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Cyclical Implications of Optimal Labor Contracts in the Equilibrium Search Model*

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Abstract The optimal contract under moral hazard is embedded in a standard equilibrium search model. Under standard assumptions, we show that when firms cannot perfectly observe workers' productivity, the optimal contract takes the form of a standard debt contract. When this contract is embedded in the standard model, the calibrated model can possibly generate a more rigid wage and more volatile employment than the standard model. However, for the model to capture the observed cyclicality of employment, the largest portion of wage should be paid as a base wage which is constant irrespective of worker's performance. When the sources of wage rigidity are investigated, we find that the introduction of optimal contract setting---the absence of bargaining power and the limited incentive effect at the optimum---plays most important roles. This paper provides an endogenous mechanism for real wage rigidity even under the presence of performance-based pay, induced by moral hazard problem.

Keywords Equilibrium Search Model, Debt Contract, Rigid Wage, Volatile Employment

JEL Classification E24, J41, J64

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1. Introduction

In U.S. data, one of the stylized facts in aggregate labor-market fluctuations is that hours worked move a lot without a corresponding movement in wages. To match this fact, a standard neoclassical model (e.g., Lucas and Rapping, 1969, Kydland and Prescott, 1982, King, Plosser, and Rebelo, 1988) requires a high intertemporal substitution elasticity of labor supply that is not supported by estimates from the micro data (MacCurdy ,1981, Altonji, 1986)[1].

An economy with search frictions has been developed (e.g., Mortensen and Pissarides, 1994) as an alternative to the Walrasian neoclassical construct[2]. While these models proved successful in addressing various issues in the labor market, they are not capable of generating the observed volatility in labor-market quantities, such as unemployment and vacancy, in response to plausible productivity shifts. Shimer (2005) argues that the inability to generate volatile quantities in these models is mainly due to the flexible movement of wages in response to a productivity shock. In the standard matching model, wages are determined by Nash bargaining between the firm and the worker. As aggregate productivity improves, wages increase rapidly and absorb most of the additional rent from the positive shock, eliminating the incentive to create more vacancies. Shimer (2004), Hall (2005), and Gertler and Trigari (2009) introduced wage rigidity—derived from the Nash equilibrium of bilateral auction or staggered bargaining—and showed that the volatility of unemployment and vacancies increases as wages become more rigid. In these studies, however, wage is assumed to be sticky in rather an ad hoc manner.

This paper studies the wage determination as a contract problem when a risk-neutral worker has private information about his effort level. We show that the optimal contract can be characterized by a standard debt contract that may exhibit a less volatile wage over time than in the standard bargaining models. The debt contract gives all of the surplus minus a base wage to the firm as long as it is below some predetermined level; then the worker takes any extra rent over that level as a bonus. When this contract is embedded in the standard MP model, the economy can possibly generate a more stable wage and more volatile unemployment or vacancies in response to aggregate productivity shifts than the standard model does. When investigating the sources of wage rigidity, the introduction of contract setting—implying the absence of bargaining and the

[1] Moreover, some argue that the optimization behavior of a worker is not consistent with the movement of wage or hours (Mankiw,Rotemberg, and Summers, 1985).

[2] Early research that embedded search frictions into an otherwise standard RBC model includes Merz (1995) and Andolfatto (1996).

limited incentive effect at the optimum—plays most important roles. While this wage rigidity can be preserved even under varying workers' outside options and the performance-based wage scheme motivated by the moral hazard problem, the model's amplification in unemployment and vacancy fluctuations seems not to be robust. Main limitation of the model is that it can generate the observed magnitude of employment volatility only if the base wage, which is exogenously fixed here, is high enough to cover almost all the fraction of worker's reservation wage.

By replacing the Nash bargaining with an explicit contract arrangement, this model deviates from the standard model in two main dimensions: absence of worker's bargaining power and information asymmetry. There are some good empirical reasons justifying the replacement of a bargaining with a bilateral contract arrangement. In most industrialized countries, particularly the U.S., the individual employment contracting has been becoming the more predominant type of contractual arrangement than collective bargaining between the delegates of firms and workers. In many industrialized countries, the union share of the labor force and the number of union memberships has fallen steadily, and recently—in 2007—the unionization rate in the U.S. is barely about 12 percent. Moreover, some recent studies (for example, Dinardo and Lee, 2004) suggest that the union effect on wage and employment, if any, is quite small in the U.S. There are lots of evidences for the weak bargaining power of workers even in some European countries, such as France and England, too(Cahuc et al., 2002, Van Reenen, 1996).

Another reason is related to the practical contractual convention in the labor market. Usually firms, not workers, are in the place to offer and enforce the terms and conditions of labor contracts. So, most jobs are offered on what amounts to a take-it-or-leave-it basis, with employees playing only a little part in negotiation process. Therefore, in most cases, the only choice available for workers hoping for a wage raise is just to quit and move onto a better-paying job or to stay in their current job if there are no better outside options. This is consistent with the fact that the volatility of job-changers' wages is much higher than that of stayers' wages (Bils, 1985, Beaudry and DiNardo, 1995).

Meanwhile, once the contract setting under private information is introduced into wage determination, another source of wage fluctuations emerges. Because firms cannot observe workers' effort level or investment in firm-specific human capital, firms should design a certain kind of performance-based wage contract in order to induce an optimal effort or investment level from workers. Still less the case of risk-neutral workers, even when workers are risk-averse, the fixed wage is no longer optimal due to the lack of incentive provision. In other words, moral hazard in a contract setting creates another source of wage fluctuations over time. One of the main innovative findings in this paper is that the firm's marginal gain from giving more incentives by paying an outcome-based bonus (incentive effect) is never larger than the firm's marginal loss from giving up that bonus (value-of-firm effect) at the optimum. It is because, at the optimum, the participation constraint for worker limits the ability of firm to discipline workers enough to make full use of its incentive scheme. Thereby, firms had better take all of the additional surplus under a positive shock, rather than sharing those with workers for providing them with more incentives. Thus, in response to a productivity change, the real wage rigidity still can survive even under the performance-based wage scheme motivated by the moral hazard.

