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Prices and the Velocity of Money in a Model with Search-Based Monopolistic Competition*

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Abstract This paper presents a search-theoretic model to investigate the role for consumers' trade-or-not-to-trade decisions in the interaction between nominal prices and the frequency of monetary trade. The general message of the results is that it yields a positive relationship between them. A rise in prices, which cuts down the value of money, leads consumers to spend their money more quickly. Moreover, there could exist positive feedback: it is found that more frequent shopping of consumers raises the prices posted by profit-maximizing producers or determined in some alternative environments. This positive relationship provides a theoretical account for a positive effect of expansionary monetary policy on production, which is not channeled by capital-money portfolio decisions. Around the unique monetary steady state of the model, an increase in the money supply normally raises nominal prices and the frequency of monetary trade at the same time. This implies that it has not only negative intensive effect but also positive extensive one on production.

Keywords money, search, price setting, velocity, extensive margin

JEL Classification E31, C78

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1 Introduction

This paper studies the relationship between nominal prices and the velocity of money in a search-theoretic model. In the model, sellers post the nominal prices of their own products and then meet random buyers, whose preferences are heterogenous and private information, one by one. Here the sellers, who take others' pricing as given, have to cut down their posting prices if they want to raise their sales. They are under search-based monopolistic competition in the sense that their products are differentiated by search costs as well as satisfaction levels of the buyers.

Definitely, the velocity of money depends on the extensive margin of monetary trade, or its frequency, as well as its intensive margin, or the terms of trade. It is also doubtless that these two margins are linked to some extent. Here it may be useful to mention the famous story about the 1923 German hyperinflation: "Workers were paid twice a day, and given half-hour breaks to rush to the shops with their satchels, suitcase, or wheelbarrow, to buy something, anything, before their paper money halved in value yet again." (The Economist, December 31st 1999, p. 94)

Nevertheless, traditional studies that use competitive equilibrium models have made little account of the extensive margin. In fact, it is incoherent to capture the margin, which means the variability of delay in trade, while assuming frictionless markets. Since the demand for real money balances in these models are not affected by the frequency of monetary trade, the velocity of money has been treated as nothing but a summary static.[1]

A search-theoretic approach makes the velocity of money a more meaningful concept since it is explicit about the extensive margin. In a search-theoretic model, the probability of monetary trade directly affects individual decisions including demand for real money balances. However, even in this literature, most of the models has paid little attention to the

[1]In a cash-in-advance model, real money balances bind total quantity of the cash goods purchased within a period as if they were traded only once in each period. Money-in-the-utility-function models as well as some transaction cost models assume that real money balances able to substitute the time which people spend in shopping. However, they depict nothing about changes in the number of shopping within a period that yield this substitution.

opposite direction, or the way in which the terms of monetary trade and the value of money affect the frequency of monetary trade. This means that the frequency of monetary trade in these models is not actually endogenous. For example, in Lagos and Wright (2005), the generalized Nash bargaining mechanism makes the frequency of monetary trade equal to the probability of a single coincidence, which is exogenously given. Hence it is hard to say that such a model fully captures the variability of the frequency of monetary trade even though Wang and Shi (2006) show that this variability is important for explaining the variability of the velocity in the US data.[2]

Clearly, a model to study endogenous trade frequency should have individual decisions that are not only influenced by but also directly affect it. In existing models with the endogenous frequency, it is usual that sellers' search (or market participation) decisions assume this position (e.g., Faig and Jerez, 2005; Rocheteau and Wright, 2005; Wang and Shi, 2006). As is standard in labor market matching models, the market tightness in these models is determined by the free entry condition that drives sellers' profits to zero.[3] Notice that, in this framework, a fall in the value of money drives out some sellers, and hence it lowers down the frequency of monetary trade tightening the market against consumers.

This paper focuses on another kind of individual decisions also directly linked with the frequency of monetary trade: buyers' to-trade-or-not-to-trade decisions. These decisions determine the conditional probability of trade for each product given price, or the extensive margin of the demand for it, and this probability affects its price. Reversely, changes in nominal prices also affect the frequency of trade. Notice that the buyers' decisions are affected by the cost of money holding, which is necessary for buyers to purchase substitutes in the future. A rise in nominal prices, or equivalently, a fall in the value of money, shortens

[2]Equation (24) in Lagos and Wright (2005) explicitly shows how the velocity of money in their model $1/L$ is determined by the exogenous frequency of monetary trade σ and the endogenous real money balance $z(q)$. Notice that, if the centralized market sector were dropped from the model, the velocity of money would be fixed at σ. In contrast, the endogenous frequency of monetary trade in the model of this paper allows the velocity of money to vary without the sector. Thanks to an anonymous referee for this comment.

[3]In many of these models such as Faig and Jerez (2005) and Wang and Shi (2006), agents choose search intensity instead. However, since each agent in those models is a large decision-making unit, its intensity decision can be interpreted as nothing but its members' participation decisions.

the voluntary delay in trade and raises the velocity.[4] These all together imply a two-way channel through which a rise in the velocity raises nominal prices and vice versa.

In order to focus on interactions through this extensive channel, like Trejos and Wright (1995) and Shi (1995), this paper normalizes the units of money traded at one time to one by assuming that agents in the model hold no more than one indivisible unit of money. In addition, it focuses on an economy in which all trades are monetary. Hence the velocity of money in the model is equivalent to the frequency of trade. Also, the reciprocals of prices in the model actually have the same meaning to real money balances since they are equal to quantities traded of real commodities at one time. However, these real balances are by no means associated with capital-money portfolio decisions, which are emphasized in standard inflation studies, since the model gets rid of capital for simplicity.

For the same purpose, the model assumes seller's price posting with buyers' heterogenous preferences. This framework is not prevailing in the search theory literature, but still quite a number of studies adopt its different versions (e.g., Green and Zhou, 1998; Camera and Winkler, 2003; Curtis and Wright, 2004; Faig and Jerez, 2005). Moreover, as explicitly shown for some benchmark cases, main findings in this environment have no reason to qualitatively change as long as buyer's trade decisions channel interactions between prices and the velocity of money in the qualitatively same way.

Most of the model predictions around its unique monetary steady state highlight the roles for the extensive channel. Changes in the money supply lead to nominal prices and the acceptance rate of monetary trade moving in the same direction. The effect of progress in the matching technology on the velocity is ambiguous in the direction since it lowers down prices by facilitating competition and the fall in nominal prices makes buyers less likely to spend money. The effect of progress in the production technology on total output is also qualitatively ambiguous even in case that it increases quantities produced for each trade.

