer to a one-shot prisoner's dilemma than they are to the indefinitely repeated prisoner's dilemma with high continuation probabilities. In addition, long-term relationships and networks of interactions must start somewhere by someone who meets a stranger and tries to figure out whether to trust him or not.

The indirect evolutionary method contributes explaining how cooperation might evolve in one-shot dilemmas by separating preferences from fitness. The evolution of cooperation in these one-shot interactions depends on the reciprocators who are rational, but not strictly self-interested. Reciprocators are different from cooperators; reciprocators prefer mutual cooperation and they are willing to cooperate if others also do so. But as a rational decision maker they take into account the specifics of an action situation to achieve mutual cooperation and to protect themselves from getting exploited by egoists. As a second mover they cooperate if and only if the first mover cooperate; they are trustworthy but not naive. As first movers reciprocators rely on their perception of the second mover's preferences. If the information about the second mover's preference is available, they cooperate unless the second mover is an egoist. If the information is not available they cooperate if and only if they believe that the population at large are trustworthy enough. Unless we assume that the information about others' preference is always available, the reciprocators may not overtake the entire population. But insofar as the information is available, by means of signals and symbols, with some positive probability, the reciprocators will comprise a significant proportion of a population in the evolutionary equilibrium.

Appendix

A1: Derivation of the vector derivatives

Complete Information Condition: Using the payoffs to each of the three types shown in Table 1 and applying them to the replicator functions defined in Section 4, we can calculate the proportions of altruists and reciprocators at time $t+1$.

$$
\begin{aligned}
\gamma_{t+1} &= \frac{(1-x)(\delta+\gamma)\gamma}{x-2x\delta-2x\gamma+\delta+\gamma}, \text{ and} \\
\delta_{t+1} &= \frac{(\frac{1}{2}(1-\gamma-\delta)+(1-x)(\delta+\gamma))\delta}{x-2x\delta-2x\gamma+\delta+\gamma}.
\end{aligned}
$$

Then, the derivatives $\dot{\delta}$ and $\dot{\gamma}$ can also be calculated as follows.

$$
\begin{aligned}
\dot{\gamma} &= \gamma_{t+1}-\gamma_t = \frac{(1-x)(\delta+\gamma)\gamma-\gamma(x-2x\delta-2x\gamma+\delta+\gamma)}{x-2x\delta-2x\gamma+\delta+\gamma} \\
&= \frac{-x\gamma(1-\gamma-\delta)}{x+(\delta+\gamma)(1-2x)}, \text{ and} \\
\dot{\delta} &= \delta_{t+1}-\delta_t = \frac{(\frac{1}{2}(1-\gamma-\delta)+(1-x)(\delta+\gamma))\delta-\delta(x-2x\delta-2x\gamma+\delta+\gamma)}{x-2x\delta-2x\gamma+\delta+\gamma} \\
&= \frac{\delta(1-2x)(1-\gamma-\delta)}{x+(\delta+\gamma)(1-2x)}.
\end{aligned}
$$

Incomplete Information Condition:

Equilibrium zone I: All of the three types cooperate in the first mover's position. Reciprocators and altruists cooperate in return in the second mover's position, but egoists defect. The proportions of altruists and reciprocators at time $t+1$ can be calculated as follows.

$$
\begin{aligned}
\gamma_{t+1} &= \frac{\gamma(\delta+\gamma+1)(1-x)}{2(\delta+\gamma)(1-x)+(1-\gamma-\delta)}, \text{ and} \\
\delta_{t+1} &= \frac{\delta(\delta+\gamma+1)(1-x)}{2(\delta+\gamma)(1-x)+(1-\gamma-\delta)}.
\end{aligned}
$$

Then, the derivatives $\dot{\delta}$ and $\dot{\gamma}$ are

$$\begin{aligned}
\dot{\gamma} &= \gamma_{t+1} - \gamma_t = \frac{-x\gamma(1-\gamma-\delta)}{(1+\gamma+\delta)-2x(\delta+\gamma)}, \text{ and} \\
\dot{\delta} &= \delta_{t+1} - \delta_t = \frac{\delta(\delta+\gamma+1)(1-x) - \delta(2(\delta+\gamma)(1-x)+(1-\gamma-\delta))}{2(\delta+\gamma)(1-x)+(1-\gamma-\delta)} \\
&= \frac{-x\delta(1-\gamma-\delta)}{(1+\gamma+\delta)-2x(\delta+\gamma)}.
\end{aligned}$$

