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Spectral Analysis of the Term Structure of Interest Rates: Habit Formation vs. Recursive Utility in a DSGE Model*

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Abstract We examine whether a new Keynesian DSGE model can generate yields that match the spectral properties of the actual yields. For this purpose, we consider two competing specifications for utility in a DSGE model: habit formation and recursive utility. We find that the yields generated from the DSGE model with either specification for utility can match the spectral properties of the actual yields reasonably well. However, it seems fair to say that each specification has some individual disadvantages. In the case of habit formation, we need very persistent and large shocks to explain the time-varying term premia. The DSGE models with recursive utility are not free of problems, either. They have difficulty in generating either a sufficient yield curve slope or positive (negative) correlations between nominal interest rates (spreads) and output.

Keywords Term Structure of Interest Rates, DSGE Model, Habit Formation, Recursive Utility, Spectral Properties

JEL Classification E4, G1

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1. Introduction

Recent developments in macroeconomics and finance have led to a proliferation of new generation small-scale economic models, generally referred to as new Keynesian models. These new models, embedding imperfect competition and nominal rigidities in a dynamic general equilibrium economy, attempt to explore empirical issues such as the relationship between asset prices, including interest rates, and the macroeconomy over business cycles. The models have yielded many genuinely new insights into the role of asset prices over the business cycle.

From the perspective of finance, the issue of the term structure of interest rates has been analyzed in a variety of both theoretical and empirical setups, and the consequent debates are still very controversial in many respects. However, the recent DSGE models are successful in generating yields that match the mean and the variance of the term structure of interest rates in the data reasonably well. For example, Wu (2006) and Bekaert et al. (2010) derive the optimal conditions of a new Keynesian DSGE model and combine them with an affine term structure model using lognormal and/or loglinear assumptions. They then show that the models can generate yields that match the actual yields well.[1] Some authors go one step further to show that the time-varying term premia generated from a DSGE model can account for rejections of the expectations hypothesis. Ravenna and Seppala (2007), for instance, simulate the term structure of interest rates with the time-varying risk premia based on the third-order approximations of the equilibrium conditions for a DSGE model and show that their model can successfully account for rejections of the expectations hypothesis. There are pros and cons to their approach. Although the model can generate the time-varying term and/or risk premia, which are widely regarded as responsible for

[1] However, these authors did not estimate the time-varying risk or term premia. Recently, Hordahl et al. (2007) and Doh (2007) used the second-order approximations of DSGE models in deriving the term structure of interest rates. Hordahl et al. (2007) still have a constant risk premium, while Doh (2007) does the Bayesian estimation of the model and derives the time-varying risk premia. However, he has difficulty in matching the mean of the term structure and the volatility of inflation at the same time. The mean is too low unless the rate of inflation is excessively volatile.

the failure of the expectations hypothesis, it has a critical weakness. Rudebusch, Sack and Swanson (2007) and Rudebusch and Swanson (2008) are very critical of Ravenna and Seppala (2007), pointing out that one has to set the autoregressive coefficients and/or standard deviations of some of the shocks at some arbitrarily big numbers to generate the first and second moments of the term structure of interest rates as shown in the data. Rudebusch and Swanson (2008) show further that a DSGE model with habit formation cannot explain the movements of financial variables without the model's ability to fit the macroeconomic data being compromised. In contrast to Ravenna and Seppala's (2007) model with habit formation, Rudebusch and Swanson (2009) argue that a DSGE model with recursive utility can generate the time-varying term premia without such problems.

In this paper, we examine whether the DSGE models can generate yields that match the spectral properties of the actual yields. Specifically, we attempt to evaluate the current generation DSGE model with various frictions employing Watson's (1993) measure of fit and explore the contributions of habit formation preference as well as recursive utility preference to the dynamic properties of the term structure of interest rates across the frequency domain. Habit formation and recursive utility have attracted much attention in the asset pricing literature, since the pricing kernels consistent with these utility functions successfully generate risk premia matching those observed in the actual data. For instance, Constantinides (1990) shows that the habit formation specification can successfully solve for the risk premium puzzle raised by Mehra and Prescott (1985), while Epstein and Zin (1991) and Weil (1989) show that the recursive utility specification can also do so.

In regard to the term structures of interest rates, Rudebusch, Sack and Swanson (2007) note that there are three competing specifications for analyzing the term premia: habit formation, recursive utility, and heterogeneous agent models.[2] The

[2] Their focus was on examining the relationship between the changes in the term premia and the macroeconomy, not on dicriminating among the competing specifications.

literature does not, of course, reach any consensus on the appropriate DSGE specification. We will take up the first two specifications for the utility function and explore which specification performs better in terms of the spectral properties of interest rates.

This paper is in the same spirit as Beaubrun-Diant (2006) and Jung (2007). Jung (2007) examines the performance of the open economy sticky price model across the frequency domain, while Beaubrun-Diant (2006) examines if the RBC model with habit formation and capital adjustment costs can match the spectral properties of the equity return, risk-free rate, the Sharpe ratio, etc., across all frequencies. Although this paper examines the performance of the DSGE model across the frequency domain as in Beaubrun-Diant (2006), it goes beyond Beaubrun-Diant (2006) and differs from his paper in several ways. First, we consider the new Keynesian DSGE model, which does a good job of explaining the dynamic properties of the inflation rate, whereas Beaubrun-Diant (2006) considers the flexible price RBC model, which explains the cyclical movements of the inflation rate poorly. Second, we focus on the term structure of interest rates, while Beaubrun-Diant (2006) leaves out the term structure of interest rates in the spectral analysis. Third, we consider the recursive utility of Epstein and Zin (1991) and Weil (1989), as well as habit formation, while Beaubrun-Diant (2006) does not consider the former.

We set up a full-fledged new Keynesian DSGE model with two variants of preference: habit formation and recursive utility. We then evaluate the model's performance across the frequency domain, employing Watson's (1993) measure of fit. In the case of habit formation, we consider the internal habit model adopted by Ravenna and Seppala (2007). In the case of recursive utility, we consider two specifications of the uncertainty resolution: early resolution of uncertainty as well as late resolution of uncertainty.

As an empirical metric, we employ Watson's (1993) relative mean square approximation error (RMSAE) to judge the fit of the model. We find that the yields generated from the DSGE model with either specification for preference match the spectral properties of the actual yield data reasonably well, and that the approximation errors are not severe across the frequency domain. Therefore, in order to discriminate between the models, we need to look at other of their features, such as the correlations between their generated yields and/or spreads and output. Simulations fail to resolve the debates about which is the appropriate DSGE model. The DSGE model with recursive utility is no exception. DSGE models with recursive utility have difficulties in generating either a sufficient yield curve slope or positive (negative) correlations between nominal interest rates (spreads) and output.

