

The Role of Temporary Disputes in Solving the Hold-up Problem^{*}

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Abstract The paper shows that allowing a simple choice such as initiating temporary disputes can cure the hold-up problem. This is despite that the specific investments are two-sided, and the nature of the investment is cooperative, the case in which the hold-up problem is the most difficult to overcome. It is because the “dispute” option generates multiple equilibria for the ex post renegotiation game, and they can be conditioned to enforce efficient investments.

Keywords Hold-up problem, Incomplete Contracts, Temporary Disputes

JEL Classification D23

^{*} This paper is a revitalized and significantly revised version of the third chapter of my dissertation, which has been gaining dusts in my drawer for quite a while. This work was supported by Konkuk University.

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I. Introduction

Since the pioneering works of Klein, Crawford, and Alchian (1978) and Williamson (1979), the hold-up problem has come to be accepted as a fundamental element in predicting contractual and organizational structures. The hold-up problem is that if two parties who make relationship specific investments may renegotiate the division of the surplus later, they tend to under-invest anticipating that they will not receive full marginal benefits. Two factors are crucial for this classic problem--contractual incompleteness and asset specificity. Incomplete contract arises due to bounded rationality or high cost of writing contingencies.¹ In particular, economists have focused on the case that information is shared by contracting parties but unavailable to the court. It is then impossible to devise a contingent contract that guarantees ex post efficiency. Allowing renegotiation can solve this problem effectively but introduces a new problem by making existing clauses of the contract less powerful. In particular, the concern about being forced to accept disadvantageous terms in renegotiation reduces the expected returns, leading to underinvestment. Specificity of assets is crucial here because, if the investment also increases the returns obtainable outside the relationship, a party would be able to claim full returns by credibly threatening to opt out.²

Contractual and institutional issues surrounding this problem have produced a huge amount of literature. Most notably, the problem has been considered as the primary cause for the way the structure of ownership affects the transaction efficiency (Williamson, 1979; Grossman

¹ See Hart (1995, Ch.4) and Segal (1999) for more discussions on the foundations of incomplete contracting approach. As Schmitz (2001) criticizes, however, there is no clear definition of what really constitutes an incomplete contract.

² As Rogerson (1992) puts it, the hold-up problem arises because the two elements suggest somewhat conflicting requirements for the nature of the contractual structure governing the transaction. Contractual incompleteness suggests that the contract should be flexible enough to accommodate the realization of uncertain parameters, whereas asset specificity requires the somewhat rigid nature of the contract to avoid the ex post opportunism.

and Hart, 1986; Hart and Moore, 1990). Other researchers have tried to determine the minimal set of contractible variables required to avoid this problem. For example, Chung (1991) and Aghion, Dewatripont, and Rey (1994) obtain efficiency results when the initial contract can impose certain restrictions on the ex post renegotiation procedure to allocate bargaining power. However, applicability of this line of research is clearly limited to special cases. Second, some researchers try to apply mechanism design theory to design efficient ex ante contracts. Such task is shown to be possible if contracting parties are able to design sophisticated mechanisms and commit not to renegotiate (Rogerson, 1992; Hermalin and Katz, 1993; Che and Hausch 1999). It has been criticized however that neither of these assumptions is quite realistic. The third line of research assumes some given renegotiation structures and tries to find relevant efficiency conditions. For example, Noldeke and Schmidt (1995) considers a non-cooperative bargaining game in which total surplus is allocated to one party. Then they show that simple option contracts can achieve efficiency. Edlin and Reichelstein (1996) and Che and Hausch (1999) adopt a Nash bargaining game for renegotiation. The former shows that efficiency is achievable if a certain separability condition is satisfied.³ The latter, on the other hand, proves that even if an arbitrarily complex contract can be written, efficiency cannot be achieved when investments are “cooperative” in the sense that a party’s investment increases the other party’s valuation.

As the literature review shows, understanding the nature of investments and renegotiation process is the key in studying the hold-up problem. Often ignored are the facts that relationship specific investments have long term effects, and real life bargaining is not as simple as the theory generally describes. For example, the famous General Motors-Fisher Body example that led to the initial idea of hold-up problem was about building a plant adjacent to GM. Another classic example of Joskow (1985) involves coal mines and electric utility plants

³ The effect of investments and the effect of the state of the world should enter the valuation functions in an additively separable manner

that are constructed right at the mouth. Relationship specific benefits from such investments should last as long as the plants operate. On the other hand, papers dealing with the hold-up problem often assume the simplest form of renegotiation process such as ‘take-it-or-leave-it’ offer or Nash bargaining game. The common feature of such games is that equilibrium is unique, and therefore the division of the surplus is independent of history.

By contrast, this paper considers a case in which the relationship specific investments generate long term benefits, and parties have short term alternatives that could be used to draw better terms in renegotiation. This makes important difference in the role of outside options. In general, the bargaining literature assumes that the parties are required to terminate their relationship once they choose outside options (trade with someone else, take another job, and so on). Then, the well known ‘outside option principle’ states that a player’s outside option payoff works only as a constraint in the sense that the bargaining outcome is not affected as long as it provides more payoff than the outside option. However, in the current context, the parties who had previously taken the outside options may come back at later periods and agree for a new contract. Then outside options affect the outcome of the bargaining game by moving threat points. Taking this short term outside option or not is up to each player, but the availability of such option generates multiple subgame perfect equilibria in the renegotiation stage. Then it is possible to construct an equilibrium that conditions renegotiation outcomes according to the levels of specific investments *ex ante*. This additional flexibility in contract design alleviates the hold-up problem and may eliminate the problem altogether if certain parameter conditions are met. This result is derived in the context of two-sided investments encompassing the case of cooperative investments, which is known to be the most difficult case to achieve efficiency.

