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The Welfare Effect of Sorting in the Labor Market: A Search-Theoretic Analysis

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Abstract We construct a labor-barter economy under search friction, introducing quality heterogeneity and information asymmetry into the economy. Under this environment, we examine the welfare effect of a sorting device. The main modifications to the standard sorting model are that i) an agent's type is endogenously determined by his optimal behavior and ii) rejecting trade is an option available for agents, so a "lemon market" problem may possibly emerge. Our findings are as follows: First, sorting can prevent the labor market from collapsing even under a serious information asymmetry by providing a more favorable condition for good-type workers. Second, sorting has a positive welfare effect, as opposed to the standard sorting models, by disciplining bad-type workers and encouraging trade. Finally, introducing a sorting device can significantly reduce the equilibrium unemployment rate by alleviating the informational friction and creating a positive externality, which prevents a possible coordination failure caused by that friction.

Keywords Sorting, Information Asymmetry, Discipline Effect, Trade-Enhancing Effect, Coordination Failure

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1. Introduction

Generally, signalling and screening models about education have some unique welfare implications, which are different from the traditional human capital model. The "sorting" model[1] predicts that, in most cases, total social welfare of the separating equilibrium is lower than that of the pooling equilibrium.[2] When this sorting is applied to schooling decisions, it means that workers decide to stay in school longer not because it would enhance their human capital, but because it would send firms a signal that they are better workers than the uneducated, being qualified for a higher wage. So without any productivity effects of education, the net gain of the educated is solely redistributed from the uneducated, incurring a net social loss by the additional cost of schooling; consequently, the separating (sorting) equilibrium can have a lower social return than the pooling (non-sorting) equilibrium.

In this paper, we show that this welfare implication of the sorting model can be revised by modifying its fundamental assumptions: the fixed type and the no “lemon market” assumptions. First, in the standard sorting model, a worker’s unobserved characteristics (or his type) are given as a model environment and do not change endogenously. But it is a more plausible assumption that workers can also determine their own types endogenously by investing in education. In fact, every level of schooling activities has both a human capital effect and a signalling effect; most of the studies on sorting, like in Spence(2002), do not exclude the cases where education enhances human capital and also serves as a signal. However they do not

[1] Strictly speaking, the signalling model and the screening model are separate, not combined, models even though they both share a common object of study. In the signalling model, workers send a signal in order to give firms information about unobserved characteristics associated with a work performance. In the screening model, a firm will demand a signal from workers and screen them according to that signal. In fact, this signalling and screening occurs simultaneously, and both have some dynamic feedback from each other. Weiss (1995) referred to both signalling and screening as “sorting”. In this paper, I use the term “sorting model” to refer to both signalling and screening models.

[2] It is not always true. There are some cases that sorting itself contributes to welfare enhancement; for example, sorting can make a job matching more efficient when individuals differ in comparative skills. See Stiglitz (1975).

endogenize the agent's type so that agents can choose their own type by investing in education while trading off the cost and benefit of signalling. With some evidence that a higher level of education, such as college or graduate school, seems to have more of a signalling effect than lower levels, such as primary or secondary education,[3] it would be a more realistic assumption that workers can be formed into good or bad workers as a consequence of their performance in the lower level of education; thus, considering possible signalling costs and the future labor market state, they can choose their own type, on which they base their signalling decision later. In this respect, this paper's model combines two extreme hypotheses about the function of education: the human capital hypothesis and the signalling hypothesis. In addition to that, this model is differentiated from other studies on sorting in that it allows for the endogenous determination of human-capital investment, which is affected by the cost and benefit of signalling.

Second, in a pooling equilibrium, a good-type worker always "sells his labor" at a lower price than its real value—the marginal productivity of labor. This implicit tax incurred by the imperfect information subsidizes bad workers. But this assumption rules out the possibility that a good worker rejects an employment opportunity because the worker's labor is undervalued in the pooling equilibrium; or in other words, good-type workers might prefer remaining unemployed because an expected wage is below their reservation wage. In this respect, as in the example of the "lemon market" in Akerlof (1970), there always exists a possibility of a market collapse under information asymmetry. The standard sorting model does not say anything about this possibility; here, we relax this "no-lemon-market" assumption of the standard sorting model so that workers can choose whether to trade or not, based on the net expected benefit from the trade. Note that this model is a generalization of the lemon market

[3] Later, I will discuss some empirical evidence to support this.

of Akerlof (1970), since agents can choose not only whether to trade or not, but also in which probability to trade by selecting mixed strategy. Once the rejection of trade is allowed, the social cost of information asymmetry gets bigger in the pooling equilibrium. There is room for welfare improvement by introducing an information-revealing device such as sorting. Aside from the human capital effect, this assumption provides another independent underpinning for welfare enhancement of a sorting economy because the presence of a sorting mechanism prevents market collapse and encourages trade.

With these two main modifications of assumptions, we construct a labor-barter economy with search friction, very similar to the "coconut economy" of Diamond (1982). Then, we introduce quality heterogeneity and information asymmetry into this model á la Williamson and Wright (1994). Given these, the objective of this paper is to measure the welfare effect of sorting in the labor market. More specifically, we investigate how durable the economy with sorting is against the informational friction and which welfare implications sorting has under information asymmetry. Our findings are that, first, sorting can prevent the labor market from collapsing even under a serious informational friction by providing a more favorable condition for good-type workers. Second, sorting has a positive welfare effect, as opposed to the standard sorting models, by disciplining bad-type workers and encouraging trade. Finally, the introduction of a sorting device can reduce the equilibrium unemployment rate significantly by alleviating the informational friction and creating a positive externality, which prevents the possible coordination failure caused by that friction.

The main implications of this paper can be summarized as follows: First, even though sorting itself does not contribute to human capital accumulation, it raises social welfare because it disciplines potential bad-type workers and encourages people to be good-type workers. Also, sorting has an independent welfare-improving effect by alleviating the information problem and creating a positive externality, which consequently extend job opportunities.