In Section 2, we present a canonical new Keynesian DSGE model with two competing specifications for utility: habit formation and recursive utility. In Section 3, we introduce the relative mean square approximation errors (RMSAEs) developed by Watson (1993), which are the empirical metric for evaluating the spectral properties of the yields generated from DSGE models. In Section 4, we present the spectral properties of the yield data and discuss the spectral densities of the yields generated from the theoretical model. Section 5 then concludes.

2. Model

The model economy consists of households, firms, and the monetary authority. Firms that produce and sell differentiated products are assumed to have monopoly power over prices. The paper introduces sticky prices along the lines of Calvo (1983).

2.1. Benchmark DSGE Model

2.1.1. Households

The economy is composed of a continuum of infinitely lived individual households.[3] A representative household chooses consumption, leisure, and portfolios to maximize its lifetime objective given by

$$\max E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{e^{D_t} C_{t+i}^{1-\sigma_C}}{1-\sigma_C} + \frac{\chi_M}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi_N \frac{N_{t+i}^{1+\sigma_N}}{1+\sigma_N} \right] \tag{1}$$

subject to the intertemporal budget constraint

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t N_t}{P_t} + \frac{M_{t-1}}{P_t} + \frac{R_{t-1} B_{t-1}}{P_t} + T_t, \tag{2}$$

where β, σ_C, and σ_N represent the discount factor, the degree of risk aversion or inverse of the elasticity of the intertemporal substitution, and the inverse of the elasticity of work effort with respect to the real wage, respectively. Here C_t is consumption, M_t money, N_t work hours, B_t one-period bond purchases, R_t the one-period gross nominal interest rate, P_t the price level, T_t the lump-sum transfer at time t, and b, χ_M, χ_N parameters. The preference shock D_t follows an $AR(1)$ process given by

$$\log D_t = (1-\rho_D) \log D + \rho_D \log D_{t-1} + \varepsilon_{Dt}, \ -1 < \rho_D < 1, \tag{3}$$

where $E(\varepsilon_{Dt}) = 0$ and ε_{Dt} is i.i.d. over time.

The first-order conditions for the household's maximization problem are given by

$$C_t^{-\sigma_C} = \beta E_t \left[C_{t+1}^{-\sigma_C} \frac{R_t}{\pi_{t+1}} \right], \tag{4}$$

[3] We do not use an index for an individual household, since all households solve the identical optimization problem facing the same aggregate variables.

$$\frac{\chi_N N_t^{\sigma_N}}{C_t^{-\sigma_C}} = \frac{W_t}{P_t}, \tag{5}$$

$$\frac{\chi_M \left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma_C}} = \frac{R_t}{1+R_t}, \tag{6}$$

and the budget constraint (2). Equation (4) is the Euler equation derived from the first-order conditions for consumption and bond holdings. Equation (5) relates the marginal disutility of labor hours to the real wage rate. Equation (6) says that the marginal rate of substitution between real money balances and consumption equals the intertemporal relative price of money or the opportunity cost of holding money, $\frac{R_t}{1+R_t}$.[4]

The composite consumption good index is given by

$$C_t = \left[\int_0^1 C_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1, \tag{7}$$

where ϵ is the elasticity of demand for the intermediate good z. Intratemporal cost minimization for achieving C_t implies that

$$C_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} C_t \tag{8}$$

and

$$P_t = \left[\int_0^1 P_t(z)^{1-\epsilon} dz\right]^{\frac{1}{1-\epsilon}}. \tag{9}$$

2.1.2. Firms

We suppose that there is a continuum of firms producing differentiated goods, and that each firm indexed by z, $0 \leq z \leq 1$, produces its product with a constant return

[4] Assuming that the monetary authority implements the Taylor rule, the money market equilibrium condition is redundant in this economy.

to scale, i.e., a concave production technology:

$$Y_t(z) = A_t N_t(z), \tag{10}$$

where A_t is the technology process at period t, and $Y_t(z)$ and $N_t(z)$ are the output and total labor input of the zth firm, respectively. We assume that the productivity shock follows an $AR(1)$ process as $\log A_t = (1-\rho_A)\log A + \rho_A \log A_{t-1} + \varepsilon_{At}, \quad 0 < \rho_A < 1$, where $E(\varepsilon_{At}) = 0$ and ε_{At} is i.i.d. over time. Cost minimization leads to

$$w_t = \text{mc}_t A_t, \tag{11}$$

where $\text{mc}_t \equiv \frac{MC_t}{P_t}$ is the real marginal cost in period t.

Next, we consider a canonical Calvo-type staggered price setting. Firms adjust their price infrequently and the opportunity to adjust follows an exogenous Poisson process. We suppose that only the fraction $(1-\xi_P)$ of the firms sets the new price, P_t^*, which is the optimally determined price in period t, independently of past history, while the other ξ_P fraction of firms does not set the price.

The reoptimizing firms' profit maximization problem is given by

$$\max_{P_t(z)} \sum_{i=0}^{\infty} (\xi_P)^i E_t \left[q_{t,t+i} \frac{(P_t(z) - MC_{t+i})}{P_{t+i}} Y_{t,t+i}(z) \right] \tag{12}$$

subject to

$$Y_{t,t+i}(z) = \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\epsilon} Y_{t+i}, \tag{13}$$

where $q_{t,t+i} = \beta^i \frac{MUC_{t+i}}{MUC_t}$ is the real stochastic discount factor applied to the period between t and t+i, MC_{t+i} the nominal marginal cost at time t+i, and $Y_{t,t+i}(z)(= C_{t,t+i}(z))$ the firm's demand curve for its product at time t+i.

If we plug equation (13) into the objective function to eliminate $Y_{t,t+i}(z)$, the optimization problem can then be rewritten as

$$\max_{P_t(z)} \sum_{i=0}^{\infty} (\xi_P)^i E_t \left[q_{t,t+i} \frac{(P_t^*(z) - MC_{t+i})}{P_{t+i}} \left[\frac{P_t^*(z)}{P_{t+i}} \right]^{-\epsilon} Y_{t+i} \right].$$

The first-order condition for $P_t^*(z)$ implies that

$$P_t^*(z) = \frac{\epsilon}{\epsilon - 1} \frac{E_t[\sum_{i=0}^{\infty} (\xi_P)^i \frac{q_{t,t+i}}{P_{t+i}} MC_{t+i} Y_{t,t+i}(z)]}{E_t \left[\sum_{i=0}^{\infty} (\xi_P)^i q_{t,t+i} \left[\frac{Y_{t,t+i}(z)}{P_{t+i}} \right] \right]}. \tag{14}$$

Since each firm z that reoptimizes its price, $P_t^*(z)$, in period t faces the same problem, we can denote $P_t^*(z) = P_t^*$. Note that the price index is given by

$$\begin{aligned} P_t &= \left[\int_0^1 P_t(z)^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}} \qquad (15) \\ &= \left[\int_0^{\xi_P} P_t(z)^{1-\epsilon} dz + \int_{\xi_P}^1 P_t(z)^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}} \\ &= \left[\xi_P P_{t-1}^{1-\epsilon} + (1 - \xi_P) P_t^{*(1-\epsilon)} \right]^{\frac{1}{1-\epsilon}}. \end{aligned}$$