[4] In this direction, the channel is quite similar with the extensive real balance effect in Shi (1999). However, they have fundamental differences. Real balances in his model are buyers' choices associated with their own market participation decisions. Here buyers take prices, or real balances, as given and make ex-post to-trade-or-not-to-trade decisions. They are basically the same in case that buyer's take-it-or-leave-it offer is the pricing mechanism but not in general.

Since a fall in nominal prices makes buyers less likely to accept a monetary trade, more quantities produced for each trade could imply lower frequency of trade.

The positive relationship between the price level and the velocity of money occurs even across different kinds of equilibria. All other things being equal, the price level in the unique steady state with buyer's take-it-or-leave-it offer is lower than that in any steady state with seller's price posting. However, the former steady state has lower frequency of trade than that in the latter. Also, the unique steady state with seller's free entry has the price level and the frequency of trade both lower than those in any steady state with a fixed number of sellers.

The rest of the paper is organized as follows. Section 2 describes the model economy with buyers' heterogenous preferences and seller's price posting. Section 3 characterizes its monetary steady state and compare it with ones in alternative environments. Section 4 discusses the results of qualitative analysis. Section 5 gives concluding remarks. All proofs of the lemmas and the propositions are in the Appendix.

2 The Model

2.1 The Environment

There is a continuum of infinitely lived agents with total mass equal to one. Time is continuous, and all of the agents discount future utility at the rate $r > 0$. There also exists a continuum of different perishable goods.

The agents are different in preferences. Agent i derives utility $z_{ij}u(q)$ from consuming q units of good j, where $z_{ij} \in [0, 1]$ represents his satisfaction with this particular good. This level of satisfaction is determined by a random shock. At the initial time and every time after he consumes, each agent draws the preference shocks independently and identically distributed with distribution function $F \in \mathbb{C}^2$. That is, for every agent, the proportion of the goods with which his satisfaction level $\tilde{z} \leq z$ is given by $F(\tilde{z})$ at any point in time. Assume $F' > 0$ and $F'' \geq 0$. In what follows, we simplify the presentation by writing

$u(q) = q$. This is merely a normalization without loss in generality: quantities of the goods are measured in utils rather than physical units.

Besides the real commodities, there exists an object called money. Money has no intrinsic value but is perfectly storable. The stock of this fiat money is exogenously given and constant over time. Initially, a fraction $M \in (0, 1)$ of the agents are each endowed with one unit of money, and the rest of them are each endowed with a production opportunity. A new production opportunity arrives at an agent every time after he consumes.

Each agent must first fix his production set to produce. Then, by spending a production opportunity, an agent can produce any quantity of one good in his production set instantly. However, in order to restrict attention on market activities, here it is assumed that autarkic production allows no surplus, and that an agent must consume others' products to receive a new opportunity of production for market sales. In addition, to focus on issues other than specialization, it is also assumed that production for a marketable good requires full specialization, or a singleton production set. To each agent, the good available for this specialization is exogenously given, and it does not change over time.[5]

Let each agent be indexed by the single good he can produce and sell. That is, an agent whose marketable good is j is called agent j. The marketable goods of the agents are uniformly distributed all over the goods. For every agent, it costs $c(q)$ utils to produce q units of his marketable good, where $c \in \mathbb{C}^2$, $c' > 0$, $c'' > 0$, and $c(0) = \lim_{q \to 0} c'(q) = 0$. Also, there exists $\hat{q} > 0$ such that $c(\hat{q}) = \hat{q}$.

Bilateral Trade and Price Posting

Agents who want to trade meet in a decentralized market according to the random pairing process described below. They are anonymous, and this makes no punishment for their

[5] In the literature, it is usual that the agents in a model are prohibited from consuming their own products in order to avoid complications for considering the possibility of autarky. However, by assuming the same production technology for market sales and autarkic consumption, we can study the binary choice between specialization for market participation and autarky as in Shi (1997b). Also, like Camera, Reed, and Waller (2003), by assuming that every agent is assigned to production for one particular good in which he is most productive and allowing each one to choose his production set for market sales with a positive measure, we can extend the model to study producer's decision of how much to specialize.

reneging on debt able to be enforced. Thus, exchange must be quid pro quo. In addition, since the goods are perishable (and production is costly), they are produced only for immediate sales or autarkic consumption. Hence only fiat money qualifies for a medium of exchange.

For simplicity of analysis, it is assumed that money is indivisible. An agent with one unit of money gets a market production opportunity if and only if he spends all of his money. An agent with a market production opportunity gets one unit of money if and only if he spends it up. Therefore, at any point in time, there could exist only three types of agents: buyers each with one unit of money but no market production opportunity, sellers each with a market production opportunity but no money, and autarkists out of the market.

Since we are interested in studying a pure monetary economy in which all exchanges are monetary, barter is ruled out by assuming directed search to the other side of the market. The sellers stand waiting for their trading partners, so they never meet each other. Let n_t denote the mass of sellers per buyer at time t, which indicates the market tightness for the buyers. Under constant return to scale matching technology, the arrival rate at which each buyer meets a seller is given by $\alpha(n_t)$ and the rate to each seller is given by $\alpha(n_t)/n_t$, where $\alpha' > 0$, $\alpha'' < 0$, $\alpha(n) \leq \min\{n, 1\}$, $\alpha(0) = 0$, and $\lim_{n\to\infty} \alpha(n) = 1$.

In determination of the terms of trade, sellers set prices. The seller in each trade match makes a take-it-or-leave-it offer how much to produce his marketable good in exchange for one unit of money. However, he cannot observe the preference shocks drawn by the buyer. That is, he does not know how much the buyer likes his product. Also, there is no way for any information about buyer's preferences to be credibly revealed (except possibly by the buyer refusing to trade). Since the sellers are not able to set discriminating prices, setting prices either in advance or after meeting a specific buyer does not make any difference. Let the sellers post prices in advance.

If agent j is a seller at time t, he posts the name of his marketable good j and its nominal price p_{jt}. This menu implies that he offers to produce $q_{jt} = 1/p_{jt}$ units of good j in exchange for one unit of money. When a buyer meets agent j and see the menu, he chooses whether he spends his money on consuming the good or refuses to trade and walk away.

In summary, the sequence of events for a representative agent who initially hold money is as follows. At the very beginning of time, he draws the preference shocks and becomes a buyer. Subsequently, he begins his search process meeting sellers pairwise and randomly over time. Each time he meets a seller, he checks her menu. If he accepts the trade, production and consumption take place after which he draws new preference shocks and becomes a seller. Then he post his menu and starts meeting sellers pairwise and randomly over time. If a buyer accepts the trade, production and consumption take place after which he becomes a buyer again.

Let distribution function Q_t characterize the distribution of prices at time t. That is, $Q_t(q) = \Pr\{\tilde{q}_t \leq q\}$, where $\tilde{q}_t$ is q_{jt} of random seller j. Also, let v_{it} denote the loss of spending one unit of money for buyer i, or that of his becoming a seller, and define $V_t(v) = \Pr\{\tilde{v}_t \leq v\}$, where $\tilde{v}_t$ is v_{it} of random buyer i.

