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Product Announcement and Reputation in a Cheap Talk Game

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Abstract Firms keep introducing new products in markets. In making a buy-or-wait decision, consumers rely on information provided by firms. In a cheap-talk game model (i.e., Crawford and Sobel (1982)), we identify a simple informative equilibrium, in which firms make a discrete announcement, either low or high quality, even though product quality space is continuous. We characterize the equilibrium properties and evaluate its welfare effects.

Keywords Product announcements, Cheap talk, Reputation

JEL Classification L1, D8

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1. Introduction

While they are developing new products, firms make announcements on specifications and functions of these new products. Based on these announcements, consumers make a buy-or-wait decision. Consumers can make better purchasing decisions based on these announcements. Also, firms can earn more profits by inducing consumers wait for their products. Thus, there are incentives for firms and consumers to exchange information on new products. Since, by exchanging information, consumers and firms with R&D projects can have win-win situation, their interests coincide with some degree. However, since firms can make more profits by inducing more consumers to wait for their products, they might have an incentive to overstate the quality of the new products. In this regard, consumers and producers interests are misaligned with some degree.

In this article we explore whether we can have an informative equilibrium in which some informations are transmitted from the firm side to the consumer side. By adopting a cheap-talk game model, (i.e. Crawford and Sobel (1984)) we explore how information exchanges can occur. In a cheap-talk game, in which an announcement itself is costless, we show that an informative equilibrium exists only if reputation concerns are large enough. In equilibrium, even though product quality space is continuous, a firm partitions its quality announcements into two discrete intervals, high and low quality intervals: If the quality level of the new product is above the cut-off quality level, the firm announces that the new product is of high quality; Otherwise, the firm announces that the new product is of low quality. We show that the length of the good-quality interval is longer than that of the low-quality one, which implies that there are more uncertainties when the firm announces a high quality product than the firm announces a low quality product. Also, the length of the high-quality interval is too long in terms of consumer surplus maximization, which indicates that the firm is too lenient in announcing high quality.

This paper is closely related to our two companion papers, Choi, Kristiansen, and Nahm (2010a, 2010b). Choi, Kristiansen, and Nahm (2010a) analyze how information transmission occurs in a finitely repeated cheap talk game. We characterize the equilibrium in which semi-separation of types takes place; types higher than a specific cut-off point behave honestly, while types below the cut-off point always lie. Thus both lying and truth-telling are observed in the equilibrium. With this equilibrium we show that product pre-announcements always increase consumers' ex ante surplus. Choi, Kristiansen, and Nahm (2010b) show that the same welfare effect exists in the market in which consumers' coordination is important.

In this paper we assume reputation effects as part of our model, whereas Choi, Kristiansen, and Nahm (2010a, 2010b) endogenously derive reputation effects from a finitely repeated game. In this paper, a firm's payoff explicitly includes a reputation value, which depends on consumers' ex-post surplus from waiting for the new product. An advantage of our approach is that we can characterize several properties of the equilibrium, where a firm's announced quality space is continuous.

Our paper is also related to several other papers analyzing strategic information transmission in either signaling models or cheap talk models. In these models, a sender sends a message, and a receiver infers the sender's type from the message/signal and then takes her own optimal action based on the inference. The receiver's action affects the sender's payoff as well as the receiver's. However, since the receiver's payoff is not perfectly aligned with the sender's, the sender has an incentive to modify the message. If sending a message incurs costs, it is a signaling model. Otherwise, it is a cheap talk model.

In a cheap talk game, Crawford and Sobel (1982) show that under some conditions a very simple equilibrium exits: the sender partitions its type space into multiple intervals with some lengths and randomly chooses its message within a specific interval; thus, after receiving the message, the receiver knows only which interval the sender's type belongs to, but not the exact type. We build on the Crawford and Sobel model to analyze the product preannouncement and reputation effects.

The paper is organized as follows. In section 2, we build a cheap-talk game model in which a firm makes an announcement on product quality, and consumers Bayesian update their beliefs on the product quality. In section 3, we construct an informative equilibrium in which the firm's announcement has some information contents. We characterize the equilibrium and evaluate its welfare properties. Conclusion follows.

