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Coalitionally strategy-proof voting rules for separable weak orderings

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Abstract In the framework of multiple objects choice (e.g. candidate selection, membership, and qualification problems) with separable preferences (weak ordering), we characterize a family of voting rules that satisfy coalitional strategy-proofness, which requires that no group of agents can benefit simultaneously by jointly misrepresenting their preferences.

Keywords Coalitional strategy-proofness; Strategy-proofness; Separable preferences; Voting rule.

JEL Classification D70, D71, D72

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1 Introduction

We study "social choice rules" that are not manipulable through coalitional preference misrepresentations, in the framework of multiple objects choice (e.g. candidate selection, membership, and qualification problems). There is a finite set of objects and a subset should be chosen collectively reflecting preferences of agents. Any subset, including the empty set, is a feasible social alternative. In this model, preferences are called *separable* when each object can be categorized into the three types, goods, bads, or nulls. An object is a good when adding it to any subset is always preferred to subtracting it from the same subset. It is a bad if the preference relation is reversed and a null when the two are indifferent. This domain of separable preferences in this model is a well-known example where we can escape from the negative result known as the Gibbard-Satterthwaite Theorem (Gibbard 1973, Satterthwaite 1975), which states that any social choice function that is not manipulable by a preference misrepresentation of a single agent and of which the range contains at least three alternatives is dictatorial. This requirement of non-manipulability is known as strategy-proofness.

Barberà, Sonnenschein, and Zhou (1991) characterize the family of strategy-proof and fully ranged (meaning the range contains all alternatives) social choice rules, which they call "voting by committees". Their result is based on the assumption that preferences are linear as well as separable. Thus there is no indifferent pair of alternatives. Ju (2003) extends this family of strategy-proof rules on the domain of all separable preferences, linear or not. Call the extended family Family Φ^*. He shows that rules in Family Φ^* are the only "voting rules" satisfying both strategy-proofness and a mild condition "null-independence".[1]

However, extending the existence result of Barberà, Sonnenschein, and Zhou (1991) for possibly non-linear preferences no longer holds when strategy-proofness is strengthened into *coalitional strategy-proofness* requiring that social choice rules should not be manipulable by preference misrepresentations by a "group" of agents. It is well known that coalitional strategy-proofness and the full-range condition together imply efficiency. Ju (2003) shows that serially dictatorial rules are the only rules in Family Φ^* satisfying strategy-proofness and efficiency. Serially dictatorial rules are examples of dictatorial rules. When preferences are non-linear, the dictator may have indifferent best alternatives. Serial dictatorship, then, gives another person the priority to choose from the best alternatives of the dictator. When this person with the second priority after the dictator has multiple best alternatives, then the priority goes on to the next person, and so on. It can be shown that any serially dictatorial rule can be manipulated, for example, by the dictator and the person with the third priority so that the manipulation makes the person with the third priority better off, while making the person with the second prior-

[1] Theorem 1 in Ju (2003). Voting rules are the rules that depend only on which objects are goods, bads, or nulls rather than the full information of preferences.

ity worse off (this will be shown more formerly later). Thus the whole argument above shows that there is no rule in Family Φ^* satisfying both coalitional strategy-proofness and the full-range condition. This means, using the above mentioned charaterization result in Ju (2003), that there in no voting rule satisfying coalitional strategy-proofness, the full-range condition, and null-independence.

To satisfy coalitional strategy-proofness, therefore, it is necessary to drop the full-range condition. No earlier study identifies coalitionally strategy-proof rules in this model. Although it is already expected that the family of coalitionally strategy-proof rules will be quite restricted, we believe that it is important to understand, as completely as possible, what those rules are in this model. Our main result pins down, among rules in Family Φ^*, the complete family of coalitionally strategy-proof rules.