The idea that multiple equilibria in *ex post* renegotiation game can cure the hold-up problem has been used in other papers, too. Ellingsen and Johannesson (2004) recognize that if

the bargaining stage has multiple equilibria, some of which may give a high enough payoff to sustain efficient investment. However, their work is mostly focused on lab experiments and the role of social preferences such as inequity aversion and altruism. On the other hand, Evans (2008) is closely related to this paper. He models renegotiation as an infinite-horizon non-cooperative bargaining game and shows that a simple option contract that gives authority to one party to set the terms of trade and gives the other party a non-expiring option to trade at these terms can generate efficiency. His setting is quite different from the current paper. For example, he studies one shot transaction without outside option, and the default terms of trade are set ex post rather than ex ante. However, the logic of the equilibrium is essentially identical to the current paper as it relies upon the multiplicity of equilibria in the renegotiation game.⁴

One additional point to clarify is the role of ‘long term relationship’ in the current paper. This is modeled here as an infinitely repeated deliveries with the possibility of breakdown. Readers might suspect that achieving efficiency relies upon the Folk Theorem, but this is not so. Specific investments are made only once in this paper, therefore the model is not in the repeated game setting.⁵ Furthermore, once a new contract is signed in renegotiation, the terms of trade continue to be in effect indefinitely; therefore repeated interaction does not have any disciplinary role. Similarly to this paper, Castaneda (2006) studies a case in which investments are made only once, and a repeated relationship follows. However, his model essentially studies a firm’s make-or-buy decision, and the equilibrium contract is non-exclusive in the sense that the buyer has an option to walk away from the relationship.⁶

The rest of the paper is organized as follows: Section II presents the basic structure of

⁴ This idea seems to have been developed independently by these papers, but the original idea of the current paper dates back to Kwon (1997), which well precedes both Ellingsen and Johannesson (2004) and Evans (2008).

⁵ Halonen (2002) uses repeated games setting to look at the investment incentives provided by reputational concerns.

⁶ Another related paper is Che and Sakovicz (2004) in which trade occurs only once, but investments can be made gradually or with delay.

the model. Section III is devoted to the detailed description of the ex post bargaining procedure. Section IV derives and discusses sufficient conditions for the existence of an efficient equilibrium. Section V is the concluding remark.

II. Model

The timing of the events is as follows. At Stage 0 two risk-neutral agents (let us call them the seller s and the buyer b following convention) devise a contract for the supply of a fixed amount of good (normalized to 1) for indefinite periods. The initial contract may specify only the price p of the good, and it is the only relevant term verifiable by the court. It is also assumed that there is no third party to act as a financial wedge between the two.⁷ They may arrange signing payments at this stage to achieve the appropriate division of expected surplus from the trade. At Stage 1, each agent chooses the level of investment; $i_s (\geq 0)$ for the seller and $i_b (\geq 0)$ for the buyer. At Stage 2, nature moves and decides the value of θ , a random variable that affects the value of trade between the two parties and their alternative trading opportunities. At this stage, ex post valuation, the level of investments and θ become all observable to both parties.

The trade may occur at Stage 3, which repeats for indefinite periods of time. The initial contract, however, is not necessarily in force. Either the parties may trade at the contract price, or at least one of the parties may refuse to trade. If they rescind the original contract as a result, they can start renegotiating the terms of the initial contract or break the relationship. The court is assumed to be unable to determine who is responsible for the breach. Therefore, the trade always occurs voluntarily.

⁷ Hart and Moore (1988), Rogerson (1992) point out that "Groves-type" mechanism could induce the first best efficiency if this were the case.

It is also assumed that one or both sides could decide to start a temporary “dispute,” a choice that incurs financial damages for both parties. If one of the parties starts the dispute, the other party cannot avoid it, and it ends only if both parties want to. An example is a wasteful lawsuit that does nothing much other than burning of time and money for both sides. A strike or a lockout would do the similar role in the labor contracting context

Renegotiation could take many periods, and the parties can either trade with third parties or continue to dispute until a new contract is made.⁸ For simplicity, we assume that there is no additional cost of dispute other than the loss of payoffs from outside options.⁹ Once a new contract is made, it stays in force unless another renegotiation becomes necessary, which would never be the case in the current context. Finally, the time spent between Stage 0 and the beginning of Stage 3 is assumed to be negligible or properly discounted according to the Stage 3 price. The renegotiation procedure will be described in more detail in the next section.

A state in Stage 3 is defined by $\omega = (i_s, i_b, \theta)$, where $\omega \in \Omega$, the set of possible states. For any period, the buyer's utility from receiving the good is $u^b(\omega)$, and the seller's cost from supplying it is $c^s(\omega)$. When no trade occurs, both parties get the payoff of zero. For the trade with third parties, let $V^b(\omega)$ denote the buyer's utility and $V^s(\omega)$ the seller's. All these functions- $u^b(\omega)$, $c^s(\omega)$, $V^b(\omega)$, $V^s(\omega)$ -are assumed to be time independent. Therefore, their values are fixed for Stage 3. This assumption might seem unrealistic given that the relationship is long term but makes the model easily tractable without distracting us with technicalities. It is also assumed that the buyer and the seller possess a common discount factor of $\delta \in (0,1)$. Let us denote the state M_t of period t at Stage 3 to be “Y” if any contract

⁸ A party may choose to end a dispute in one period and begin another in later periods without agreeing to a new contract, although such a strategy is not a part of equilibrium we consider.

⁹ Or it may be modeled that the buyer and the seller trade with third parties while in dispute, but these “outside” payoffs are lost as the cost of dispute. Remaining payoffs need not be zero; they could be either positive or even negative. As long as the payoffs for dispute periods are strictly smaller, the analysis of the paper remains unchanged.