Yun (2005) shows that unless the steady state inflation rate equals 0, there is a relative price dispersion(Δ_t):

$$\Delta_t = \int_0^1 (\frac{P_t(z)}{P_t})^{-\epsilon} dz, \tag{16}$$

which follows

$$\Delta_t = (1 - \xi_P) \left(\frac{1 - \xi_P \pi_t^{\epsilon-1}}{1 - \xi_P} \right)^{-\frac{\epsilon}{1-\epsilon}} + \xi_P \pi_t^{\epsilon} \Delta_{t-1}. \tag{17}$$

2.1.3. Monetary Policy

Monetary policy follows the familiar Taylor-type rule:

$$R_t = \rho_r R_{t-1} + (1 - \rho_r)[b_y x_t + b_\pi E_t \frac{\pi_{t+1}}{\pi^*}] + \varepsilon_{rt}, \tag{18}$$

where x_t represents the output gap at time t and ε_{rt} is a normally distributed, mean-zero shock which is serially uncorrelated.

2.2. Extended Model: Habit Formation vs. Recursive Utility

In this subsection, we will present an extended model by considering habit formation preference as well as recursive utility preference in the household's problem.

2.2.1. Habit Formation

We consider an internal habit model, and the household's intertemporal objective function is given by

$$\max E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\frac{e^{D_t}}{1-\sigma_c} (C_{t+i} - hC_{t+i-1})^{1-\sigma_c} - \frac{1}{1+\sigma_N} (N_{t+i})^{1+\sigma_N} \right] \right\}, \tag{19}$$

where h denotes the degree of habit formation.

The asset pricing equation is given by

$$1 = \beta E_t \left[\frac{MUC_{t+1} R_t}{MUC_t \pi_{t+1}} \right], \tag{20}$$

where MUC_t denotes the marginal utility of consumption at time t and is given by

$$MUC_t = (C_t - hC_{t-1})^{-\sigma_c} - \beta h \left(C_{t+1} - hC_t\right)^{-\sigma_c}. \tag{21}$$

Equation (20) will be used in our asset pricing to generate the bond prices and interest rates.

2.2.2. Recursive Utility

We consider a recursive utility of the following form:

$$Max \;\; V_t = \left\{(1-\delta)\left((e^{D_t}C_t)^{\nu}(1-N_t)^{1-\nu}\right)^{\frac{1-\gamma}{\theta}} + \delta\left(E_t(V_{t+1}^{1-\gamma})\right)^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}} \tag{22}$$

subject to

$$\frac{B_{t+1}}{P_t} = \frac{R_t B_t}{P_t} + \frac{W_t N_t}{P_t} - C_t, \tag{23}$$

where δ is the discount rate, $\theta = \frac{1-\gamma}{1-\frac{1}{\Psi}}$, γ represents the degree of risk aversion, Ψ represents the intertemporal elasticity of substitution, and ν measures the relative importance of consumption vis-a-vis leisure. When the parameter $\theta = 1$, we have the standard power utility (constant relative risk aversion utility).

The asset pricing equation is given by

$$1 = \delta E_t\left[\frac{\lambda_{t+1}\Omega_{t+1}R_t}{\lambda_t\Omega_t\pi_{t+1}}\right], \tag{24}$$

where

$$\lambda_t = (e^{D_t})^{\frac{\nu(1-\gamma)}{\theta}} C_t^{\frac{\nu(1-\gamma)}{\theta}-1}(1-N_t)^{\frac{(1-\nu)(1-\gamma)}{\theta}} \tag{25}$$

and

$$\frac{\Omega_{t+1}}{\Omega_t} = \frac{V_{t+1}^{\frac{(1-\gamma)(\theta-1)}{\theta}}}{\left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{\theta-1}{\theta}}}. \tag{26}$$

2.2.3. Asset Pricing Equation

We can rewrite the asset pricing equations, (20) and (24) as

$$1 = E_t\left[Q_{t,t+1}R_t\right], \tag{27}$$

where $Q_{t,t+1}$ is called the nominal stochastic discount factor and is defined appropriately depending upon the specification for the utility function.

We can derive the one-period and the N-period yields to maturity from the following relationships, respectively:

$$R_t^1 = [E_t(Q_{t,t+1})]^{-1}, \tag{28}$$

and

$$R_t^N = [E_t(Q_{t,t+1}Q_{t+1,t+2}\cdots Q_{t+N-1,t+N})]^{-1}. \tag{29}$$

Following Ravenna and Seppala (2007) and Rudebusch and Swanson (2009), we do the third-order approximations of the equilibrium conditions in order to have time-varying term premia, and then generate the yields data using the Dynare++ version 1.3.6.

3. Spectrum Analysis

In order to do the spectrum analysis, we first estimate a VAR on the simulated series.[5] The implied autocovariance generating function is then used to calculate the measure of fit.

Following Watson (1993), Beaubrun-Diant (2006) and Jung (2007), we use the relative mean square approximation error as a metric to judge the fit of the model. Let $\mathbf{x}_t, \mathbf{y}_t, \mathbf{u}_t$ be the vector of variables from the model, corresponding data, and the error, respectively. More specifically,

$$\mathbf{u}_t = \mathbf{y}_t - \mathbf{x}_t \tag{30}$$

[5] We consider two sets of four-variable VARs. The first set includes the 3-month rate, and 1-, 3-, and 5-year rates, and the second set the first difference of the 3-month rate, the 1-year rate - the 3-month rate, the 3-year rate - the 1-year rate, and the 5-year rate - the 1-year rate.

If $\mathbf{x}_t$ and $\mathbf{y}_t$ are jointly covariance stationary, then

$$A_u(z) = A_y(z) + A_x(z) - A_{xy}(z) - A_{yx}(z), \quad (31)$$

where $A_u(z)$ is the autocovariance generating function (ACGF) of $\mathbf{u}_t$, $A_{xy}(z)$ the cross ACGF between $\mathbf{x}_t$ and $\mathbf{y}_t$, and so on.

Watson (1993) shows that the relative mean square approximation error (RMSAE) is a lower bound (analogous to a lower bound on 1-R^2 from a regression) of the variance of $\mathbf{u}_t$ without imposing any restrictions on $A_{xy}(z)$. The relative mean square approximation error (RMSAE) is given by

$$\Gamma_j(w) = \frac{[A_u(z)]_{jj}}{[A_y(z)]_{jj}}, z = e^{-iw}, \quad (32)$$

where $[A_u(z)]_{jj}$ and $[A_y(z)]_{jj}$ are the jth diagonal elements of $[A_u(z)]$ and $[A_y(z)]$, respectively. The RMSAE is the lower bound of the variance of the error relative to the variance of the data for each frequency, with a small RMSAE implying a high goodess of fit of the model. $\Gamma_j(w)$ can be larger than one; i.e., the R^2 of the model can be negative.