2 Model and basic concepts

A society $N \equiv \{1, \cdots, n\}$ of agents with $n \geq 3$ needs to choose objects from a finite set A. There is no constraint in the choice and so any subset is an *alternative*. Thus the set of alternatives is the power set 2^A. Each agent $i \in N$ has a *separable* preference R_i that is a weak ordering over 2^A such that for all $x \in A$ and all $X \subseteq A \backslash x$,[2] $[X \cup x]\ P_i\ X$ if and only if $x\ P_i\ \varnothing$; $[X \cup x]\ I_i\ X$ if and only if $x\ I_i\ \varnothing$, where P_i and I_i denote the corresponding strict and indifference relations, respectively. Let $\mathcal{S}$ be the set of separable preferences. A *social choice rule*, or simply, a *rule*, is a function $\varphi\colon \mathcal{S}^N \rightarrow 2^A$ mapping each preference profile into a *single* alternative.

Barberà, Sonnenschein, and Zhou (1991) study rules that are not manipulable through preference misrepresentations by a single agent. To state the condition formally, a rule φ is *strategy-proof* if for all $R \in \mathcal{S}^N$, all $i \in N$, and all $R'_i \in \mathcal{S}$, $\varphi(R)\ R_i\ \varphi(R'_i, R_{-i})$.[3] We refer readers to Barberà (2001) and Thomson (2001) for extensive surveys of literature on *strategy-proofness*. Barberà, Sonnenschein, and Zhou (1991) consider separable preferences that are linear (no indifference between two distinct alternatives). They identify a great variety of *strategy-proof* rules, called, "voting by committees", and characterize this family on the basis of *strategy-proofness* and the "full-range condition".[4] When all separable preferences, linear or non-linear, are admissible, Ju (2003) identifies a bigger family, including voting by committees, of *strategy-proof* rules. These rules make the decision on each object, according to a predetermined set of ordered pairs of disjoint groups, called a "power structure". To give a formal definition, we first define basic concepts. An object $x \in A$ is a *good for* $R_i \in \mathcal{S}$ if $x\ P_i\ \varnothing$.[5] It is a *bad* if $\varnothing\ P_i\ x$. It is a *null* if $x\ I_i\ \varnothing$. For each

[2] For convenience, we denote each singleton $\{x\}$ by x.

[3] We use notation, R, R', $\bar{R}$, $\bar{R}'$, *etc.* for elements in $\mathcal{S}^N$. Following standard notational convention, we denote i's component of R by R_i and the list of all others' preferences by R_{-i}.

[4] The full-range condition says that the range of rules should be the set of alternatives, or 2^A.

[5] Note that by *separability*, $x\ P_i\ \varnothing$ if and only if for all $X \subseteq A \backslash x$, $[X \cup x]\ P_i\ X$.

$R \in \mathcal{S}^N$ and each $x \in A$, let $N_x^G(R)$ be the set of agents for whom x is a good and $N_x^B(R)$ the set of agents for whom x is a bad. Let $\mathfrak{C}^* \equiv \{(C_1, C_2) \in 2^N \times 2^N : C_1 \cap C_2 = \varnothing\}$ be the set of all pairs of disjoint groups of agents. A *power structure associated with* $x \in A$, denoted by $\mathfrak{C}_x \subseteq \mathfrak{C}^*$, is a (possibly empty) set of pairs of disjoint groups such that for all $(C_1, C_2) \in \mathfrak{C}_x$, if $(C_1', C_2') \in \mathfrak{C}^*$ is such that $C_1' \supseteq C_1$ and $C_2' \subseteq C_2$, then $(C_1', C_2') \in \mathfrak{C}_x$. This condition is called *power monotonicity*, or briefly, *P-monotonicity*, and is needed for *strategy-proofness.*

Definition. A rule φ is in *Family* Φ^* if there exists a profile of power structures $(\mathfrak{C}_x)_{x \in A}$ such that for all $R \in \mathcal{D}$ and all $x \in A$, $x \in \varphi(R)$ if and only if $\left(N_x^G(R), N_x^B(R)\right) \in \mathfrak{C}_x$. In this case, we say that φ is associated with $(\mathfrak{C}_x)_{x \in A}$.

Our main objective is to identify members of *Family* Φ^* which are not manipulable through preference misrepresentations by any coalition. To state this axiom formally:

Coalitional strategy-proofness. For all $R \in \mathcal{S}^N$, all $N' \subseteq N$, and all $R'_{N'} \in \mathcal{S}^{N'}$, if $\varphi(R'_{N'}, R_{-N'})\ P_i\ \varphi(R)$ for some $i \in N'$, then $\varphi(R)\ P_j\ \varphi(R'_{N'}, R_{-N'})$, for some $j \in N'$.