(original or renewed) is in force, “F” if the “dispute” is going on, and “O” if outside options are taken by the parties. Then the buyer's payoff from the trade can be written as follows:

$$\sum_{t=1}^{\infty} \delta^{t-1} U_t^b(\omega, M_t) - i_b$$

where $U_t^b(\omega, Y) = u^b(\omega) - p_t$ (1)

$$U_t^b(\omega, O) = V^b(\omega)$$

$$U_t^b(\omega, F) = 0.$$

On the other hand, the seller's payoff from the trade is expressed as:

$$\sum_{t=1}^{\infty} \delta^{t-1} U_t^s(\omega, M_t) - i_s$$

where $U_t^s(\omega, Y) = p_t - c^s(\omega)$ (2)

$$U_t^s(\omega, O) = V^s(\omega)$$

$$U_t^s(\omega, F) = 0.$$

Let us assume that $u^b(\omega)$, $c^s(\omega)$, $V^b(\omega)$, $V^s(\omega)$ are all positive, bounded, and differentiable for any possible state $\omega \in \Omega$. The following assumption ensures that investments are specific and not valuable outside the relationship:

Assumption 1

V^s, V^b are independent of (i_s, i_b) .

Now, let us define $S(\omega) \equiv u^b(\omega) - c^s(\omega)$ and $\bar{S}(\omega) \equiv \max[S(\omega), V^s(\omega) + V^b(\omega)]$.

We assume that the expected surplus $E_{\theta} \bar{S}(\omega)$ exists for each (i_s, i_b) and is strictly increasing in i_s as well as i_b , and that the unique solution exists for the following problem:

$$\max_{i_s, i_b} \left[E_\theta \frac{\bar{S}(\omega)}{1-\delta} - i_s - i_b \right]. \quad (3)$$

Let (i_s^*, i_b^*) be the first best investment levels that solve the maximization problem. To avoid a trivial case, we assume that the resulting surplus from investing (i_s^*, i_b^*) is strictly positive.

The structure of the model is quite standard but general enough to incorporate the most difficult cases for the hold-up problem. In particular, investments are two-sided, and u^b and c^s each varies with the set of (i_s, i_b) . This implies that the model includes both the self-investment and the “cooperative” investment case of Che and Hausch (1999). The only major difference is that the trade is not limited to one-time transaction, and the gains from specific investments are expected to occur for indefinite periods. An important implication from this change is that the outside option can be now used as a *threat point* in the bargaining. Even though choosing an outside option eliminates the current period's surplus, it does not mean that the parties cannot come back next period and renegotiate. For example, workers with specific skills and experiences do not lose them easily while they are working elsewhere for certain periods of time. A cost advantage of coal mine located near an electricity generating plant does not vanish even if the mine supplies the coal to other firms temporarily.

III. The Renegotiation Game

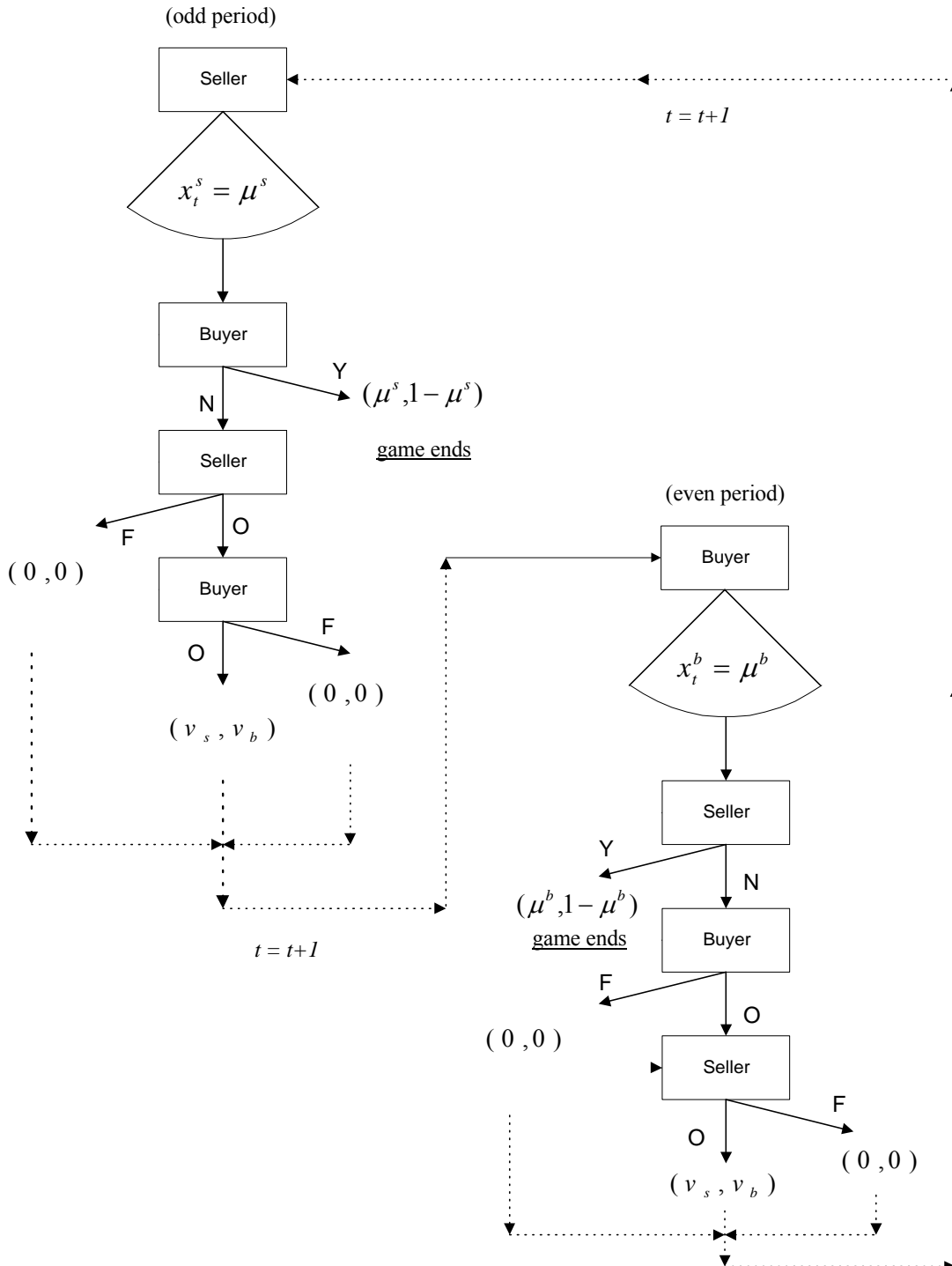
In describing the renegotiation game, we only consider the case that $S \geq V^s + V^b$. If this is not the case, one of the parties must be better off by breaking up the relationship and take the outside option forever, and it is efficient, too. For notational convenience, the payoffs for the parties are normalized by multiplying $1 - \delta$, and we denote $v^s \equiv V^s/S$ and $v^b \equiv V^b/S$.