Previous macro-studies approaching wage determination as a contracting process includes Ramey and Watson (1997), Boldrin and Horvath (1995), Kennan (2003), Menzio (2004), Shimer and Wright (2004), Rudanko (2009). However, most studies do not explore the cyclical implications of those contract arrangements under information asymmetry. In particular, it is worth comparing this model with Shimer and Wright (2004). Both have much in common, in that they model the wage determination process as a bilateral contract problem under asymmetric information problem. In both model, firms offer optimal wage contract to maximize their profit under the constraint that the contract induces workers to supply effort, because workers have private information about their effort level, as well as it guarantees workers above market-determined reservation utility. Another common thing between both models is that, under optimal contract, workers get paid a constant level of compensation irrespective of their outcome; in our model, a minimum level of base wage is exogenously imposed while, in Shimer and Wright (2004), it is endogenously derived as a kind of severance pay. Meanwhile, in Shimer and Wright (2004), search is competitive since it is directed toward different kinds of jobs while matching occurs at random and every match is same ex-ante in our model. Furthermore, Shimer and Wright (2004) examines the bilateral asymmetric information in the sense that only firm can observe match-specific productivity. So, optimal wage contract should be incentive compatible so that firms may reveal observed state truthfully. This incentive-compatibility necessitates that a constant level of severance pay should be paid to worker, independent of the productivity realization. However, Shimer and Wright (2004) do not investigate the cyclical implication of their wage contract, which is one of our main interest.

This paper is organized as follows. Section 2 describes the model environment and defines its equilibrium. Section 3 derives a standard debt contact as an optimal contract and examines its comparative statics. In Section 4, the model is calibrated to the data. We examine the cyclical properties of the calibrated model. In Section 5, we investigate some sources of wage rigidity in the model. Section 6 concludes the paper.

2. Model Environments

This framework is a variation of the Mortensen and Pissarides stochastic matching model (Mortensen and Pissarides, 1994, Pissarides, 2000). However, once worker and firms are matched randomly, they do not bargain each other; a wage contract is offered by the profit-maximizing firm and workers behave competitively under the given wage contract. This contract arrangement is an approximation to the case when workers have no or very weak bargaining power as calibrated in Hagedorn and Manovskii (2008). There exists information asymmetry problem; that is, a worker's effort level affecting output is known only to the worker, so that the firm cannot observe it directly, but can observe only ex-post gross output reflecting the joint effect of both a realized effort and an unobserved idiosyncratic productivity. This information asymmetry forces firms to induce desirable efforts from the worker by designing an optimal labor contract.

2.1. Firms and Workers

In the model economy, there are a continuum of risk-neutral, infinitely lived workers and firms, each with unit mass. All agents discount future payoffs at the rate $0 < \beta < 1$. Firms have constant returns to scale technology in labor as in the standard model. Here, the main difference is that marginal productivity depends also on the level of a worker's effort. Gross productivity (y) is a function of a stochastic productivity shock and a worker's effort level.

$$y(e, a, x) = f(e) + ax \tag{2.1}$$

where $f(e)$ is an increasing, concave function of effort level e and a, x are an aggregate productivity shock and an idiosyncratic (job-specific) shock each.

The aggregate shock follows a stationary Markov Process and the idiosyncratic shocks are an i.i.d. random process. To be concrete, the logarithm of a follows an AR(1) process with normal innovation.

$$\log a_t = (1 - \rho_a)\overline{loga} + \rho_a \log a_{t-1} + \epsilon_{at} \tag{2.2}$$

where $\epsilon_{at} \sim N(0, \sigma_a)$. This process approximates the conditional distribution of $a^{'}$ given a, which is denoted by $P(a^{'}|a)$. x is an i.i.d. log-normal random variable. $\log x_t \sim N(\mu_x, \sigma_x)$, $\forall t$. $H(x_t)$ is a cumulative distribution function of shock x_t.

A matched firm operates production and pays a contract wage based on observed gross productivity. Then, workers are exogenously separated from the match at a constant rate δ. The operating firm's value at the end of current period is given by,

$$J(a, x) = y(e, a, x) - w(y(e, a, x)) + \beta(1 - \delta) \int_{a'} \int_{x'} J(a^{'}, x^{'}) dH(x^{'}) dP(a^{'}|a) \tag{2.3}$$

where $w(y)$ is a contract wage depending on gross productivity y. $H(x^{'})$ and $P(a^{'}|a)$ are cumulative distribution functions of idiosyncratic shock x and aggregate shock a, conditional on a previous level of the shock. To hire a worker, a firm must maintain an open vacancy at flow cost c. An open vacancy is matched to a searching worker with a probability $q(\theta)$ depending on the vacancy-unemployment rate $\theta_t = \frac{v_t}{u_t}$. Free entry drives the expected present value of an open vacancy (V) to zero.

$$c = \beta q(\theta) \int_{a'} \int_{x'} J(a^{'}, x^{'}) dH(x^{'}) dP(a^{'}|a) \tag{2.4}$$

Unlike in the standard framework, a worker's utility u^W relies not only on the compensation from work $w(y)$ but also on the effort level e he exerts in production.

$$u^W(w(y(e, a, x)), e) = w(y(e, a, x)) - \phi(e) \tag{2.5}$$

where the disutility function $\phi(e)$ satisfies $\phi^{'}(e) > 0$, $\phi^{''}(e) > 0$. Workers can either be unemployed or employed. An unemployed worker gets flow utility z from non-market activity and searches for a job. A searching worker is matched to an open vacancy with a probability $p(\theta)$. The value of the unemployed worker (U) in the current period is given by,

$$U(a) = z + \beta[p(\theta) \int_{a'} \int_{x'} W(a^{'}, x^{'}) dH(x^{'}) dP(a^{'}|a) + (1 - p(\theta)) \int_{a'} U(a^{'}) dP(a^{'}|a)] \tag{2.6}$$

The value of the employed worker (W) in the current period is then,

$$W(a,x) = w(y(e,a,x)) - \phi(e) + \beta[(1-\delta)\int_{a'}\int_{x'} W(a',x')dH(x')dP(a'|a) + \delta\int_{a'} U(a')dP(a'|a)] \tag{2.7}$$

Given a wage contract, An operating worker chooses an effort level that maximizes his expected utility in each period.