2.2 Value Functions and Decisions

This study focuses on stationary equilibria, or steady states, in which the aggregate states are time invariant: for all t, $n_t = n$, $Q_t = Q$, and $V_t = V$. In addition, we are interested in monetary ones, in which $n > 0$, $Q(0) < 1$, and $V(0) < 1$.

Each individual agent takes the aggregate states, which are determined in equilibrium, as given. Given n, Q, and V, let V_m^j and V_0^j denote the stationary expected lifetime utility of buyer j and that of seller j respectively. Notice that constant $\alpha(n)$ and $\alpha(n)/n$ imply Poisson arrival rates. The Bellman equation for buyer j has asset pricing representation

$$\rho^b V_m^j = \int\int \max\left\{\tilde{z}\tilde{q} - V_m^j + V_0^j, 0\right\} dF(\tilde{z})\, dQ(\tilde{q}), \tag{1}$$

where $\rho^b = r/\alpha(n)$ represents the degree of search friction for the buyers.

Suppose that buyer i meets seller j. Clearly, the buyer's decision about whether to accept or reject the seller's offer has a reservation property: he accepts if and only if $z_{ij}q_j \geq$

$v_i = V_m^i - V_0^i$. Therefore, the value function for seller j is given by

$$\rho V_0^j = \max\left\{\max_q \int \left[1 - F\left(\tilde{v}/q\right)\right]\left[V_m^j - V_0^j - c\left(q\right)\right] dV\left(\tilde{v}\right), 0\right\}, \tag{2}$$

where $\rho = n\rho^b$ represents the degree of search friction for the sellers. Clearly, $V_0^j > 0$ if and only if $V_m^j > V_0^j$. Also, $V_m^j > 0$ if and only if $V_0^j > 0$.

Assume that agent j is in the market only when $V_m^j > 0$ and hence $V_0^j > 0$. Then, all of the market participants have the same V_m and V_0. See the Appendix for the proof. This implies that V degenerates to $v > 0$ in equilibrium. Hence, for any monetary stationary equilibrium, we have

$$\rho V_0 = \max_q \left[1 - F\left(v/q\right)\right]\left[v - c\left(q\right)\right]. \tag{3}$$

The following lemma claims that every seller has the same optimal price for any value of money v able to constitute a monetary equilibrium.[6]

Lemma 1 *For every $v \in (0, \hat{q})$, there exists unique solution $q^\star \in \left(v, c^{-1}\left(v\right)\right)$ to the optimization problem in (3) such that*

$$\left(v/q^{\star 2}\right) F'\left(v/q^\star\right)\left[v - c\left(q^\star\right)\right] = \left[1 - F\left(v/q^\star\right)\right] c'\left(q^\star\right). \tag{4}$$

Hence Q also degenerates to $q > 0$ in equilibrium, and for any monetary stationary equilibrium, we have

$$\rho^b V_m = \left[1 - F\left(v/q\right)\right]\left[E\left(v/q\right) q - v\right], \tag{5}$$

where $E\left(z\right) = \mathbb{E}\left[\tilde{z} \mid \tilde{z} \geq z\right]$. Given optimal q, (3) and (5) yield

$$\rho v = \rho\left(V_m - V_0\right) = \left[1 - F\left(v/q\right)\right]\left[nE\left(v/q\right) q - \left(n+1\right) v + c\left(q\right)\right]. \tag{6}$$

Lemma 2 *For every $q \in (0, \hat{q})$, there exists unique $v \in (0, q)$ that satisfies (6).*

[6] The assumption of $F'' \geq 0$ is sufficient but not necessary for the uniqueness. Moreover, it has nothing to do with the existence of the optimal solution. Hence it might be less contrived to simply focus on symmetric equilibria without such restriction. However, this assumption on preference shocks is no more than standard one on the demand to yield the diminishing marginal revenue. Notice that the the LHS and RHS of (4) represent the marginal revenue and cost respectively. Here it might be also interesting to study price dispersion as in Curtis and Wright (2004) without any assumption for the uniqueness, but it is not the subject of this paper.

3 Monetary Stationary Equilibrium

3.1 Price Posting Equilibrium

It is clear that autarky is a unique non-monetary stationary equilibrium for this economy. In monetary stationary equilibrium, every agent, who is initially a buyer or a seller, does not leave the market since $V_m > 0$ and $V_0 > 0$. In addition, exchanges do not affect the total mass of the buyers and that of the seller. Hence $n = 1/M - 1$ in any monetary stationary equilibrium.

Focus on buyer's reservation satisfaction level $\epsilon = v/q$ rather than the value of money v, and let $\Xi = (0, \hat{q}) \times (0, 1)$. Manipulation of (4) and (6) yields

$$\phi(q, \epsilon) = \rho\epsilon - [1 - F(\epsilon)][nE(\epsilon) - (n+1)\epsilon + \hat{c}(q)] = 0, \tag{7}$$

$$\psi(q, \epsilon) = \epsilon F'(\epsilon)[\epsilon - \hat{c}(q)] - [1 - F(\epsilon)]c'(q) = 0, \tag{8}$$

where $\hat{c}(q) = c(q)/q$ represents the average cost of producing q utils. The partial derivatives of ϕ and ψ with their signs are listed in the Appendix.

A price posting equilibrium, which is monetary and stationary, is defined as a pair of $(q^*, \epsilon^*) \in \Xi$ that simultaneously solves $\phi(q^*, \epsilon^*) = 0$ and $\psi(q^*, \epsilon^*) = 0$ together with $n = 1/M - 1$ and $\rho = rn/\alpha(n)$. In an equilibrium, the economy has the price level $P(q^*) = 1/q^*$, and the flow rate of exchange, or the velocity of money,

$$T(\epsilon^*) = \alpha(1/M - 1)M[1 - F(\epsilon^*)]. \tag{9}$$

Notice that the flow rate of exchange depends on *the acceptance rate of trade* $1 - F(\epsilon^*)$, which represents buyers' to-trade-or-not-to-trade decisions, as well as the flow rate of bilateral meeting $\alpha(1/M - 1)M$.