4. Data and Results

The yields data are those used by Piazzesi and Schneider (2006) and many others. The 3-month yield are from the CRSP Fama risk-free rate file. The 1- to 5-year yield data is from the CRSP Fama-Bliss discounted bond files. We use the end-of-quarter monthly data as the quarterly data for the period between 1952:II and 2005:IV. The model yields are simulated 1000 times with 215 periods for each simulation.

Table 1 reports the parameter values used in the simulations for the different models. We use the same parameter values as Ravenna and Seppala for the DSGE model with habit formation, since Ravenna and Seppala show that the yields generated using these parameter values can match the mean and the standard deviation of the actual yields reasonably well and can satisfactorily explain the failures of the expectations hypothesis. In the case of the model with recursive utility, we report two cases of parameter values. The first case we refer to as Recursive Utility model 1, where we use the same parameter values as Ravenna and Seppala, to see what happens with utility specifications different from theirs. In the second case, which we call Recursive Utility model 2, we use parameter values different from those in Ravenna and Seppala. Since the appropriate parameter values may differ when a different utility specification is used in a DSGE model, we want to see what happens with parameter values different from theirs. We also consider a canonical base line new Keynesian(NK) model where no habit formation is allowed. We report this case in order to see what happens without the more sophisticated utility specifications: habit formation and recursive utility.

Note that for Recursive Utility model 1, we set the value of the risk aversion parameter to be $\gamma = 7.5$ and that of the intertemporal elasticity of substitution (hereafter IES) parameter to be $\psi = 1.5$. In Recursive Utility model 2, we set the value of the risk aversion parameter to $\gamma = 2.45$, and the value of the IES parameter to $\psi = 0.4$. These two sets of parameter values happen to represent cases of early and late resolution of uncertainty, respectively, as we will see later. The values of the risk aversion parameter, γ, are similar to the values commonly used in the (homogeneous agent) expected utility literature, but much smaller than those used in much of the recursive utility literature. For example, Tallarini (2000) considers the case of $\gamma = 100$, while Bansal and Yaron (2004) set $\gamma = 10$, Piazzesi and Schneider (2006) have $\gamma = 59$, and Chen, Favilukis and Ludvigson (2007) estimate γ to be $17 \sim 60$. In the case of the values of the IES parameter, ψ, they are in the range of the values mostly used in the literature. For example, Piazzesi and Schneider (2006) set ψ at 1, and Bansal and Yaron (2004) set it at 1.5. Chen, Favilukis and

Ludvigson (2007) estimate it to be ≈ 2, and Binsbergen et al. (2008) estimate it at less than 1. We will also discuss later why we use large numbers for the standard deviations of the taste and monetary policy shocks in the case of the recursive utility specification.

Table 2 reports the means, the standard deviations, and the correlations with output of the actual and simulated yields. They confirm the well-known stylized facts: the yield curve is upward sloping, the volatility of the yields is slightly downward sloping, and the correlation between the yields (spreads) and output is positive (negative).[6] The mean of the yields goes up from about 5.15% for the 3-month rate to about 6.14% for the 5-year yield. The standard deviation of the yields goes slightly down, from about 2.92% for the 3-month to about 2.74% for the 5-year yield. The positive correlation between nominal interest rates and output declines from 0.22 for the 3-month rate to 0.05 for the 5-year yield, and the short- and medium-end spreads are negatively correlated with output.[7] The model yields from the DSGE model with habit formation show the same pattern except for the correlation between spreads and output. The yields from the model show slightly higher means[8] and much smaller yield volatilities across all maturities than seen in the actual data. The mean of the yields rises from about 5.47% for the 3-month rate to about 6.67% for the 5-year rate, while the standard deviation falls from about 1.74% for the 3-month rate to about 0.76% for the 5-year yield. As the maturity increases, the model generates yields with much smaller volatility. The positive correlation between nominal yields and output rises as the maturity lengthens, and the correlation between spreads and output is positive, which is contrary to the data. The model yields from the DSGE model with

[6] In the data, the positive correlation between nominal interest rates and output turns negative as the maturity increases. However, the negative correlation between spreads and output is robust.

[7] Here we call the spread between the 3- and/or 5-year rate and the 1-year rate the medium-end spread.

[8] We do not take the difference in mean seriously, since Ravenna and Seppala used a different data set, that is, the McCulluck and Kwon data. In addition, the main focus of this paper is examining whether the spectral properties of the model yields can match those of the actual data, and the difference in mean does not affect the spectral properties.

recursive utility do not generate a positive slope in the yield curve. The mean of the yields is flat at about 6.8 % $\sim$ 6.9 % for both Recursive Utility models 1 and 2. The standard deviation of the yields goes down rapidly, from about 1.47% (1.59%) for the 3-month rate to about 0.56% (0.39%) for the 5-year rate. Furthermore, the DSGE model with Recursive Utility 1 generates nominal yields (spreads) that are negatively (strongly positively) correlated with output – counter to the data. In contrast, Recursive Utility model 2 generates nominal yields (medium-end spreads) that are positively (negatively) correlated with output. The bottom three rows of Table 2 report the mean, the standard deviation, and the correlation with output of the interest rates generated from the benchmark new Keynesian(NK) DSGE model. The standard deviation and the correlation with output of interest rates generated from the benchmark new Keynesian model show qualitatively similar patterns to those from the DSGE model with habit formation. However, it has difficulty in generating either a sufficient yield curve slope or negative correlations between spreads and output.

Table 2 reveals some problems associated with the DGSE model with habit formation. First, while we find that the yields generated explain the mean and the standard deviation of the actual data reasonably well, the model is not free of criticism. Rudebusch, Sack and Swanson (2007) and Rudebusch and Swanson (2008) note that Ravenna and Seppala use extremely high persistence and volatilities of shocks to generate sizeable term premia. For example, the autoregressive coefficient for the taste shock is set to be 0.95 and its standard deviation is set to be 0.08, which is a pretty big value. The same criticism applies here, since we just use the same parameter values as Ravenna and Seppala. Another problem with the DSGE model with habit formation is that it generates procyclical short- and medium-end spreads.

Why do DSGE models with habit formation need highly persistent and large shocks to match the mean and the variance of the term structure of interest rates? We believe it is because habit formation does not really provide a good intuition as to the different behavior of interest rates with different maturities, although it can provide good intuition for solution of the equity premium puzzle. More specifically, habit increases the risk aversion parameter implicit in the model without changing the IES of the utility function. This allows for a higher equity premium. To put it differently, introduction of habit makes the marginal utility, and hence the stochastic discount factor or the intertemporal marginal rate of substitution, more volatile and the risk premia generated therefore match the observed data with relatively low variability in the consumption growth rate. However, this intuition does not provide good clues as to how bonds of different maturities should behave in response to a shock. We may thus need to set the value of some parameters, for example, the autoregressive coefficients or the standard deviations of shocks, to some arbitrarily big numbers to explain the term structure.