2.2. Vacancy, Unemployment and Matching technology

A matching technology of this economy is as follows. When the total number of unemployed workers and vacancies is u_t and v_t, the number of new matches m_t is a first-order homogeneous function of u_t and v_t as follows:

$$m_t(v,u) = \omega v_t^{\alpha} u_t^{1-\alpha} \tag{2.8}$$

where α denotes the elasticity of the number of the matched to vacancy and ω is a matching efficiency parameter. The firm's probability of filling a vacancy $q(\theta_t)$ is,

$$q(\theta_t) = \frac{m_t}{v_t} = \omega\theta_t^{\alpha-1} \tag{2.9}$$

the worker's probability of finding a job $p(\theta_t)$ is expressed as,

$$p(\theta_t) = \frac{m_t}{u_t} = \omega\theta_t^{\alpha} \tag{2.10}$$

where $\theta_t = \frac{v_t}{u_t}$. A fraction δ of workers separate from their jobs exogenously each period. In this economy, endogenous separation does not occur because productivity is unknown before production decision is made, thus a firm makes a separation decision based on expected productivity, not a realized one, which implies separation never occurs unless the expected rent from production is below zero, which is a trivial case.

Finally, unemployment evolves following the law of motion below.

$$u_{t+1} = \delta(1-u_t) - p(\theta_t)u_t \tag{2.11}$$

2.3. Information Structure

The information structure of this model is different from that of the standard model mainly in that firms should decide whether to produce or not and how to allocate outcome ex-ante—i.e., without observing the job-specific productivity and worker's effort level. Note that worker cannot observe an job-specific shock x, either. Owing to a worker's private information about his own effort level, a typical moral hazard problem arises. The timing of the event is summarized as follows.

- Aggregate productivity a is realized and commonly observed.
- a firm and a worker negotiate a new labor contract (when negotiations are broken off, the firm and the worker can choose whether to destroy a job or to lay off (get laid off) temporarily).
- Given the wage scheme, a worker set his effort level that maximizes his expected utility.
- Production is carried out.
- Gross output y is observed, and outcome is allocated by the predetermined wage rule.
- Under the chosen level of vacancy, new matches and exogenous separations occur.

Under this information structure, firms cannot observe the worker's effort level even ex post, because all that firms observe are a gross productivity y under a given aggregate productivity a. Thus, it excludes any possibility of punishing workers or firing them for their shirking. Also note that when negotiation between firm and worker fails, they can choose temporary layoff as an feasible option, even though it never happens in the equilibrium. The availability of temporary layoff affects the outside option of workers and firms and, consequently, may change the wage dynamics significantly.

2.4. Contractual Environment

The optimal contract problem between principal and agent under moral hazard has been very traditional issue. In the case of risk-neutral agents without limited liability, it is well known that a principal can induce a first-best effort level[3] with fixed rent contract, where the entire output

[3]First-best effort level denotes the optimal level when there is no moral hazard problem.

minus a fixed rent is given to an agent (Harris and Raviv, 1979). However, if either agent's wealth constraint is binding or his liability is limited for some reason, a fixed rent contract is not feasible. This kind of contract arrangement was first investigated by Innes (1990), though he focuses on financial contracts between investors and entrepreneurs when the liabilities of both are limited. He showed that a standard debt contract emerges as optimal—even if second-best—when entrepreneurs have private information about their own effort choices. Gale and Hellwig (1985) also showed that the optimal credit contract takes the form of a standard debt contract under the costly state verification—where creditor can observe the state only by paying some costs.

The optimal wage contract here is a labor-market version of Innes' financial contracts. Entrepreneurs and investors in his model correspond to workers and firms in this model. As a entrepreneur has private information about the state, only workers can observe their effort level; firms cannot. While it is a entrepreneur who chooses a contract in Innes(1990) and Gale and Hellwig (1985), firms design and offer the wage contract to worker, here. As investors are competitive in the capital market in their model, here workers are competitive in the labor market; i.e., a worker participates in production as long as his reservation wage is guaranteed. Whether it is newly matched or ongoing, workers and firms renegotiate each other every time period. Every period, firms try to design a wage contract so as to induce optimal effort for maximizing their profit. Given the offered wage contract, workers set their effort level to maximize their utilities. The firm's optimal contract problem is expressed as,

$$
\begin{aligned}
& Max_{w(y),e} \int_y \pi(y) dG(y|e) \qquad (2.12)\\
& s.\ t.\ \int_y w(y) dG(y|e) - \phi(e) \geq w^r \quad (P.C.)\\
& \int_y w(y) g_e(y|e) dy - \phi^{'}(e) = 0 \quad (I.C.)\\
& \overline{w} \leq w(y) \leq y,\ {}^{\forall}y \quad (Limited\ Liability)\\
& \pi(y+\epsilon) \geq \pi(y),\ {}^{\forall}y,\ {}^{\forall}\epsilon > 0 \quad (Monotone\ Constraint)
\end{aligned}
$$

where w^r is the reservation utility of the worker—which is endogenously determined—and $\overline{w}$ is the base wage firm should guarantee at the minimum. Here, $G(y|e)$ is a cumulative distribution function of gross productivity y conditioned on effort and $g(y|e)$ is its density. $\pi(y)$ is firm's profit, $\pi(y) = y - w(y)$. (P.C.) and (I.C.) denote the participation constraint and incentive compatibility constraint. Note that worker's maximization is incorporated into firm's problem as the (I.C.)

constraint.