Since Spence (1973) and Stiglitz (1975) have laid the foundation for the sorting hypothesis, there have been a great number of empirical studies examining the relevance and the relative importance of a sorting behavior in real labor market. While there is, to the best of my knowledge, no theoretical study investigating the welfare effect of sorting under search friction in the labor market, many search-theoretic models have dealt with issues related to this paper's model—such as coordination failure, the universal acceptability as a trade-enhancing mechanism, and the discipline effect under quality uncertainty, etc. For example, Diamond (1982) shows that multiple equilibria may arise when strategic complementarities exists[4] and that those equilibria can be Pareto-ranked, and consequently there might be a role for a stabilization policy, creating positive externalities and inducing agents to coordinate on the "good" equilibrium among them. Cooper and John (1988) generalize Diamond's model in the context of trade externalities and strategic complementarities. On the other hand, the signal in this model plays a similar role to money in the monetary search model because sorting speeds up trade by the universal acceptability of the signal as money does in those models by its universal acceptability. Kiyotaki and Wright (1993) show that money can raise the welfare of economy by mitigating the "absence of double coincidence of wants" problem. The model has been extended to explore the welfare role of money under the qualitative uncertainty of commodity by Williamson and Wright (1994). They show that the discipline effect of money imposes an effective cash-in-advance constraint on agents with a bad commodity, leading to improvement in welfare even though the private-information problem about the quality of commodities is very severe. In this paper, we try to combine these seemingly unrelated features of those

[4]Strategic complementarities denote the situation that an increase in the action of one player increases the marginal return to the other player's action. See Cooper and John (1988).

search models in order to have better understanding of the welfare effect of sorting in the labor market.

This paper is organized as follows. In the next section, we review the welfare effect of sorting under the standard sorting model. We see that sorting in the model has a negative welfare effect. In Section 3, we construct a labor-barter economy with search friction and information asymmetry. Then, in Section 4, we compare the sorting equilibrium with the non-sorting equilibrium in terms of welfare. Section 5 interprets the model, comparing it to real-world labor markets, and discusses the model's implications for unemployment policy.

2. The Standard Sorting Model and Its Welfare Implications

We review the welfare effect of sorting in the standard sorting model. The discussion here is basically from Spence (1973). There is a continuum of infinitely lived workers, whose population is normalized to 1. Any individual in the population can be described by a single property, productivity n. Suppose that there are only two types of risk-neutral workers in the economy: a good type and a bad type. Bad-type and good-type workers are denoted by their productivity levels, n_1 and n_2 ($n_1 < n_2$). One half of the population consists of bad-type workers and the other half consists of good-type workers. A worker knows his own type, but firms cannot observe the type of workers. Assume that there is a signalling process that is available at a cost and that the signalling cost is inversely correlated with workers' productivity:

$$C(n) = \frac{z}{n} \tag{2.1}$$

Here, z denotes the level of education and let's assume that it is fixed to constant z^* for simplicity's sake. Say that workers' productivity and the signalling cost have the following relationship:

$$w^* > n_2 - \frac{z^*}{n_2} > n_1 > n_2 - \frac{z^*}{n_1} \tag{2.2}$$

where $w^* = \frac{1}{2}(n_1+n_2)$, i.e., w^* denotes the average productivity of this economy. Under this condition, we now establish that there exist two equilibria depending on a firm's conditional belief about a worker's type.

The first equilibrium is a pooling equilibrium. Suppose that firms believe that if a worker is educated, he should be a good type with a probability of 1 and that if a worker is not educated, there is a 0.5 probability that he is a good type and a 0.5 probability that he is a bad type. Under this firm's belief, each type of worker makes the optimal decision about education. A good-type worker chooses not to be educated because the net income from education $n_2 - \frac{z^*}{n_2}$ is lower than that without education w^*. A bad-type worker also prefers not being educated because the net income from education $n_2 - \frac{z^*}{n_1}$ is way below the income level without education w^*. So, nobody chooses to be educated, and every worker gets paid w^* in this equilibrium. This confirms firms' conditional belief about workers' productivity, meaning that this equilibrium is stable.

The second equilibrium is a separating equilibrium. Suppose that firms believe that if a worker is educated, he should be a good type with a probability of 1 and that if a worker is not educated, he should be a bad type with a probability of 1. Under this firm's belief, a good-type worker had better be educated because the net income from education $n_2 - \frac{z^*}{n_2}$ is higher than that without education n_1. A bad-type worker chooses not to be educated because the net income from education $n_2 - \frac{z^*}{n_1}$ is below the income level without education n_1. So, only good-type workers are educated and bad-type workers are never educated in this equilibrium. This also confirms firms' conditional belief about workers' productivity, meaning that this equilibrium is also stable.

But when the private return is compared across these equilibria, the separating equilibrium is unambiguously Pareto inferior to the pooling equilibrium because, in the separating equilibrium, both types of workers have lower incomes than in the pooling equilibrium. If we compare the social welfare of each equilibrium, with education costs incurred, welfare in the separating equilibrium $W^S = 0.5(n_1 + n_2 - \frac{z^*}{n_2})$ is lower than welfare in the pooling equilibrium $W^P = 0.5(n_1 + n_2)$. In this case, both types of workers become poorer in the separating equilibrium due to a high cost of education. But even if the education cost is lower, satisfying $w^* < n_2 - \frac{z^*}{n_2}$, so that a good-type worker always gets better by signalling and no pooling equilibrium exists, the separating equilibrium is still inferior to the pooling equilibrium in terms of social welfare unless there are other social benefits from signalling.

Here, we can notice how the two main assumptions of the standard sorting model function. First, workers' type is fixed and exogenously determined, so the fraction of each type of worker is constant. Additionally, firms cannot observe this type of worker. It is a typical example of a "adverse selection" problem if there is no information-revealing device, such as sorting mechanism. Second, in the pooling equilibrium, good-type workers remain in the market even though their labor is undervalued relative to its true productivity. But, under this circumstances, there always exists the incentive for a worker to give up selling his labor and shut down trade, as in the lemon market. In the next section, we modify these two assumptions and integrate that modification into the labor-barter economy with search friction á la Williamson and Wright (1994).

3. Model

This model is along the lines of Diamond's coconut model in which agents cannot consume products of their own labor; they must trade with other agents for their consumption and there is a search friction in meeting other traders. On the other hand, this model is a variant of Williamson and Wright (1994) in which there are different types of agents, and when agents meet each other, they cannot perfectly identify the other trader's type perfectly. One difference from Williamson and Wright's (1994) model is that agents in this model trade their labor itself instead of the product of their labor. But the main difference is in the objective of the model, because we confine our attention to the welfare role of sorting in a labor market rather than that of money in a commodity market.