The results listed in Table 2 show that the DSGE model with recursive utility is not free of problems, either. We find it has difficulty in generating a sufficiently upward sloping yield curve. Furthermore, it generates much smaller volatility than that characterizing the actual yields. These results do not depend on the combinations of the risk aversion parameter, γ, or the IES parameter, ψ. Note that we set $\gamma = 7.5$ and $\psi = 1.5$ in Recursive Utility model 1 and $\gamma = 2.45$ and $\psi = 0.4$ in Recursive Utility model 2. One advantage of the recursive utility model in solving the equity premium puzzle is that it separates the intertemporal substitution (consumption smoothing over time) from the coefficient of risk aversion (consumption smoothing over different states of nature). In addition, the recursive utility specification provides a good intuition as to the slope of the term structure of interest rates. Binsbergen et al. (2008) discuss how the slope of the term structure is affected by the relationship between the relative magnitude of the risk aversion and the inverse of the IES. If $1/\psi > \gamma$, as is the case in Recursive Utility model 2, agents prefer late resolution of uncertainty and buy short-term bonds and roll them over, instead of buying long-term bonds. As a result, the yield curve is upward sloping. If $1/\psi < \gamma$, as is the case with Recursive Utility model 1, agents prefer early resolution of uncertainty. They prefer to buy long-term rather than short-term bonds. As a result, the yield curve slopes downward. However, after many simulations we have found that this mechanism lacks the strength to generate a sufficient yield curve slope, which in view of theory is surprising.

Is it then impossible to generate a sufficient yield curve slope from a DSGE model with recursive utility? The answer is no; we can generate a sufficient yield curve slope from a DSGE model with recursive utility, but only at a cost. We need arbitrarily big numbers for the standard deviations of the taste and monetary policy shocks, although to a lesser degree than in the case of the model with habit formation. For this reason, we do not even bother to report it.

Why do we need big numbers for the standard deviations of the taste and monetary policy shocks in this case? We can find a hint in the previous literature. To name a few, Wu (2006), Bekaert et al. (2010) and Doh (2007) show that the (temporary) monetary policy shock is the slope (and/or curvature) factor in explaining the term structure of interest rates. We need a large monetary policy shock in order to have a sufficient yield curve slope. This study has found that the standard deviation of the monetary policy shock should be as high as one percentage point to result in a sufficient yield curve slope. However, introduction of a large monetary policy shock poses at least two problems. One is that the monetary policy shock implies a negative correlation between nominal interest rates and output, contrary to the observed positive correlation between them. As the interest rate goes up, output goes down. Therefore, the correlation between nominal interest rates and output is negative. Remember that we are considering three shocks: technology, taste, and monetary policy shocks. Among these, only the taste shock implies a positive correlation between nominal interest rates and output. That is why we need a somewhat large taste shock along with a somewhat large monetary policy shock to generate a sufficient yield curve slope and explain the positive correlation between nominal interest rates and output. The second problem is that inflation and/or short-term rates become too volatile as the shocks become too volatile.

Our results are complementary to those of Gregory and Voss (1991). They examine if the utility function with habit formation and/or recursive utility can explain the failure of the expectations hypothesis observed in the Canadian data and show that neither non-expected utility nor habit formation is able to duplicate satisfactorily the magnitude or the variability of the risk premia. In order to generate the observed large risk premia in the model, they require that (counter-factually) bond prices be more volatile than observed in the data.

Table 3 reports the model's goodness of fit, that is, the relative mean square approximation errors (RMSAEs) of the yields generated from the DSGE models with different utility specifications. In order to see if the approximation errors are severe throughout the frequency domain or are confined to particular frequencies, we report the RMSAEs on three frequency bands: $[\pi/3, \infty]$ representing high frequencies, $[\pi/16, \pi/3]$ representing business cycle frequencies, and $[0, \pi/16]$ representing low frequencies. In the case of the model with habit formation, the RMSAEs are in the range of 25% to 47% for the level of yields at all frequencies. These results imply that the approximation errors are, to say the least, not very severe throughout the frequency domain. The RMSAEs are slightly smaller at high and business cycle frequencies than at low frequencies. Errors with variance on the order of 14-40% (10-39%) of the magnitude of the variance of the series are necessary for the components of various yields with periods in the 6 to 32-quarter range (with periods shorter than 6 quarters). In the case of spreads, all of the RMSAEs except for that of the short-end spread are much less than one. The biggest RMSAE is 51% for the first difference of the 3-month yield at low frequency. Most RMSAEs are somewhere around 10% for the medium-end spreads. By comparison, Jung (2007) shows that many of the RMSAEs are greater than one in the case of foreign exchange rates. We can thus say that the yields and spreads generated from the DSGE model with habit formation match the spectral properties of the actual yields and spreads reasonably well.

In the case of the DSGE model with recursive utility, the situation is more or less the same. The RMSAEs are in the ranges of 34% to 65 % and 35% to 75% for the yield levels for Recursive Utility models 1 and 2, respectively. These results imply that the approximation errors are not very severe throughout the frequency domain. The RMSAEs are slightly smaller at the high and/or business cycle frequencies than they are at low frequencies. Errors with variance on the order of 8-47% (13-49%) of the magnitude of the variance of the series are necessary for the components of various yields with periods shorter than 6 quarters (with periods in the 6-32-quarter range) for Recursive Utility model 1. Errors with variance on the order of 8-47% (14-52%) of the magnitude of the variance of the series are necessary for the components of various yields with periods shorter than 6 quarters (with periods in the 6-32-quarter range) for Recursive Utility model 2. In the case of the spreads, all of the RMSAEs except that for the short-end spread are also much less than one. The biggest RMSAE is 50% for the first difference of the 3-month yield at low frequency. Most of the RMSAEs are somewhere around 10% for the medium-end spreads, whatever the parameter specification is. We can thus say that the yields and spreads generated from the DSGE model with Recursive Utility models 1 and 2 also match the spectral properties of the actual yields and spreads reasonably well.

The last column of Table 3 reports the relative mean square approximation errors (RMSAEs) of the yields generated from the benchmark new Keynesian DSGE model. Again, implying a pretty good fit of the model, most of the RMSAEs are much smaller than 1 – in the range of 15% to 50% for the level of yields at all frequencies. These results imply that the approximation errors are not severe throughout the frequency domain. Similarly to the case of the model with habit formation, the RMSAEs are slightly smaller at high and business cycle frequencies than at low frequencies. Errors with variance on the order of 11-36% (8-37%) of the magnitude of the variance of the series are necessary for the components of various yields with periods in the 6-32-quarter range (with periods shorter than 6 quarters). In the case of the spreads, all RMSAEs are also much less than one. In particular, it is noticeable that they are not as bad as Ravenna and Seppala's specification for habit formation in matching the spectral properties of the short-end spread. The worst RMSAE for the short-end spread is about 75% at business cycle frequency. Most of the RMSAEs are again somewhere around 10-20% for the medium-end spreads. We can thus say that the yields and spreads generated from the benchmark new Keynesian DSGE model match pretty well the spectral properties of the actual yields and spreads.