In the beginning of the Stage 3, both parties may start renegotiating the price of the good. The institutional mechanism governing the renegotiation is assumed to be as follows: the seller and the buyer alternate in making offers (which will be described by the seller's share of total surplus from now on) over discrete time periods ($t \in \{1, 2, \dots\}$). In each odd-numbered period, the seller proposes a new contract P_t yielding the share of μ_t to the seller. The buyer responds (R_t) by either accepting the offer (Y) or rejecting it (N). If the buyer accepts the offer, the renegotiation process is over, and the seller delivers the good under the new contract. If the buyer rejects the offer, the seller must then decide whether to “dispute” (F) or choose the outside option for that period (O). Even if the seller chooses the outside option, however, the buyer can always change the state by declaring to dispute. In every even-numbered period, the buyer in turn makes a new offer, and the seller decides whether to accept it, and so forth.¹⁰

Once the contractual state (M_t) of the period is determined, corresponding actions are taken, time advances for one period and the discount rate applies. Without loss of generality, let us assume that the bargaining begins in Period 1 (the seller's turn to offer) which starts Stage 3. Representative two periods of this game is described in Figure 1:

¹⁰ In making decision to initiate a dispute, which party makes the first move is unimportant. For example, the results remain the same if they simultaneously make their moves.

Figure 1: Representative periods of the renegotiation game.



Note that the bargaining procedure can potentially last for infinite periods. This bargaining process is a modified version of the bargaining game by Rubinstein (1982) and is

similar to the one analyzed by Fernandez and Glazer (1991).¹¹ Rubinstein's model assumes that two parties bargain over a fixed amount that can be shared only when the final agreement is reached, whereas the current model assumes there is a positive sum to bargain over in every period. This feature makes the current model closer to the repeated game structure, but more important distinctions come from the role of the outside option and the decision of “dispute.”

We adopt the notion of subgame perfect equilibrium to analyze this extensive form game with two completely informed agents. There are multiple subgame perfect equilibria in this game, and it is this multiplicity that derives the main results of the paper. For a benchmark, let us denote $\underline{\mu} \equiv 1/(1+\delta)$. This is the share of the seller (first proposer) in Rubinstein's original game with the common discount factor. Here, it is easy to see that this outcome corresponds to the case where both the buyer and the seller are committed to dispute in every period in which the ongoing contract is unsatisfactory.

In what follows, we will first characterize three of this game's subgame perfect equilibria. These are all Pareto-efficient such that the agreement is made in the first period, and production is made at every period (which is efficient by the assumption $S \geq V^s + V^b$). We continue to characterize these equilibria by the seller's surplus shares.

Lemma 1

There is a subgame perfect equilibrium in which the agreement of the seller's share $\bar{\mu}$ is reached in the first period, where

$$\bar{\mu} = \underline{\mu} + \frac{\delta v^s - v^b}{1 + \delta}. \tag{4}$$

¹¹ Fernandez and Glazer(1991) studies a bargaining situation in which the union may engage in strike during wage negotiation. Their model can be considered as a simplified version of the current renegotiation game.

Proof. See the Appendix.

It is noteworthy that $\bar{\mu}$ can be also written as

$$\bar{\mu} = v^s + \frac{1 - v^s - v^b}{1 + \delta}. \quad (5)$$

That is, $\bar{\mu}$ is equal to v^s plus the solution of the original Rubinstein game in which the size of pie is $1 - v^s - v^b$. This is because (v^s, v^b) becomes the *impasse* point of this game once we exclude the possibility of “dispute.” We can also see that $\bar{\mu}$ cannot be an equilibrium if $v^s + v^b > 1$, because, if that is the case, either the buyer or the seller can be better off by choosing the outside options.

Using this lemma, the next proposition shows that there exist at least two subgame perfect equilibria in this game:

Proposition 1

- (i) Let $\tilde{\delta} = \sqrt{\frac{v^s}{v^s + v^b}}$. Then for all δ with $\tilde{\delta} \leq \delta < 1$, there is a subgame perfect equilibrium in which an agreement of $\tilde{\mu}$ is reached in the first period, where

$$\tilde{\mu} = \underline{\mu} + \frac{\delta v^s}{1 + \delta}. \quad (6)$$

- (ii) Let $\hat{\delta} = \sqrt{\frac{v^b}{v^s + v^b}}$. Then for all δ with $\hat{\delta} \leq \delta < 1$, there is a subgame perfect equilibrium in which an agreement of $\hat{\mu}$ is reached in the first period, where

$$\hat{\mu} = \underline{\mu} - \frac{v^b}{1 + \delta}. \quad (7)$$

Proof. See the Appendix.

The reason that this game has multiple equilibria is that the existence of “dispute” option makes it possible for players to use punishment that incurs losses for both parties.¹² A party who threatens would endure the short-term loss expecting the increase in future payoffs. In the original Rubinstein game, however, any choice that strictly improves the present period payoff (for example, not rejecting the offer) will terminate the game, making the threat of not taking the single-period best strategy incredible. By contrast, whether to choose “dispute” affects the present period payoff of the current game without terminating the game. In part (i), it is only the seller who utilizes this additional leverage, while only the buyer utilizes it in part (ii). In this respect, these equilibria also describe the maximal payoff each party can get from the negotiation game.

