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Business Cycle and Asset Prices: A Computable General Equilibrium Analysis with Agency Costs and Habit Formation*

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Abstract For the purpose of explaining both business cycles and asset returns, we examine a real business cycle (RBC) model with habit-augmented preferences and *endogenous* costs of adjusting the capital stock. Following the agency-cost model of Carlstrom and Fuerst (1997), capital adjustment costs are affected by the level of entrepreneur's net worth such that an increase in net worth (following a positive productivity shock) lowers agency costs associated with external financing, and hence makes it easier to expand the capital stock. Along with the restricted labor supply, the model resolves the asset pricing puzzles of the consumption-based model in the sense that the implied stochastic discount factor (or pricing kernel) reaches the Hansen-Jagannathan(1991) volatility bound. Further, this improvement in the asset pricing dimension is achieved without reducing its business cycle performance such as output and consumption volatility. This is in a sharp contrast to the standard RBC model with the reduced-form adjustment cost technology where sufficiently low supply elasticity of capital (or persistently high capital adjustment costs) is required to generate the equity premium at the expense of low output volatility. Here, the capital supply is highly elastic with respect to Tobin's *q* under the plausible calibrations of the structural parameters affecting endogenous capital adjustment costs. The sluggish behavior of net worth, as a shifter of the capital supply curve, is the key mechanism by which capital adjustment is delayed, hampering consumption smoothing desired by households with habit persistence preferences. The agency-costs model reveals that a small curvature in the capital adjustment cost function, viewed as crucial for understanding the fluctuations in Tobin's *q*, can be also consistent with both the historical equity premium and the key business cycle facts.

Keywords agency cost, habit formation, asset price, business cycle

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1. Introduction

Recent studies in macroeconomics have directed their efforts to explain asset returns in production economies where output and consumption choices are determined as agents' optimal response to shocks. They include Lettau and Uhlig (2000, 2002), Jermann (1998), Boldrin, Christiano, and Fisher (2001). A rich set of empirical findings on asset returns is used to discriminate various candidate models of macroeconomic fluctuations, or often called real-business-cycle (RBC) models.

The key question is: what version of the RBC model can explain both business cycles and asset returns? Boldrin, Christiano, and Fisher (2001) suggests three modifications of the standard RBC model (e.g. Hansen 1985) to improve the asset pricing implications without reducing its business cycle performance: (i) habit persistence in preferences; (ii) features of technology that hamper the ability to use variations in leisure or labor supply for consumption smoothing; and (iii) features that lower the elasticity of capital supply in a transient manner. The inclusion of habit persistence in preferences alone turns out to be very difficult to account for substantial risk premium in production economies.[1] This is because endogenous consumption choices become much smoother with habit formation. The literature has found that, in addition to habit, features (ii) and (iii) of technology are required to explain the equity premium by reducing household's ability to smooth consumption.

This paper considers the agency-cost model of Carlstrom and Fuerst (1997) with habit persistence preferences, as a candidate to explain both business cycles and the asset

[1] Constantinides (1990) has shown that once a habit is added to the standard model with power utility and lognormal distribution, the equity premium puzzle of Mehra and Prescott (1985) is resolved. More recently, Campbell and Cochrane (1999) consider a different habit formation model which avoids a high and volatile risk-free rate. However, these models specify an exogenously given consumption process and use the first-order necessary condition of a representative consumer to derive the asset-pricing implications.

returns.[2] Here, the capital adjustment costs are modeled in a less restrictive fashion than the standard reduced-form approach in Jermann (1988) and Boldrin, Christiano, and Fisher (2001). Capital adjustment costs (in the form of agency costs) are endogenized such that an increase in entrepreneurial net worth, following a positive technology shock, reduces the agency costs associated with external financing and hence makes it easier to expand the capital stock. Endogenous agency costs are crucial for the model to generate a close match with business cycle facts, including strongly autocorrelated output growth. For example, changes in agency costs give rise to a significant propagation mechanism: impact of productivity shocks are propagated by inducing households to delay their investment decisions until agency costs are at their lowest several periods after the initial shock. [3]

We first examine the model with flexible labor supply. As elaborated in Lettau and Uhlig (2000) and Boldrin, Christiano, and Fisher (2001), households with habit-augmented preferences fulfill consumption smoothing by adjusting labor-leisure hours. This then implies a negligible volatility of the implied stochastic discount factor, reaching far below the Hansen-Jagannathan (1991) volatility bound (or the HJ bound in short). The model would further require some form of restricted labor supply to explain the equity premium puzzle.

Along with the restricted labor supply, the model resolves the asset pricing puzzles

[2]Bernanke and Gertler (1989) first developed a general equilibrium model with incomplete asset markets in which entrepreneurs' investment must partially rely on external finance (in addition to internal funds or net worth) which is limited because of the agency costs involved in the financial contract. Calstrom and Fuerst (1997) builds on Bernanke and Gertler (1989) by constructing a calibrated, computable general equilibrium model in which endogenous agency costs enhance propagation of productivity shocks by affecting entrepreneurial net worth.