As in Gale and Hellwig (1985) and Innes (1990), We assume here that the liability of the worker is limited. However, the difference is that it is not bound by zero wealth constraint of agent; a wage contract should warrant some "non-negative" level of base wage for a worker. The base wage is defined as the constant portion of real wage that is independent of the realized outcome, excluding contingent bonuses, incentives, and other performance-based pays and fringes, etc. Thus, it is not necessarily related to legal minimum wage or some kind of minimum cost-of-living. According to this definition of the base wage, we will calibrate the ratio of the base wage to worker's reservation wage to match the empirical ratio of variable pay to total payrolls. Finally, we assume the monotonicity of a contract[4], which means a firm's profit should be a non-decreasing function in outcome.

However, the level of base wage should be an endogenous variable that firms can choose optimally. Even though paying some base pay to workers, irrespective of their performance, is commonly observed in reality, it is not easy to justify theoretically why firms choose to pay some non-negative level of base wage or to which level it should amount to, if they do. There are only a few theoretical justifications for the optimality of paying some level of base pay. Due to the necessity of inducing an investment in firm-specific human capital and two-sided information asymmetry between firm and worker, an up-or-out contract can be optimal, thus it leads firms to paying some minimum level of wage, when hiring workers, or severance pay when firing them(Kahn and Huberman, 1988, Shimer and Wright, 2004).

Nonetheless, since the level of the base wage plays a very crucial role in amplifying the shock responses of vacancy and unemployment in later section, we admit that imposing constant base wage exogenously is very critical limitation of this model. Despite the exogeneity of base wage, an inefficient separation never occurs in the model economy since the ex-ante contract wage in the model always lies in the bargaining set as suggested by Hall (2005). First, by the participation constraint (P.C.) in equation (2.12), the ex-ante expected wage is at least as high as the worker's reservation wage level in every match($E(w(y)) \geq w^r$). Second, to make sure that a contract wage do not exceed the firm's before-wage surplus, we calibrate the distributions of match-specific and aggregate productivity shocks so that the net (after-wage) surplus of operating firms J may be above zero even when the worst states are realized. Thus, irrespective of the level of base wage

[4]For the rationale of this assumption, see Innes (1990).

and the realization of shocks, a contract wage never takes all the rents from the match($^{\forall}a, x$ $J(a,x) > 0$) ex-post as well as ex-ante. Moreover, to evaluate the effect of changes in base wage level from a neutral point of view, we explore a wide range of base wage level for our quantitative exercises in later section, in addition to calibrating it to a single value from the data moment.

2.5. Model Equilibrium

Definition 1. *A equilibrium of the model economy consists of a wage rule $w(y)$, effort level e, vacancy level v, and unemployment level u such that:*

1. *The wage rule $w(y)$ is optimal policy for (2.12) and that rule induces the optimal effort level e from worker.*

2. *Firms post vacancies to the level of v satisfying (2.4).*

3. *Workers choose the effort level e in order to maximize their expected utility, i.e., satisfying (I.C) of (2.12).*

4. *The unemployment level u evolves following (2.11) under a given $\theta = \frac{v}{u}$.*

Once optimal wage rule is set up, the value of a filled job is represented by the following bellman equation,

$$J(a,x) = y(e(a),a,x) - w(y(e(a),a,x)) + \beta(1-\delta)\int_{a'}\int_{x'} J(a',x')dH(x')dP(a'|a) \tag{2.13}$$

By applying the Contraction Mapping Theorem, we can solve for value function $J(a,x)$. Then, using free entry condition (2.4), we can solve for the equilibrium labor-market $v-u$ ratio θ as a function of aggregate productivity a. This ratio θ renders the general equilibrium which represents the outcome of the worker's utility-maximizing effort choice and firm's profit-maximizing policy on vacancy posting and contract design under the predetermined unemployment level.

3. Optimal Contract in a Search Economy

In this section, we derive the optimal contract solution for firm's problem (2.12) and, under a regular parametrization, examine the cyclical implications of the optimal contract. Here, we put more emphasis on its underlying intuition why a debt contract must be chosen as an optimal one. Mathematical proofs will follow at the Appendix.

3.1. Optimal Contract under Moral Hazard

Another distinguishing feature of our contract arrangement is that a temporary layoff is possible as an alternative option when negotiation between firm and worker fails. This assumption is not arbitrary in the sense that firms and workers would voluntarily choose that option in order to maximize their expected payoff, as will be proven by Lemma 1 in the Appendix. This temporary layoff is assumed to be costless and enforceable, which means that once the worker gets laid off, he should go back to the original job in the next period even though he has another job offering. This enforceability seems to be justified when we considers some of the evidence: Katz and Meyer (1990) reported that the recall rate of temporary layoff amounts to about 80 percent.

Allowing for a temporary layoff plays important role in characterizing different wage dynamics from the standard model. Lemma 1 shows that the reservation utility of the worker is the same as the flow utility from non-market activity when a temporary layoff is available. Intuitively, when negotiation fails, workers and firms prefer agreeing temporary lay-off to separating permanently, because firms and workers is reluctant to pay additional search cost. It implies that what determines the reservation wage of a worker is how much a worker is going to get when he is temporarily laid off—flow utility from non-market activity z while holding the same opportunity of renegotiation in next period as if the negotiation would not fail—, not how much a worker can expect from walking out of table and searching for a new job. Hall and Milgrom(2005) challenged the standard assumption that the threat point of bargainers are their outside option; in other words, payoff that they can get by quitting the negotiation and search for another job opportunity. They argue that the rational threat point is what bargainer can get by delaying the bargaining, not terminating it because "the bargainers have a joint surplus, arising from search friction, that glue them together". Thus, a threat to permanently terminate the bargain is not credible and the relative bargaining power depends on the relative delaying cost of both sides. If a temporary layoff is interpreted as an outcome of delaying a bargain, Lemma 1 draws out the same point.