Lemma 1 and Proposition 1 do not present all the equilibria in this game. In fact, as in Fernandez and Glazer (1991), we may construct a continuum of equilibria including inefficient ones. It will be shown later that the existence of an inefficient equilibrium can be used to strengthen our main result. For our purpose, we present a subset of equilibria in the following proposition:

Proposition 2

For all δ with $\max[\tilde{\delta}, \hat{\delta}] \leq \delta < 1$,

- (i) Any share of $\mu' \in [\hat{\mu}, \tilde{\mu}]$ can be generated as an efficient subgame perfect equilibrium with the agreement reached in the first period.
- (ii) There are inefficient subgame perfect equilibria that have disputes for the first T

¹² If a party can punish the other with no loss of its own, any equilibrium strategy should include the threat of punishment, hence there is no multiplicity.

periods followed by an agreement of $m = \delta^{-T} \hat{\mu}$ and

$$1 - \delta^{-T-1}(1 - \tilde{\mu}) < m \leq 1 - \delta^{-T}(1 - \tilde{\mu}). \quad (8)$$

Proof. See the Appendix.

The first part is derived using the fact that both parties would accept the given division of surplus unless deviation leads to a larger payoff. By prescribing the off-the-equilibrium strategies that give the minimal payoffs to the deviator, any division that is between the minimum and the maximum share can be supported as a subgame perfect equilibrium. The similar argument can be applied to generate inefficient equilibria, as long as the equilibrium set of strategies provide payoffs greater than or equal to the guaranteed minimums. Notice that m in part (ii) amounts to the lowest discounted share ($\hat{\mu}$) the seller can get, while $1 - m$ is the closest to the lowest share the buyer can get given the discrete discounting. That is, part (ii) describes an equilibrium that provides the lowest payoffs for ‘both’ parties.

IV. Contracts with Efficient Investments

Once we have shown that the ex post bargaining procedure has multiple equilibria, it is easy to construct an equilibrium with efficient specific investments. In fact, we may be able to build more than one equilibrium set of strategies that induce the first best levels of investment. Two such examples are described below in Proposition 3, in which we show that an ex ante agreement with no initial contract could achieve both the efficient levels of investments and ex post efficiency. We first present two distinct set of sufficient conditions to achieve efficiency. Let us partition Ω into Ω_0 and Ω_1 such that $\Omega_0 = \{\omega \in \Omega \mid S(\omega) < V^b(\omega) + V^s(\omega)\}$, and

$\Omega_1 = \{\omega \in \Omega \mid S(\omega) \geq V^b(\omega) + V^s(\omega)\}$. An indicator function $\lambda : \Omega \rightarrow \{0,1\}$ is defined such that $\lambda(\omega)$ is equal to one if $\omega \in \Omega_1$ and zero otherwise.

Assumption 2: For all investment levels of i_s, i_b

$$E_\theta \left[\lambda(\omega) \{S(i_s^*, i_b^*; \theta) - S(i_s, i_b^*; \theta) - V^s(\theta)\} \right] \leq 0 \quad (9)$$

$$\text{and } E_\theta \left[\lambda(\omega) \{S(i_s^*, i_b^*; \theta) - S(i_s^*, i_b; \theta) - V^b(\theta)\} \right] \leq 0. \quad (10)$$

Assumption 3: For all investment levels of i_s, i_b

$$E_\theta \left[\lambda(\omega) \{S(i_s^*, i_b^*; \theta) - S(i_s, i_b^*; \theta) - \delta^{-1}V^b(\theta)\} \right] \leq 0 \quad (11)$$

$$\text{and } E_\theta \left[\lambda(\omega) \{S(i_s^*, i_b^*; \theta) - S(i_s^*, i_b; \theta) - \delta V^s(\theta)\} \right] \leq 0. \quad (12)$$

Being in the state of Ω_1 implies that the transaction between the two parties is the optimal outcome. Since we will consider the case in which there is no initial contract, it also means that the ex post bargaining should occur in the state of Ω_1 . Next, V^s (or V^b , respectively) is the one-period outside option payoff for the seller (or buyer), but it also indicates the strength of the per-period punishment that could be imposed on the seller (or buyer) by invoking a dispute. Therefore, Assumption 2 implies that the punishment a party could receive for a violation is not too small compared with the party's influence on the total expected surplus with the specific investment.

On the other hand, one may consider a case in which engaging in dispute is the *status quo*. That is, if both parties invest efficiently, they play as if there were no outside option. In this case, the punishment is not made by invoking dispute but by the deviator who caves in and allows the other party to use the outside option as the bargaining leverage. In such a case, V^s

(or V^b) indicates the strength of the per-period allowance the buyer (or seller) can provide to the other party. Therefore, Assumption 3 is similar to Assumption 2 in that it imposes the requirement that the punishment is relatively powerful.¹³

Finally, we state the main proposition of the paper:

Proposition 3

Under Assumption 2 or 3, the followings hold:

- (i) For any discount rate δ with $\max[\tilde{\delta}, \hat{\delta}] \leq \delta < 1$, a contract can be made such that it supports the first best levels of investments from both parties as a subgame perfect equilibrium.
- (ii) There exists $\delta^* (\geq \max[\tilde{\delta}, \hat{\delta}])$ such that for all δ with $\delta^* \leq \delta < 1$, there is a subgame perfect equilibrium in which investing the first best level is optimal for each player regardless of the investment levels.

Proof. See the Appendix.

The optimal levels of investments are induced because the renegotiation game has multiple equilibria, and therefore the outcomes can be conditioned to punish the party who does not follow the equilibrium strategy. As we have discussed, the result depends upon the relative sizes of the punishment and the contributions from the specific investments. The punishment here is the foregone outside option payoffs by engaging in a dispute. These payoffs can be also considered as the value of trade not affected by the specific investments. Therefore, we may

¹³ In the negotiation game, the seller enjoys the first proposer's advantage a la Rubinstein, but both parties are equal in choosing of dispute. The reason that the discount rate is included only in Assumption 3 is related to the fact that the 'anchor' equilibrium in that case is same as the original Rubinstein game.

conclude that *relative* size of asset specificity is important.