2. Model

There are two reports. In period one, consumers can buy either a currently available product or wait for a new product. A competitive market supplies the current product at a fixed price.[1] For simplicity, we assume that the price is zero.[2] By buying the current

[1] Since the product is supplied by a competitive market, the price is determined by marginal costs. The price of the current product does not depend on the new product's R&D prospect. However, if the current product is supplied by a firm with a market power, then the firm selling the current one might adjust its price strategically. See GERLACH (2004) for the analysis of the game in which an

product, consumers get v net surplus per period. A different consumer has a different value of v, thus v is a consumer's type. v is distributed between zero and one. Consumers can use the product for two periods. For simplicity, we assume that there is no time discount, $\delta=1$.[3]

A firm is developing a new and advanced product that will be introduced in period two. We assume that because of switching costs, consumers cannot buy both current and new products. The new product brings a higher value to consumers than the currently available one does. The value of buying the new product is denoted by $v+q$, and q denotes the size of the improvement.

There are uncertainties on the R&D results. Depending on the R&D results, the quality of the product would be different. q is a random variable uniformly distributed between zero and one.[4] Consumers get $v+q$ surplus from the new product. Consumers, if they wait in period one, would buy the new product in period two.

In summary, consumers' payoffs are as follows,

$$U = v+\delta v=2v, \text{ if she buys the current one in period 1}$$

$$= \delta(v+q) = v+q, \text{ if she waits to buy the new one in period 2}$$

Thus, the consumer surplus of buying the new one over buying the current one is $q-v$. It is optimal for consumers to wait until the second period only if q is larger than v. If

incumbent selling a current product adjusts its price strategically based on an entrant's new product R&D announcements.

[2] Even though we assume that the price is positive, we still get the same qualitative results.

[3] Even though we assume that $\delta<1$, we still get the same qualitative results.

[4] The results are easily generalized to a general distribution.

consumers know the value of q, then only types lower than q would wait. Since consumers do not know the exact value of q, consumers need to make an expectation of q in period one. Since q is uniformly distributed between zero and one, consumers' prior expectation of q is 0.5.

The firm that undertakes an R&D for the new product observes the value of q and makes an announcement on the value of q. Thus, q becomes the firm's type. Then consumers update their belief on q based on the firm's announcement and make decisions whether to wait or not. In period two, consumers who have waited in period one would buy the new product. The firm introducing a new product earns more profits when more consumers wait. We assume that the firm earns π per waiting consumer.

Consumer types v are continuously distributed, but consumers have two discrete choices, either 'waiting for the new one' or 'buying the current one.' By monotonicity, consumers' strategies in equilibrium are characterized by cut-off properties. s denotes the cut-off point in consumer types. That is, types higher than s ($v > s$) would buy the current product in period one, while types lower than s ($v < s$) would wait for the new product. Thus, since v is uniformly distributed between zero and one, the measure of consumers who wait in period one is s, while the measure of consumers who buy the current productin period 1 is 1-s.

The payoff of the firm introducing the new product consists of two parts; the first part measures its profits from inducing consumers to wait, and the second part measures its 'reputation' value. The firm makes an announcement on q to maximize the sum of its profits and the reputation value.

Since the firm earns π per waiting consumer by selling its product, its profits from selling the product is $s\pi$ when consumers' types lower than s wait. We model the reputation value function explicitly. The firm's reputation value depends on consumer's experience of the product. For a consumer of type v, the consumer's additional surplus of waiting and buying new one over buying the current one is q-v. After the value of q is realized, consumer types whose values of v are higher than q are disappointed since they could have gotten the larger surplus by buying the current one instead of waiting. They get a negative surplus by waiting.

However, for consumer types whose values are lower than q, they are satisfied with waiting since waiting and buying the new one provides the higher surplus than buying the current one. They get a positive surplus by waiting.