Thus if a rule is coalitionally strategy-proof, then there can be no coalition and a preference misrepresentation by coalition members that can make every member weakly better off and some strictly better off; that is, there are no $N' \subseteq N$ and $R'_{N'}$ such that for all $i \in N'$, $\varphi(R'_{N'}, R_{-N'})\ R_i\ \varphi(R)$ and for some $j \in N'$, $\varphi(R'_{N'}, R_{-N'})\ P_j\ \varphi(R)$. We refer readers to Moulin (1993) for a survey of literature on *coalitional strategy-proofness. Family* Φ^* includes a wide spectrum of rules. There are a number of "nice" rules such as the "plurality", "unanimity" rules treating agents "symmetrically". On the other hand, there are other rules treating agents "asymmetrically". Here are most extreme cases. For all $i \in N$, all $R_i \in \mathcal{S}$, and all $\mathcal{X} \subseteq 2^A$, let $Max[R_i \colon \mathcal{X}]$ be the set of all best alternatives for R_i in $\mathcal{X}$. A rule φ is *dictatorial* if there exists $i \in N$ such that for all $R \in \mathcal{S}^N$, $\varphi(R) \in Max\left[R_i \colon 2^A\right]$. Let π be a permutation on N, interpreted as a priority ranking of agents: for each $k \in N$, the person with the k^{th} priority is given by $\pi(k)$. Let $R \in \mathcal{S}^N$. Let $M^1(R, \pi) \equiv Max\left[R_{\pi(1)} \colon 2^A\right]$. For all $k \in \{2, \cdots, n\}$, let $M^k(R, \pi) \equiv Max\left[R_{\pi(k)} \colon M^{k-1}(R, \pi)\right]$. A rule φ is *serially dictatorial with respect to* π if for all $R \in \mathcal{S}^N$ and all $k \in N$, $\varphi(R) \in M^k(R, \pi)$. Thus, the serially dictatorial rule with π allows the person with the highest priority $\pi(1)$ to choose his best alternatives. If there are multiple best alternatives for $\pi(1)$, the person with the next priority $\pi(2)$ is allowed to choose out of these alternatives. This process continues until the end of the priority. For example, when $N \equiv \{1, 2, 3\}$, for each $i \in N$, $\pi(i) = i$, and R is such that $N_a^G(R) = \{1\}$, $N_b^G(R) = \{2\}$, $N_c^G(R) = \{3\}$, $N_a^B(R) = \{2\}$, $N_b^B(R) = \{3\}$, $N_c^B(R) = \emptyset$. Then $M^1(R, \pi) = Max[R_1 \colon 2^A] =$

$\{\{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$,[6] $M^2(R,\pi) = Max[R_2\colon M^1(R,\pi)] = \{\{a,b\}, \{a,b,c\}\}$,[7] and $M^3(R,\pi) = Max[R_3\colon M^2(R,\pi)] = \{\{a,b,c\}\}$.[8] So the serial dictatorship selects $\{a,b,c\}$.

3 Results

We first show that *coalitional strategy-proofness* is too restrictive, when combined with the full-range condition. This is because, in our choice problems, the two conditions imply "efficiency". To define this axiom formally, a rule φ satisfies *efficiency* if for all $R \in \mathcal{S}^N$ and all $X \subseteq A$, if $X \; P_i \; \varphi(R)$ for some $i \in N$, $\varphi(R) \; P_j \; X$ for some $j \in N$. On the domain of linear separable preferences, Barberà, Sonnenschein, and Zhou (1991) show that dictatorial rules are the only *strategy-proof* and *efficient* rules (Theorem 4 in their paper). Therefore, since the full-range condition and *coalitional strategy-proofness* imply *efficiency*, only dictatorial rules remain. On the domain of all separable preferences, Ju (2003) shows that serially dictatorial rules are the only *strategy-proof* and *efficient* rules in *Family* Φ^* (Theorem 2 in his paper). Thus, if a rule in *Family* Φ^* satisfies both *coalitional strategy-proofness* and the full range condition, then it is *serially dictatorial.* However, serial dictatorship violates *coalitional strategy-proofness.* To explain this, consider the serially dictatorial rule associated with permutation π. Without loss of generality, assume $\pi(1) = 1$, $\pi(2) = 2$, and $\pi(3) = 3$. Let R be the profile in which all objects other than a are null for all agents, a is a null for agent 1, a is a good for agent 2, and a is a bad for agent 3. Then a will be chosen.[9] But if agents 1 and 3 make a coalition and report jointly (R'_1, R_3) where R'_1 is such that all objects except a are nulls and a is a bad, then a will not be chosen making 3 better off, without making 1 worse off. Therefore, we obtain:

Proposition 1. *No rule in Family Φ^* satisfies* coalitional strategy-proofness *and the full-range condition.*

Now we drop the full-range condition. To fully characterize *coalitional strategy-proof* rules in *Family* Φ^*, we first establish five useful lemmas. Let φ be associated with $(\mathfrak{C}_x)_{x\in A}$. We say φ is *discriminating in x* if $\mathfrak{C}_x$ is non-empty and $(\varnothing, N) \notin \mathfrak{C}_x$. Given a power structure $\mathfrak{C}_x$, a pair $(C_1, C_2) \in \mathfrak{C}_x$ is *minimal in* $\mathfrak{C}_x$ if there exists no $(C'_1, C'_2) \in$

[6] For agent 1, a is a good and b, c are nulls. Thus he is indifferent whether b or c are accepted or not.

[7] Since b is a good for R_2, agent 2 prefers accepting b. Since c is a null, he is indifferent whether c is accepted or not.

[8] Since c is a good for R_3, agent 3 prefers accepting c.

[9] For example, let $A = \{a,b\}$. Then $M^1(R,\pi) = Max[R_1\colon 2^A] = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$ because all objects are nulls for R_1. $M^2(R,\pi) = Max[R_2\colon M^1(R,\pi)] = \{\{a\}, \{a,b\}\}$ because a is a good for R_2. $M^3(R,\pi) = Max[R_3\colon M^2(R,\pi)] = \{\{a\}, \{a,b\}\}$. Thus the serial dictatorship must choose either $\{a\}$ or $\{a,b\}$, in whichever case a is chosen. When R'_1 is reported, $M^1(R'_1, R_{\{2,3\}}, \pi) = M^2(R'_1, R_{\{2,3\}}, \pi) = \{\emptyset, \{b\}\}$, $M^3(R'_1, R_{\{2,3\}}, \pi) = Max[R_3\colon \{\emptyset, \{b\}\}] = \{\emptyset, \{b\}\}$, and so the serial dictatorship must choose either $\emptyset$ or $\{b\}$ and a is not chosen.

$\mathfrak{C}_x \backslash (C_1, C_2)$ such that $C_1' \subseteq C_1$ and $C_2' \supseteq C_2$. A *minimal* pair (C_1, C_2) is a *minimalization of* (C_1', C_2') if $C_1 \subseteq C_1'$ and $C_2 \supseteq C_2'$.

Lemma 1. *Let* φ *be the* coalitionally strategy-proof *rule associated with* $(\mathfrak{C}_x)_{x \in A}$. *For all* $x \in A$, *if a pair* (C_1, C_2) *is minimal in a power structure* $\mathfrak{C}_x$, *then* (i) $|C_1| \leq 1$ *and* (ii) *either* $C_2 = \varnothing$ *or* $C_2 = N \backslash C_1$.

Proof. Let φ, $x \in A$, and (C_1, C_2) be given as above.

