

## Welfare Effect of Labor Union in a General Equilibrium Efficient Contract Model<sup>\*</sup>

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**Abstract** This paper analyzes the effects of labor union in the steady state of an overlapping generation economy when a union and a firm determine wage and employment through an efficient contract. We find that when the wage set in the bargaining is the same as the competitive level, the steady state equilibrium through the efficient contract is the same as the steady state of competitive equilibrium. We also find that an increase in the bargaining power of the union improves the welfare of the representative generation in the new steady state with a sacrifice of the current old generation. Moreover, the command optimum maximizing the welfare of the representative generation in the steady state can be achieved with the efficient contract when the bargaining power of the union is maximal so that profit of the firm becomes zero.

**Keywords** labor union, efficient contract, general equilibrium.

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## 1. Introduction

Effects of labor union have often been analyzed in partial equilibrium models (see, e.g., McDonald and Solow (1981), Oswald (1985, 1993) and Pencavel (1985)).<sup>1</sup> In these models, efficiency has been investigated in an environment where firms and labor unions bargain over wages and employment. The right-to-manage contract where labor unions and firms bargain over wages and firms choose employment has been shown inefficient relative to the case of simultaneous bargaining over wages and employment in the form of efficient contracts. However, partial equilibrium models have limits in analyzing macroeconomic consequences of the existence of labor union.<sup>2</sup> This study analyzes, in an overlapping generation general equilibrium model, effects of labor union on interest rate, capital stock, output level, welfare of the representative generation and so on.

We first derive the competitive equilibrium in which the representative worker and the representative firm behave in a competitive way and compare it with the command optimum which maximizes the welfare of the representative generation in the steady state. The existence of profit in the competitive equilibrium creates suboptimality comparing with the command optimum because profit guarantees positive return on investment and hence equilibrium capital level is lower than the golden rule level. Thus, the introduction of the labor union can reduce suboptimality by decreasing profit of the firm.

We find that when the wage is set at the competitive equilibrium level, the steady state equilibrium through the efficient contract may be the same as the steady state without labor union. We also find that the welfare of the representative generation in the steady state increases in the bargaining power of the union because the increase in wage income and the decrease in dividend income lead to more savings. Moreover, the command optimum maximizing the welfare of the representative generation in the steady state may be achieved with the efficient contract when the bargaining power of

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<sup>1</sup>Espinosa and Rhee (1989) present a dynamic model where repeated bargaining occurs and find that employment is likely set on the contract curve even in the right-to-manage contract model due to reputation, implying that the efficient contract level of employment is sustainable. However, the feature of partial equilibrium analysis makes it hard to examine the macroeconomic effects of labor union in the economy.

<sup>2</sup>Some macroeconomic implications have been examined in partial equilibrium models (see, e.g., Pencavel (1985) and Blanchard and Fischer (1989, Chapter 9)). In the models, it is focused whether the existence of labor union can produce, in response of disturbances, smaller fluctuations in real wages and larger fluctuations in employment.

the union is maximal so that profit of the firm becomes zero.

The rest of this paper is organized as follows. Section 2 introduces an overlapping generation model and analyzes the steady state equilibrium when all the markets are competitive. It also analyzes the command optimum defined as the steady state which maximizes the welfare of the representative generation. In Section 3, we modify the model introduced in Section 2 to incorporate efficient bargaining on the determination of wage between the union and the firm and examine effects of labor union in the steady state. Section 4 concludes the paper.

## 2. Competitive Equilibrium and Command Optimum

In this section, we introduce an overlapping generation economy and analyze the steady state equilibrium and the command optimum steady state of the economy. Except the labor input, only one good exists in the economy. A part of the good is consumed and the remaining part is used as capital input to produce the good in the next period. A representative agent is born in each period and lives for two periods. The utility function of the representative agent who is born in period  $t$  is  $u(c_{1t}) + v(c_{2t})$ , where  $c_{1t}$  is his first period consumption and  $c_{2t}$  is his second period consumption. We assume that the agent supplies a unit of labor exogenously in the first period but cannot supply labor in the second period. We also assume that  $u(\bullet)$  and  $v(\bullet)$  are differentiable, increasing, and strictly concave and that  $\lim_{c \downarrow 0} u'(c) = \lim_{c \downarrow 0} v'(c) = \infty$  and  $\lim_{c \rightarrow \infty} u'(c) = \lim_{c \rightarrow \infty} v'(c) = 0$ .