Consider $\epsilon^\phi : [0, \hat{q}] \to [0, 1]$ such that $\phi\left(q, \epsilon^\phi(q)\right) = 0$ and $q^\psi : [0, 1] \to [0, \hat{q}]$ such that $\psi\left(q^\psi(\epsilon), \epsilon\right) = 0$. By Lemma 1 and 2, both are well-defined, and $q^\psi(\epsilon) < \hat{c}^{-1}(\epsilon)$ for every ϵ. By the implicit function theorem, $\epsilon^{\phi\prime} = -\phi_q/\phi_\epsilon > 0$ and $q^{\psi\prime} = -\psi_\epsilon/\psi_q > 0$. In addition, since $\phi(0, 0) = -n\mathbb{E}[\tilde{\epsilon}]$ and $\phi(\hat{q}, 1) = \rho$, $\epsilon^\phi(0) = \underline{\epsilon} > 0$, $\epsilon^\phi(\hat{q}) < 1$, and $\lim_{q \to 0} \epsilon^{\phi\prime}(q) = 0$. Also, $\psi(0, 0) = \psi(\hat{q}, 1) = 0$ and $\lim_{\epsilon \to 0} q^{\psi\prime}(\epsilon) = 0$.

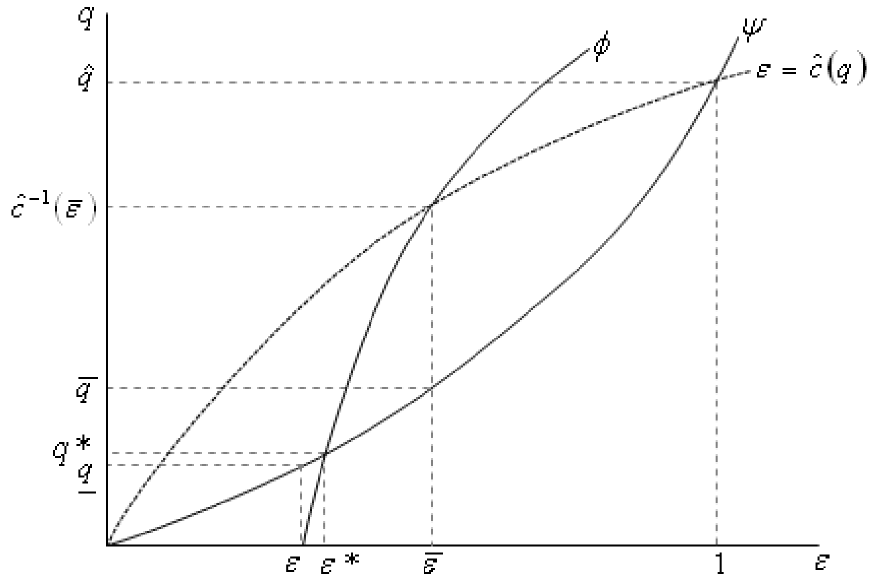

Figure 1: monetary steady state

Lemma 3 *There exists unique* $\bar{\epsilon} \in (0,1)$ *such that* $\phi\left(\hat{c}^{-1}\left(\bar{\epsilon}\right), \bar{\epsilon}\right) = 0$.

Let $\underline{q} = q^{\psi}\left(\underline{\epsilon}\right)$ and $\bar{q} = q^{\psi}\left(\bar{\epsilon}\right) = \hat{c}^{-1}\left(\bar{\epsilon}\right)$. Figure 1 graphically shows equilibrium (q^*, ϵ^*). Clearly, there exists a price posting equilibrium, and it lies in $\left(\underline{q}, \bar{q}\right) \times \left(\underline{\epsilon}, \bar{\epsilon}\right)$. There could exist multiple symmetric equilibria. However, we can rule out such possibility by assuming convex $\hat{c}$, increasing c'', or $c\left(q\right) = \gamma q^{\theta}$, where $\theta > 1$.

Proposition 1 *There exists a price posting equilibrium. It is unique if the costs of production satisfies* $q\hat{c}''\left(q\right) > c'\left(q\right) - \hat{c}\left(q\right)$ *for* $q \in \left(\underline{q}, \bar{q}\right)$.

3.2 Benchmark Cases

Buyer's Take-It-or-Leave-It Offer

Here it might be noticeable that $\left(\hat{c}^{-1}\left(\bar{\epsilon}\right), \bar{\epsilon}\right)$ is a monetary stationary equilibrium under a different pricing mechanism. In the literature, for practice, it is usually assumed that the buyer in a trade match makes take-it-or-leave-it offer to the seller with the full bargaining power. This means that the buyers extract the whole trading surplus and the sellers produce their reservation quantity, which makes them indifferent between trading or not. That is, given the value of money v, every seller produces q such that $v = c(q)$ having $V_0 = 0$. Given this q, the value of money v becomes V_m in (5). Define

$$\varphi\left(\epsilon;\rho^b\right) = \rho^b\epsilon - [1 - F(\epsilon)]\,[E(\epsilon) - \epsilon]\,. \tag{10}$$

A buyer's price-setting equilibrium is defined as a pair of $(\bar{q}, \bar{\epsilon}) \in \Xi$ that satisfies $\varphi\left(\bar{\epsilon};\rho^b\right) = 0$ and $\bar{\epsilon} = \hat{c}(\bar{q})$ together with $\rho^b = r/\alpha\,(1/M - 1)$. The proof of the following proposition comes directly from Lemma 3 and the fact that price posting equilibrium $\epsilon^* < \bar{\epsilon}$.

Proposition 2 *There exists a unique buyer's price-setting equilibrium. The price level in it is lower than that in any price posting equilibrium, but its frequency of trade is also lower.*

This finding and many others below highlight the role for changes in the equilibrium value of money. If we hold to the extensive margin of trade at the firm level, it might be surprising that lower prices are associated with lower frequency of trade. However, since lower steady-state nominal prices mean stably higher value of money, buyers become less willing to spend their money with higher reservation satisfaction level. This makes trade less likely to take place. Intuitively, we can say that buyers do not trade so often as before when they become able to purchase more in each trade.

Free Entry

Now consider the case that, given mass M of the buyers, the mass of the sellers is endogenously determined by free entry. As in market structure studies, this can be thought the long-run case of monopolistic competition.

In (3), for every $v \in (0, \hat{q})$, $V_0 = 0$ if and only if $\alpha(n)/n = 0$, or equivalently $n = \infty$. Hence the zero profit condition implies the infinite mass of the sellers.[7] Given this q, the

[7] For more formal discussion, we can assume that the sellers have to pay fixed search (or market participation) cost $\delta > 0$ as in standard labor market matching models. Then, free entry implies $\hat{V}_0 = V_0 - \delta = 0$, and it can be shown that the sequence of price posting equilibria converges to the free entry equilibrium as δ approaches 0 from above. Here I consider only sufficiently small δ. However, since this paper is not mainly interested in sellers' search decisions, it looks enough to take account of an extreme case in which there exists the infinite number of the sellers per buyer.