Equilibrium Zone II: The proportions of altruists and reciprocators at time $t+1$ are

$$\begin{aligned}
\gamma_{t+1} &= \frac{2(\delta-\delta x+\gamma-x\gamma)}{(1-2x)(\delta^2+\delta\gamma+\delta+2\gamma)+2x}, \text{ and} \\
\delta_{t+1} &= \frac{\delta(2\delta-3\delta x+2\gamma-3x\gamma+x)}{(1-2x)(\delta^2+\delta\gamma+\delta+2\gamma)+2x}.
\end{aligned}$$

The derivatives are

$$\begin{aligned}
\dot{\gamma} &= \gamma_{t+1} - \gamma_t = \frac{2(\delta-\delta x+\gamma-x\gamma) - \gamma((1-2x)(\delta^2+\delta\gamma+\delta+2\gamma)+2x)}{(1-2x)(\delta^2+\delta\gamma+\delta+2\gamma)+2x} \\
&= -\frac{\gamma(1-\gamma-\delta)(2x+2x\delta-\delta)}{2x+(1-2x)(2\gamma+\delta+\delta\gamma+\delta^2)}, \text{ and}
\end{aligned}$$

$$\begin{aligned}
\dot{\delta} &= \delta_{t+1} - \delta_t = \frac{\delta(2\delta-3\delta x+2\gamma-3x\gamma+x) - \delta((1-2x)(\delta^2+\delta\gamma+\delta+2\gamma)+2x)}{(1-2p)(\delta^2+\delta\gamma+\delta+2\gamma)+2p} \\
&= -\frac{\delta(1-\gamma-\delta)(x+2x\delta-\delta)}{2x+(1-2x)(2\gamma+\delta+\delta\gamma+\delta^2)}.
\end{aligned}$$

Equilibrium zone III: The proportions of altruists and reciprocators at time $t+1$ are

$$\begin{aligned}
\gamma_{t+1} &= \frac{\gamma(\delta-\delta x+2\gamma-2x\gamma)}{(1-\gamma-\delta)(2x-2x\gamma+2\gamma)+\delta(2x-3x\gamma+2\gamma)+\gamma(\delta-\delta x+2\gamma-2x\gamma)} \\
&= \frac{\gamma(\delta-\delta x+2\gamma-2x\gamma)}{2x+\gamma(\delta+2)(1-2x)}, \text{ and} \\
\delta_{t+1} &= \frac{\delta(2x-3x\gamma+2\gamma)}{(1-\gamma-\delta)(2x-2x\gamma+2\gamma)+\delta(2x-3x\gamma+2\gamma)+\gamma(\delta-\delta x+2\gamma-2x\gamma)} \\
&= \frac{\delta(2x-3x\gamma+2\gamma)}{2x+\gamma(\delta+2)(1-2x)}.
\end{aligned}$$

The derivatives $\dot{\delta}$ and $\dot{\gamma}$ are:

$$\begin{aligned}
\dot{\gamma} &= \gamma_{t+1} - \gamma_t = \frac{\gamma(\delta - \delta x + 2\gamma - 2x\gamma) - \gamma(2x + 2\gamma(1-2x) + \delta\gamma(1-2x))}{2p + 2\gamma(1-2x) + \delta\gamma(1-2x)} \\
&= \frac{\gamma(\delta - \delta x + 2x\gamma - 2x + 2\delta x\gamma - \delta\gamma)}{2x + \gamma(1-2x)(2+\delta)}, \text{ and} \\
\dot{\delta} &= \delta_{t+1} - \delta_t = \frac{\delta(2x - 3x\gamma + 2\gamma) - \delta(2x + 2\gamma(1-2x) + \delta\gamma(1-2x))}{2x + 2\gamma(1-2x) + \delta\gamma(1-2x)} \\
&= \frac{\gamma\delta(x - \delta(1-2x))}{2x + \gamma(1-2x)(2+\delta)}.
\end{aligned}$$

A2: Extension to simultaneous game

Under complete information, only reciprocators exist in the attractor. Under incomplete information either both reciprocators egoists exit or only egoists exit in the attractors. In the attractors with both reciprocators and egoists under incomplete information, the two types are not behaviorally distinguished. That is, both defect always . There is a level of information q, for a given x, such that the attractor features both reciprocators and egoists.