There exists a representative firm with production function  $y = F(n, k)$ , where  $y$ ,  $n$  and  $k$  denote output, labor input and capital input, respectively. We assume that  $F(\cdot)$  is differentiable, increasing and strictly concave and that  $F(\cdot)$  satisfies the Inada condition,  $\lim_{n \downarrow 0} F_n(n, \bar{k}) = \lim_{k \downarrow 0} F_k(\bar{n}, k) = \infty$  and  $\lim_{n \rightarrow \infty} F_n(n, \bar{k}) = \lim_{k \rightarrow \infty} F_k(\bar{n}, k) = 0$  for all  $\bar{n}$  and  $\bar{k}$ . We also assume that the production function exhibits decreasing returns to scale in the region of interest. Thus, firms can earn positive profit in the competitive equilibrium.<sup>3</sup> Capital depreciates at rate  $\delta$ . Shares of

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<sup>3</sup>Some studies such as Basu and Fernald (1995) and Burnside et. al. (1995) empirically estimate the returns to scale in the U.S. industries and find that the assumption of decreasing returns to scale is not implausible.

the firm are traded in the stock market and the shareholder receives profit as dividend. We assume that all the markets are competitive.

The young agent supplies labor and receives wage. He consumes the good and decides savings in the form of renting capital to the firm and purchasing shares of the firm. The old agent receives capital with interest and dividend from shares and sells shares. let  $\kappa$  denote capital supplied by the young generation which will be used for next period production,  $z$  denote stock holdings,  $q$  denote stock price,  $w$  denote wage,  $r$  denote interest rate and  $\pi$  denote profit of the firm. Price of the good is normalized to be 1. Then, the agent born at  $t$  solves the following problem in competitive markets:

$$\max_{c_{1t}, c_{2t}, \kappa_t, z_t} u(c_{1t}) + v(c_{2t}), \quad (P1)$$

subject to

$$c_{1t} + \kappa_t + q_t z_t = w_t, \quad (1)$$

$$c_{2t} = (1 + r_{t+1})\kappa_t + (q_{t+1} + \pi_{t+1})z_t. \quad (2)$$

The intertemporal first order condition for the problem (P1) is

$$\frac{u'(c_{1t})}{v'(c_{2t})} = 1 + r_{t+1}. \quad (3)$$

The firm solves the following problem:

$$\max_{n_t, k_t} F(n_t, k_t) - w_t n_t - (r_t + \delta)k_t. \quad (P2)$$

The first order conditions for the problem (P2) are the followings:

$$F_n(n_t, k_t) - w_t = 0, \quad (4)$$

$$F_k(n_t, k_t) - (r_t + \delta) = 0, \quad (5)$$

where  $F_n$  and  $F_k$  denote  $\partial F / \partial n$  and  $\partial F / \partial k$ , respectively.

Then, the firm's profit can be calculated as

$$\pi_t = F(n_t, k_t) - w_t n_t - (r_t + \delta)k_t. \quad (6)$$

The equilibrium conditions for goods market, labor market, capital market and stock market, respectively, are the followings:

$$c_{1t} + c_{2t-1} + \kappa_t = F(n_t, k_t) + (1 - \delta)k_t, \quad (7)$$

$$n_t = 1, \quad (8)$$

$$k_{t+1} = \kappa_t, \quad (9)$$

$$z_t = 1. \quad (10)$$

We confine our analysis to the steady state, dropping the time subscripts. Our assumptions on the production function guarantees that the firm demands positive

amount of capital and  $\pi > 0$  in a steady state equilibrium. Moreover,  $r > 0$  in a steady state equilibrium because the agent would purchase shares of the firm rather than rent capital to the firm if  $\pi > 0$  and  $r \leq 0$ . Because the agent rents capital to the firm and purchases shares of the firm at the same time in a steady state equilibrium,

$$q = \frac{\pi}{r}. \tag{11}$$

We have 10 variables,  $n, k, c_1, c_2, \kappa, z, q, w, r, \pi$  and 11 equations (1)-(11). However, due to the Walras' law, we can drop one of Equations (7)-(10). Assume that after dropping one of Equations (7)-(10), the simultaneous equation system has a unique solution and denote the solution values by superscript  $c$ , for example  $n^c$ .<sup>4</sup>