[3]As documented by Cogley and Nason (1993, 1995) and Rotemberg and Woodford (1996), most quantitative business cycle models in the real-business-cycle (RBC) tradition share the feature that model-generated output data exhibit dynamic properties nearly identical to those of the underlying exogenous shocks, so that economic mechanisms play a minimal role in propagating shocks. Among the other models of endogenous propagation mechanisms are the factor-hoarding model of Burnside, Eichenbaum, and Rebelo (1993) and Burnside and Eichenbaum (1996), labor market search models of den Haan, Ramey, and Watson (1997), Andolfatto (1996), and Merz (1995).

of the consumption-based model in the sense that the implied stochastic discount factor (or pricing kernel) reaches the HJ volatility bound. This improvement in the asset pricing dimension is achieved without reducing its business cycle performance such as output and consumption volatility. More importantly, the results do *not* depend on a considerably inelastic supply of capital with respect to Tobin's q: instead, the implied capital supply is highly responsive to a change in the price of capital. This is in a striking contrast to the findings based on the reduced-form capital adjustment costs technology such as Jermann (1998) which has to rely on the lower bound in the range of empirically plausible estimates of the supply elasticity of capital (or upper bound in the capital adjustment costs). In order to hamper consumption smoothing by restricting the investment variations, the model requires substantially inelastic labor supply curve.

How can the model generate the equity premium without relying on a highly inelastic supply of capital? The key is the role of net worth as a "sluggish shifter" of the relatively flat capital supply curve. The immediate impact of a positive technology shock is to increase net worth slightly as the shock raises the entrepreneur's wage and rental income, which then shifts out the capital supply curve. However, net worth's increase is limited by the initially fixed level of entrepreneurial capital. Subsequently, as the increased demand for capital raises the price of capital, pushing up the return to internal fund and hence causing net worth to rise, the capital supply curve shifts further out. This delayed shift of the capital supply curve due to a sluggish adjustment of net worth is analogous to a delayed response of labor-leisure hours in the labor hoarding model by Burnside, Eichenbaum, and Rebelo (1993) or the labor market search model by Merz (1995). Essentially, the delayed response of investment disturbs consumption smoothing desired by the households with habit persistence preferences, so that the implied stochastic discount factor reaches the HJ volatility bound. Therefore, a more general model of capital adjustment costs is shown to be consistent with both the asset returns and the key business cycle facts.

The paper is organized as follows. Section 2 describes the agency-cost model of Carlstrom and Fuerst (1997) modified to include habit formation in preferences. Section 3 calibrates the model economy following Carlstrom and Fuerst (1997) and Campbell and Cochrane (1999). The quantitative implications for the business cycle and asset pricing are presented in section 4 by working out the log-linearized system of equations characterizing competitive equilibrium. Section 5 summarizes the paper with a few remarks and the derivation of supply elasticity of capital follows in the Appendix.

2. Model

We modify the agency-cost model of Carlstrom and Fuerst (1997) to include habit formation as in Campbell and Cochrane (1999). In this section, we briefly describe the key features of our model specification and calibration. There is a continuum of agents with unit mass in the model economy. The fraction η of population are entrepreneurs, and $1-\eta$ are households. The entrepreneurs engage in producing the investment good. If a household wishes to purchase capital, it must fund entrepreneurial projects which are subject to agency problems. Entrepreneurs receive their external financing in the form of financial contract with households via intermediaries, called capital mutual funds (CMFs). The economy is also populated with numerous firms producing the single consumption good. Assume that these firms are owned by households and they are not subject to any agency problems.

2.1. The financial contract

In any given period t, the financial contract consists of an entrepreneur (as a borrower) with net worth, $n_t > 0$, and a household (as a lender) with resources that she may wish to lend to the entrepreneur. The contract is assumed to be static in the sense that it lasts only one period. The entrepreneur has access to a stochastic technology that contemporaneously transforms i_t consumption goods into $\omega_t i_t$ units of capital where ω_t is an i.i.d. random variable (across time and across enterpreneurs) with distribution Φ, density ϕ, a nonnegative support, and a mean of unity. Assume that ω_t is private information of the entrepreneur: others can observe ω_t only at a monitoring cost of $\mu \in [0,1]$ per unit of of consumption goods invested.

In order to make a moral hazard problem due to the asymmetric information relevant, assume that an entrepreneur's net worth (for internal financing) is sufficiently small that the entrepreneur should get some external financing from households and hence bear some agency costs. An entrepreneur who borrows $(i_t - n_t)$ consumption goods agrees to repay $(1 + r_t^k)(i_t - n_t)$ capital goods to the lender. The entrepreneur will default if the realization of ω_t is low: that is, if $\omega_t < (1 + r_t^k)(i_t - n_t)/i_t \equiv \bar{\omega}_t$. Assuming the pure strategy contract with nonstochastic monitoring as in Williamson (1987), the lender will monitor the project outcome only if the entrepreneur defaults, in which case it will confiscate all the returns from the project. Note that the contract is completely defined by the pair $(i_t, \bar{\omega}_t)$, which then implies the lending interest rate, $(1+r_t^k) \equiv \bar{\omega}_t i_t/(i_t - n_t)$.

The optimal contract is characterized by the following first-order conditions:

$$q_t \left[1 - \Phi(\bar{\omega}_t)\mu + \phi(\bar{\omega}_t)\mu \frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \right] = 1 \tag{1}$$

$$i_t = \left[\frac{1}{1 - q_t g(\bar{\omega}_t)} \right] n_t \tag{2}$$

where q_t denotes the price of capital at the end of t and $f(\bar{\omega}_t)$ and $g(\bar{\omega}_t)$ are respectively the fraction of the expected net capital output received by the entrepreneur and the lender (or household):

$$f(\bar{\omega}_t) \equiv \int_{\bar{\omega}_t}^{\infty} \omega_t \Phi(d\omega_t) - [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t$$

$$g(\bar{\omega}_t) \equiv \int_{0}^{\bar{\omega}_t} \omega_t \Phi(d\omega_t) - \Phi(\bar{\omega}_t)\mu + [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t$$