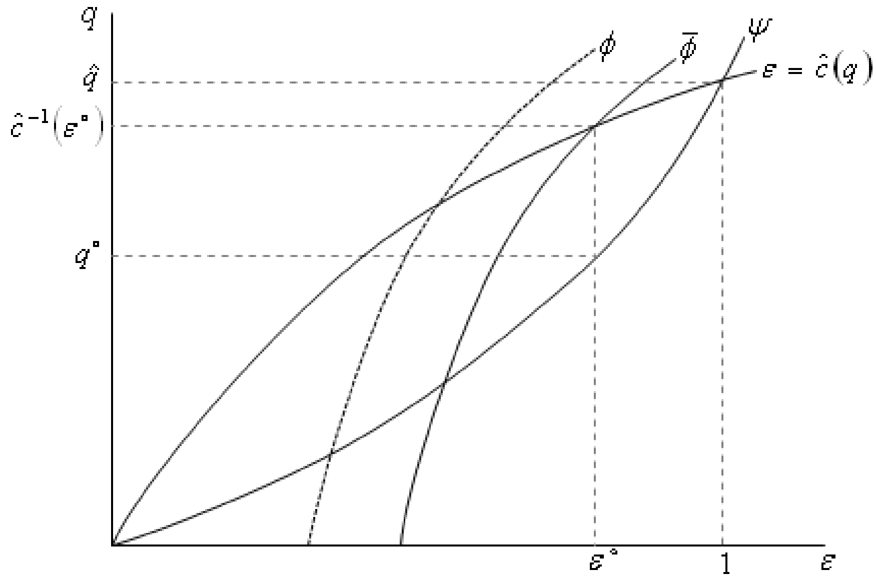

Figure 2: monetary steady state with free entry

value of money v becomes V_m in (5) together with $\rho = r$. Define

$$\bar{\phi}(q, \epsilon) = \varphi(\epsilon; r) - (1/n)[1 - F(\epsilon)][\epsilon - \hat{c}(q)] \tag{11}$$

and $\epsilon^{\bar{\phi}} : [0, \hat{q}] \to [0, 1]$ such that $\bar{\phi}\left(q, \epsilon^{\bar{\phi}}(q)\right) = 0$. Since $r\epsilon - \bar{\phi}(q, \epsilon)$ is concave on ϵ and $r < \rho^b$ for every $n < \infty$, $\epsilon^{\bar{\phi}}(q; n) > \epsilon^{\phi}(q; n)$ for every $(q, n) \in [0, \hat{q}] \times \mathbb{R}_{++}$.

A free entry equilibrium is defined as a pair of $(q^{\circ}, \epsilon^{\circ}) \in \Xi$ that satisfies $\varphi(\epsilon^{\circ}; r) = 0$ and $\psi(q^{\circ}, \epsilon^{\circ}) = 0$. Figure 1 graphically shows equilibrium $(q^{\circ}, \epsilon^{\circ})$. The proof of the following proposition comes directly from Proposition 2 and the fact that ϵ^{ϕ} is bounded above by $\epsilon^{\bar{\phi}}$.

Proposition 3 *There exists a unique free entry equilibrium. The price level in it is lower than that in any price posting equilibrium with a finite mass of the sellers, but its frequency of trade is also lower.*

4 Qualitative Results

Now let me examine the effects of changes in the money supply and technology around the unique price posting equilibrium.

4.1 Money Supply

First consider the effects of a change in M, total mass of money, or the fraction of the population holding money. As in other models with indivisible money, the market tightness $n = 1/M - 1$ channels every effect of a change in the money supply to equilibrium (q^*, ϵ^*). An increase in the money supply lowers down the arrival rate to the buyers $\alpha(n)$ but raises that to the seller $\alpha(n)/n$. Also notice that, even if equilibrium ϵ^* would remain unchanged, it directly affects the velocity of money $T(\epsilon^*; M)$.

Proposition 4 *An increase in the money supply raises the price level and the acceptance rate of trade level if and only if*

$$[1 - F(\epsilon^*)][E(\epsilon^*) - \epsilon^*] > r[\alpha(n) - n\alpha'(n)]\alpha(n)^{-2}\epsilon^* \tag{12}$$

for equilibrium ϵ^* together with $n = 1/M - 1$. It raises the velocity of money if additionally $\alpha(n) - n\alpha'(n) \geq \alpha'(n)$ holds.

Notice that the left hand side of (12) represents the effect of the relative change in the arrival rates on the value of money while its right hand side represents the effect of the absolute change in the degree of search friction ρ. Thus it is natural to assume that the condition holds as long as we think it is not desirable to emphasize the latter too much when studying the effects of a change in the money supply. For the same reason, it is also natural to assume that the second condition in Proposition 4 holds.

For every price level, an increase in the money supply cuts down the value of money and buyers' reservation satisfaction levels if and only if the condition in (12) holds. For every posting price of each product, the fall in the value of money raises the probability of its sales, or increases the demand for it in the extensive margin. Hence the sellers raise the prices.[8] Graphically, the ϕ curve shifts to the left in Figure 1. The ψ curve remains unchanged since

[8] The fall in the value of money also lowers down the marginal revenues of the sellers from sales in the intensive margin. Hence it looks like it is possibility that they cut down the prices of their products. However, as discussed in Section 1, prices in a indivisible-money model actually mean the reciprocals of real balances, or quantities traded of the products at one time. Hence the fall in the marginal revenues in the intensive margin raises prices in the model since it makes the sellers produce less for each trade.

seller's choice of q is not directly affected by the market tightness. Notice that a change in the money supply leads the price level and the acceptance rate of trade to move in the same direction anyhow without depending on whether the condition in (12) holds or not.

Clearly, Proposition 4 is also true for the buyer's price-setting equilibrium even though the sizes of the effects are generically different. Money is neutral in the free entry equilibrium since ϵ° such that $\varphi(\epsilon^{\circ}; r) = 0$ does not depend on n.

4.2 Technology

Two kinds of technology work in this economy: one is the matching technology that determines α and the other is the production technology characterized by c.

Matching Technology

Consider the effects of progress in the matching technology, which implies a rise in the arrival rate $\alpha(n)$ for every level of the market tightness n. Given prices, higher frequency of meeting in the market implies shorter delay in the use of money. Hence, for every price level, it raises the value of money and buyers' reservation satisfaction levels. The rest of discussion is not different from the previous one in the last subsection but only directions are opposite. In Figure 1, the ϕ curve shifts to the right.

Proposition 5 *As the matching technology progresses, both of the price level and the acceptance rate of trade go down. Its effect on the velocity of money is ambiguous.*

Perhaps surprisingly, there exists a possibility that lower degree of search friction induces lower frequency of trade. Again, this sheds light on the equilibrium effect due to changes in the value of money channeled by buyers' to-trade-or-not-to-trade decisions. A fall in the degree of search friction surely raised the frequency of trade in the economy if the acceptance rate would not change. However, it lowers down the rate in equilibrium since it raises the value of money for the reason described above. Hence its overall effect on the frequency of trade becomes ambiguous.