On the other hand, part (ii) shows that our result is quite strong in the sense that it does not have to rely on knife-edge Nash optimality (I invest efficiently because you do). From the proof, it is easy to see that the additional requirement for the discount rate is not too stringent and imposed only to deal with the discrete discounting. This result is derived because the ex post negotiation game is flexible enough to generate an inefficient outcome in which both parties are punished at the same time.

At this point, one might ask whether an ex ante contract could have any role in our analysis. In general, even a simple form of such a contract is very helpful to achieve the first best. Although Assumption 2 or 3 is sufficient to guarantee the first best outcome without any initial contract, an initial contract of a price p with a signing payment can relax these assumptions in a great deal. This is because, by taking an appropriate contract price, we can control the set of states in which renegotiation occurs. Then, Assumptions 2 and 3 could be replaced with conditions defined on a proper subset of Ω_1 . This additional instrument can be used to increase the likelihood that the ex post bargaining occurs only when the available punishments are powerful enough to guarantee the efficient outcome. This is in contrast to Che and Haush (1999) in which initial contract is useless for cooperative investments unless parties can commit not to renegotiate.

V. Concluding Remarks

This paper has shown that, if the renegotiation procedure allows temporary disputes between parties, even the strongest form of hold-up problem may disappear. The model is built around the case in which asset specificity lasts for long term, and trade occurs repeatedly. The result however could be easily extended to a case in which trade occurs only once. What is necessary is that each party has an extra option to reduce the total surplus a little without delaying the game. This is because we rely on the multiplicity of equilibria in the renegotiation game to draw efficient investments, and the repetition of trades has no extra role.

Like the previous literature that achieves the efficiency result, this paper does not refute the hold-up problem entirely but shows that, in some cases, contracting parties may be able to solve the problem of opportunism themselves. As Coase (1988) points out, there are companies such as A.O. Smith, which was not bothered by high asset specificity in building its plant and has been enjoying amicable relationship with GM for fifty years. As Hackett (1994) and Ellingsen and Johannesson (2004) have found, experimental studies show that people seem to care the supposedly sunk investments and coordinate to reward those who invested more. Despite the restrictive features of our model, it is believed that the kind of arrangement described in this paper is quite general and applicable in a wide range of environments.

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Appendix

Proof of Lemma 1

Let $\bar{\eta} = \delta \underline{\mu} + \frac{v^s - \delta v^b}{1 + \delta}$. The pair of strategies that generate the subgame perfect equilibrium is as follows. Neither player ever chooses to dispute. The seller offers $\bar{\mu}$ in every odd-numbered period t and, in every even-numbered period τ , responds to the buyer's offer¹⁴ by

$$R_{\tau}^s = Y \quad \text{if } \mu_{\tau}^b \geq \bar{\eta}$$

$$N \quad \text{otherwise.}$$

The buyer's strategy is to offer $\bar{\eta}$ in every even-numbered period τ and, in every odd-numbered period t , to reply to an offer by

$$R_t^b = Y \quad \text{if } \mu_t^s \leq \bar{\mu}$$

$$N \quad \text{otherwise.}$$

Given these strategies, it never pays to take the “dispute” option because only the present period payoff is lost by doing so. Then, in each period, the parties face essentially the same kind of decision problem they would face in the original Rubinstein game except that the no agreement payoff is (v^s, v^b) . Therefore, we obtain the subgame perfect equilibrium.

Q. E. D.

¹⁴ An offer by i in period j is denoted by seller's share μ_j^i .

Proof of Proposition 1

Let $\tilde{\eta} = \delta \underline{\mu} + \frac{v^s}{1+\delta}$. The following pair of strategies is a subgame perfect equilibrium.

In every odd-numbered period t , the seller offers $\tilde{\mu}$. If this offer is rejected, the seller declares “dispute” for the period. In an even-numbered period τ , given an offer from the buyer, the seller's response is

$$R_{\tau}^s = Y \quad \text{if } \mu_{\tau}^b \geq \tilde{\eta}$$

$$N \quad \text{otherwise.}$$

When the seller is given the choice of taking “dispute” after the buyer chooses the outside option, she does not choose to dispute. That is, the seller always conforms to the buyer's decision to trade outside for the current period.

The buyer's strategy is to offer $\tilde{\eta}$ in even-numbered periods. In odd-numbered periods, the buyer's response is

$$R_t^b = Y \quad \text{if } \mu_t^s \leq \tilde{\mu}$$

$$N \quad \text{otherwise.}$$

The buyer never chooses to dispute. If the seller deviates at any point, the strategies call for both parties to play thereafter according to the strategies described in Lemma 1. Since the strategies of Lemma 1 yield an outcome that is supported as a subgame perfect equilibrium, we do not need to check further deviation possibilities for any continuation game starting with the seller's deviation.

To show that these strategies yield a subgame perfect equilibrium, we need to show that the strategies are Nash equilibrium for every subgame. First, consider a subgame starting

with an offer by the seller for any period. Given that the buyer uses the strategy described above, the maximum possible payoff for the seller from deviation is $v^s + (1 - \delta)^{-1} \delta \bar{\eta}$ which is obviously less than $(1 - \delta)^{-1} \tilde{\mu}$. Next, in the same period, the seller chooses not to dispute after the buyer's rejection if $v^s + (1 - \delta)^{-1} \delta \bar{\eta} \leq (1 - \delta)^{-1} \delta \tilde{\eta}$. This condition is satisfied if and only if $\delta^2 \geq \frac{v^s}{v^s + v^b}$, therefore we take $\tilde{\delta} = \sqrt{\frac{v^s}{v^s + v^b}}$.

On the other hand, given that the seller uses the strategies described above, the buyer is assured of the share $1 - \tilde{\mu}$ by accepting the seller's offer. If this could be a part of the equilibrium strategies, $1 - \tilde{\mu} \geq \delta(1 - \tilde{\eta})$ should hold because the buyer can reject the offer, go through one period of dispute, and offer $\tilde{\eta}$ at the next period successfully. This inequality holds by definition. Similar arguments straightforwardly apply to subgames starting with an offer by the buyer. Finally, since it never pays to dispute unless doing so affects the future payoffs, the strategies that prescribe a party not to choose "dispute" should be the best in any subgame with no further condition.