Equation (1) implicitly defines $\bar{\omega}(q_t)$, the critical value of the entrepreneur's capital output productivity (as an increasing function of q_t) below which the entrepreneurial investment projects default. Substituting this into (2) defines the implicit function $i(q_t, n_t)$, total investment of consumption goods into the entrepreneurial capital technology. The new-capital supply function is then given by $I^S(q_t, n_t) \equiv i(q_t, n_t)\{1-\mu\Phi[\overline{\omega}(q_t)]\}$ which increases with both q_t and n_t. In particular, an increase in net worth is to decrease the need for external financing and hence lower agency costs, boosting the level of capital production for a given price of capital. The level of net worth (n_t) and the price of capital (q_t) will be determined as a part of competitive general equilibrium which consists of (i) optimizing behaviors of households, firms, and entrepreneurs; and (ii) market-clearing conditions.

2.2. Households

Households are infinitely lived and, following Campbell and Cochrane (1999), households' preferences are featured with habit formation:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - L_t; X_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - X_t)^{1-\psi} - 1}{1 - \psi} + \nu(1 - L_t) \right] \qquad (3)$$

where E_0 is the expectations operator conditional on information available at $t = 0$, $\beta \in (0, 1)$ is the discount factor, c_t is household consumption, and L_t is labor supply at t

with the time endowment normalized to unity. Further, $\{X_t\}_{t=0}^{\infty}$ denotes a (stochastic) sequence of habits which is regarded as exogenous by the individual households and tied to the stochastic sequence of household consumption $\{c_t\}_{t=0}^{\infty}$. Let s_t denote the surplus consumption ratio given by:

$$s_t = \frac{c_t - X_t}{c_t}$$

which is assumed to follow the linear process in the logarithmic form:

$$\ln s_t = (1-\phi)\ln s + \phi \ln s_{t-1} + \left(\frac{1}{s} - 1\right)(\Delta \ln c_t - g) \tag{4}$$

Here, $\Delta \ln c_t = \ln c_t - \ln c_{t-1}$, $\phi \in (0,1)$, g is the average consumption growth rate, $g = E(\Delta \ln c_{t+1})$, and s is the steady state surplus consumption ratio.

In any given period t, households sell their labor to firms at a wage rate of w_t, rent their previously accumulated capital to the firms at a rental rate of r_t, purchase consumption at the price of unity (i.e., consumption is the numeraire), and purchase new capital goods at a price of q_t at the end of period t via CMFs. Households' optimal choices are summarized by the following first-order conditions:

$$-\frac{U_L(t)}{U_c(t)} = w_t \tag{5}$$

$$q_t U_c(t) = \beta E_t \left\{ U_c(t+1) \left[q_{t+1}(1-\delta) + r_{t+1} \right] \right\} \tag{6}$$

Households supply labor according to (5), while their purchase of capital goods is determined by the equation (6) where $\delta \in (0,1)$ is the rate of capital depreciation. With functional form for the households' preferences speficied in (3), above two conditions can be written as:

$$\nu (s_t c_t)^{\psi} = w_t$$

$$q_t = \beta E_t \left\{ \left(\frac{s_{t+1}}{s_t} \frac{c_{t+1}}{c_t} \right)^{-\psi} [q_{t+1}(1-\delta) + r_{t+1}] \right\}$$

2.3. Firms

The firms produce the consumption good using a constant-returns-to-scale production function given by:

$$Y_t = \theta_t F(K_t, H_t, H_t^e) = \theta_t K_t^{\alpha_1} H_t^{\alpha_2} (H_t^e)^{1-\alpha_1-\alpha_2}$$

where Y_t is aggregate output of the consumption good, K_t denotes the aggregate capital stock (including entrepreneurial capital), H_t and H_t^e denote respectively aggregate labor supply of households and entrepreneurs. Further, θ_t denotes the technology (or productivity) shock which is assumed to follow the stochastic process given by:

$$\theta_t = (1-\rho) + \rho\theta_{t-1} + \nu_t \tag{7}$$

where ν_t is a serially uncorrelated random variable with standard deviation σ_ν, $\rho \in (0,1)$ is the autocorrelation coefficient, and the nonstochastic steady state of θ is unity.

Competition in the markets for production inputs implies that wage and rental rates are equal to their respective marginal products:

$$r_t = \theta_t F_1(K_t, H_t, H_t^e), \;\; w_t = \theta_t F_2(K_t, H_t, H_t^e), \;\; \text{and } x_t = \theta_t F_3(K_t, H_t, H_t^e) \tag{8}$$

where x_t is the wage rate for entrepreneurial labor.

2.4. Entrepreneur

Agency costs associated with external financing by entrepreneurs imply that the return to internal funds is greater than the return to external funds. In order to prevent entrepreneurs from postponing consumption to quickly accumulate sufficient capital for a complete self-financing ($i = n$) and bearing no agency costs, it is assumed that entrepreneurs discount the future more heavily than households do. That is, entrepreneurs' preferences are given by

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_t^e$$

where $\gamma \in (0, 1)$ denotes additional rate of discounting and c_t^e denotes an entrepreneur's consumption at t.

The entrepreneur raises internal funds (or net worth) by inelastically supplying unit endowment of labor to firms, selling undepreciated capital to a CMF for consumption goods, and renting capital to firms. The net worth of the entrepreneur (in consumption units) is given by

$$n_t = x_t + z_t[q_t(1-\delta) + r_t] \tag{9}$$

where z_t denotes the capital holdings of the entrepreneur at the *beginning* of period t. Risk neutrality and the higher return from internal funds imply that the entrepreneur will always choose to put his entire net worth into the investment project.