Now, we consider the command optimum which maximizes the representative agent's utility in steady state, assuming that the social planner does not discount the utility of future generations. When there is no discount for the future, several different specifications of the objective have been proposed. (See, e.g., Rubinstein (1979).) The simplest one is the time-average criterion, where the social planner's object is to maximize  $\lim_{T \rightarrow \infty} (v(c_{2,-1}) + \sum_{t=0}^{T-1} (u(c_{1t}) + v(c_{2t})) + u(c_{1T})) / T$  which is  $u(c_1) + v(c_2)$  in the steady state. Thus, the command optimum can be derived by solving the following problem:

$$\max_{c_1, c_2, k} u(c_1) + v(c_2) \tag{P3}$$

subject to

$$c_1 + c_2 + k = F(1, k) + (1 - \delta)k. \tag{12}$$

First order conditions of the problem (P3) can be summarized as

$$\frac{u'(c_1)}{v'(c_2)} = 1, \tag{13}$$

$$F_k(1, k) - \delta = 0. \tag{14}$$

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<sup>4</sup>Conditions for existence and uniqueness of the steady state equilibrium in an overlapping-generations model are established when the production function exhibits constant returns to scale. (See, e.g., Galor and Ryder (1989).) If the production function exhibits constant returns to scale, then in a competitive equilibrium, there is no profit and stock price is zero. Existence and uniqueness of the steady state equilibrium in our model are not guaranteed. However, we could obtain an equilibrium in a numerical analysis with CRRA utility functions  $u(c) = v(c) = \frac{c^{1-\gamma}}{1-\gamma}$  ( $\gamma > 0$  and  $\gamma \neq 1$ ) and a generalized Cobb-Douglas production function  $F(n, k) = n^\alpha k^\beta$  ( $\alpha, \beta > 0$  and  $\alpha + \beta < 1$ ) for various values of the parameters  $\alpha, \beta$  and  $\gamma$ .

Equations (12)–(14) determine consumptions  $c_1$  and  $c_2$  and capital input  $k$ . Our assumptions on production function and utility function guarantee the existence of the unique solution of the problem (P3), which is also the unique solution of the simultaneous equation system (12)–(14). We denote the solution values by superscript  $o$ , for example,  $c_1^o$ .

As the (modified) golden rule implies, Equations (3) and (5) coincide with Equations (13) and (14), respectively, if  $r = 0$ . This observation suggests that the suboptimality of the competitive equilibrium stems from the fact that the interest rate  $r$  is not equal to the social planner's discount rate 0. The young agent has two ways of savings. One is renting capital to the firm and the other is purchasing shares of the firm. He receives interest from capital and dividends from stocks. In a steady state equilibrium with both assets valued, the interest rate is positive and the equilibrium is suboptimal because profit of the firm is positive. In the next section, we will examine whether the interest rate and hence the degree of suboptimality decreases as profit decreases due to the bargaining between labor union and the firm.

### 3. Efficient Contract between Labor Union and Firm

In this section, we modify the model introduced in the previous section to incorporate a contract on wage determination between the labor union and the firm. Though there are competing models for collective bargaining, we adopt the efficient contract model. The agent (who is also the labor union) and the firm set wage and labor input level (0 or 1). Given  $r$ , the agent and the firm agree on a wage–employment pair which is Pareto optimal to the two parties among all wage–employment pairs, expecting optimal choice of  $k$  by the firm after the bargaining. Thus, efficiency of the efficient contract does not mean efficiency of the resulting general equilibrium. In our model, a wage–employment pair is Pareto optimal to the two parties among all wage–employment pairs if and only if  $n = 1$ . Let  $w^e$  denote the wage set in the bargaining.<sup>5</sup> Then, the bargaining outcome consists of