To prove (i), suppose by contradiction that there are two agents i, j in C_1. Let $R \in \mathcal{S}^N$ be such that all objects other than x are nulls for all agents and $\left(N_x^G(R), N_x^B(R)\right) = (C_1 \backslash i, C_2)$. Since (C_1, C_2) is minimal, $(C_1 \backslash i, C_2) \notin \mathfrak{C}_x$ so $x \notin \varphi(R)$. Now if agent i announces R_i' in which x is a good, then $\left(N_x^G(R_i', R_{-i}), N_x^B(R_i', R_{-i})\right) = (C_1, C_2)$. Thus $x \in \varphi(R_i', R_{-i})$, agent i being indifferent but agent j being better off. This contradicts *coalitional strategy-proofness.*

To prove (ii), suppose by contradiction that $C_2 \neq \varnothing$ and $C_2 \neq N \backslash C_1$. Then there exist $i \in C_2$ and $j \in N \backslash (C_1 \cup C_2)$. Let $R \in \mathcal{S}^N$ be such that all objects other than x are nulls and $\left(N_x^G(R), N_x^B(R)\right) = (C_1, C_2)$. Then $x \in \varphi(R)$. Now if agent j announces R_j' in which x is a bad, then $\left(N_x^G\left(R_j', R_{-j}\right), N_x^B\left(R_j', R_{-j}\right)\right) = (C_1, C_2 \cup j)$. Since (C_1, C_2) is minimal, $x \notin \varphi(R)$, agent i being better off and agent j being indifferent. This contradicts *coalitional strategy-proofness.* ■

It follows directly from Lemma 1 that:

Lemma 2. *Let* φ *be the* coalitionally strategy-proof *rule associated with* $(\mathfrak{C}_x)_{x \in A}$. *For all* $x \in A$, *if a pair* (C_1, C_2) *is minimal in a power structure* $\mathfrak{C}_x$, *then* $(C_1, C_2) = (i, \varnothing)$ *or* $(i, N \backslash i)$ *or* $(\varnothing, \varnothing)$ *or* $(\varnothing, N)$, *for some* $i \in N$.

Lemma 3. *Let* φ *be the* coalitionally strategy-proof *rule associated with* $(\mathfrak{C}_x)_{x \in A}$. *For all* $x \in A$ *and all* $i, j \in N$, $(i, \varnothing) \in \mathfrak{C}_x$ *if and only if* $(j, \varnothing) \in \mathfrak{C}_x$.

Proof. Suppose by contradiction that $(i, \varnothing) \in \mathfrak{C}_x$ and $(j, \varnothing) \notin \mathfrak{C}_x$. Let $R \in \mathcal{S}^N$ be such that all objects other than x are nulls and $\left(N_x^G(R), N_x^B(R)\right) = (j, \varnothing)$. So $x \notin \varphi(R)$. If agent i announces that x is a good, then x will be chosen, making j better off and i indifferent. This contradicts *coalitional strategy-proofness.* ■

Lemma 4. *Let* φ *be the* coalitionally strategy-proof *rule associated with* $(\mathfrak{C}_x)_{x \in A}$. *For all* $x \in A$ *and all* $i \in N$, *if a pair* $(i, N \backslash i) \in \mathfrak{C}_x$, *then for all* $(C_1, C_2) \in \mathfrak{C}^*$ *with* $C_1 \neq \varnothing$ *and* $i \notin C_2$, $(C_1, C_2) \in \mathfrak{C}_x$.

Proof. Let i and (C_1, C_2) be given as above. When $i \in C_1$, by P-monotonicity, $(C_1, C_2) \in \mathfrak{C}_x$. Now suppose $i \notin C_1$. Suppose by contradiction $(C_1, C_2) \notin \mathfrak{C}_x$. Since $C_1 \neq \varnothing$, there exists $j \in C_1$. Let $R \in \mathcal{S}^N$ be such that all objects other than x are nulls and $\left(N_x^G(R), N_x^B(R)\right) = (C_1, C_2)$. So $x \notin \varphi(R)$. If agent i announces that x is a good, then x will be chosen, making j better off and i indifferent. This contradicts *coalitional strategy-proofness.* ■

Lemma 5. *Let φ be the* coalitionally strategy-proof *rule associated with $(\mathfrak{C}_x)_{x\in A}$. Suppose that φ is discriminating in $x \in A$. Then* (i) *for all non-empty $C_2 \subseteq N$, $(\varnothing, C_2) \notin \mathfrak{C}_x$;* (ii) *for all $i, j \in N$, $(i, N\backslash i)$ is minimal in $\mathfrak{C}_x$ if and only if $(j, N\backslash j)$ is minimal in $\mathfrak{C}_x$.*

Proof. To prove (i), suppose $(\varnothing, C_2) \in \mathfrak{C}_x$. Then by Lemma 2, the only minimalization of $(\varnothing, C_2)$ is $(\varnothing, N)$. This contradicts that φ is discriminating in x.