3.1. Non-sorting Economy

There is a continuum of infinitely-lived and risk-neutral agents, whose population is normalized to 1. Agents are heterogeneous in their types and there are two types of agents: a good type or a bad type. Let p be the proportion of good-type agents and $1-p$ be the proportion of bad-type agents. Because workers cannot consume the product of their own labor, agents must meet other agents and trade their labor, consuming the product from it. In each period, agents meet pairwise, at random and must decide whether or not to trade once they meet each other. Trades take place if and only if it is mutually agreeable. In any meetings between agents, there is a probability θ that one agent can identify the other's type. So, $\theta < 1$ implies a degree of information asymmetry. After one agent examines the type of the other agent, they simultaneously announce whether or not they wish to trade. If the identified type is good, agents agree to trade, and if it is bad, agents do not trade. Let Σ be the probability that an agent agrees to trade even if the type of the counterpart is not identified. Let σ be an individual's best response to the other agent's Σ; σ denotes the agent's trade strategy under information asymmetry and $\sigma < 1$ means that rejecting trade is possible, unlike in the standard sorting model. If both agents agree to trade (a trade takes place), a good-type agent produces one unit of a good commodity yielding utility u for the other agent while a bad-type agent produces one unit of a bad commodity yielding utility 0. Once the commodity is consumed, agents can choose to be either a good type with a cost of γ or a bad type with no cost. This is also the main modification of the standard assumption because workers' type is given to, not chosen by, workers in the standard sorting model.

The symmetric equilibrium of this barter economy is characterized by the pair of (p, Σ), which consists of a fraction of good-type agents p and the agent's equilibrium strategy $\Sigma = \sigma$. We look for the stationary Nash equilibria in which strategies and probabilities of meeting are time-invariant and expectations are rational. We confine our attention to the case where good-type and bad-type agents coexist ($0 < p < 1$) and trade is somewhat restricted by information asymmetry ($0 \leq \Sigma < 1$) because our objective is to see the welfare role of sorting in the presence of serious information asymmetry.

The maximization problem of each agent is described as the following value functions. Let V_g be the value at the end of a period for a good type agent and V_b for a bad type agent. Let $W = Max(V_g - \gamma, V_b)$ represents the value function for an agent who just consumed the commodity from trade and is in position to decide whether to be a good type or a bad type.

$$\begin{aligned} rV_g = & \quad \theta p(\theta + (1-\theta)\Sigma)(u + W - V_g) \\ & +(1-\theta)Max_{\sigma}\sigma[p(\theta + (1-\theta)\Sigma)(u + W - V_g) + (1-p)(W - V_g)] \end{aligned} \tag{3.1}$$

$$rV_b = p(1-\theta)\Sigma(u + W - V_g) \tag{3.2}$$

Equation (3.1) shows that the flow return to a good-type agent, rV_g, consists of these three terms: The first term is the probability of meeting an identified good-type agent, θp, multiplied by the probability that the other agent is willing to trade, $\theta + (1-\theta)\Sigma$, and multiplied by the expected net gain from trade, $u + W - V_g$. The second term is the probability that the agent cannot identify the other agent, $1-\theta$, multiplied by the expected gain from choosing optimal the σ. The expected gain (or loss) consists of one from trade with a good-type agent, $p(\theta+(1-\theta)\Sigma)(u+W-V_g)$, and the other from trade with a bad-type agent, $(1-p)(W-V_g)$. Equation (3.2) shows that the flow return to a bad-type agent, rV_b, comes from the expected gains, $u + W - V_g$, when agents meet a good-type agent who does not identify its type and agrees to trade, which happens with the probability $p(1-\theta)\Sigma$.

There are three types of potential equilibria: case 1 ($p = 1, \Sigma = 1$), case 2 ($0 < p < 1, \Sigma = 1$), and case 3 ($0 < p < 1, 0 < \Sigma < 1$). Here, we focus on case 3 because we are only interested in the case of serious informational friction.[5] Any σ is the best response if an agent is indifferent about agreeing or disagreeing to trade with an unidentified agent. By (3.1), it implies,

$$p = \frac{\gamma}{\gamma + [\theta + (1-\theta)\Sigma](u-\gamma)} \tag{3.3}$$

Any $0 < p < 1$ is the best response if $V_g - \gamma = V_b$. Using this and (3.3), we can solve for the equilibrium Σ:

[5] The case 3 is an unique equilibrium if θ is sufficiently low. See Williamson and Wright (1994).

$$\Sigma = \frac{(\theta-\gamma)\theta(u-\gamma) - r\gamma}{(1-\theta)[\gamma + (1-\theta+r)(u-\gamma)]} \tag{3.4}$$

One can show that $0 < \Sigma < 1$ and, therefore the equilibrium (case 3) exists, if and only if θ satisfies the following relationship:[6]

$$0.5r + \frac{0.5}{u-\gamma}\sqrt{r^2(u-\gamma)^2 + 4r\gamma(u-\gamma)} < \theta < \frac{(1+r)u}{(2u-\gamma)} \tag{3.5}$$

The welfare of this non-sorting economy is expressed as

$$Z_{non} = p(V_g - \gamma) + (1-p)V_b \tag{3.6}$$

3.2. Sorting Economy

Now, we are ready to introduce the sorting mechanism to model above. In this section we demonstrate that with a severe information problem, there can exist sorting equilibria under the circumstances in which there are no non-sorting equilibria and that even if a non-sorting equilibrium exists there may also exist a sorting equilibrium that renders a higher welfare. Now, we have another type of agents, neither good nor bad, delivering a signal. Let's refer to that type of agent as a signal type. Let s and V_s be the fraction of signal-type agents in a total population and their value. This type of agent is always identifiable. We keep our attention only to equilibria where a signal is universally accepted with no additional cost by good-type agents, implying $V_s > V_g$, because, if it is not, we cannot characterize any separating equilibria at all. In addition, this universal acceptability can also be interpreted in an alternative way that an agent with a signal is more acceptable to a good-type agent than an agent without

[6] For the derivation, see Williamson and Wright (1994).

it, because agents screen other agents' type by their signal. Note that, even for a good-type agent, there are some opportunity costs in acquiring a signal since he can produce nothing while he is carrying a signal.

Since we are interested mainly in the demand side of the signal—whether each type of agent chooses to send a signal or, in the alternative interpretation, how acceptable an agent with signal is to each type of non-signal agents—we assume that the signal-type agent always agrees to trade when the other's type is not identified. Thus, with the universal acceptability of the signal, when a good-type agent meets a signal-type agent, their decisions are quite trivial; both always agree to trade. Meanwhile, when a bad-type agent wants to trade for a signal, there are some disutility costs with a magnitude of δ; therefore, a bad-type agent has a non-trivial decision problem about whether to acquire a signal or not. Let Ω be the probability that a bad-type worker agrees to trade for a signal. Thus, a low Ω implies that a bad type agent is more reluctant to trade for a signal; consequently, agents with a signal are more likely to trade with a good type, rather than with a bad type, so that they can expect a higher gain from trade than those without a signal. In this sense, Ω can also be interpreted as a degree of screening. Thus, we implicitly assume that the degree of screening is closely related to how willing each type of agents is to acquire a signal, where both are parameterized by Ω. To interpret it in the real-world terms, when less bad-type workers are likely to acquire a signal,—low Ω—more firms are willing to employ the worker with a signal and pay more wages—in the model, which means that agents with a signal have a better chance to trade with a good type than those without a signal—because now the signal becomes a better indicator showing that its carrier is a good type. Finally, we also restrict our attention to equilibria with $0 < p < 1$, as in the non-sorting economy.