Proposition 5 is also true for the buyer's price-setting equilibrium. The free entry equilibrium is not affected by progress in the matching technology since there always exist the infinite mass of the sellers in the market.

Production Technology

Consider the effects of progress in the production technology, which implies a fall in the total cost $c(q)$ and the marginal cost $c'(q)$ for every quantity produced q. It is clear that, if the value of money would not change, each seller would cut down the price of his product when the progress takes place. Lower marginal costs make them produce more in exchange for one unit of money. Graphically, the ψ curve in Figure 1 rotates upward. However, given quantity produced in exchange for one unit of money, lower production cost makes the sellers better off. For every price level, this lowers down the value of their becoming buyers, and it also makes the buyers more willing to spend their money with lower reservation satisfaction levels. The ϕ curve rotates to the left. After all, the relative size of these two effects determines the equilibrium effects not only quantitatively but also qualitatively.

For more formal discussion, let me introduce parameter γ such that $c_{\gamma}(q;\gamma) > 0$ and $c_{q\gamma}(q;\gamma) > 0$ for every $q > 0$. It can be easily shown that $\phi_{\gamma} < 0$ and $\psi_{\gamma} < 0$. See the proof of Proposition 6. Notice that $q_{\gamma}^{\psi} = -\psi_{\gamma}/\psi_{q} < 0$ or $\epsilon_{\gamma}^{\psi} = -\psi_{\gamma}/\psi_{\epsilon} > 0$, where $\epsilon^{\psi} = q^{\psi-1}$, formalizes the former effect, and that $\epsilon_{\gamma}^{\phi} = -\phi_{\gamma}/\phi_{\epsilon} > 0$ or $q_{\gamma}^{\phi} = -\phi_{\gamma}/\phi_{q} < 0$, where $q^{\phi} = \epsilon^{\phi-1}$, formalizes the latter. The following proposition claims that $\partial q/\partial\gamma < 0$ if and only if $\epsilon_{\gamma}^{\psi} > \epsilon_{\gamma}^{\phi}$ in the equilibrium, and that $\partial\epsilon/\partial\gamma < 0$ if and only if $\left|q_{\gamma}^{\psi}\right| > \left|q_{\gamma}^{\phi}\right|$ in it. Progress in technology progress lowers down the price level if it does not raise the value of money too much. It facilitates trade if it does not cut down the price level too much.

Proposition 6 *Technology progress in production lowers down the price level if and only if*

$$\begin{aligned}&\left[nE(\epsilon^{*}) - c_{q}(q^{*};\gamma) + \hat{c}(q^{*};\gamma)\right]\left[\epsilon^{*}\hat{c}_{\gamma}(q^{*};\gamma) + (1-F(\epsilon^{*}))\,c_{q\gamma}(q^{*};\gamma)\right] \\ &> \epsilon^{*}F'(\epsilon^{*})\,\hat{c}_{\gamma}(q^{*};\gamma)\left[2\epsilon^{*} + c_{q}(q^{*};\gamma) - \hat{c}(q^{*};\gamma) + \epsilon^{*}F''(\epsilon^{*})\,(\epsilon^{*} - \hat{c}(q^{*};\gamma))\right] \qquad (13)\end{aligned}$$

in the equilibrium. It raises the frequency of trade if and only if

$$\begin{aligned}&\epsilon^{*}\hat{c}_{\gamma}\left(q^{*};\gamma\right)\left[\epsilon^{*}F'\left(\epsilon^{*}\right)\left(c_{q}\left(q^{*};\gamma\right)-\hat{c}\left(q^{*};\gamma\right)\right)+\left(1-F\left(\epsilon^{*}\right)\right)q^{*}c_{qq}\left(q^{*};\gamma\right)\right]\\&>\left[c_{q}\left(q^{*};\gamma\right)-\hat{c}\left(q^{*};\gamma\right)\right]\left[\epsilon^{*}\hat{c}_{\gamma}\left(q^{*};\gamma\right)+\left(1-F\left(\epsilon^{*}\right)\right)c_{q\gamma}\left(q^{*};\gamma\right)\right].\end{aligned} \tag{14}$$

Notice that, in the buyer's price-setting equilibrium, changes the production costs do not directly affect seller's maximized profits since $V_0 = 0$ anyhow. Progress in the production technology always lowers down the frequency of trade as well as the price level. In Figure 1, the ϕ curve remains unchanged us while the $\epsilon = \hat{c}(q)$ curve rotates upward. However, its effect on the flow rate of trade is ambiguous. Its effects on the free entry equilibrium follow from the same arguments, and they are qualitatively same to those on the buyer's price-setting equilibrium.

5 Concluding Remarks

This paper has investigated how consumers' trade-or-not-to-trade decisions channel the interaction between prices and the frequency of monetary trade. The general message of the results is that this channel yields a positive relationship between them. A rise in prices, which cuts down the value of money, leads consumers to spend their money more quickly. The existence of such a channel looks quite natural, but it has not been discussed in existing studies including search-theoretic ones.

Moreover, there could exist positive feedback in case that sellers have a control over the prices of their products. If something that derives up nominal prices in general happens, a fall in the real value of money makes buyers more likely to accept trade for given prices. This implies that the fall in the value of money does not only cut down seller's marginal revenues in its intensive margin but also raises them in its extensive margin. The former makes sellers lower down price but also produce less for each trade reducing real balances demanded, and the letter makes them raise prices. Hence both amplify the original changes in real money balances.

The positive relationship revealed in this paper provides a theoretical account for a positive effect of expansionary monetary policy on production, which has not been addressed in existing studies. Around the unique monetary steady state of the model, changes in the money supply lead to the price level and the acceptance rate of monetary trade moving in the same direction, and in usual conditions, an increase in the money supply raise both. This shows a positive effect of money growth on frequency of monetary trade and its positive extensive effect on production, which is not channeled by capital-money portfolio decisions, even though the rise in the price level implies its negative intensive effect. In addition, the results about the effects of changes in the matching and production technology implies that the output-price long-run relationship could vary with different types of technology progress.

In this paper, it is also shown that seller's profit maximization is sufficient but not a necessary feature for the positive relationship between the price level and the frequency of monetary trade. Even if sellers have no surplus from trade due to buyer's take-it-or-leave-it offer or seller's free entry, zero profits yield the qualitatively same relationship between them. Most findings in the model with seller's profit maximization remain valid for those cases.

This paper intended to build a simple model to highlight one aspect in a monetary economy. Hence many other aspects important for more comprehensive analysis are left to future research. Clearly, one of the most necessary steps for such analysis is to relax the assumption of the unit upper bound on individual money holding with indivisible money by following either Shi (1997a) or Lagos and Wright (2005).[9] Then, there are many extensions that may be worth pursuing in future research. Possible features that might be introduced by the extensions include labor and capital market, neoclassical production, monetary and productivity shocks, and so on.