If we add up the two groups, the sum of consumer extra surplus among consumers buying the new product is $\int_0^s (q-v)dv$ when the consumer cut-off point is s. The reputation effect depends on the size of $\int_0^s (q-v)dv$. For instance, for a given value of q, the reputation effect is maximized when $s = q$. If the firm wants to maximize only the reputation effect, it is optimal for it to announce true value of q.[5]

In summary, the firm's payoff is $bs\pi + \int_0^s (q-v)dv$. The weight of the profits part over the reputation part is measured by b, which is positive. If b is zero, then the firm just maximizes the sum of consumer surplus, and the firm's incentive is perfectly aligned with the consumers' incentive. Since b is positive, the firm's incentive is misaligned with the

[5] If the firm's reputation depends on the difference between the true value of q and the announced q (message), then the game becomes a signaling game which is interesting, but beyond the scope of our analysis.

consumers' incentives. If b is large, the firm puts more weights on its profit and has a higher incentive to maximize its short-term profits.

The timing of the moves are as follows; in period one, the firm developing a new product observes the value of q and makes an announcement (m) on q. First-period consumers infer the value of q from the announcement and decide whether to wait for the new product or not. In period two, the firm introduces the new product, and consumers having waited buy the new product. The equilibrium concept is Perfect Bayesian Equilibrium: The consumers' belief is derived from the firm's strategy in equilibrium; all players' strategies are optimal given the belief.

3. Analysis

The firm developing a new product observes the value of q and makes an announcement (m) on q. However, since the firm can manipulate the message, the announced quality can be different from the true value of q. After knowing the announced level of q, consumers update their belief on the value of q. $E(q|m)$ denotes the expected value of q conditional m.

Consumers each can maximize their own payoffs by either waiting if $v < E(q|m)$ or buying the current one otherwise. The consumer cut-off points is set such that $s = E(q|m)$.

Given the cut-off point s, the firm's payoff becomes $bs\pi + \int_0^s (q-v)dv$. For analysis, suppose that the firm can choose s. Let us solve for the firm's optimal level of s given q. Then, the first-order-condition is as follows.

$$\text{Max}_{s}\ bs\pi + \int_0^{s^*} (q - v)dv$$

$$\text{F.O.C}\quad b\pi + q - s = 0$$

Solving the first order condition provides the firm's optimal s^*, $s^* = b\pi + q$ given q. Thus, if the firm can choose s given q, then it would choose $s = b\pi + q$. If consumers were naïve enough to believe the firm's announcement as a face value, then the firm wants to inflate its quality by $b\pi$.

The consumer welfare maximizing cut-off point is q, which differs from the firm's optimal cut-off point ($q+b\pi$) by $b\pi$. Thus, the firm has an incentive to inflate its value of q, and $b\pi$ measures the incentives misalignment between the firm and consumers.

However, as long as consumers are rational enough to know that the firm would inflate its quality; rational consumers would not take the announced quality as a face value and discount it. In turn, as long as the firm knows that rational consumers would discount it, the firm would consider the discount when it makes an announcement. We want to analyze whether we can have an informative equilibrium in this situation.

Since it is a cheap talk game, we always have a pooling equilibrium in which there is no information transmits between consumers and the firm. For instance, no matter what the level of q is, the firm chooses message randomly between zero and one. Thus, the message does not have any informative contents. So it becomes optimal for consumers to discard the message and expect q to be 1/2. In turns, since consumers discard the message, it becomes optimal for the firm to send any message randomly.

However, following Crawford and Sobel (1982), we find a condition under which there exists a partially pooling Perfect Bayesian Equilibrium. Let us build an informative equilibrium in which the firm's announcement has some informative contents. The following procedure is analogue to Crawford and Sobel (1982).

Suppose first that the firm would divide the quality space between zero and one into two intervals; good and bad one. If the quality level is above q_0, or q is between q_0 and one, the firm announces that the new product is of high quality. Otherwise, the firm announces that the new product is of low quality.[6] q_0 becomes the cut-off point in the quality space.

Given the firm's announcement strategy, consumers' belief Bayesian updating are as follows: when the firm announces a high quality product, consumers expect the new quality level to be between q_0 and one. Thus, $E(q|m=\text{high})$ becomes $(q_0+1)/2$, and consumers whose types are lower than $(q_0+1)/2$ would wait for the new product. Thus, the size of consumers waiting becomes $(q_0+1)/2$; When the firm announces a low quality, consumers expect the quality of the new one to between zero and q_0, and $E(q|m=\text{low})$ becomes $q_0/2$, and consumers whose types are lower than $q_0/2$ would wait for the new product. The size of consumers waiting becomes $q_0/2$.