In this sorting economy, an agent's optimization problem is characterized by the following

Bellman equations:

$$rV_g = s(V_s - V_g) + (1-s)\theta p[\theta + (1-\theta)\Sigma](u-\gamma)$$
$$+(1-s)(1-\theta)Max_\sigma \sigma\{p[\theta + (1-\theta)\Sigma](u-\gamma) - (1-p)\gamma\} \quad (3.7)$$

$$rV_b = (1-s)p(1-\theta)\Sigma u + s(1-\theta)Max_\omega \omega(V_s - \delta - V_b) \quad (3.8)$$

$$rV_s = (1-s)\theta p(u - \gamma + V_g - V_s)$$
$$+(1-s)(1-\theta)[p(u-\gamma+V_g-V_s) + (1-p)\Omega(-\gamma+V_g-V_s)] \quad (3.9)$$

Equation (3.7) shows that the net return to a good-type agent consists of the following three terms. The first term is the probability of meeting a signal-type agent, s, multiplied by the expected net gain from trade, $V_s - V_g$. The second term is the probability that the agent will meet another good-type agent he identifies, $(1-s)\theta p$, multiplied by the probability that the other agent also agrees to trade, $\theta + (1-\theta)\Sigma$, and multiplied by a net gain from trade $u-\gamma$. The third term is the probability that the agent will meet a non-signal type agent who is not identified, $(1-s)(1-\theta)$, multiplied by the expected net gain from choosing the optimal σ. The expected net gain consists of one from trade with a good-type agent, $p(u-\gamma+V_g-V_s)$, and the other from trade with a bad-type agent, $(1-p)\Omega(-\gamma+V_g-V_s)$. Likewise, Equation (3.8) shows that the expected return to a bad-type agent consists of the following two terms. One is the expected net gain from trade with a good-type agent, $(1-s)p(1-\theta)\Sigma u$. The other is the expected net gain from the trade with a signal-type agent, $s(1-\theta)Max_\omega \omega(V_s-\delta-V_b)$, depending on the optimal signalling choice, ω. Equation (3.9) represents that the expected return of a signal-type agent consists of gains from the trade with an identified good-type agents, $(1-s)\theta p(u-\gamma+V_g-V_s)$, and gains from the trade with an unidentified non-signal type agents, $(1-s)(1-\theta)[p(u-\gamma+V_g-V_s)+(1-p)\Omega(-\gamma+V_g-V_s)]$.

Table 1. Candidate Sorting Equilibria

$0<\Phi<1$	$\Omega=0$ Perfect Screening	$\Omega=\Phi$ Imperfect Screening
$\Sigma=0$	Equilibrium 4	Equilibrium 3
$\Sigma=\Phi$	Equilibrium 2	Equilibrium 1

The equilibria in which we are interested are summarized in Table 1. We exclude the case of no screening, which is $\Omega=1$. Then, as in the case of the non-sorting economy, we also exclude the case where there is no restriction in trade, which is $\Sigma=1$, since we only focus on the situation where information asymmetry is so severe that it restrains trade. Furthermore, it can be shown that whenever there is a sorting equilibrium with $\Sigma=1$, there also exists a non-sorting equilibrium which entails a higher level of welfare. Since we are interested in equilibria where the sorting mechanism potentially leads to an increase in welfare, we do not pursue this case.

For expositional purposes, we will characterize equilibrium 1, which is the case with $0<\Sigma<1$ and $0<\Omega<1$. Any $\sigma\in(0,1)$ is the best response if an individual is indifferent about trading or not trading with an unidentified agent, from (3.7), which implies

$$p[\theta+(1-\theta)\Sigma](u-\gamma)=(1-p)\gamma \tag{3.10}$$

From this we can express p as a function of Σ, which leads to

$$p(\Sigma) = \frac{\gamma}{\gamma + [\theta + (1-\theta)\Sigma](u-\gamma)} \tag{3.11}$$

Now, $0 < p < 1$, $0 < \Omega < 1$ implies $V_g - \gamma = V_b$ and $V_s - \delta = V_b$. Substituting these two conditions and (3.11) into the Bellman equations (3.7)-(3.9), we can solve for equilibrium Σ, Ω also:

$$\Sigma = \frac{s(\delta-\gamma) + (1-s)\theta^2 p(u-\gamma) - r\gamma}{(1-s)p(1-\theta)(u-(u-\gamma)\theta)} \tag{3.12}$$

$$\Omega = \frac{(1-s)p(u-\delta) - r\delta - (1-s)p(1-\theta)\Sigma u}{(1-s)(1-\theta)(1-p)\delta} \tag{3.13}$$

As we can see, equilibrium p is always between 0 and 1. So, with given parameters, we can calculate the range of equilibrium θ under which both $0 < \Sigma < 1$ and $0 < \Omega < 1$ hold. And the welfare of this sorting economy (Z_s) is given by

$$Z_s = sV_s + (1-s)p(V_g - \gamma) + (1-s)(1-p)V_b \tag{3.14}$$

4. Numerical Results

4.1. Parameterization

Parameters used in the numerical exercise[7] are summarized in Table 2. Williamson and Wright (1994) have done similar numerical exercises to examine the welfare role of money under the quality uncertainty as in this model. We adopt the same value of r, θ as in Williamson and Wright (1994) for the welfare calculation. Note that this level of θ implies quite a severe

[7]Because this parameterization is just for a numerical execise aiming to confirm some normative implications of sorting, not a calibration exercise for a positive analysis, what matters the most is not whether those parameter values are consistent with a real-world labor market, but whether those same values of parameters are consistently applied to every case and whether or not they are viable .

information problem because the case 3 mentioned in the previous chapter is the unique non-sorting equilibrium at this level of θ.