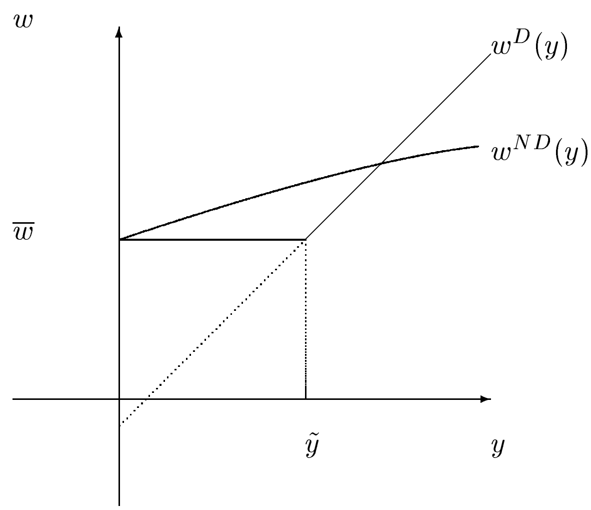
Now, I will discuss the intuition why any solution to (2.12) contains a debt-type contract (for a firm). Suppose there is an optimal non-debt contract $w^{ND}(y)$ and consider alternative debt contract $w^D(y)$ that satisfies the condition below,

$$w^D(y) = y - \pi^D(y), \quad \pi^D(y) = \min(y, \tilde{y}), \quad 0 < \tilde{y} < \bar{y}$$

$$E_y(w^D(y(e^{ND},a,x)) - w^{ND}(y(e^{ND},a,x))) = 0 \tag{3.1}$$

which means that given the same effort level e^{ND}, the debt wage contract $w^D(y)$ gives the same expected compensation as the non-debt contract $w^{ND}(y)$. Note that, without loss of generality, we assume $\overline{w} = 0$ in the firm's problem (2.12) for simplicity. This debt contract gives all of the surplus to the firm as long as it is below some threshold level $\tilde{y}$, then the worker takes any extra rent over that level as a wage. The figure below illustrates the non-debt optimal contract $w^{ND}(y)$ and the debt contract $w^D(y)$ in a general case, $\overline{w} > 0$.

Non-debt contract $w^{ND}(y)$ and debt contract $w^D(y)$



By Lemma 2 in the Appendix, we can show that $e^{ND} < e^D$, where e^{ND} and e^D means a unique optimal effort level under the non-debt contact and the debt contract. It implies that under the same expected compensation, there always exist a debt contract that induces more effort from workers than any non-debt contract.

This can be well understood intuitively. Since the worker bears the total cost of effort while some of the benefits of marginal effort are shared with firms, he will choose an effort level that is less than a first-best one. This implies that the firm should choose a contract that commits the worker to the highest possible effort level by permitting him to reap as much of the first-best surplus as possible. Since higher effort increases the probability weight on high-profit outcomes, with a contract that gives the worker maximum payoffs in high-profit states, the worker is induced to choose maximum effort. Note that the debt contract provides workers with maximum payoffs in high-profit states among any feasible contract satisfying the limited liability and monotone constraint.

By Lemma 3 and Proposition 1 in the Appendix, we show that there exists an optimal contract and it takes the form of a standard debt contract. Thus, now we can restrain the functional form of the optimal contract to a debt contract. Now, the firm's problem is reduced to how to choose the optimal threshold level, $\tilde{y}$.

3.2. Characterization of a Parametric Optimal Contract

In this section, we derive a parameterized version of optimal contract with some specified functional forms and then examine its cyclical properties. There are two functions to be specified: the production function and the disutility function in effort ($f(e)$ and $\phi(e)$).

$$f(e) = \log e \qquad \textit{and} \qquad \phi(e) = Be^2, \quad B > 0.$$

Over the proper domain of x, it is easy to show that assumption (A1)-(A3) is satisfied. By picking up reasonable B, there exists some $\tilde{y}$ satisfying (A4). Furthermore, the convexity of the disutility function guarantees the existence and uniqueness of an optimal effort level.

Firm's problem is characterized by optimization under two constraints (the participation constraint and incentive compatibility constraint). We can substitute worker's first order condition for the incentive compatibility constraint as in the firm's problem (2.12).

First, by substituting specified functional forms, the participation constraint can be written as follows

$$a(\int_{\tilde{x}}^{\bar{x}} xh(x)dx - \tilde{x}(1 - H(\tilde{x}))) \geq Be^2 - \overline{w} + z \quad (P.C.)$$

where $\tilde{x}$ is a net output threshold satisfying $\tilde{y} = \log e + a\tilde{x}$ for given e, a and $H(x)$ and $h(x)$ is the cumulative and probability distribution function of an idiosyncratic shock x. From the incentive compatibility constraint,

$$e = \sqrt{\frac{1 - H(\tilde{x})}{2B}} \tag{3.2}$$

Substituting the second equation into the first one, we can modify the participation constraint to the inequality with respect to $\tilde{x}$ only.

$$a\int_{\tilde{x}}^{\bar{x}} xh(x)dx - (a\tilde{x} + \frac{1}{2})(1 - H(\tilde{x})) \geq z - \overline{w} \tag{3.3}$$

Now the firm's problem is reduced to maximization with respect to $\tilde{x}$ as follows.

$$Max_{\tilde{x}} \int_y (y - w^D(y;\tilde{x}))dG(y|e(\tilde{x})) \tag{3.4}$$

$$s.\ t.\ a\int_{\tilde{x}}^{\overline{x}} xh(x)dx - (a\tilde{x} + \tfrac{1}{2})(1 - H(\tilde{x})) \geq z - \overline{w} \tag{3.5}$$

Once the solution to this problem determines $\tilde{x}$, then the optimal effort level can be picked by (3.2).

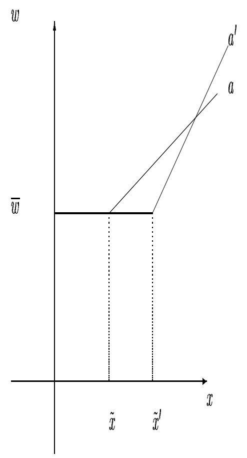
Now, let's turn to some comparative statics. The main interest is the effect of the aggregate productivity shock a and base pay level $\overline{w}$ on output threshold $\tilde{x}$ and optimal effort e.