[9] If the model could accommodate the intensive margin of trade, I guess that it would reinforce the positive relationship between prices and the velocity of money. Higher prices make agents need to pay more in each transactions, so each unit of money might be spent more quickly. However, it seems that the overall effect of monetary policy on production should be newly investigated together with the introduction of capital. Here it might be noticeable that a strong dichotomy emerges in Lagos and Wright (2005) and its extensions considered in Aruoba and Wright (2003). Aruoba, Waller, and Wright (2007) shows that this kind of dichotomy is broken by introducing capital-money portfolio decisions into the framework. This paper shows that endogenous trade frequency could be another source that could break it.

Appendix

Proof of identical V_m and V_0

It is clear that all of the market participants have the same V_m and V_0 if and only if they have the same $v = V_m - V_0$. Suppose that there exists agent i and agent j such that $v_i > v_j$. Then, as long as $\left(V_m^i, V_0^i, V_m^j, V_0^j\right) \in \mathbb{R}_{++}^4$, (1) implies $V_m^i < V_m^j$, and (2) implies $V_0^i > V_0^j$. This yields $V_m^i - V_0^i < V_m^j - V_0^j$, which contradicts that $v_i > v_j$. ■

Proof of Lemma 1

First notice that $0 < v < \hat{q}$ implies $v > c(v)$ and hence $v < c^{-1}(v) < \hat{q}$. Define

$$\Pi(q) = [1 - F(v/q)][v - c(q)].$$

Clearly, (4) is equivalent to $\Pi'(q^\star) = 0$. Notice that $\Pi(q) = 0$ for $q \le v$, $\Pi(c^{-1}(v)) = 0$, and $\Pi(q) < 0$ for $q > c^{-1}(v)$. Also, we have $\lim_{q \to v+} \Pi'(v) = \lim_{z \to 1-} F'(z)[1 - c(v)/v] > 0$ and $\Pi'(c^{-1}(v)) = -[1 - F(v/c^{-1}(v))] c'(c^{-1}(v)) < 0$. Therefore, there exists optimal solution $q^\star \in (v, c^{-1}(v))$ such that $\Pi(q^\star) > 0$, $\Pi'(q^\star) = 0$, and $\Pi''(q^\star) < 0$. In addition, since

$$\begin{aligned}\Pi''(q) = &-2\left(v/q^3\right) F'(v/q)[v - c(q) + qc'(q)] \\ &- \left(v^2/q^4\right) F''(v/q)[v - c(q)] - [1 - F(v/q)] c''(q) < 0\end{aligned}$$

for every $q \in (v, c^{-1}(v))$, this optimal solution is unique. ■

Proof of Lemma 2

Let $\epsilon = v/q$. Notice that (6) with $v \in (0, q)$ is equivalent to (7) with $\epsilon \in (0, 1)$. Fix q. Since $\phi(q, 0) = -n\mathbb{E}(\tilde{\epsilon}) - \hat{c}(q) < 0$ and $\phi(q, \epsilon) = \rho > 0$ for $\epsilon \ge 1$, a solution exists in $(0, 1)$. Suppose that there exist more than one solutions. Then, there exists at least one solution $\epsilon^\star \in (0, 1)$ such that $\phi_\epsilon(q, \epsilon^\star) \le 0$. This implies $\epsilon^\star > \hat{c}(q)$ since

$$\phi_\epsilon(q, \epsilon) = \rho + (n + 1)[1 - F(\epsilon)] - F'(\epsilon)[\epsilon - \hat{c}(q)]$$

Then $\phi_\epsilon(q, \epsilon) < 0$ for every $\epsilon^\star < \epsilon < 1$ since

$$\phi_{\epsilon\epsilon}(q, \epsilon) = -(n + 2) F'(\epsilon) - F''(\epsilon)[\epsilon - \hat{c}(q)] < 0$$

This contradicts $\phi(q,1) > 0$. Therefore, $\epsilon^\star \in (0,1)$ such that $\phi(q,\epsilon^\star) = 0$ is unique. ■

Partial Derivatives of ϕ and ψ

In the proof of Lemma 2, ϕ_ϵ is shown, and the uniqueness of the solution implies that $\phi_\epsilon > 0$ around every $(q,\epsilon) \in \Xi$ such that $\phi(q,\epsilon) = 0$. Also, since $c'' > 0$ implies $c' > \hat{c}$,

$$\phi_q(q,\epsilon) = -[1 - F(\epsilon)]\, q^{-1} [c'(q) - \hat{c}(q)] < 0.$$

The partial derivatives of ψ are given by

$$\begin{aligned}\psi_q(q,\epsilon) &= -\epsilon F'(\epsilon)\, q^{-1} [c'(q) - \hat{c}(q)] - [1 - F(\epsilon)]\, c''(q) < 0,\\ \psi_\epsilon(q,\epsilon) &= F'(\epsilon)\{2\epsilon + c'(q) - \hat{c}(q)\} + \epsilon F''(\epsilon)[\epsilon - \hat{c}(q)].\end{aligned}$$

Since $\psi(q,\epsilon) = 0$ implies $\epsilon > \hat{c}(q)$, $\psi_\epsilon > 0$ around for every $(q,\epsilon) \in \Xi$ such that $\psi(q,\epsilon) = 0$.

Proof of Lemma 3

Define

$$\varphi(\epsilon) = (1/n)\,\phi\left(\hat{c}^{-1}(\epsilon), \epsilon\right) = \rho^b \epsilon - [1 - F(\epsilon)][E(\epsilon) - \epsilon].$$

Notice that $\varphi(0) = -\mathbb{E}(\tilde{\epsilon}) < 0$, $\varphi(1) = \rho^b > 0$, and for every $\epsilon \in (0,1)$, $\varphi'(\epsilon) = \rho^b + [1 - F(\epsilon)] > 0$. Therefore, there exists $\bar{\epsilon} \in (0,1)$ such that $\varphi(\bar{\epsilon}) = 0$. ■

Proof of Proposition 1

First induce a function

$$\Phi(\epsilon) = \phi(0,\epsilon) - [1 - F(\epsilon)]\, \hat{c}\left(q^\psi(\epsilon)\right).$$

An equilibrium is $\left(q^\psi(\epsilon^*), \epsilon^*\right)$ such that $\epsilon^* \in (\underline{\epsilon}, \bar{\epsilon})$ and $\Phi(\epsilon^*) = 0$. Since $\phi(0,\underline{\epsilon}) = 0$ and $q^\psi(\underline{\epsilon}) > 0$,

$$\Phi(\underline{\epsilon}) = -[1 - F(\underline{\epsilon})]\, \hat{c}\left(q^\psi(\underline{\epsilon})\right) < 0.$$

Also, since $\phi(0,\bar{\epsilon}) = \bar{\epsilon}[1 - F(\bar{\epsilon})]$ and, for every $\epsilon \in (0,1)$, $\epsilon > \hat{c}\left(q^\psi(\epsilon)\right)$,

$$\Phi(\bar{\epsilon}) = [1 - F(\bar{\epsilon})]\left[\bar{\epsilon} - \hat{c}\left(q^\psi(\bar{\epsilon})\right)\right] > 0.$$

Thus, there exists at least one monetary equilibrium as graphically shown in Figure 1.