The technical details can be found in the author's working paper (Ahn, 2002). Egoists always defect and altruists always cooperate under both complete and incomplete information conditions. Under complete information, reciprocators cooperate against altruists and reciprocators but not against egoists. Under incomplete information, reciprocators cooperate if the combined proportion of reciprocators and altruists is large enough ($\delta + \gamma \geq \frac{x}{\alpha}$) and defect otherwise. Under complete information, altruists get the smallest objective payoff under all possible distribution of types whenever egoists are present. After altruists are wiped out of the population, reciprocators do better than the egoists. The reason is that while egoists always get the mutual defection payoff of x, reciprocators can get the mutual cooperation payoff of $1-x$ when they are matched with another reciprocator. When two reciprocators are matched with each other, there are two equilibria (D,D) and (C,C). Insofar as the reciprocators play (C,C) with positive probability, the evolution favors the reciprocators over the egoists. The attractor ($\delta = 1$) absorbs all other steady states such as continuum of ($\delta + \gamma = 1$) and ($\gamma = 0$). Around the neighborhoods of steady states ($\delta + \gamma = 1$; $\delta \geq \frac{x}{1-2x}$) reciprocators do better than the egoists. Around the neighborhoods of steady states ($\delta + \gamma = 1$; $\delta < \frac{x}{1-2x}$) egoists do better than the reciprocators and these steady states initially move to other population states with ($\gamma = 0$). In these population states, the reciprocators do strictly better than the egoists insofar as they play (C,C) with positive probability when match with another reciprocator.

Under incomplete information, egoists do at least as well as reciprocators for all possible distribution of types. The steady states that consist only of reciprocators and altruists ($\gamma + \delta = 1$) are vulnerable to egoists' invasion since egoists defect while both cooperators and reciprocators cooperate. All population states with $(\gamma = 0, \delta < \frac{x}{\alpha})$ are attractors because both reciprocators and egoists defect and altruists cannot invade.

If $\frac{q}{1-q} < \frac{x}{1-2x}$ a mixed population $(\delta = \frac{x}{\alpha}, \gamma = 0)$ is an attractor along with the homogenous population of reciprocators $(\delta = 1)$. If $\frac{q}{1-q} \geq \frac{x}{1-2x}$, the only attractor is $(\delta = 1)$. Reciprocators do better than egoists under mixed information condition if the combined proportion of reciprocators and altruists are small. This is because, the reciprocators protect themselves from getting exploited by egoists under incomplete information (by defecting) while reaping the benefits of mutual cooperation under complete information. Therefore, a population state with egoists only $(\delta = \gamma = 0)$ or with egoists and small proportions of reciprocators $(0 < \delta < \frac{x}{\alpha}, \gamma = 0)$ are unstable and in which the proportion of reciprocators increases. But as he proportion of the reciprocators gets large enough for them to cooperate under incomplete information condition, there emerge two possibilities. When the proportion of reciprocators is very large $(\delta \geq \frac{x(1-q)}{(1-2x)q})$ the reciprocators more than compensate their loss in the incomplete information game by the gains they obtain in the complete information game. Thus, all the population states with $(\delta \geq \frac{x(1-q)}{(1-2x)q})$ evolve towards the attractor $(\delta = 1)$. On the other hand, if $(\delta < \frac{x(1-q)}{(1-2x)q}$ and $\delta + \gamma \geq \frac{x}{\alpha})$ there are not enough reciprocators and altruists combined for the reciprocators to recover, in the complete information game, their loss incurred in the incomplete information game. Of course, if $\delta + \gamma < \frac{x}{\alpha}$, there does not exist a Bayesian equilibrium of incomplete information game in which reciprocators cooperate, the reciprocators do as well as the egoists in the incomplete information game, do strictly better than egoists in the complete information game and the proportion of reciprocators increases, and the population evolves towards the attractor with mixed reciprocators and egoists.

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