Corollary 1 in the Appendix means that in the optimal solution, the threshold level of the wage contract will get higher (which means more workers would get only base pay) when an aggregate productivity or a base pay is rising. It is because the marginal increase of firm's saved wage when it raises the threshold level $\tilde{x}$ (value-of-firm effect) always weakly dominates a firm's loss from raising threshold that is incurred by a deteriorated incentive for worker (incentive effect). If the participation constraint is binding, the value-of-firm effect should be strictly larger than the incentive effect at the constrained optimum. If the constraint is not binding, both are supposed to equal by the Marginal Principle. Thus, the value-of-firm effect always weakly dominates the incentive effect. When a surplus to share increases—e.g. when an aggregate productivity goes up or a more base pay is prepaid so that an additional amount of wage to pay gets smaller—the value-of-firm effect gets higher, then firms had better raise the threshold level in order to take extra rent over a worker's reservation wage rather than provide more incentive by lowering it. This is the case as long as it does not harm the worker's incentive so much to the extent of decreasing the firm's after-wage profit—which never happens at the optimum.

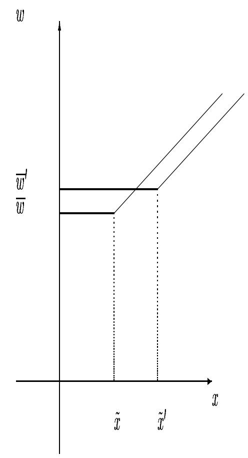
The figure below illustrates Corollary 1 by describing a change in the optimal threshold when aggregate productivity goes up from a to a'. Firm absorbs all of additional surpluses from the positive shock, so that real wage varies less volatile. The effect of base wage level $\overline{w}$ on the form of contract is illustrated in the right panel. Given total expected pay, the increase in base wage lessens the share of variable pay (performance-based portion of wage). According to Corollary 1, it raises the threshold and makes wage scheme even closer to a fixed wage, and irresponsive to productivity shock. As a base wage approaches a reservation level of worker, it becomes more like a fixed wage scheme, though it cannot become perfectly fixed due to incentive considerations.

Illustration of Corollary 1

Change in a



Change in $\overline{w}$



4. Quantitative Results

4.1. Calibration

Table 1 summarizes the calibrated parameters for model simulation. To begin with, A time unit is set to one quarter and the average output per worker in the economy—pinned down by the mean of logged idiosyncratic shock μ_x—is scaled up to fit the labor share of the optimal contract model to its empirical counterpart (0.65 on average over the sample period). For the non-market utility z, under the chosen steady-state level of average output, the conventional replacement ratio (40% of average output) is applied to the standard Nash bargaining model (Shimer, 2005). The exogenous separation rate is set to 0.1 based on Abowd and Zellner's(1985) measurement from 1972-1982 data (3.42 percent per month). It implies that a job lasts for about 2.5 years on average. The discount factor β is 0.99, which implies an annual interest rate of about 4 percent. The elasticity of the number of the matched to unemployment (1-α) is fixed at 0.6, the median value of the estimates that Petrongolo and Pissarides (2001) have reported. In the standard Nash bargaining model, We set a worker's bargaining power parameter η to the same value, 0.6. As proven by Hosios (1990), it enables the decentralized equilibrium to achieve Pareto optimality. For the logged aggregate productivity shock, its persistence (0.95) and standard deviation (0.008) of innovation is chosen following Prescott (1986). Chang (2000) calibrated the distribution of worker's skill level using cross-sectional wage data from the Panel Study of Income

Dynamics(PSID). The chosen value 0.58 of the standard deviation for the logged idiosyncratic shock (σ_x) is based on his calibration results with the assumption that the cross-sectional wage difference in the PSID sample mainly reflects differences in job-specific productivity.

In both the standard model and the optimal contract model, the matching technology parameter ω is set up in order to pin down the steady-state unemployment level to 0.11. Considering that there exists no out-of-labor-force population in the model, this value is based on a real effective basis, which is grounded in the Shimer's(2005) observation that the average unemployment rate during 1951-2003 was 5.67 percent plus the findings of Blanchard and Diamond (1990) that the magnitude of inflow from out-of-the-labor force to employment is almost as large as that of inflow from unemployment[5]. The vacancy posting cost c is normalized to one. Under the normalized level of vacancy posting cost, the ratio of vacancy posting cost to average labor productivity is 0.588 in the benchmark optimal contract model, which is very similar to Hagedorn and Manovskii's (2008) calibration (0.584). Note that the steady state value of market tightness θ is close to unit[6] in a monthly basis under the normalized level of vacancy posting cost, which is the same as Shimer's (2005) normalization. The effort-disutility parameter B are normalized to one half.

Here, parameter γ, which characterizes the base wage level, needs to be explained in details. γ is the ratio of base wage (which is a fixed compensation independent of production outcome) to worker's reservation wage, that is, $\frac{\overline{w}}{w^r} = \gamma$. By Lemma 1, worker's reservation wage w^r equals non-market utility z in the optimal contract model. So here denominator of γ is fixed over time[7]. As mentioned before, the level of base wage basically should be derived as an optimal choice of contracting firms. Notwithstanding, as the best alternative, here we explore the cyclical implications of a considerable range of base wage. The range of γ is set so as to fit the lower bound of the range to the legal minimum wage level of data. Mankiw et al. (1992) have estimated the share of return to human capital in total labor income by using the fact that the minimum wage has averaged about 30 to 50 percent of the average wage in manufacturing in the U.S. If we choose median value (0.4) of this range[8], with a rough labor share of total output (0.6) and calibrated

[5]Main results of the model are not sensitive to the calibrated level of steady-state unemployment rate.

[6]For example, when γ, the base wage ratio which will be explained below, is 0.95, the steady state value of monthly θ is 1.167.