Table 2. Parameter Values for Numerical Exercise

	r	γ	u	δ	θ	s
Value	0.01	0.01	0.02	0.017	0.2	0.2

If we see γ as a flow cost for accumulating human capital, which is the same as the flow return from the human capital under the capital market competitiveness, assuming that all of the return for capital is equal regardless of its type, we can set γ equal to the real interest rate, r. We pick up the value of u and δ satisfying $u = 2\gamma$ and $\delta = 1.7\gamma$; thus, $\delta - \gamma = 0.7\gamma$ is an additional cost of signalling for a bad-type agent.[8] This model parameterization ($u = 2\gamma$ and $\gamma = r$, leading into $u = \gamma + r$) implies constant returns to scale technology under the competitive factor market, and the same shares for two inputs—physical and human capital—because the compensations for both inputs, γ and r, dissipate total output, u, and each compensation is same. For calculating the range of equilibrium θ for each equilibria, we use $s = 0.2$, meaning that 20% of agents on average send a signal in the model economy. It is very hard to calibrate this ratio from data because signalling is such an abstract and broad concept that telling whether some degrees of education functions as a signal in the real-world labor market is almost impossible. Note that we set $u > \gamma$, and $u > \delta$ for excluding the trivial cases. We conceive an economy where a signal is universally accepted by a good-type agent, implying $V_s > V_g$, and there are some screenings, implying $V_s - \delta \leq V_b$, and good types and bad types coexist in the economy, implying $V_g - \gamma = V_b$. From these conditions, we can see that $\delta > \gamma$

[8]When we examine some other values of δ—ranging from $\delta = 1.1\gamma$ to $\delta = 1.9\gamma$—there is no significant change in the main results.

should hold because

$$V_g < V_s \leq V_b + \delta = V_g - \gamma + \delta \Rightarrow \gamma < \delta \tag{4.1}$$

4.2. Durability of Sorting Economy

The table below shows the range of equilibrium θ for the non-sorting economy and the sorting economy (equilibrium 1 and 2 in Table 1).[9] As we see in the table, the sorting economy is more durable against severe information asymmetry than the non-sorting economy is.

Table 3. Range of θ for Non-sorting and Sorting Economy

	Non-sorting Economy	Sorting Economy	
		Equilibrium 1	Equilibrium 2
Range of θ	(0.11, 0.67)	(0.01, 0.19)	(0.19, 0.64)

Without sorting, no equilibrium can be supported if information asymmetry is too severe: for instance, if $\theta < 0.11$. It is because, if θ is too low, being a bad type and taking a chance of cheating other agents are so profitable that any equilibrium where some agents choose to be a good type ($0 < p$) cannot be supported. On the other hand, even when θ is sufficiently low, the availability of the sorting mechanism makes it more favorable to be a good type than in the case without that mechanism. Now, a good-type agent has more options other than hoping to identify another good-type agent or taking a chance on meeting another unidentified good-type agent with a probability of σ. A good-type agent can choose to acquire the signal without any additional cost and wait for another good-type agent to appear. Because of the

[9] We choose these two sorting equilibria because the other two equilibria trivially support any θ as an equilibrium value, no matter how low it is. As mentioned, $s = 0.2$ for each sorting equilibrium.

universal acceptability of the signal, an agent with a signal is more likely to trade with another good-type agent than a good-type agent is in the non-sorting economy. Specifically, under the non-sorting economy, the probability that a good-type agent meets and trades with another good-type agent is $p(\theta + (1-\theta)\Sigma)^2$. But, under the sorting economy, the probability that an agent with a signal meets and trades with another good-type agent is $(1-s)p$. So, unless s is too high, an agent with a signal is more likely to meet a good agent than a good-type agent is in the non-sorting economy. This is similar to the separating equilibrium of the standard sorting model where a worker sending a signal is paid according to his true productivity because firms screen workers by their signals. The sorting mechanism provides this favorable condition for good-type agents; as a result, good-type agents can survive even under severe information asymmetry in which they would not survive otherwise. To conclude, a sorting device makes the economy more sustainable against information problem by providing more favorable conditions for good-type agents.

4.3. The Welfare Effect of Sorting

Figure 1 illustrates the positive welfare effect of the sorting mechanism compared to the non-sorting economy. As shown in the figure, the existence of a sorting device enhances the welfare of the economy relative to that of the non-sorting economy, which is contrary to the standard sorting model. For all the ranges of s, the welfare of the sorting economy is higher than that of the non-sorting economy and the difference between them is maximized on some plausible level of s, around 0.4.

This welfare improvement can be explained by the following two factors. First, the introduction of a sorting device has a differential welfare effect for each type of agents. A sorting device imposes a great deal of discipline on a bad-type agent as well as benefits on a good-type agent. It takes a significant cost (δ) for a bad-type agent to acquire a signal, contrary to a good-type agent. This discipline becomes more harsh as s increases because signalling prevails so much that the chance for a bad-type agent to cheat a good-type agent—to be subsidized by information asymmetry—is getting thinner. This discipline effect of the sorting device discourages agents from being a bad type and exhibiting opportunistic behavior. Figure 2 shows the share of bad-type agents in equilibrium 1 and 2 for some ranges of s. As s is moderately high, the share of bad agents is significantly lower than in the non-sorting economy. Note that there are some ranges of s where the share of bad types is higher than in the non-sorting economy. This is because introducing a sorting device has both direct and indirect effects. The direct effect is that sorting encourages good-type agents and disciplines bad-type agents. Once a good-type agent entertains the favorable conditions and, as a consequence, workers become more willing to be good types, it also indirectly increases the value of bad-type agents because now they have more chances to meet a good-type agent and cheat him. Moreover, if more good-type agents are showing up, then an unidentified agent is more likely to be acceptable, which also encourages bad-type agents to take a chance of cheating. Nonetheless, this indirect effect is predominated by the direct effect as s is increasing, because this high s imposes serious discipline on bad-type agents.

Second, sorting has a positive effect on the frequency of trade. This trade-enhancing effect consists of two components: one comes from the universal acceptability of the signal and the other from a positive externality caused by the discipline effect of sorting. Due to the universal acceptability of the signal, sorting itself raises the probability of trading. In the non-sorting economy, a good-type agent faces the informational friction, measured by θ, hindering him from trading with an unidentified agent. In the sorting economy, rather than reluctantly taking a risk of being cheated by a bad agent, a good-type agent now can find a signal-type agent who wants his labor, selling it for a signal, and then find another good-type agent with a higher probability, trading his signal for consumption goods. As the introduction of money resolves the problem of "double coincidence of wants" and improves trade[10], so the presence of a sorting device facilitates trade under information asymmetry. Meanwhile, once a sorting device disciplines bad-type agents and begins to drive them out as mentioned above, it increases the value of being a good-type agent; in other words, a decrease in the number of bad-type agents causes a positive externality. As a good-type agent now becomes more optimistic about the quality of his trade partner, so a good-type agent is more likely to agree to trade with an unidentified agent, which implies the higher Σ. Once other good-type agents increase their probability of accepting the unidentified agent, it is also optimal for an agent to raise his own probability of accepting (strategic complementarity), which increases the Σ of the whole economy more quickly. Figure 3 confirms this fact. Over the whole range of s, the level of the equilibrium Σ is higher than that of the non-sorting economy and is increasing in s unless it is too high. Note that the equilibrium Σ as a function of s has a peak point. Under low levels of s, the indirect effect mentioned above retards the driving-out effect against bad-type agents. Under sufficiently high levels of s, most good-type agents have already acquired a signal, so the remaining agents are highly likely to be bad types; thus, agents have no reason to accept an unidentified agent. To see how much sorting encourages trade, Figure 4 presents the ratio of realized matches to total matches.[11] Sorting obviously boosts the frequency of trade even under serious information asymmetry. The frequency of trade drastically increases until s reaches about a half, then it keeps some steady level. Note that this high frequency of

[10] See Kiyotaki and Wright (1993) for the welfare effect of money in the presence of the double-coincidence problem.