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<sup>5</sup>We may introduce a parameter which represents the bargaining power of labor union and let the wage set in the contract increase in the parameter. However, for simplicity, we identify an increase in the bargaining power of labor union with an increase in the level of the wage set in the contract.

$$n = 1, \tag{15}$$

$$w = w^e. \tag{16}$$

Given  $w^e$ , the representative agent's problem is the same as the problem (P1) in the previous section. The firm's problem is also the same as the problem (P2) in the previous section except that the firm chooses only  $k$  in the current efficient contract model though it chooses both  $n$  and  $k$  in the previous section. Profit of the firm under efficient contract is calculated by Equation (6) in the previous section.

Equilibrium conditions for all the market but the labor market are the same as those in the previous section. Equation (15) in the efficient contract model agrees with Equation (8) which is the equilibrium condition for the labor market in the previous section.

In sum, we have the same 10 variables  $n, k, c_1, c_2, \kappa, z, q, w, r, \pi$  as in the previous section, and 11 equations (1)–(3), (5)–(11) and (16). Due to the Walras' law, we can drop one of Equations (7), (9) and (10). Thus, the only difference between the simultaneous equation system under efficient contract and that in the previous section is the difference between Equation (4) and Equation (16). Assume that the simultaneous equation system in this section has a unique solution and denote the solution values by superscript e, for example,  $n^e$ .<sup>6</sup>

Now, we analyze effects of labor union. Theorem 1 will show that when the wage is set at the level of a competitive equilibrium so that  $w^e = w^c$ , the steady state equilibrium with union is the same as that without union.

**Theorem 1**     If the wage in the efficient contract is set at the same level of the competitive equilibrium, then the steady state equilibrium with union is the same as that without union.

**Proof**     By the definition, the 10 values with superscript c satisfy Equations (1)–(11). If  $w^e = w^c$ , then the 10 values with superscript c satisfy Equations (1)–(3), (5)–(11) and (16). Therefore, if  $w^e = w^c$ , then the 10 values with superscript c agree to those with superscript e.  $\square$

Effects of a change in the bargaining power of the labor union on the economy can be analyzed as follows. Relying on the Walras' law, exclude Equation (7) from the simultaneous equation system. Using 5 equations (6) and (8)–(11) to remove 5 variables

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<sup>6</sup>We could obtain an equilibrium in the numerical analysis with CRRA utility functions and a generalized Cobb–Douglas production function. (See footnote 4.)

$n$ ,  $\kappa$ ,  $z$ ,  $q$  and  $\pi$  in 4 equations (1)-(3) and (5), we have the following 4 equations with 5 variables  $r$ ,  $k$ ,  $c_1$ ,  $c_2$  and  $w$ :

$$c_1 + k + \frac{F(1, k) - w - (r + \delta)k}{r} = w, \quad (17)$$

$$c_2 = (1 + r)k + \frac{1 + r}{r}(F(1, k) - w - (r + \delta)k), \quad (18)$$

$$\frac{u'(c_1)}{v'(c_2)} = 1 + r, \quad (19)$$

$$F_k(1, k) - (r + \delta) = 0. \quad (20)$$

By the implicit function rule, we can derive the following results from Equations (17)-(20):

$$\frac{dr}{dw^e} = \frac{F_{kk}(r(1+r)u'' + r(1+r)^2v'')}{A} < 0, \quad (21)$$

$$\frac{dk}{dw^e} = \frac{r(1+r)u'' + r(1+r)^2v''}{A} > 0, \quad (22)$$

$$\frac{dc_1}{dw^e} = \frac{r^2(1+r)^2v'' + r^2F_{kk}v'}{A} > 0, \quad (23)$$

$$\frac{dc_2}{dw^e} = \frac{r(1+r)(-F_{kk}v' + ru'')}{A}, \quad (24)$$

Where  $A = r^2F_{kk}v' + r^2(1+r)^2v'' - (1+r)(F - \delta k - w^e)F_{kk}v'' + (r^2 - (F - \delta k - w^e)F_{kk})u'' < 0$ . Moreover, we can get the following result from Equations (23) and (24):

$$\frac{d(u + v)}{dw^e} = \frac{r^2(1+r)((1+r)^2v''v' + F_{kk}(v')^2 + v'u'')}{A} > 0. \quad (25)$$

The results in Equations (21), (22) and (25) are summarized in Theorem 2.