[7]Later on, we will extend this benchmark model to the case that worker's reservation wage is time-varying

[8]For the sample period(1964Q1-2003Q4), the cross-sectional median of minimum/average wage ratio has averaged 0.42.

z (40 percent of average output flow in the standard model), it leads to the lower bound of γ at about 0.6[9]. To see how the market operates when the base wage converges to the reservation wage level, we set its upper bound up to $\gamma = 0.985$.

To check how plausible the level of γ is, we calculate the model-implied share of variable pay in total payroll and compare it to data. As γ rises up, the base wage takes more portion of total payroll, while the bonus part of pay is getting relatively smaller. According to the Hewitts survey in 2007 and 2009, they reported that actual spending on variable pay as a percentage of total payroll was 11.8 percent in 2007, up from just 8.8 percent in 2003, which is the end of sample period. In the long run, it has almost doubled in 15 years, from 6.4 percent in 1994 to 11.2 percent in 2009. Table 5 presents the share of variable pay in total payroll from the model by γ level. That share is decreasing as γ level goes up. The share when γ is 0.95 is closest to the data in 2003. Moreover, considering the sharp upward trend of performance-related pay, that share in the sample period was likely to be lower than the recent level the survey reported. So, we conclude that data for the sample period indicate a considerably high level of γ, way higher than the benchmark model (γ=0.9) has[10].

As a digression, we discuss the relation between our calibration and Hagedorn and Manovskii(HM)'s method. As they pointed out, the key of the amplification mechanism of their calibrated model is the size of profit and the wage elasticity, which is determined by the steady-state ratio of non-market utility to output per worker, $\frac{z}{p}$ in their notation ($\frac{z}{y}$ here)—the so-called replacement ratio. They calibrate the ratio to match the measured accounting profit and wage elasticity of data, of which the value was 0.955. They argue that the lower the difference between p and z is, the higher the elasticity of market tightness (θ) to aggregate productivity is, and the lower the elasticity of wage becomes. Therefore, the measured small profit implies that $p - w$ is small and only moderately procyclical wages mean that $w - z$ is small, which leads to small $p - z$. Small profit implies that it changes significantly in percentage terms in response to even a small change in productivity. Less procyclical wage implies that wages respond inertly and absorb only the smaller portion of the additional rent from the positive shock. Also on the labor supply side, low

[9]The base wage to non-market utility ratio, conditional on the chosen ratio of minimum wage to average wage, can be calibrated as follows. When y and Y are output per worker and total output in the steady state, $\frac{\underline{w}}{y} = \frac{\underline{W}L}{pY} = \frac{\underline{W}}{W} * \frac{WL}{pY} = 0.4 \times 0.6$. Then $\gamma = \frac{\frac{\underline{w}}{y}}{\frac{z}{y}} = \frac{0.4*0.6}{0.4} = 0.6$

[10]Using the $\gamma = 0.9$ as a benchmark enables us to take the more conservative position about the amplification performance of our model, since a higher γ implies more stable wage and, consequently, more volatile vacancy and unemployment.

rents from being employed relative to the non-market utility imply that workers are willing to separate or search for a new job more frequently in response to a small shock in productivity. In both cases, firm's incentives to post vacancies also respond strongly to a change in productivity.

In our optimal contract model, the steady-state average wage is almost the same level as z, since the contract wage covers only worker's reservation level (apart from a small stable amount of reward for worker's effort) in equilibrium and the reservation wage level equals z by Lemma 1. Because the average output per worker is calibrated to fit the model's steady-state ratio of wage to output per worker to the empirical labor share, about two thirds, the replacement ratio $\frac{z}{y}$ in our optimal contract economy is also around two thirds[11]. It implies that the replacement ratio in our optimal contract model is much closer to Shimer's calibrated value 0.4, way below HM's 0.955. Therefore, the amplification mechanism in our model mainly works through the stable wage movement around its reservation level (small $w-z$) resulted from absence of worker's bargaining power, not the smaller profit (small $p-w$ in HM's notation) which is one of the main mechanisms in the HM.

4.2. Computation

For aggregate and idiosyncratic shocks, the point grid is constructed. The range of the grid is $\pm 3SD$ around its mean. The number of the grid for a and x is 9 (101 for impulse response analysis) and 1001. The reason why there are a large number of x grid points relative to a is that the effect of the optimal debt contract is highly dependent on the choice of its threshold. The law of motion for aggregate shocks is approximated to the discrete state space by the method suggested by Tauchen (1986).

We draw one thousand periods of aggregate shocks from their approximated law of motion given the initial value. Then, given the aggregate state, we solve for the model so that the time series of labor quantities and wage can be generated. By repeating this simulation one hundred times, we can generate one hundred samples of time series with each length of one thousand periods. For each sample, we calculated sample moments of interest and then averaged these moments across samples.

[11] Even under the same level of non-market utility z and the mean of productivity shocks μ_x, the steady-state replacement ratio $\frac{z}{y}$ in the optimal contract model can be different from the same steady-state ratio in the standard Nash bargaining model, which is calibrated to 0.4 here. The gap between them originates from the fact that the production technology in the optimal contract model additionally includes a worker's effort level, in contrast to that of the standard model.

4.3. Results

Table 2 summarizes the key statistics of quarterly U.S. data. I used the quarterly employment data from the Current Population Statistics(CPS) survey, real wage data (total private average hourly earnings in 1982 dollars) from the Current Employment Statistics(CES) survey, and vacancy data(help-wanted advertising index) from the Conference Board. The time period of the data is from 1964Q1 to 2003Q4. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

As is well known, vacancy is highly pro-cyclical, and unemployment is counter-cyclical. They are strongly negatively correlated with each other. Vacancy and unemployment are ten times more volatile than a productivity shock. In contrast, as Table 6 shows, real wages are almost acyclical relative to employment or vacancy, so that real wages' positive correlation with productivity is much weaker than predicted by the standard RBC model. In addition, wage varies too little compared with other quantity variables. The standard deviation of real wages is almost the same as that of a productivity shock and so no amplification like that in other quantity variables is found.