Beaubrun-Diant (2006) finds that the approximation errors in explaining the equity premia concentrate mainly on the high frequencies, while those in explaining the risk-free rate concentrate mainly on the low frequencies. However, the results in Table 3 suggest that the approximation errors in explaining the yields and spreads (with the exception of the short-end spread) are not very bad throughout the frequency domain, whatever the utility specification is.

Figures 1A-D show the spectral properties of the actual yields (and spreads), the model yields (and spreads), and the errors to reconcile the model and the data.[9] In the case of the yields, the spectra of the model yields are generally smaller, especially at low frequencies, than those of the actual yields – regardless of the utility specification (see Figure 1A for habit formation and 1C for Recursive Utility model 2, respectively). In the case of the spreads, the spectra of the model short-end spreads are bigger than

[9] Here, we do not plot the spectra of the yields generated from the Recursive Utility model 1 to save space, since it is not superior to Recursive Utility model 2 in terms of the spectral properties and it generates negative (positive) correlations between yields (spreads) and output – counter to the data.

those of the actual short-end spreads, and the spectra of the model medium-end spreads are more or less similar to those of actual medium-end spreads, whatever the utility specification is (see Figure 1B for habit formation and 1D for Recursive Utility model 2, respectively).

In sum, in terms of the spectral properties it is not easy to differentiate between the performances of the two competing specifications for utility. It seems fair to say that each specification has some individual disadvantages. In the case of habit formation, we need very persistent and large shocks to explain the time-varying term premia. In the case of the recursive utility, we need a very large monetary policy shock along with a large taste shock to explain the time-varying term premia depending upon the specification for recursive utility. Simulations based on the DSGE models with recursive utility turn out to have difficulty in generating either a sufficient yield curve slope or positive (negative) correlations between nominal interest rates (spreads) and output.[10]

5. Concluding Remarks

In the recent macro-finance literature, two competing specifications for utility have been widely employed to explain the term structure of interest rates: habit formation and recursive utility. We have examined whether we can discriminate between the performances of these two specifications for utility by investigating which one can better match the spectral properties of the term structure of interest rates. In terms of the spectral properties, we have found that the performance of each utility specification is more or less the same, suggesting that we should look at other criteria in order to discriminate between them.

We have thus examined the mean, the standard deviation, and the correlations

[10] We tried to do simulations of Rudebusch and Swanson's (2009) model with long-run inflation risk based on the parameter values reported in their Table 3. Unfortunately, the model failed to converge in dynare++.

with output of the interest rates generated from each specification for utility. The previous criticisms of the DSGE model with habit formation are well-known. Those criticisms are, first, that we need very persistent and large shocks in order to generate the time-varying term premia, and, second, that a DSGE model with habit formation cannot explain the movements of financial variables without the model's ability to fit the macroeconomic data being compromised. Our simulations reveal that the DSGE models with recursive utility are not free of problems, either. They have difficulty in generating either a sufficient yield curve slope or positive (negative) correlations between nominal interest rates (spreads) and output. Depending upon the specification for recursive utility, to explain the time-varying term premia, we need a very large monetary policy shock along with a large taste shock (although to a lesser degree than with habit formation).

This paper has some limitations. One and probably the most serious of its limitations is the fact that its results are based on parameter values that are calibrated, not estimated. It would be interesting to estimate the parameters along the lines of Chen, Favilukis and Ludvigson (2007) and Binsbergen et al. (2008), to examine which specification, between habit formation and recursive utility for preferences, can better explain the movements of the yield curve.

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Table 1: Parameter Values

Parameter	Habit Formation*	Recursive Utility 1	Recursive Utility 2	Benchmark NK Model
α	0	0	0	0
β	0.99	0.99	0.99	0.99
h	0.8			0
ϵ	11	11	11	11
σ_C	2.5			2.5
σ_N	0.5			0.5
γ		7.5	2.45	
ψ		1.5	0.4	
ρ				
ξ_P	0.75	0.75	0.75	0.75
ρ_r	0.9	0.9	0.8	0.9
ρ_π	1.5	1.5	1.5	1.5
ρ_y	0	0	0	0
ρ_a	0.9	0.9	0.9	0.9
ρ_D	0.95	0.95	0.5	0.95
ρ_G				
ν		0.3204	0.4235	
π_{ss}	1.0075	1.0075	1.0075	1.0075
σ_{ε_a}	0.0035	0.0035	0.0035	0.0035
σ_{ε_D}	0.08	0.08	0.08	0.08
σ_{ε_r}	0.003	0.003	0.003	0.003
σ_{ε_G}				

* Ravenna and Seppala (2007)

Table 2: Mean, Standard Deviation (s.d.), and Correlation with Output (corr.)

	R3	R12	R24	R36	R48	R60	R12-R3	R36-R12	R60-R12
					Data				
mean (%)	5.15	5.56	5.76	5.93	6.06	6.14	0.41	0.38	0.59
s.d. (%)	2.92	2.92	2.88	2.81	2.78	2.74	0.45	0.54	0.79
corr.	0.22	0.20	0.14	0.10	0.07	0.05	-0.12	-0.56	-0.55
					Habit Formation*				
mean (%)	5.47	6.41	6.62	6.65	6.67	6.67	0.95	0.22	0.26
s.d. (%)	1.74	1.30	0.97	0.89	0.82	0.76	0.68	0.55	0.72
corr	0.15	0.25	0.32	0.36	0.38	0.39	0.12	0.07	0.01
					Recursive Utility 1				
mean (%)	6.77	6.79	6.79	6.79	6.78	6.78	0.02	0.00	-0.01
s.d. (%)	1.48	1.09	0.83	0.70	0.62	0.56	0.49	0.49	0.65
corr.	-0.74	-0.64	-0.50	-0.40	-0.32	-0.27	0.78	0.83	0.82
					Recursive Utility 2				
mean (%)	6.92	6.93	6.93	6.93	6.93	6.93	0.01	0.01	0.01
s.d. (%)	1.59	1.09	0.74	0.57	0.46	0.39	0.59	0.57	0.74
corr.	0.05	0.19	0.22	0.21	0.20	0.18	0.21	-0.16	-0.18
					Benchmark NK Model				
mean (%)	6.66	6.69	6.70	6.71	6.71	6.71	0.02	0.02	0.03
s.d. (%)	1.73	1.38	1.15	1.02	0.92	0.84	0.52	0.53	0.71
corr.	0.03	0.17	0.28	0.34	0.37	0.39	0.33	0.21	0.13

* Ravenna and Seppala (2007)

Rxx represents the yield to maturity of xx months.