Given consumers' optimal response, let us check whether the firm's strategy can be optimal. $\Pi(m;q)$ would represent the firm's payoffs for the firm with quality level q sending message=H or L. The firm's payoffs are as follows,

[6] Instead, suppose that when q is above the threshold level q_0, the firm announces high quality and does not make any announcement on q otherwise. Then, if the firm does not make any announcement, consumers consider the new product to be of low quality.

$$\Pi(m{=}H;q) \; = \mathrm{b}\pi\tfrac{q_0+1}{2} + \int_0^{\frac{q_0+1}{2}} (q - v)dv$$

$$\Pi(m{=}L;q) = \mathrm{b}\pi\,\tfrac{q_0}{2} + \int_0^{\frac{q_0}{2}} (q - v)dv\,.$$

If we find q_0 such that $\Pi(m{=}H;q) = \Pi(m{=}L;q)$, then by monotonicity, it is optimal for firm types $q > q_0$ to announce high quality one, and it is optimal for firm types $q < q_0$ to announce low quality one. By monotonicity, the firm's strategy can be represented by the cut-off property q_0. As long as b is lower than $\frac{1}{4\pi}$, the cut-off quality level q_0 exists, and q_0= ½-2bπ. It constitutes a Perfect Bayesian Equilibrium.

Proposition 1. If b is lower than $\frac{1}{4\pi}$, there is a Perfect Bayesian Equilibrium with the cut-off strategy, such that

(a) if q is lower than $½\text{-}2b\pi$, then the firm announces that the new product quality is of low quality; if q is higher than ½-$2b\pi$, then the firm announces that the new product quality is of high quality.

(b) after the announcement of low quality, only consumer types lower than 1/4-$b\pi$ wait; after the announcement of high quality, only consumer types lower than 3/4-$b\pi$ wait.

Proof. See the Appendix.

b measures the weight of profits over reputation concerns in the firm's payoff. We can have an informative equilibrium when b is low enough. That is, in order to have an informative equilibrium, reputation concerns should be large enough.

Choi, Kristiansen, and Nahm (2010a, 2010b) also derive an informative equilibrium. However, there is one important difference between these papers and the current paper. In Choi, Kristiansen, and Nahm (2010a, 2010b), a firm introduces new products twice. When the firm makes a product announcement on the first-cycle product, the firm knows the chance of developing high-quality product in the second cycle. Thus, firm types are in two dimensions, the quality of the first-cycle new product and the prospect of having a high quality product in the second cycle.[7] These papers show that a higher type (i.e. a firm with a higher chance of bringing a high quality product in the future) behaves more honestly than a lower type does. However, in this paper firms' type is just one dimension, q (product quality). Also, in Choi, Kristiansen, and Nahm (2010a, 2010b), product quality is discrete, either high or low. However, in this article, product quality is continuous between zero and one. The firm endogenously partitions the continuous quality space into two intervals, low and high intervals.

We explore several properties in the informative equilibrium described in Proposition 1. When the firm announces a low quality, q is between zero and $1/2\text{-}2b\pi$, and the length of the low-quality interval is $1/2\text{-}2b\pi$. However, when the firm announces a high quality one, q is between $1/2\text{-}2b\pi$ and one, and, the length of the high-quality interval is $1/2+2b\pi$. Since the interval of high quality is longer than that of low quality by $4b\pi$, we

[7] A firm's type is in a two dimension, product quality and the prospect of a high quality product in the second cycle.

have that Var (q| m=H)>Var(q| m=L) i.e., there is more uncertainty on q when high-quality is announced than when low-quality is announced.

Proposition 2. Under Bayesian updating, consumers face higher uncertainties when the firm announces high-quality than they do when the firm announces low-quality.