At the end of the period, those entrepreneurs who are still solvent make their consumption decision based on the following first-order condition:

$$q_t = \beta\gamma E_t \left[[q_{t+1}(1-\delta) + r_{t+1}] \left\{ \frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right\} \right] \tag{10}$$

This equates the benefit of current consumption with the future return on internal funds: the term in braces, which is greater than one, is the gross expected return on internal funds. It is this additional return that encourages entrepreneurs to accumulate capital even though they discount the future more heavily than households. Further, the law of motion of aggregate entrepreneurial capital stock, Z_t, is obtained by aggregating the entrepreneurs' budget constraints:

$$Z_{t+1} = \{\eta x_t + Z_t[q_t(1-\delta) + r_t]\} \left[\frac{f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)}\right] - \frac{\eta c_t^e}{q_t} \tag{11}$$

Finally, the CMF intermediates capital purchases between households (as buyers) and entrepreneurs (as sellers). On behalf of households, it lends resources to an infinite number of entrepreneurs, so that it can exploit the law of large numbers to ensure a certain return to the household regardless of the entrepreneurs' investment outcomes. That is, an expenditure of q_t consumption goods guarantees one unit of capital. The households' demand-for-capital schedule is given by the equation (6), whereas the new-capital supply schedule is given by $I^S(q,n) \equiv i(q,n)\{1 - \mu\Phi[\overline{\omega}(q)]\}$ as an outcome of the financial contract between households and entrepreneurs. Recall that the supply schedule is an increasing function of net worth (n).

2.5. Competitive equilibrium

Given the distribution Φ on idiosyncratic productivity across entrepreneurs and the technology process (7), a (recursive) competitive equilibrium is defined by decision rules for K_{t+1}, Z_{t+1}, $H_t, q_t, n_t, i_t, \bar{\omega}_t, c_t^e, c_t$, and s_t as stationary functions of $(K_t, Z_t, s_{t-1}, c_{t-1}, \theta_t)$ which satisfy the first-order conditions for optimal financial contract, (1) and (2), the first-order conditions for households' choices, (5) and (6), along with (4) describing the law of motion of habit, the entrepreneur's net-worth (9), the first-order condition for entrepreneur's consumption decision (10), and the law of motion of aggregate entrepreneurial capital stock (11). Further, they should satisfy the following market-clearing conditions in the two labor markets, a consumption-goods market, and a capital goods market, respectively:

$$H_t = (1-\eta)L_t \qquad H_t^e = \eta \tag{12}$$

$$(1-\eta)c_t + \eta c_t^e + \eta i_t = Y_t \tag{13}$$

$$K_{t+1} = (1-\delta)K_t + \eta i_t[1-\Phi(\bar{\omega}_t)\mu] \tag{14}$$

Finally, let the period-t aggregate consumption be denoted C_t. Then,

$$C_t = (1-\eta)c_t + \eta c_t^e.$$

2.6. Asset returns

The model economy where final-goods producing firms rent capital from households, is formally equivalent to a setup where these firms own the capital, while households own equity in the firms. Therefore, the return to household capital in our model is equivalent to the equity return. Since households do not face any credit constraint in their intertemporal consumption decision, returns and prices of any asset held by household should satisfy the standard asset pricing equation such as (6) in section 2.2. This can be compactly written as

$$1 = E_t\left(m_{t+1} \cdot R_{t+1}^k\right). \tag{15}$$

where $R_{t+1}^k = [q_{t+1}(1-\delta) + r_{t+1}]/q_t$ is the gross return on equity holding and $m_{t+1} = \beta U_c(t+1)/U_c(t)$ is the intertemporal marginal rate of substitution or the stochastic discount factor for this economy.

Although there is no explicit trading of risk-free asset in the model economy, we can infer the shadow risk-free rate by using the standard asset pricing equation:

$$1 = E_t\left(m_{t+1} \cdot R^f_{t+1}\right)$$

where R^f_{t+1} is the risk-free rate. Noting that the risk-free rate is known ahead of time, it can be taken out of the conditional expectation so that $1 = E_t(m_{t+1}R^f_{t+1}) = E_t(m_{t+1})R^f_{t+1}$. Hence, the risk-free rate is given by

$$R^f_{t+1} = \frac{1}{E_t(m_{t+1})}. \tag{16}$$

The asset pricing quations (15) and (16) then imply

$$\begin{aligned} 1 &= E_t(m_{t+1} \cdot R^k_{t+1}) = E_t(m_{t+1}) \cdot E_t(R^k_{t+1}) + \mathrm{cov}_t(m_{t+1}, R^k_{t+1}) \\ &= \frac{E_t(R^k_{t+1})}{R^f_{t+1}} + \mathrm{cov}_t(m_{t+1}, R^k_{t+1}), \end{aligned}$$

which can be rearranged as

$$E_t(R^k_{t+1}) - R^f_{t+1} = -R^f_{t+1}\mathrm{cov}_t(m_{t+1}, R^k_{t+1})$$

The left-hand side of the above equation is the equity premium. Equity, whose return tends to covary negatively with marginal utility of consumption (covary positively with consumption), makes consumption more volatile and hence should promise higher expected returns in order to induce equity holdings. The above equation can be further modified to

$$E_t(R^k_{t+1}) - R^f_{t+1} = -\frac{\sigma_t(m_{t+1}) \cdot \sigma_t(R^k_{t+1}) \cdot \rho_t(m_{t+1}, R^k_{t+1})}{E_t(m_{t+1})}$$