To prove the uniqueness of equilibrium, notice that

$$\epsilon\Phi'(\epsilon) - \Phi(\epsilon) = [1 - F(\epsilon)]\left[nE(\epsilon) - \epsilon\hat{c}'\left(q^{\psi}(\epsilon)\right)q^{\psi\prime}(\epsilon)\right]$$

since $\psi\left(q^{\psi}(\epsilon), \epsilon\right) = 0$, and that

$$\Phi'(\epsilon) = \phi_{\epsilon}\left(q^{\psi}(\epsilon), \epsilon\right) + \phi_{q}\left(q^{\psi}(\epsilon), \epsilon\right)q^{\psi\prime}(\epsilon)$$

together with, for (q, ϵ) such that $\phi(q, \epsilon) = \psi(q, \epsilon) = 0$,

$$\begin{aligned}
\phi_{\epsilon}(q, \epsilon) &= (\epsilon q)^{-1}[1 - F(\epsilon)][nE(\epsilon) - c'(q) + \hat{c}(q)], \\
\phi_{q}(q, \epsilon) &= -q^{-1}[1 - F(\epsilon)][c'(q) - \hat{c}(q)].
\end{aligned}$$

Also, notice that $q\hat{c}' = c' - \hat{c}$. For ϵ such that $\Phi(\epsilon) = 0$, we have

$$\begin{aligned}
\Gamma(\epsilon) &= \epsilon[1 - F(\epsilon)]^{-1}\left[1 + c'\left(q^{\psi}(\epsilon)\right) - \hat{c}\left(q^{\psi}(\epsilon)\right)\right]\Phi'(\epsilon) \\
&= \left[\phi_{q}\left(q^{\psi}(\epsilon), \epsilon\right)nE(\epsilon) + \epsilon\phi_{\epsilon}\left(q^{\psi}(\epsilon), \epsilon\right)\hat{c}'\left(q^{\psi}(\epsilon)\right)\right] \\
&= \left[c'\left(q^{\psi}(\epsilon)\right) - \hat{c}\left(q^{\psi}(\epsilon)\right)\right]\left[\left(1 - q^{\psi}(\epsilon)\right)nE(\epsilon) - c'\left(q^{\psi}(\epsilon)\right) + \hat{c}\left(q^{\psi}(\epsilon)\right)\right].
\end{aligned}$$

If $q\hat{c}'' > c' - \hat{c}$, or equivalently $c'' - \hat{c}' > 0$, $\Gamma' < 0$ since $q^{\psi\prime} > 0$ and $E' < 0$.

Suppose that the solution to $\Phi(\epsilon) = 0$ is not unique. There exists no continuum of the solutions since $\Gamma' < 0$ implies that $\Phi''(\epsilon) < 0$ around a solution. Hence there must exist an odd number of the solutions including ϵ^* such that $\Phi'(\epsilon^*) \leq 0$ and $\epsilon^{**} > \epsilon^*$ such that $\Phi'(\epsilon^{**}) \geq 0$. But this contradicts $\Gamma' < 0$. ■

Proof of Proposition 4

Clearly, $dn/dM = -(1/M^2) < 0$. Notice that the implicit function theorem yields

$$\begin{bmatrix} \partial q/\partial n \\ \partial \epsilon/\partial n \end{bmatrix} = -D^{-1}\begin{bmatrix} \psi_{\epsilon} & -\phi_{\epsilon} \\ -\psi_{q} & \phi_{q} \end{bmatrix}\begin{bmatrix} \phi_{n} \\ \psi_{n} \end{bmatrix},$$

where $D = \phi_q\psi_{\epsilon} - \phi_{\epsilon}\psi_q$. In the proof of Proposition 1, it is shown that $\Phi'(\epsilon) > 0$, or equivalently $D > 0$, holds around the unique equilibrium. Also,

$$\phi_n(q, \epsilon; n) = r[\alpha(n) - n\alpha'(n)]\alpha(n)^{-2}\epsilon - [1 - F(\epsilon)][E(\epsilon) - \epsilon]$$

and $\psi_n(q,\epsilon;n)=0$. Therefore, $\partial q/\partial n>0$ and $\partial\epsilon/\partial n>0$ if and only if the condition in (12) holds since $\psi_\epsilon>0$ around the equilibrium and $\psi_q<0$. Since

$$(n+1)^2\, dT/dn = -\left[n\alpha-(n+1)\,\alpha'\right][1-F]-(n+1)\,\alpha F'\partial\epsilon/\partial n,$$

it is clear that $\partial\epsilon/\partial n>0$ together with $\alpha-n\alpha'\geq\alpha'$ is sufficient for $dT/dn<0$. ■

Proof of Proposition 5

Notice that progress in the matching technology lowers down ρ only. Since $\phi_\rho(q,\epsilon;\rho)=\epsilon$ and $\psi_n(q,\epsilon;n)=0$, $\partial q/\partial\rho=-D^{-1}\psi_\epsilon\phi_\rho<0$ and $\partial\epsilon/\partial\rho=D^{-1}\psi_q\phi_\rho<0$. The first part of the proposition is proved. Clearly, the progress raises $T(\epsilon)$ for every ϵ. However, since equilibrium level of ϵ goes up, the result is ambiguous in its direction. ■

Proof of Proposition 6

Notice that

$$\phi_\gamma(q,\epsilon;\gamma)=-\left[1-F(\epsilon)\right]\hat{c}_\gamma(q;\gamma)<0,$$
$$\psi_\gamma(q,\epsilon;\gamma)=-\epsilon\hat{c}_\gamma(q;\gamma)-\left[1-F(\epsilon)\right]c_{q\gamma}(q;\gamma)<0.$$

By the implicit function theorem, since $D>0$, it can be easily shown that $\partial q/\partial\gamma<0$ if and only if $\psi_\epsilon\phi_\gamma<\phi_\epsilon\psi_\gamma$, and that $\partial\epsilon/\partial\gamma<0$ if and only if $\psi_q\phi_\gamma>\phi_q\psi_\gamma$. ■

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