Table 3: Relative Mean Square Approximation Errors

Cycles (Quarters)	Variables	Habit Formation*	Recursive Utility 1	Recursive Utility 2	Benchmark NK Model
all	R3	0.25	0.34	0.35	0.25
	R12	0.29	0.42	0.45	0.30
	R36	0.40	0.57	0.66	0.42
	R60	0.47	0.65	0.75	0.50
-6	R3	0.10	0.08	0.10	0.08
	R12	0.10	0.12	0.08	0.12
	R36	0.31	0.35	0.33	0.29
	R60	0.39	0.47	0.47	0.37
6-32	R3	0.14	0.13	0.14	0.11
	R12	0.16	0.20	0.15	0.15
	R36	0.33	0.40	0.39	0.29
	R60	0.40	0.49	0.52	0.36
32-	R3	0.29	0.40	0.41	0.28
	R12	0.33	0.48	0.53	0.34
	R36	0.42	0.61	0.71	0.44
	R60	0.49	0.68	0.79	0.52
all	ΔR3	0.16	0.13	0.17	0.13
	R12-R3	0.91	0.58	0.74	0.61
	R36-R12	0.10	0.11	0.15	0.11
	R60-R12	0.09	0.12	0.12	0.10
-6	ΔR3	0.16	0.08	0.08	0.08
	R12-R3	0.91	0.48	0.54	0.48
	R36-R12	0.10	0.23	0.43	0.24
	R60-R12	0.09	0.16	0.29	0.18
6-32	ΔR3	0.47	0.32	0.54	0.32
	R12-R3	1.12	0.70	0.91	0.75
	R36-R12	0.12	0.12	0.17	0.11
	R60-R12	0.10	0.12	0.13	0.12
32-	ΔR3	0.51	0.50	0.50	0.49
	R12-R3	0.90	0.50	0.70	0.50
	R36-R12	0.05	0.07	0.04	0.06
	R60-R12	0.07	0.11	0.07	0.08

* Ravenna and Seppala (2007)
Rxx represents the yield to maturity of xx months.

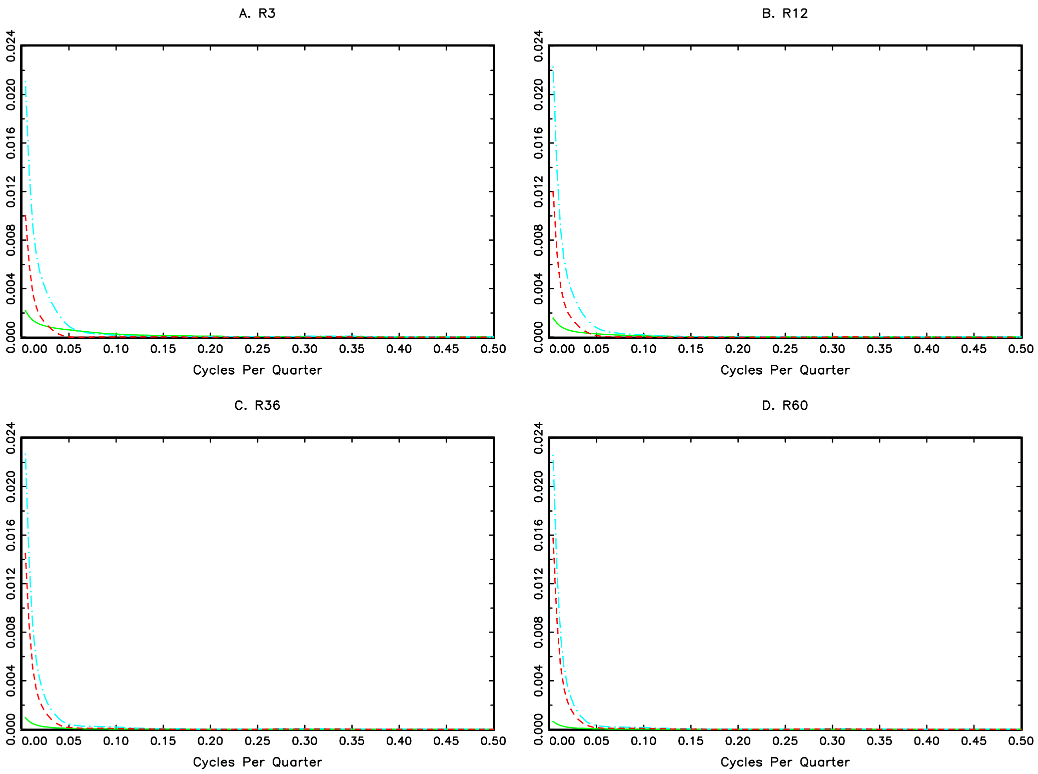

Figure 1A Spectra from the DSGE model with Habit Formation (Ravenna and Seppala (2007))
(solid lines(green)-model; dots & dashes(blue)-data; dashes(red)-error)