To prove (ii), suppose by contradiction that $(i, N\backslash i)$ is minimal in $\mathfrak{C}_x$. By Lemma 4, $(j, N\backslash\{i,j\}) \in \mathfrak{C}_x$. Possible minimalization from $(j, N\backslash\{i,j\})$ are $(j, N\backslash\{i,j\})$, $(j, N\backslash j)$, $(\varnothing, N\backslash\{i,j\})$, $(\varnothing, N\backslash i)$, $(\varnothing, N\backslash j)$, and $(\varnothing, N)$. Since φ is discriminating in x, $(\varnothing, N) \notin \mathfrak{C}_x$. Hence by Lemma 2, $(j, N\backslash j)$ is minimal in $\mathfrak{C}_x$. ■

We now define five types of critical power structures. Note that when the power structure of an object is empty, the object is never chosen and that when $(\varnothing, N)$ is minimal in the power structure, the object is always chosen. A power structure is *indiscriminating* if it is empty or $(\varnothing, N)$ is minimal. A power structure satisfies *unanimity-only* if the set of minimal pairs is $\{(i, \varnothing) : i \in N\}$.[10] It satisfies *unanimity-only** if the set of minimal pairs is $\{(\varnothing, \varnothing)\}$. A power structure satisfies *all-veto* if the set of minimal pairs is $\{(i, N\backslash i) : i \in N\}$.[11] It satisfies *all-veto** if the set of minimal pairs is $\{(i, N\backslash i) : i \in N\} \cup \{(\varnothing, \varnothing)\}$.

We are now ready to prove the main result.

Theorem 1. *A rule in Family Φ^* satisfies* coalitional strategy-proofness *if and only if it is discriminating in at most one object and the power structure of this object satisfies unanimity-only or unanimity-only* or all-veto or all-veto*.*

Proof. We only prove the non-trivial direction. Let φ be the *coalitionally strategy-proof* rule associated with $(\mathfrak{C}_x)_{x\in A}$. Let A^* be the set of all objects in which φ is discriminating. Note that for all $x \in A\backslash A^*$, x is either chosen always or never chosen. Let $\bar{A} \subseteq A\backslash A^*$ be the set of all objects that are always chosen.

We first show $|A^*| \leq 2$. Suppose by contradiction $|A^*| \geq 3$. We embed φ in the model with the set of objects, A^*, as follows. Let ψ be the rule associating with each preference profile $R|_{2^{A^*}}$,[12] $\varphi(R) \cap A^*$. Note that by definition of *Family* Φ^*, for all R, R' with $R|_{2^{A^*}} = R'|_{2^{A^*}}$, $\varphi(R) \cap A^* = \varphi(R') \cap A^*$. Thus, ψ is well-defined. Since φ satisfies *coalitional strategy-proofness*, so does ψ. Since φ is discriminating in all objects in A^*, so is ψ. Thus the range of ψ is 2^{A^*}, that is, the full-range. Hence ψ is *efficient.* Since

[10] Here we use the term "unanimity" because to accept a, everyone must agree that a is not a bad.

[11] Here we use the term "veto" because everyone can veto the rejection of a by claiming it as a good.

[12] For all R_i, $R_i|_{2^{A^*}}$ is the reduced ordering of R_i on 2^{A^*}. For all R, $R|_{2^{A^*}}$ is the profile of such reduced orderings.