Since the correlation coefficient $\rho_t(m_{t+1}, R^k_{t+1})$ cannot be greater than 1, we get

$$\left|\frac{E_t(R^k_{t+1}) - R^f_{t+1}}{\sigma_t(R^k_{t+1})}\right| \leq \frac{\sigma_t(m_{t+1})}{E_t(m_{t+1})}. \tag{17}$$

The left-hand side represents the Sharpe ratio. As elaborated in Cochrane and Hansen (1992), the post-war quarterly data in the U.S. (e.g. real value-weighted NYSE return and Treasury Bill return) show that the historical Sharpe ratio has been around 0.25. Therefore, in order for a model to be able to explain the historical equity premium, $\sigma_t(m_{t+1})/E_t(m_{t+1})$ implied by the model should be at least 0.25, which is referred to as the Hansen-Jaganathan (1991) volatility bound (we call "HJ bound" below). With $E_t(m_{t+1}) = 1/R^f_{t+1}$ sufficiently close to one with quarterly model specification[4], the equity premium requires $\sigma_t(m_{t+1})$ to be not much smaller than 0.25. We will use this implication below to check whether the model economy can reasonably account for the historical equity premium.

3. Calibration

First of all, standard choices are made for the following parameters: $\beta = 0.99$, $\delta = 0.02$, $\rho = 0.99$, and $\sigma_\nu = 0.01$. These parameter values are along the line with Boldrin, Christiano, and Fisher (2001) and Jermann (1998). We also try $\rho = 0.995$ to examine the effect of a highly persistent technology shock. A household's capital share is set as $\alpha_1 = 0.36$ and labor share as $\alpha_2 = 0.6399$: the latter implies an entrepreneurial labor share of 0.0001. The entrepreneur's labor share needs to be positive to guarantee that each entrepreneur always has at least some labor income as a part of her net worth.

[4]When we did simualtion, we checked this magitude and found that it was always very close to one. (i.e., around 0.99.)

As for the monitoring technology, Altman (1984) estimates the sum of direct and indirect bankruptcy costs at about 20 percent of total firm assets. Since bankruptcy can be viewed as the entrepreneur being closed and his assets being liquidated, another measure of bankruptcy costs could be liquidation costs. Alderson and Betker (1995) use data from Chapter 11 proceedings to calculate liquidation costs equal to approximately 36 percent of firm assets. Carlstrom and Fuerst (1997) set the monitoring cost at the low end of the $[0.2, 0.36]$ range: $\mu = 0.25$.

Further, they assume that the distribution Φ is lognormal with a mean of unity and a standard deviation of σ_ω. The two parameters, σ_ω and γ (additional discount factor for entrepreneur), are calibrated using the two measures of default risk: (i) a quarterly bankruptcy rate of 0.974 percent (Fisher 1994), and (ii) an annual risk premium of 187 basis points measured by the average spread between the prime rate and the 3-month commercial paper rate. Matching these two empirical measures with the model's bankuptcy rate, $\Phi(\bar{\omega}_t)$, and a risk premium associated with a loan of one consumption good, $[q_t(1 + r_t^k) - 1]$, implies $\sigma_\omega = 0.207$ and $\gamma = 0.947$.

Campbell and Cochrane (1999) use the postwar quarterly data in the U.S. to calibrate ϕ in the habit process. Serial correlation parameter ϕ is chosen to match the serial correlation of log price-consumption ratios. Since there is a strong seasonality in quarterly price-dividend ratios (and hence deseasonalized data are artificially smooth), ϕ is calibrated to match the quarterly serial correlation coefficient implied by annual postwar data: $0.88^{1/4} = 0.97$. The risk-aversion parameter ψ is set as 2.372, which is the value chosen by Campbell and Cochrane (1999). Following Campbell and Cochrane (1999) and Lettau and Uhlig (2002), we choose a steady-state value for the surplus consumption ratio s of 0.05. Finally, the utility-weight-on-leisure parameter ν is chosen so that in the steady state each household devotes 30% of their time to market activities (i.e. $L = 0.3$).

4. Results

We analyze the business cycle dynamics with the log-linearized system of equations characterizing competitive equilibrium with $g = 0$ for the habit equation (4) since we examine a stationary economy. These equations can be solved using the method of undetermined coefficients with the techniques in Uhlig (1999). In order to save notations, let the letters (defined earlier as the level of variables) now denote log-deviations from steady state. The state of the economy in any given period t is given by the vector $[K_t, Z_t, s_{t-1}, c_{t-1}, \theta_t]$. The solution for this dynamic system is a linear vector function:

$$[K_{t+1}, Z_{t+1}, H_t, q_t, n_t, i_t, \bar{\omega}_t, c_t^e, c_t, s_t] = f(K_t, Z_t, s_{t-1}, c_{t-1}, \theta_t).$$

Further, the general-equilibrium solution for the household consumption (c_t) is used to compute the implied equity premium in terms of the volatility of stochastic discount factors (m_{t+1}) or HJ volatility bound as in (17). Notice that the HJ volatility bound is derived from the property that the equity premium is determined by the covariance between equity return $\left(R_{t+1}^k\right)$ and stochastic discount factors (m_{t+1}). That is, following Lettau (2003), we first solve the model using a log-linear approximation which essentially expresses the equilibrium relationship among variables in terms of their first moments. We then solve the implied equity premium in terms of the volatility of stochastic discount factors which depends on the preferences of the agents, the elasticity of consumption with respect to the technology shock, and the variance of the underlying technology shock.