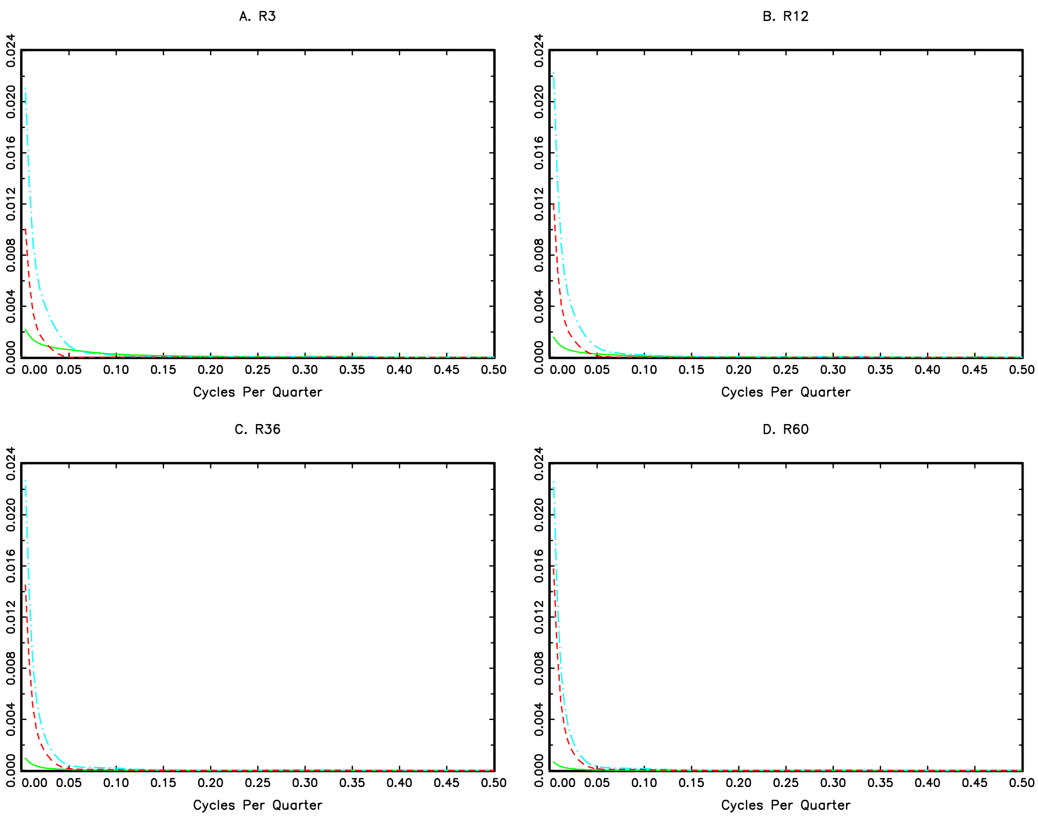

Figure 1B Spectra from the DSGE model with Recursive Utility 2
(solid lines(green)-model; dots & dashes(blue)-data; dashes(red)-error)

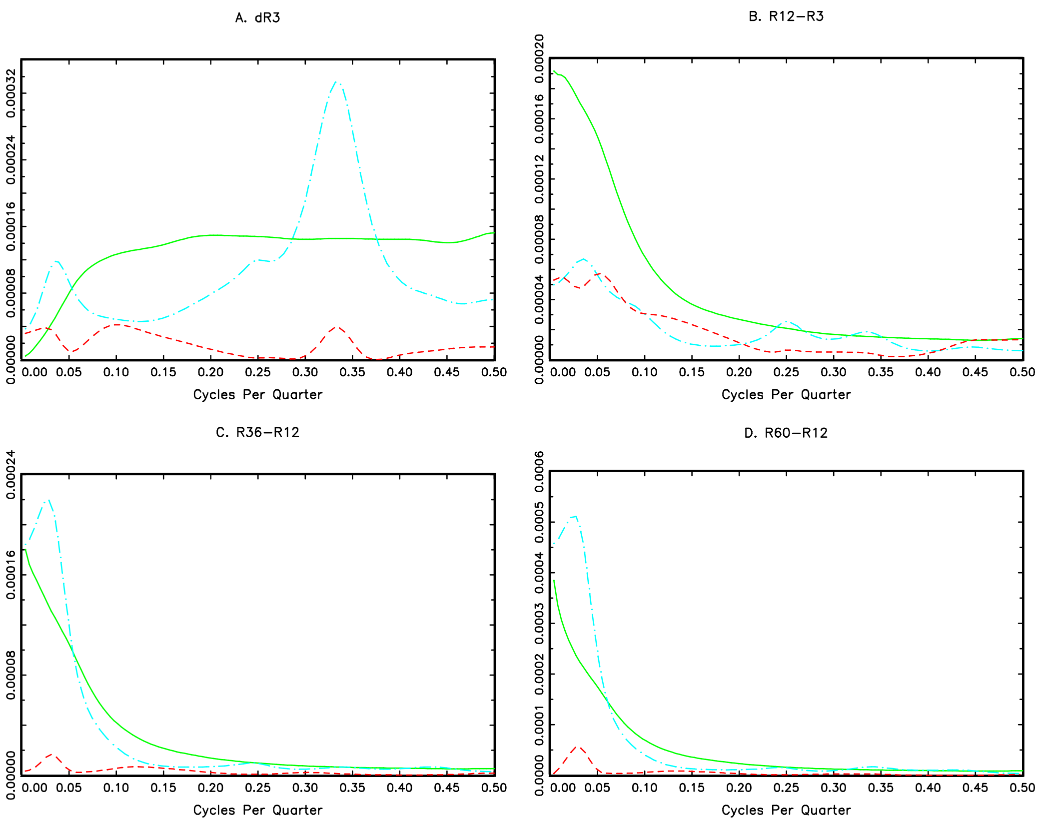

Figure 1C Spectra from the DSGE model with Habit Formation (Ravenna and Seppala (2007))
(solid lines(green)-model; dots & dashes(blue)-data; dashes(red)-error)

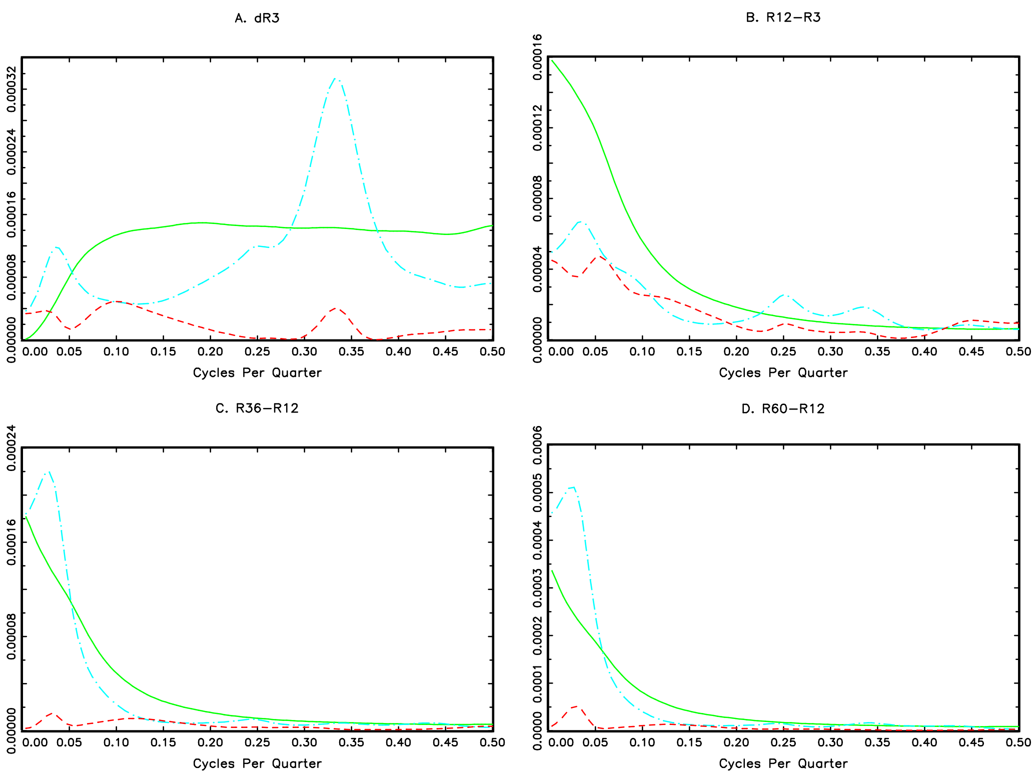

Figure 1D Spectra from the DSGE model with Recursive Utility 2
(solid lines(green)-model; dots & dashes(blue)-data; dashes(red)-error)