[11] Realized matches denote the matches in which both agents agree to trade. For details on this concept, see the next section.

trade also reflects the effect resulting from the universal acceptability of the signal.

These two welfare-enhancing factors are related to the differences in assumptions from the standard sorting model. Unlike in the standard model, agents' type and each type's share in the economy is endogenously determined by the optimizing behavior of agents. Thus, when the sorting device is in effect, imposing some discipline on bad-type agents, it encourages human-capital accumulation, leading to a decrease in the number of bad-type agents after all. Another difference is the assumption on the trade structure. In the standard model, a good-type worker always sells his labor even if his labor is undervalued because of information asymmetry. So the standard model is equivalent to assuming $\Sigma = 1$ under any circumstance in this model. However, if one's labor is evaluated lower than its true productivity due to the information problem, the lemon market may emerge; that is, the market may possibly collapse. In this model, we allow for this possibility by imposing $\Sigma < 1$. Under this condition, the introduction of a sorting device provides a roundabout through which agents can avoid the collapse of market and keep trading despite a severe information problem. Moreover, once the discipline effect of the sorting device drives out bad-type agents, it leads to an increased probability of accepting Σ, which means the extension of the opportunity to trade.

Figures 5 and 6 show how important the extension of trade opportunity is for enhancing welfare. Figure 5 shows the welfare of the economy in the equilibria with $\Sigma = 0$ (Equilibrium 3 and 4 in Table 1). In these equilibria, the opportunity to trade with an unidentified agent is perfectly shut down, which implies that the market is collapsing because of information asymmetry (as in the lemon market). With a perfectly collapsing market, the sorting economy has even lower welfare than the non-sorting economy. It implies that it is no use driving out bad-type agents if the market is perfectly collapsing due to the pessimistic belief about agents' quality; in other words, there is little independent effect of discipline without any trade-enhancing effect.

Figure 6 decomposes the welfare effect into two components: one from the discipline effect and the other from the trade-enhancing effect of sorting. The dotted line denotes the welfare of the sorting economy in the equilibrium 1 and 2 when the share of bad-type agents is restricted to equal that of the non-sorting economy. Thus, in this case, sorting has only trade-enhancing effects. Next, we decompose this trade-enhancing effect into those two components; the starred line denotes the welfare of the sorting economy in the equilibrium 1 and 2 under the additional restriction that the accepting probability, Σ, is equal to that of the non-sorting economy. In this case, sorting improves trade only by the universal acceptability of the signal, while driving out bad-type agents does not contribute at all in creating a positive externality or extending opportunities to trade. When we compare the welfare of the sorting economy in the restricted case (dotted line) with that in the unrestricted case (solid line), we can see that a relatively larger amount of welfare improvement comes from the trade-enhancing effect of sorting than from the discipline effect. In fact, the trade-enhancing effect takes account of all the improvement in welfare at the lower level of s and gets dampened as s increases, while the discipline effect gets more striking at the higher level of s. This is because the share of bad-type agents starts getting smaller than that in the non-sorting economy only at the higher level of s, as we see in Figure 2. As shown in Figure 4, sorting remarkably raises the frequency of trade mainly when s is relatively low. By comparing the dotted line and the starred line, we can guess how much the increase in the accepting probability, Σ,—as a result of the positive externality—improves welfare. The trade-enhancing effect due to the increase of the accepting probability is dominant when s is low and gets smaller as s is increasing. At the higher level of s, the higher Σ induces only a smaller additional welfare gain because there remain only a few good-type agents.

Another implication is that the welfare of the sorting economy does not improve if s is too high. It is a result of the unit-inventory assumption, which is common in many search models. The assumption means that an agent carrying a signal cannot simultaneously become other types of agents ready for production, whether it is a good type or not. Thus, if there are too many agents with a signal, it crowds out good-type agents as well, leading to a overall decrease in social welfare. This assumption is commonly made for the simplicity of analysis, especially in many money search models, but in this model, it has empirical justification. If we think that the signalling in the model corresponds to schooling in the real world, it can be quite a natural assumption because most schooling should be finished before people go out for a job in the real world.

5. Interpretation of the Model and Its Unemployment Policy Implications

Since this model is quite abstract and somewhat stylized, so we try to interpret it in the context of the real-world labor market and explore a few implications for unemployment policy. Let's go over how the model parameter and environment can be interpreted in a real-world context.

Good-type or bad-type agents can be regarded as newcomers in the labor market. Good-type agents can be seen as potential job-seekers who finished relatively successful primary and secondary education and accumulated some levels of human capital needed for getting a job and maintaining it. Investing in human capital takes some time and psychic costs, which is parameterized by γ here. We see in the model that if good-type agents are traded—a trade in the model corresponds to an employment contracting in the real labor market—they produce a valuable commodity with a worth of u. Bad-type agents are potential job-seekers who did a poor job in their primary and secondary schooling, having failed to accumulate any human capital. Thus, there is no investment cost incurred, and they cannot function well in their job even if luckily employed. Likewise, a bad-type agent in the model produces a commodity, but it is worthless for other traders.

When job-seekers are matched to a job, employers try to screen their applicants by examining applicants' observable characteristics. But, under no circumstances can workers' types be perfectly identified due to the presence of some unobserved characteristics that affect job performance. Thus, there always exists certain amount of information asymmetry about the type of job applicants. The model parameterizes this degree of information asymmetry by $\theta < 1$, implying that a low θ means a low observability of workers' characteristics. Once an applicant's type is not identified, an employer must make a critical decision about whether or not to hire him. Analogously, agents choose the optimal probability to accept an unidentified agent, Σ, in the model. Once the employer decides to hire an applicant, the applicant works and performs according to his type.