Therefore, we have proved that there is a subgame perfect equilibrium in which the seller offers $\tilde{\mu}$ in the first period of renegotiation and the buyer accepts instantly. This is also the equilibrium in which the seller obtains the maximum share, because the condition $1 - \tilde{\mu} \geq \delta(1 - \tilde{\eta})$ defines the lower bound for the buyer as in the original Rubinstein game.

Next, since there is no asymmetry between the buyer and the seller in the current game except that the seller makes the first offer, it is straightforward to show that the equilibrium that yields the largest payoff to the buyer also exists. The corresponding share is $\hat{\mu}$ and the threshold

discount rate is $\hat{\delta} = \sqrt{\frac{v^b}{v^s+v^b}}$. In this equilibrium, as long as $\hat{\delta} \leq \delta < 1$, the seller offers $\hat{\mu}$ in the first period of renegotiation and the buyer accepts it instantly. Only the buyer chooses to dispute when he is up to make an offer and the offer is rejected by the seller.

Q. E. D.

Proof of Proposition 2

A formal proof involves tedious repetitions of the steps similar to the proof of Lemma 1 and Proposition 1. To make the proof simple and more intuitive, we will instead take a less formal approach and describe how to construct equilibria based on the results obtained so far.

First, a subgame perfect equilibrium that yields $\mu' \in [\hat{\mu}, \tilde{\mu}]$ for the seller can be constructed in

the following way. Let $\delta' = \max \left[\sqrt{\frac{v^s}{v^s+v^b}}, \sqrt{\frac{v^b}{v^s+v^b}} \right]$. In the first period, the seller offers μ' . If

this offer is rejected, the seller starts “dispute” for the period. The buyer accepts the offers less than or equal to μ' and never disputes. From the second period on, both parties play strategies according to their plays in the first period. That is, if any deviation has been occurred by the

seller in the first period, they play the equilibrium strategies of Proposition 1 that give the maximum share to the buyer ($\hat{\mu}$). If any deviation has been occurred by the buyer in the first

period, they play the equilibrium strategies of Proposition 1 that give the maximum share to the

seller ($\tilde{\mu}$). To prove the subgame perfectness, we only need to show that the first period strategy is optimal for both parties, because they play subgame perfect equilibrium strategies in any continuation game followed by deviations. For the seller, the maximum possible payoff

from deviation is $v^s + (1-\delta)^{-1} \delta \hat{\eta}$, and this is clearly less than non-deviation payoffs

$(1-\delta)^{-1} \mu'$ and $(1-\delta)^{-1} \delta \tilde{\eta}$. For the buyer, deviation is unprofitable because

$$1 - \mu' \geq \delta(1 - \hat{\eta}).$$

Next, we construct the inefficient subgame perfect equilibrium in part (2). Take T such that it is the minimal integer that satisfies $\delta^T \geq \hat{\mu} + 1 - \tilde{\mu}$. Without loss of generality, we assume that T is even-numbered. First, let us describe the strategies in the equilibrium path. In any period κ before $T + 1$ ($1 < \kappa \leq T$), the seller chooses to dispute whenever she is up to. She makes non-serious offers (proposing the share of one for her) and accepts only the offers that give her at least $\tilde{\mu}$ (the maximal share the seller can get). In period $T + 1$, she offers $\delta^{-T} \hat{\mu}$. On the other hand, the buyer also chooses to dispute in any period κ before $T + 1$. He makes non-serious offers (one for him) and accepts only the offers that give him at least $1 - \hat{\mu}$ (the maximal share the buyer can get). In period $T + 1$, he accepts the offers that give him at least $1 - \delta^{-T} \hat{\mu}$.

Next, we describe the off-the-equilibrium path strategies. Suppose the buyer is the one who deviates first, then from the next period on, both players follow the strategies described in Proposition 1(i) (the worst equilibrium for the buyer). Likewise, if the seller is the first deviator, both players follow the strategies described in Proposition 1(ii) (the worst equilibrium for the seller) from the next period on.

Let us verify this set of strategies is a subgame perfect equilibrium. Notice that given the strategies the maximum payoff the seller can get by deviation is $\hat{\mu}$, and the same payoff for the buyer is $1 - \tilde{\mu}$. Since off-the-equilibrium path strategies constitute subgame perfect equilibria for continuation games by Proposition 1, we only need to show that both players get at least the maximum deviation payoffs by following the equilibrium set of strategies. In the equilibrium path, no agreement is made until period $T + 1$, and dispute occurs in every period.

In period $T + 1$, the seller offers $\delta^{-T} \hat{\mu}$ and, the offer is accepted by the buyer. This yields the discounted payoff of $\hat{\mu}$ for the seller and $\delta^T - \hat{\mu} (\geq 1 - \tilde{\mu})$ for the buyer. Therefore, the equilibrium set of strategies is optimal for both players.

Q. E. D.

Proof of Proposition 3

Suppose the buyer and the seller play the following set of strategies. No initial contract is made at Stage 0 except for the side payments to adjust the levels of expected payoffs. At Stage 1, both parties make the first best levels of investments. Before the beginning of the Stage 3 θ is realized, and the state $\omega = (i_s, i_b, \theta)$ is fixed. If $\omega \in \Omega_0$, they break the relationship with each other and take outside options forever. If $\omega \in \Omega_1$, they begin negotiating the price of the good to adjust each party's share of the surplus.