Let us evaluate how the cheap-talk affects ex ante consumer surplus. For a benchmark, suppose that there is no announcement on q. Then, the consumers' belief on q is the same as the prior belief. Since q is uniformly distributed between zero and one, the expected value of q is 1/2. Thus, only types lower than ½ wait until period two. Without the preannouncement, the expected ex ante total consumer surplus is

$$\int_{1/2}^{1} 2v dv + \int_0^1 \int_0^{1/2} (q+v) dv dq .$$

In the informative equilibrium described in Proposition 1, the firm announces high-quality with probability 1/2+2$b\pi$ and low-quality with probability 1/2-2$b\pi$. Thus, the expected ante total consumer surplus in the informative equilibrium is

$$q_0[\int_0^{q_0} \int_0^{\frac{1}{2}q_0} (q+v) \tfrac{1}{q_0} dv dq + \int_{\frac{1}{2}q_0}^{1} 2v dv] + (1-q_0)[\int_{q_0}^{1} \int_0^{\frac{1}{2}(q_0+1)} (q+v) \tfrac{1}{(1-q_0)} dv dq + \int_{\frac{1}{2}(q_0+1)}^{1} 2v dv] \quad (1)$$

,where q_0=1/2-2$b\pi$.

We can show that the first part of (1) is higher than the second part.

Proposition 3. Ex ante, the quality announcement by the firm enhances the total consumer surplus.

Proof. See the Appendix.

Without an announcement, the distribution of the quality is between zero and one. However, with an announcement, consumers know at least whether the quality is either on the high or low sides, which helps them make better decisions. This result is consistent with Choi, Kristiansen, and Nahm (2010a, 2010b), which show that ex ante consumer welfare is higher under announcement than under no announcement even though firms might intentionally mislead consumers.

In the cheap-talk equilibrium, the total consumer surplus is as follows,

$$q_0[\int_0^{q_0} \int_0^{\frac{1}{2}q_0} (q+v)\tfrac{1}{q_0}\,dvdq + \int_{\frac{1}{2}q_0}^{1} 2vdv] + (1-q_0)[\int_{q_0}^{1} \int_0^{\frac{1}{2}(q_0+1)} (q+v)\tfrac{1}{(1-q_0)}\,dvdq + \int_{\frac{1}{2}(q_0+1)}^{1} 2vdv]\,.$$

The total consumer surplus depends on q_0, and q_0 is endogenously derived by the reputation value. Let us evaluate q_0 in terms of maximizing consumer surplus.

Suppose that the social planner can choose q^*, the boundary between the good and the bad quality. That is, the social planner asks the firm to make a high-quality announcement only if q is above q^*. Then, when the firm announces high-quality, consumers expect q to be between q^* and 1. Given q^*, the consumer total surplus is as follows,

$$q^*[\int_{\frac{1}{2}q^*}^{1} 2vdv + \int_0^{q^*} \int_0^{\frac{1}{2}q^*} (q+v)\tfrac{1}{q^*}dvdq] + (1-q^*)[\int_{\frac{1}{2}(q^*+1)}^{1} 2vdv + \int_{q^*}^{1} \int_0^{\frac{1}{2}(q^*+1)} (q+v)\tfrac{1}{(1-q^*)}dvdq] .$$

Let us calculate the optimal q^*, which maximizes ex ante the total consumer surplus. We find that optimal q^* is ½.

Since q_0 derived from the cheap-talk game is ½ -2$b\pi$, the interval of low quality is shorter than the optimal length, and the interval of high quality is longer than the optimal length.

Proposition 4. In terms of consumer welfare, the length of the high quality interval in the informative equilibrium is higher than the optimal length. That is, the firm is too lenient in announcing high-quality in terms of consumer welfare.

Proof. See the Appendix.

So far we have built an equilibrium in which the number of partition is two, high or low. However, as b gets smaller, we can have more partitions in the product announcement, which implies that more information is transmitted to consumers when the firm has more reputation concerns.

4. Conclusion

In a cheap-talk game model we identify an informative Perfect Bayesian equilibrium, in which a firm's announcement has information contents, and consumers Bayesian update their beliefs on products based on the announcement. Even though the quality space is continuous, the firm partitions the space into two intervals and makes a discrete product announcement, high or low quality product. Consumers face more uncertainty on quality level when the firm announces high quality. We show that the length of the high quality interval is too long in the comparison with the consumer welfare maximizing interval length.