4.1. Flexible labor supply

In Table 2 we present the results on the business cycle and stochastic discount factors along with the corresponding data in the U.S. where σ_X denotes standard deviation of variable X in percentage terms. The model-implied business cycle statistics and the standard deviation of the model implied stochastic discount factors are the average of the respective moments calculated respectively from the Hodrick-Prescott filtered data and raw (or "unfiltered") data over 100 simulations each 200 quarters long.

The business cycle data in the second column are replicated from Boldrin, Christiano, and Fisher (2001).[5] The volatility figure (σ_m) in the second column is the historical Sharpe ratio (i.e. HJ volatility bound) in the post-war U.S. data. Finally, columns denoted with "No Habit" display statistics calculated from the model without habit formation in preferences such as Carlstrom and Fuerst (1997).

First, Table 2 shows that the Carlstrom-Fuerst (1997) model ("No Habit") captures basic stylized facts of business cycles reasonably well, whereas the implied volatitily of stochastic discount factors is very small. Hence, the model cannot explain substaintial equity premium in the data.

It is now well known that models with households maximizing time separable utility function cannot geneate substiantial equity premium unless the coefficient of relative risk aversion is increased to unrealistically high level. This motivated several researchers to introduce habit formation into household preference specification. The inclusion of a habit in the agency-cost model produces some interesting and novel implications for the two related issues in business cycle and asset pricing. In particular, it is notable in Table 2 that, with habit formation, household consumption becomes extremely smooth: that is, volatility of household consumption (σ_c) is not even a tenth of the corresponding volatility in the model without habit formation. The fall in volatility of *aggregate* consumption with habit is less dramatic due to the the presence of risk-neutral entrepreneurs whose consumption is hardly affected by the inclusion of a habit.

[5] The estimates (standard errors in paranthesis) are based on an updated version of the Christiano (1988) database compiled by Fisher (1997) covering the sample period 1964:1 - 1988:2.

With habit formation, σ_m increases, although the improvement is too small to to explain the historical equity premium. Intuitively, introducing a habit formation is very similar to increasing the value of coefficient of relative risk aversion to an appropriately high level. As households are (locally) very risk averse with habit, they want to have very smooth consumption. This consumption smoothing significantly cancels out the direct effect of higher risk aversion parameter on the variability of stochastic discount factors. Further, as shown in Table 2, an increase in the persistence of productivity shock ($\rho = 0.995$) appears to make hardly any difference in reaching the HJ bound.

In short, the agency-cost model with habit formation and flexible labor supply cannot explain the existence of substantial equity premium in the sense that the implied volatility of consumption-based stochastic discount factor is far below the HJ volatility bound. With habit formation, (locally) very risk averse households desire consumption smoothing by adjusting labor supply. For example, in response to a temporary positive productivity shock, households would increase leisure instead of consumption.

4.2. Fixed labor supply

In order to investigate the consumption variability with habit once agents are not able to use labor/leisure to smooth out consumption, we solve the model economy with fixed labor supply. In Table 3, it can be seen that the household consumption variability (σ_c) increases substantially. This then yields a stark increase in the volatility of stochastic discount factors, approaching close to the HJ volatility bound. As the technology shock becomes more persistent ($\rho = 0.995$), the implied discount factor finally reaches the HJ bound.

The question is whether the model resolves the equity premium puzzles at the expense of the business cycle facts. It is well known in the RBC literature (e.g. Campbell 1994) that a highly variable labor supply is needed to match the variability of output with respect to technology shocks. However, this is not necessarily the case in the model economy where technology shocks affect output through the capital investment channel (rather than the labor supply) by altering agency costs associated with financing the capital investment.

For example, as elaborated in Carlstrom and Fuerst (1997), a positive technology shock increases the return to entrepreneurs' internal funds, which causes entrepreneurial capital and hence net worth to rise. As net worth continues to grow with a somewhat persistent technology shock, entrepreneurs will continue to increase the supply of new capital (or investment) by exploiting lower agency costs associated with less use of external financing. As for the demand side, the positive shock increases marginal product of capital, which shifts out the demand-for-capital schedule. This is the agency-cost driven propagation mechanism by which the investment response to the technology shock leads to a positive autocorrelation in output change. Notice in Table 3 that the output variability is kept at a reasonably high level along with the other moments which remain relatively close to the data.[6]

The favorable results of the agency-cost model with fixed labor supply are in a sharp contrast to Lettau and Uhlig (2000) where the fixed labor supply yields consumption only a third as variable as the U.S. consumption and output half as variable as that in the U.S. This can be attributed to the absence of the propagation mechanism driven by the supply and demand for investment. Further, in Lettau and Uhlig (2000), the difference between the Habit and No Habit models are much smaller with fixed labor supply. In the agency-cost model, however, habit formation still plays a significant role in the business cycle as well as the asset pricing. For instance, the model with no habit

[6] An increase in the persistence of the shock hardly affects the output variability. As properly pointed out by Lettau and Uhlig (2000), this is because output changes are almost permanent due to the persistent shock, which is canceled out by the detrending procedure using the HP filter.

yields too high a volatility in household consumption, whereas the habit model implies a much more reasonable one.