**Theorem 2** As the wage set in the efficient contract increases, capital increases, interest rate declines and utility of the representative generation increases in the new steady state.

Theorem 2 shows that an increase in the bargaining power of the labor union improves the representative generation's welfare by decreasing the profit of the firm. The increase in the wage squeezes profit of the firm, implying a lower return on the shares, which in turn increases savings in the form of capital and decreases the interest rate. As the interest rate declines, there is a switch from the consumption in old period to young period. The lower interest rate tends to increase consumption when young



with the positive wealth effect. However, the effect on consumption when old is ambiguous since the wealth effect and the substitution effect work in an opposite direction to each other. The higher level of capital increases output and the utility of the representative agent. In sum, an increase in wage leads to a new steady state in which capital is higher, interest rate is lower and utility of the representative generation is higher. In our model, the new steady state is reached from the next period without any transition period after the wage increases, though the increase in the utility of the representative agent comes with the decrease in the current of old generation's utility.

Now, the following theorem examines whether the command optimum can be achieved through the efficient contract when the bargaining power of the union is maximal so that profit of the firm becomes zero.

**Theorem 3** Assume that the command optimum level of consumption when old is not less than the command optimum capital stock, that is  $c_2^o \geq k^o$ . Then, the command optimum can be reached if in the efficient contract, wage is set to make profit of the firm to be zero.

**Proof** Let  $w^m$  be the wage which makes  $\pi^e$  equal 0. Then, the following values satisfy the simultaneous equation system which consists of Equations (1)–(3), (5)–(11) and (16):  $n^e = 1$ ,  $k^e = k^o$ ,  $c_1^e = c_1^o$ ,  $c_2^e = c_2^o$ ,  $\kappa^e = k^o$ ,  $z^e = 1$ ,  $q^e = c_2^o - k^o$ ,  $w^e = w^m$ ,  $r^e = 0$  and  $\pi^e = 0$ . For  $q^e$  to be non-negative, we need that  $c_2^o \geq k^o$ .

□

Theorem 3 shows that the command optimum can be achieved through the efficient contract if the bargaining power of the union is maximal and the command optimum level of consumption when old is not less than the command optimum capital stock. The intuition behind Theorem 3 is as follows. When  $\pi^e = 0$ , the return on the shares is 0 and hence the interest rate should be zero, in order for both capital and shares to exist. As the (modified) golden rule suggests, when the interest rate is zero, the conditions for the command optimum are satisfied in the efficient contract equilibrium. More precisely, if  $r = 0$  and  $n = 1$ , then Equations (12)–(14) which determine the command optimum coincide Equations (7), (3) and (5) in the simultaneous equation system which determine the efficient contract equilibrium. Note that the shares with zero return play the

important role of cost-free storage of value for achieving the command optimum in the market equilibrium.

## 4. Conclusion

We have analyzed the effects of labor union in a general equilibrium efficient contract model. In our model, positive profit creates suboptimality compared with the command optimum because it leads to a lower wage income of the young and a higher dividend income of the old and hence less saving than in the command optimum. Therefore, the labor union can reduce the suboptimality by decreasing profit of the firm. We have found three results. First, when the wage set in the bargaining is the same as the competitive level, the steady state equilibrium through the efficient contract is the same as the steady state of competitive equilibrium. Second, the welfare of the representative agent increases in the bargaining power of the union because the increase in wage income and the decrease in profit lead to more saving. Last, the command optimum can be achieved with the efficient contract when the bargaining power of the union is maximal and the command optimum level of consumption when old is greater than the command optimum capital stock.

This paper has some shortcomings which we need to overcome in future research. Labor supply is exogenously given in our model. It is worth analyzing a model in which labor supply is endogenously determined. In our model, there is only one firm which is unionized and has decreasing returns to scale technology. It is also worth analyzing a two sector model in which one sector has a unionized firm with decreasing returns to scale technology and the other sector has a non-unionized firm with constant returns to scale technology.

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