In this paper, the firm makes an announcement in order to affect consumers' choices. However, when a rival firm exists, the rival firm would react to the new product quality announcement by adjusting its price or R&D plans. GERLACH (2004) analyzes the situation in which a firm selling a current product may adjust its price. It would be more interesting to analyze strategic aspects of product announcements in a cheap talk model with two different audience groups (i.e., Farrell (1989)). It might be worthwhile to investigate the government's optimal policy when by legistration the government can punish the firm if an announced quality level is too far from the actually delivered one.

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Appendix

Proof of proposition 1

The proof requires us to find q_0 such that $\Pi(m=H;\ q_0) = \Pi(m=L;\ q_0)$. That is, the firm with $q=q_0$ is indifferent between announcing low and high quality ones. Given q_0, the firm's optimal consumer cut-off point is $q_0+b\pi$. When the firm announces a high quality one, the consumer cut-off point becomes $(q_0+1)/2$. When it announces a low quality one, the consumer cut-off point becomes $q_0/2$. In order for the firm with $q= q_0$ to be indifferent between announcing low and high ones, the ideal consumer cut-off point $q_0+b\pi$ should be located at the mid-point of $(q_0+1)/2$ and $q_0/2$. Thus, we have to have $q_0+b\pi = q_0/2+1/4$. That is, $q_0=1/2- 2b\pi$. As long as b is lower than $\frac{1}{4\pi}$, the firm's cut-off point q_0 exists between zero and one. *Q.E.D*

Proof of proposition 2

Observe that

$$A= \int_{1/2}^{1} 2v\,dv + \int_0^1 \int_0^{1/2} (q+v)\,dv\,dq = \frac{3}{4} + \frac{3}{8} = \frac{9}{8}$$

$$B=q_0 B1 + (1-q_0)B2 = \frac{1}{8}(9 + q_0 - q_0^2).$$

$$B1=\int_0^{q_0}\int_0^{\frac{1}{2}q_0}(q+v)\frac{1}{q_0}dvdq+\int_{\frac{1}{2}q_0}^{1}2vdv=\frac{3q_0^{\;2}}{8}+\left(1-\frac{q_0^{\;2}}{4}\right)=1+\frac{q_0^{\;2}}{8}$$

$$B2=\int_{q_0}^{0}\int_0^{\frac{1}{2}(q_0+1)}(q+v)\frac{1}{1-q_0}dvdq+\int_{\frac{1}{2}(q_0+1)}^{1}2vdv=$$

$$\frac{3}{8}(1+q_0)^2+\frac{1}{4}(1-q_0)(3+q_0)=\frac{1}{8}(9+2q_0+q_0^2).$$

We conclude B - A= $\frac{1}{8}q_0(1-q_0)>0$ because q_0 is a probability measure. *Q.E.D*

Proof of proposition 3

If the social planner can choose q^*, then the consumer surplus is as follows.

$$q^*[\int_{\frac{1}{2}q^*}^{1}2vdv+\int_0^{q^*}\int_0^{\frac{1}{2}q^*}(q+v)\tfrac{1}{q^*}dvdq]+(1-q^*)[\int_{\frac{1}{2}(q^*+1)}^{1}2vdv+\int_{q^*}^{1}\int_0^{\frac{1}{2}(q^*+1)}(q+v)\tfrac{1}{(1-q^*)}dvdq]$$

Observe that the objective function can be simplified into

$$q^*\left[1+\frac{q^{*2}}{8}\right]+(1-q^*)\left[\frac{1}{8}(q^{*2}+2q^*+9)\right]=-\frac{1}{8}(q^{*2}-q^*-9).$$

The first order condition with respect to $\mathbf{q^*}$ is given by

$$\frac{1}{8}(1-2q^*)=0.$$

It is clear that the optimal solution is $q^* = \frac{1}{2}$. Since the equilibrium cut-off point q_0 is less than 1/2, we find that the firm is too lenient in announcing a high-quality product in terms of consumer welfare. *Q.E.D*