4.3. Supply elasticity of capital

Jermann (1998) also shows that a standard RBC model with habit formation preferences and capital adjustment costs can explain the historical equity premium as well as the key business cycle properties, although labor supply is assumed to be fixed. However, the results depend crucially on the lower bound in the range of empirically plausible estimates of the curvature of the capital adjustment costs technology or the supply elasticity of capital with respect to its price q (or upper bound in the capital adjustment costs). More specifically, the supply elasticity of capital is specified 0.23 to match a set of business-cycle and asset-returns moments. In order to hamper consumption smoothing by inhibiting the variation in capital, the model requires the substantially low elasticity of capital supply (or close to vertical capital supply curve).

Noting that the agency-cost model as considered here can be regarded as an endogenous formulation of capital adjustment costs, we can compute the corresponding supply elasticity of capital from the log-linearization of the capital supply function $I^S(q,n) \equiv i(q,n)\{1-\mu\Phi[\overline{\omega}(q)]\}$ as discussed in section 2.1:

$$I^S(\hat{q}_t, \hat{n}_t) = \hat{n}_t + \eta_q^S \hat{q}_t$$

where η_q^S denotes the supply elasticity of new capital with respect to q, evaluated at a given set parameter values in the steady state (see the Appendix for its derivation). It can be shown that the supply elasticity η_q^S is a decreasing function of (μ, σ), the monitoring cost and the degree of idiosyncratic uncertainty in entrepreneurial investment projects, respectively. This property of the supply elasticity of capital is consistent with its counterpart in the standard capital adjustment cost model in the sense that these two parameters determine the model's capital adjustment costs in the form of the agency costs. In a striking contrast to Jermann (1998), the structural parameters in the model economy (e.g. $\mu = 0.25$ and $\sigma = 0.207$) implies highly elastic capital supply: $\eta_q^S = 8.94$!

The supply elasticity η_q^S here measures the effect of percentage change in Tobin's q on percentage change in investment (I^S), instead of the effect of change in Tobin's q on change in investment rate (I^S/K) as discussed in Hayashi (1982) and Gilchrist and Himmelberg (1995). According to Christiano and Fisher (1998), the estimated relationships between investment rate (I^S/K) and Tobin's q as in Abel (1980), Eberly (1997), and Hassett and Oliner (1997) imply that the supply elasticity η_q^S (i.e., the percentage change in investment with respect to Tobin's q) should be at least in the range of $0.4 \sim 0.5$. This is the sense in which $\eta_q^S = 0.23$ as assumed in Jermann (1998) is the lower bound in the range of empirically plausible estimates of the supply elasticity of capital.

How can the model generate the equity premium without relying on a considerably inelastic supply of capital with respect to q? The key is the role of net worth, n, as a "sluggish shifter" of the capital supply curve: recall that $I_n^S(q,n) > 0$ as well as $I_q^S(q,n) > 0$. In the model economy, the immediate impact of a positive technology shock is to increase net worth slightly as the shock raises the entrepreneur's wage (w_t) and rental income (r_t): see equation (9). However, net worth's increase is limited by the initially fixed level of entrepreneurial capital (z_t). Subsequently, as the increased demand for capital pushes up the price of capital (q_t), the return to internal funds $qf/[1-qg]$ goes up causing net worth to rise: see equation (11) which is the law of motion of entrepreneurial capital stock. This sluggish behavior of net worth (or accumulated capital) leads to the delayed shift of the capital supply curve following the shock, which is similar to a delayed response of labor supply due to the labor hoarding behavior or the search frictions in the labor market.[7] Essentially, the delayed response of investment puts the "burden" of cyclical variations onto consumption. Despite habit-augmented preferences, the resulting variations in consumption enable the implied stochastic discount factor to reach the HJ volatility bound.

5. Concluding Remarks

We have attempted to explain both business cycle facts and asset returns using an RBC model which incorporates habit-augmented preferences and endogenous costs of adjusting the capital stock. Following the agency-cost model of Carlstrom and Fuerst (1997), capital adjustment costs are affected by the level of entrepreneur's net worth such that an increase in net worth (following a positive productivity shock) lowers agency costs associated with external financing, and hence makes it easier to expand the capital stock. In the presence of the restricted labor supply, the agency-cost model of capital adjustment costs is capable of resolving the key asset pricing puzzles without reducing its business cycle performance.

The sluggish behavior of net worth, as a shifter of the now relatively flat capital supply curve, is the key mechanism by which capital adjustment is delayed, hindering consumption smoothing desired by households with habit persistence preferences. The supply elasticity of capital with respect to its price (i.e., Tobin's q) is determined by the "deep" structural parameters such as the monitoring cost in the financial contract with asymmetric information. For instance, an increase in the monitoring cost decreases the price elasticity of capital supply (i.e., a steeper capital supply curve). This would then

[7] As properly noted by Carlstrom and Fuerst (1997), when the adjustment costs technology depends on the investment-capital ratio (not investment only) as in Jermann (1998), the capital supply curve also shifts out as capital begins to grow. However, there is no delayed response of investment because households internalize the effect and increase their initial investment in anticipation. Among the labor hoarding models are Burnside and Eichenbaum (1996) and Burnside, Eichenbaum, and Rebelo (1993), whereas Merz (1995) is the labor market search model.

determine movements in Tobin's q following an aggregate technology shock.

More importantly, in contrast to the standard reduced-form approach to the capital adjustment costs technology as in Jermann (1998), the results do not depend on the lowest possible value of the supply elasticity of capital with respect to Tobin's q (or maximum value of the capital adjustment cost). Instead, the capital supply is highly elastic under the plausible calibrations of the structural parameters. Hence, a small curvature in the capital adjustment cost function, viewed as crucial for understanding the fluctuations in Tobin's q, can be also consistent with the historical equity premium and the key business cycle facts.