But potential job-seekers who just finished high school have an option other than looking for a job. They can choose to enter college or even go further to graduate school. If we assume that a college education or above has no productivity effect and only contributes to sending a signal about a student's ability, the potential job-seeker's decision about attending college or not is analogous to the agent's decision problem in the model: whether to trade for a signal or not. Here we impose an assumption that primary and secondary education have some human capital effect—so it can change a worker's type—and that achievement in those earlier stages of education affects the cost of a college education, while a college education or above functions as a signal only. This assumption make it possible for workers to choose their own type by deciding how much to invest in primary and secondary education, taking into account the trade-off between the cost and benefit of college education. This assumption can also be empirically justified as follows; if we think that previously accumulated human capital has some spillover effects on the acquisition of new knowledge, as is commonly assumed in many endogenous growth models, there is no doubt that achievement in the earlier stages of education affects learning productivity in the latter stages of education. In addition, many empirical studies[12] suggest that the signalling effect of education is more striking at the higher levels of education, such as college or above, than at the lower levels of education.

By these assumptions, once potential job-seekers decide to attend college, good and bad job-seekers face different cost functions for education. Accumulated human capital makes it easier for good job-seekers to get an education than it is for bad job-seekers. So, college education imposes a relatively higher cost on bad job-seekers, which is parameterized as δ in the model. Thus, δ in the model consists of the catch-up cost for accumulating human capital, γ, plus some additional cost incurred by lack of human capital or delayed education, $\delta - \gamma$. Since bad job-seekers recognize this higher cost of college education, they choose whether or not to enter college by trading off the expected benefit against the cost of college education. This real-world decision problem is modeled as the signalling strategy of a bad-type agent parameterized by Ω.

After graduating from college or above, job-seekers now can send a signal about their ability and look for a job. Potential employers try to screen applicants by the signal they send and job-seekers with a signal tend to earn a higher wage—they either earn a higher wage in the same job or get a high-wage job. In the model, this screening process is modeled as the universal acceptability of the signal by a good-type agent and the lower probability for a bad-type agent

[12] Hartog (1984), Hungerford and Solon (1987), Belman and Heywood (1991), and Heywood (1997) found that the sheepskin effect for college education is much larger and more significant than the effect for grade school or high school. The sheepskin effect denotes the prediction that wages will rise faster with extra years of education when the extra year also conveys a certificate. Many empirical studies have estimated and tested this sheepskin effect to examine the existence and degree of signalling and screening behavior.

to acquire a signal, which is parameterized by $\Omega < 1$.

We reinterpreted the model in the context of the relation to the real-world education and labor market. Now, we want to investigate some implications of the model for unemployment policy. As we see above, whether one is employed or not in the model depends on the trading decision of agents. Suppose we do a thought experiment in which a social planner draws two agents randomly, matches them, and then lets them choose their optimal trade strategies. The social planner can calculate the probability that the match fails to trade. Let's define this probability as the unemployment rate of this economy. Figure 7 shows the unemployment rate of an economy in the non-sorting and sorting equilibria. Note that it is just the mirror image of Figure 4. Definitely, the sorting mechanism contributes to extending the opportunity to trade and to lowering the equilibrium unemployment rate. The introduction of the sorting mechanism disciplines bad-type agents so that they may tend to change their type to a good type or acquire a signal, because sending a signal increases the chance of trade—by its universal acceptability—and being a good type is more favorable under the sorting mechanism for the mentioned reason. Then, the ratio of good-type agents in the population is going up and, by a strategic complementarity, leads to a higher accepting probability Σ; this accepting probability increases even faster with the positive externality it entails. In this process, a higher frequency of trade between an increased portion of good-type and signal-type agents—once matched, the probability of trade between them is one—and a higher Σ jointly lowers the equilibrium unemployment.[13] In sum, this model implies that the unemployment caused by serious information asymmetry—in other words, the lemon market problem—can be addressed by the introduction of a sorting device, which disciplines and induces a bad-type agent to acquire a signal or change his type.

[13]This transmission mechanism of sorting effect is illustrated in Figure 8.

This model has something in common with Diamond's (1982) model in that both models emphasize the role of the positive externality under a coordination failure problem. But both models have different sources of the coordination failure. In Diamond (1982), the main reason for higher equilibrium unemployment is the coordination failure in total demand, so a demand stabilization policy is needed to create the positive externality of total demand. Meanwhile, in this model, severe information asymmetry is the reason for higher equilibrium unemployment, so some information-revealing devices, like signalling and screening, can reduce unemployment by preventing coordination failure caused by the informational friction.

6. Conclusion

We construct a labor-barter economy with search friction and information asymmetry about agents' types. Then, we introduce a sorting device so that we can investigate the welfare effect of sorting under information asymmetry. The main difference from the standard sorting model is that i) an agent's (or a worker's) type is endogenously determined by his optimal behavior and ii) the rejection of trade is a available option for agents, so the lemon market may possibly emerge. With these modifications of assumptions, contrary to the standard sorting model, the availability of a sorting device enhances the welfare of the economy by disciplining bad agents and extending the opportunity to trade. It also has some policy implications because it can reduce the equilibrium unemployment rate of the economy by alleviating the informational friction and creating the positive externality, which prevents the possible coordination failure.

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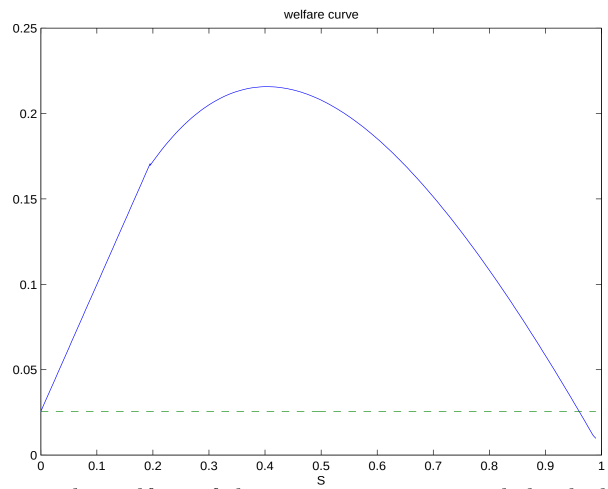
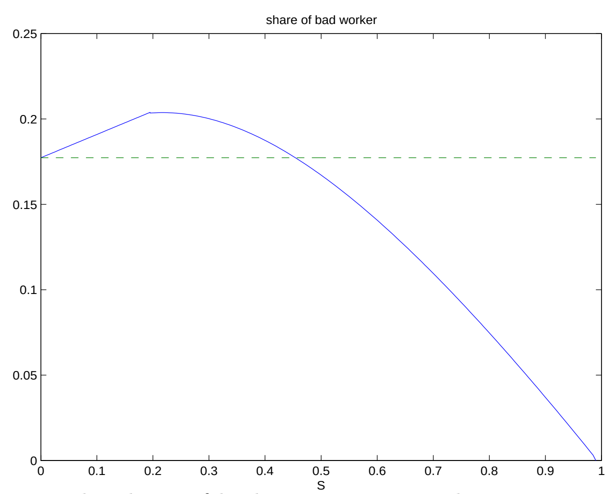
Figure 1. Welfare of Sorting Economy (Equilibria 1 and 2)