Let us consider the following contingent strategies for the negotiation subgame. If both parties have invested the first best level (i_s^*, i_b^*) , they play the strategies of the equilibrium described in Lemma 1. If the buyer deviates by investing less than the first best level, they play the strategies of the equilibrium described in Proposition 1(i). If the seller deviates by investing less than the first level, they play the strategies of the equilibrium described in Proposition 1(ii). No other types of deviation need to be considered to construct an efficient subgame perfect equilibrium, but we will consider this issue later. Obviously, this set of strategies constitutes the subgame perfect equilibria for the continuation game, once we take the threshold discount rate to be $\max \left[\sqrt{\frac{v^s}{v^s + v^b}}, \sqrt{\frac{v^b}{v^s + v^b}} \right]$. Also, if $\omega \in \Omega_0$, there must be a party who is strictly better off by taking outside option forever, so breaking the relationship is subgame perfect.

Consider the investment stage. The seller's problem at Stage 1 is

$$\begin{aligned} \max_{i_s} \left[E_\theta \frac{U^s(\omega)}{1-\delta} - i_s \right] \\ \text{where } U^s = \mu(\omega)S(\omega) \text{ if } \omega \in \Omega_1 \\ = V^s(\theta) \text{ if } \omega \in \Omega_0, \end{aligned} \quad (13)$$

and the buyer's problem is defined similarly.

For i_s^* to be the solution of the seller's problem, the following condition has to be satisfied for all $i_s \neq i_s^*$:

$$E_\theta \left[\lambda(\omega) \left\{ \bar{\mu}S(i_s^*, i_b^*; \theta) - \hat{\mu}S(i_s, i_b^*; \theta) \right\} \right] \geq (1-\delta)(i_s^* - i_s). \quad (14)$$

Recall that, since i_s^* maximizes the joint surplus, we can derive the following from (3):

$$E_\theta \left[\lambda(\omega) \left\{ S(i_s^*, i_b^*; \theta) - S(i_s, i_b^*; \theta) \right\} \right] \geq (1-\delta)(i_s^* - i_s). \quad (15)$$

Using (15), a sufficient condition for (14) to be satisfied can be expressed as

$$E_\theta \left[\lambda(\omega) \left\{ (\bar{\mu}-1)S(i_s^*, i_b^*; \theta) - (\hat{\mu}-1)S(i_s, i_b^*; \theta) \right\} \right] \geq 0. \quad (16)$$

This can be reduced to

$$E_\theta \left[\lambda(\omega) \left\{ S(i_s^*, i_b^*; \theta) - S(i_s, i_b^*; \theta) - V^s(\theta) \right\} \right] \leq 0. \quad (17)$$

Likewise, a sufficient condition for the buyer's problem is

$$E_\theta \left[\lambda(\omega) \left\{ S(i_s^*, i_b^*; \theta) - S(i_s^*, i_b; \theta) - V^b(\theta) \right\} \right] \leq 0. \quad (18)$$

Therefore, given Assumption 2, investing the first best levels is the solutions to the problems for both parties. On the other hand, we may construct another subgame perfect equilibrium in which the seller's share is $\underline{\mu}$ when both parties invest efficiently. In that case, sufficient conditions of (17) and (18) change to those of Assumption 3.

Next, let us prove that it is possible to construct an equilibrium in which investing the

first best levels is the best regardless of the investment levels of the other player. For this, off-the-equilibrium path strategies should be defined for the cases in which both players deviate. Suppose both parties follow the strategies defined in Proposition 2(ii) in the continuation game after both players have invested inefficiently. For the seller to find it optimal to invest efficiently for any investment level of the buyer, the following condition should be satisfied:

$$\text{for all } i_s, E_\theta \left[\lambda(\omega) \left\{ \tilde{\mu}S(i_s^*, i_b; \theta) - \hat{\mu}S(i_s, i_b; \theta) \right\} \right] \geq (1 - \delta)(i_s^* - i_s). \quad (19)$$

The expression inside the curly bracket can be transformed the following way:

$$\begin{aligned} & \tilde{\mu}S(i_s^*, i_b; \theta) - \hat{\mu}S(i_s, i_b; \theta) \\ &= \frac{1}{1 + \delta} \left[S(i_s^*, i_b; \theta) - S(i_s, i_b; \theta) \right] + \frac{\delta}{1 + \delta} V^s(\theta) + \frac{1}{1 + \delta} V^b(\theta) \\ &\geq \frac{1}{1 + \delta} \left[S(i_s^*, i_b; \theta) - S(i_s, i_b; \theta) \right] + \frac{\delta}{1 + \delta} \left[S(i_s^*, i_b^*; \theta) - S(i_s, i_b^*; \theta) \right] \\ &\quad + \frac{1}{1 + \delta} \left[S(i_s^*, i_b^*; \theta) - S(i_s^*, i_b; \theta) \right] \\ &= \left[S(i_s^*, i_b^*; \theta) - S(i_s, i_b^*; \theta) \right] + \frac{1}{1 + \delta} \left[S(i_s, i_b^*; \theta) - S(i_s, i_b; \theta) \right]. \end{aligned} \quad (20)$$

The first inequality is derived using (17) and (18). Notice that the first bracket in the last line in (20) is same as the surplus in the curly bracket of (15). Since the second bracket must be nonnegative, we infer that (19) is satisfied from the first best condition.

For the buyer, the situation is slightly different because the buyer's share in Proposition 2(ii) is $\delta^T - \hat{\mu}$, which is generally greater than $1 - \tilde{\mu}$. With some manipulation, the condition similar to (20) can be derived as

$$\begin{aligned} & \left[S(i_s^*, i_b^*; \theta) - S(i_s^*, i_b; \theta) \right] + \frac{\delta}{1+\delta} \left[S(i_s^*, i_b; \theta) - S(i_s, i_b; \theta) \right] \\ & - \delta^T (1-\delta) S(i_s, i_b; \theta). \end{aligned} \quad (21)$$

As δ gets larger, the third term gets arbitrarily close to zero, while the second term remains positive. Therefore, there must exist $\delta^* (\geq \delta')$ such that (21) is greater than or equal to $S(i_s^*, i_b^*; \theta) - S(i_s^*, i_b; \theta)$. Therefore, for any $\delta \geq \delta^*$, the buyer finds it optimal to invest the first best level regardless of the seller's choice.

Q. E. D.