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Appendix: supply elasticity of capital

As discussed in section 2.1 Financial contract, the period-t investment (or new-capital) supply function is defined as $I^S(q_t, n_t) \equiv i(q_t, n_t)\{1 - \mu\Phi[\overline{\omega}(q_t)]\}$ where $I_q^S(q_t, n_t) > 0$ and $I_n^S(q_t, n_t) > 0$. Log-linearization around steady state yields:

$$I^S(\hat{q}_t, \hat{n}_t) = \hat{\imath}_t - \left[\frac{\phi(\overline{\omega})\mu\overline{\omega}}{1 - \Phi(\overline{\omega})\mu}\right]\widehat{\overline{\omega}}_t \tag{A.1}$$

In order to write $\widehat{\overline{\omega}}_t$ as a function of $\hat{q}_t$, we can log-linearize (1) to obtain:

$$\widehat{\overline{\omega}}_t = \frac{1}{X}\left(\frac{1}{q}\right)\hat{q}_t \tag{A.2}$$

where

$$X = \mu\phi(\overline{\omega})\overline{\omega}\left\{1 - \frac{f(\overline{\omega})}{f'(\overline{\omega})}\left[\frac{\phi'(\overline{\omega})}{\phi(\overline{\omega})} + \frac{f'(\overline{\omega})}{f(\overline{\omega})} - \frac{f''(\overline{\omega})}{f'(\overline{\omega})}\right]\right\}.$$

Similarly, we can log-linearize (2) to obtain:

$$\hat{\imath}_t = \left[\frac{g(\overline{\omega})q}{1 - qg(\overline{\omega})}\right]\hat{q}_t + \left[\frac{qg'(\overline{\omega})\overline{\omega}}{1 - qg(\overline{\omega})}\right]\widehat{\overline{\omega}}_t + \hat{n}_t \tag{A.3}$$

Now, substituting (A.3) and (A.2) into (A.1) to obtain the log-linearized version of capital supply function:

$$I^S(\hat{q}_t, \hat{n}_t) = \hat{n}_t + \eta_q^S\hat{q}_t$$

where

$$\eta_q^S \equiv \frac{g(\overline{\omega})q}{1 - qg(\overline{\omega})} + \left[\frac{qg'(\overline{\omega})\overline{\omega}}{1 - qg(\overline{\omega})} - \frac{\phi(\overline{\omega})\mu\overline{\omega}}{1 - \Phi(\overline{\omega})\mu}\right]\frac{1}{X}\left(\frac{1}{q}\right).$$

Table 1: Parameter Calibration

Preferences		Technology (Firm)		Technology (Entre.)	
β	0.99	α_1	0.36	σ_ω	0.207
ψ	2.372	α_2	0.6399	$\Phi(\bar{\omega})$	0.00974
ϕ	0.97	δ	0.02	μ	0.25
s	0.05	ρ	0.99		
ν	$\{ \quad L = 0.3 \quad \}$	σ_ν	0.01		
γ	0.947				

Table 2: Stochastic Discount Factors and the Business Cycle: Flexible Labor

		$\rho = 0.99$		$\rho = 0.995$	
	Data	Habit	No Habit	Habit	No Habit
Sharpe ratio	0.25	0.0216	0.0178	0.0223	0.0182
σ_m		(0.0008)	(0.0007)	(0.0008)	(0.0007)
σ_Y	1.89	0.720	1.820	0.438	1.61
	(0.21)	(0.120)	(0.290)	(0.071)	(0.250)
σ_C	0.75	0.142	0.805	0.0902	0.851
		(0.010)	(0.130)	(0.010)	(0.130)
σ_c		0.055	0.869	0.058	0.909
		(0.007)	(0.100)	(0.008)	(0.110)
σ_{c^e}		18.76	34.68	11.01	27.97
		(0.970)	(1.800)	(0.570)	(1.400)
σ_C/σ_Y	0.40	0.2003	0.4433	0.2076	0.5277
	(0.04)	(0.019)	(0.007)	(0.011)	(0.0066)
σ_i/σ_Y	2.39	4.0524	2.8461	3.8824	2.5670
	(0.06)	(0.035)	(0.018)	(0.032)	(0.0184)
σ_H/σ_Y	0.80	2.8378	0.3993	5.5285	0.4894
	(0.05)	(0.1784)	(0.063)	(0.2601)	(0.0665)

Table 3: Stochastic Discount Factors and the Business Cycle: Fixed Labor

		$\rho = 0.99$		$\rho = 0.995$	
	Data	Habit	No Habit	Habit	No Habit
Sharpe ratio	0.25	0.1994	0.0268	0.2510	0.0298
σ_m		(0.0093)	(0.0011)	(0.0130)	(0.0011)
σ_Y	1.89	1.93	1.95	1.93	1.94
	(0.21)	(0.260)	(0.26)	(0.260)	(0.26)
σ_C	0.75	0.488	1.28	0.602	1.45
		(0.081)	(0.170)	(0.098)	(0.200)
σ_c		0.51	1.33	0.637	1.51
		(0.067)	(0.160)	(0.084)	(0.190)
σ_{c^e}		27.69	19.15	26.14	16.60
		(1.500)	(1.000)	(1.400)	(0.87)
σ_C/σ_Y	0.40	0.2503	0.6562	0.3094	0.7477
	(0.04)	(0.0092)	(0.003)	(0.0098)	(0.002)
σ_i/σ_Y	2.39	3.5509	2.1298	3.3324	1.8281
	(0.06)	(0.0449)	(0.008)	(0.0424)	(0.006)