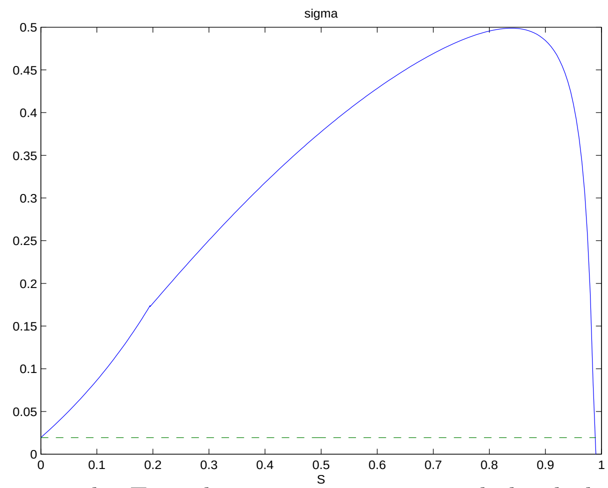
Note: The solid line denotes the welfare of the sorting economy and the dashed line denotes the welfare of the non-sorting economy.

Figure 2. Share of Bad-Type Agents



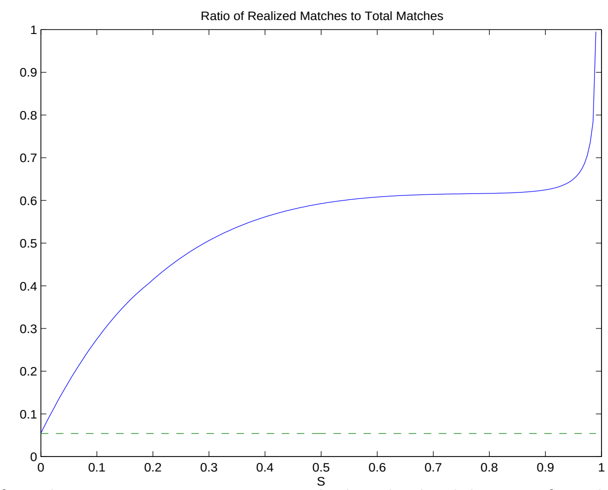
Note: The solid line denotes the share of bad-type agents in the sorting economy and the dashed line denotes that in the non-sorting economy.

Figure 3. Probability of Accepting an Unidentified Trader (Σ)



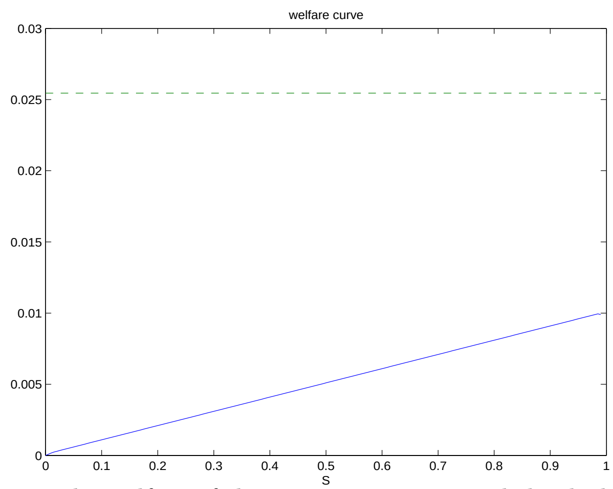
Note: The solid line denotes the Σ in the sorting economy and the dashed line denotes that in the non-sorting economy.

Figure 4. Trade-Enhancing Effect



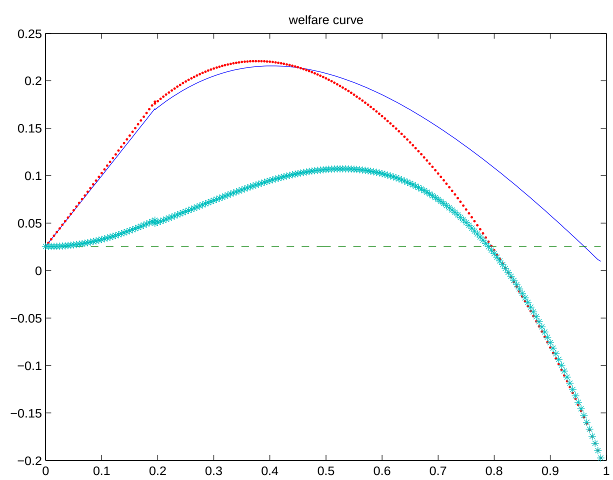
Note: The solid line is for the sorting economy and the dashed line is for the non-sorting economy.

Figure 5. Welfare of Sorting Economy (Equilibria 3 and 4)



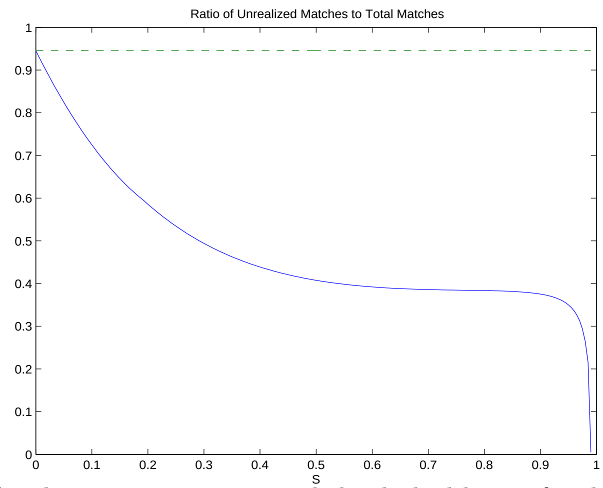
Note: The solid line denotes the welfare of the sorting economy and the dashed line denotes the welfare of the non-sorting economy.

Figure 6. Decomposition of Welfare Effect



Note: The dotted line denotes the welfare of the sorting economy with restriction that the share of bad-type agent is equal to that of the non-sorting economy. The starred line denotes the welfare of the sorting economy with an additional restriction ($\Sigma = \Sigma_{non}$), where Σ_{non} is the accepting probability of the non-sorting economy. The solid line denotes the welfare of the sorting economy without any restrictions and the dashed line denotes the welfare of the non-sorting economy.

Figure 7. Unemployment Rate



Note: The solid line is for the sorting economy and the dashed line is for the non-sorting economy.

Figure 8. Transmission Mechanism of Sorting Effect