$|A^*| \geq 3$, then by the impossibility result in Ju (2003, Theorem 2), ψ is serially dictatorial. As explained before, the serially dictatorial rule ψ violates *coalitional strategy-proofness.*

To show $|A^*| \neq 2$, suppose by contradiction $|A^*| = 2$. Let $a, b \in A^*$. By Lemma 2, there can be four types of minimal pairs (C_1, C_2) in $\mathfrak{C}_a$ or $\mathfrak{C}_b$. Note that for all $i \in N$, the minimal pair $(i, \varnothing)$ is not compatible with any other types of minimal pairs. Similarly, for the type $(\varnothing, N)$. The only two types of minimal pairs that can coexist are $(\varnothing, \varnothing)$ and $(i, N \backslash i)$. Therefore, by Lemmas 2, 3, and 5, each of $\mathfrak{C}_a$ and $\mathfrak{C}_b$ satisfies one of *unanimity-only, unanimity-only**, *all-veto,* and *all-veto**. We now show that in all such cases, φ violates *coalitional strategy-proofness.*

Case 1. Each of $\mathfrak{C}_a$ *and* $\mathfrak{C}_b$ *satisfies unanimity-only or unanimity-only**. To show this, let R and $i, j \in N$ be such that $\{a,b\} \cup \bar{A}\ P_i\ \bar{A}$, $\{a,b\} \cup \bar{A}\ P_j\ \bar{A}$, $\left(N_a^G(R), N_a^B(R)\right) = (i, j)$, and $\left(N_b^G(R), N_b^B(R)\right) = (j, i)$. Then neither a nor b will be chosen by φ, and so $\varphi(R) = \bar{A}$. If agents i, j jointly report that both a and b are goods for them, then the choice becomes $\{a,b\} \cup \bar{A}$, which are preferred to $\bar{A}$ by both agents. This contradicts *coalitional strategy-proofness.*

Case 2. Each of $\mathfrak{C}_a$ *and* $\mathfrak{C}_b$ *satisfies all-veto or all-veto**. To show this, let R and $i, j \in N$ be such that $\bar{A}\ P_i\ \{a,b\} \cup \bar{A}$, $\bar{A}\ P_i\ \{a,b\} \cup \bar{A}$, $\left(N_a^G(R), N_a^B(R)\right) = (i, N \backslash i)$, and $\left(N_b^G(R), N_b^B(R)\right) = (j, N \backslash j)$. Then both a and b are chosen by φ and so $\varphi(R) = \{a,b\} \cup \bar{A}$. Since $(i, N \backslash i)$ and $(j, N \backslash j)$ are minimal, then when i reports a as a bad and j reports b as a bad, neither one of them will be chosen and the outcome becomes $\bar{A}$. Both i and j prefer $\bar{A}$ to $\{a,b\} \cup \bar{A}$, contradicting *coalitional strategy-proofness.*

Case 3. $\mathfrak{C}_a$ *satisfies unanimity-only or unanimity-only* and* $\mathfrak{C}_b$ *satisfies all-veto or all-veto**. To show this, let R and $i, j \in N$ be such that $a \cup \bar{A}\ P_i\ b \cup \bar{A}$, $a \cup \bar{A}\ P_j\ b \cup \bar{A}$, $\left(N_a^G(R), N_a^B(R)\right) = (j, i)$, and $\left(N_b^G(R), N_b^B(R)\right) = (j, N \backslash j)$. Then $\varphi(R) = b \cup \bar{A}$. If i reports a as a good or a null and j reports b as a bad or a null, then a will be chosen and b will not be chosen, making both agents better off. This contradicts *coalitional strategy-proofness.* ■

Remark 1. Combining the above result with Theorem 1 in Ju (2003), we obtain: a voting rule satisfies coalitional strategy-proofness and null-independence if and only if it is a rule in Family Φ^* that is discriminating in at most one object and the power structure of this object satisfies unanimity-only or unanimity-only* or all-veto or all-veto*.[13]

The main result and Remark 1 show that unlike strategy-proofness, coalitional strategy-proofness has a very strong implication. Such rules decide non-trivially only over at most a single object (decisions on others objects are trivial; always accepted or always rejected) and the non-trivial decision must either be based on the unanimity principle or give everyone a sort of veto power. Our result, though it is negative, is the first charaterization

[13] We refer readers to Ju (2003) for the formal definition of voting rules and null-independence.

result with coalition strategy-proofness on the domain of all separable preferences.

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