Table 3 shows the sample moments of the time series generated by the standard MP model, where wage is determined by a Nash bargaining process. In response to a productivity change of similar magnitude in the real data, the standard MP model generates much smaller fluctuations of unemployment, vacancy, and the v-u ratio than that of real data. Furthermore, there is no propagation of a productivity shock, since the contemporary correlations between vacancy (or unemployment) and a productivity shock are close to one.

Tables 4 shows the corresponding sample moments of the time series generated by the optimal contract model. The volatility of unemployment, vacancy, and the v-u ratio is much bigger in the optimal contract model than in the standard Nash bargaining model for all ranges of γ. Figure 1 compares the impulse responses of the labor-market quantity variables to 1 SD of the productivity shock between the standard model and the optimal contract model. Those responses are also shown by the level of base pay ratio γ. For all ranges of γ, Those quantity variables respond more largely in the optimal contract model than in the standard Nash bargaining model. The optimal contract arrangement obviously tends to amplify the responses of unemployment and vacancy than the standard bargaining model does.

However, the magnitude of amplification seems not to be enough or robust. The order of amplification is substantially sensitive to the level of γ, base wage ratio. Table 4 and Figure 1 confirm that the amplification effect is getting bigger as the base pay ratio increases. According to Corollary 1, as γ approaches one, the optimal wage scheme gets much closer to a fixed wage, which leads to more volatile vacancies. However, to generate the observed magnitude of employment volatility, the level of base wage should be high enough. When the base wage covers 95 percent of the worker's reservation wage, the volatility of unemployment, vacancy, and the v-u ratio is only 3.3 times higher than the standard model, still much lower than that of the data. Only when γ reaches up to 0.985, the magnitude of the amplification in the model is comparable to—even exceeds—that of the data as seen in Table 4. This suggests that the model can generate enough amplification only if base wage covers almost all the fraction of wage. It is one of main limitations of the model that its quantitative performances are sensitive to the base wage level which is exogenously set out of the model. Furthermore, the recent upward trend of adopting performance-related pay may keep the model from generating amplification enough to be comparable to the data.

The main reason for the model's improvement on amplification is the more rigid wage movement it induces. When a productivity shock hits, the rigid wage movement gives firms the stronger incentive to adjust more vacancies, based on the expected change in future profit. Table 6 compares the data with the simulated series in terms of real wage volatility and correlation with other variables. Obviously, the optimal contract model generates more rigid wage movements compared to the standard model where the wage dynamics are basically the same as the shock dynamics. Figure 2 compares the impulse response of the model-generated real wage and labor quantities to 1 SD of productivity shock between the standard model and the optimal contract model. In the optimal contract economy, the real wage responds less to a productivity shock, and consequently vacancy and unemployment vary more volatile.

Meanwhile, compared with real data, the wage generated from the optimal contract model is 'too rigid' in fact. Its volatility is only one tenth of that of the real data. The part of reason for that is because our optimal contract model assumes that workers behave competitively in the labor market unlike the standard bargaining model, so it does not allow any wage mark-up. Moreover, the unique source of wage fluctuations in the optimal contract model is the productivity shock. Once the model allows for the wage mark-up and the various shocks, other than a productivity shock, affecting the mark-up, the fluctuations of model's contract wage may become more volatile.

Which factors make wages more rigid in the optimal contract model? The first is the allowance of a temporary layoff. With the availability of temporary layoff, as Lemma 1 in the Appendix indicates, the worker's reservation wage, which is otherwise mainly determined by worker's outside option, becomes fixed independent of the state of the labor market. That can make wages' responses to a shock more inert. Second, the more important factor is the competitiveness assumption for the worker in our contractual setting. All that firms need to care about in order to induce workers' participation is to guarantee their reservation wage—implying $w \risingdotseq w^r = z$ by Lemma 1. Unlike a highly procyclical wage in the typical bargaining model, the competitive contracting arrangement dictates that real wage vary along the business cycle only as long as the worker's outside option is changing, affected by the aggregate state of labor market.

Even though the wage scheme should be based on time-varying performances due to moral hazard, the wage rigidity can be preserved through the optimal contract's own property described earlier. As Corollary 1 shows, when a positive productivity shock hits, firm's marginal gain from raising the bonus threshold $\tilde{x}$ (value-of-firm effect) is bigger than a firm's loss from raising threshold that is incurred by a deteriorated incentive for worker (incentive effect). Thus, in the face of a positive shocks, firms had better take extra rent over a worker's reservation wage, rather than raising the worker's pay to provide more incentives.

Now, we decompose those factors to see how much the fixed reservation wage contributes to generating rigid wage. To do this, we allow the reservation wage—which was fixed to the non-market utility according to Lemma 1—to be time-varying. We conceive the economy, where a labor-market state summarized by vacancy-unemployment ratio θ affects worker's reservation wage, but worker's wage scheme is still determined by the optimal debt contract as described. In this economy, a worker's reservation wage is the wage level making the value of the employed W equal the value of the unemployed U. In other words, workers feel indifferent between being employed and being unemployed under this reservation wage.

Figure 3 presents the impulse response of vacancy and unemployment in the optimal contract economy with time-varying reservation wage. Figure 4 compares the wage dynamics between the standard model and the optimal contract model with a time-varying reservation wage. Comparing with Figure 2, We can hardly find any difference between the cases: whether the reservation wage is time-varying or not. Apart from small increase in volatility, the wage dynamics of this economy is seldom distinguished from that of the optimal contract model with the fixed reservation wage, which is the case with the volatility of vacancy and unemployment. Thus, we can conclude that model's amplification is robust, whether a worker's reservation wage is fixed or not. To sum up, the fixed reservation wage does not contribute much to generating rigid wage and volatile employment. Rather, worker's limited bargaining power featuring our contract environment plays the most significant role in the model's capability to generate rigid wages and volatile quantities.

5. Concluding Remarks

In this paper, we construct a search model augmented with contract arrangements under asymmetric information. Under standard regularity conditions, a simple debt contract form à la Innes (1990) is derived as an optimal contract, which may exhibit less volatile wage fluctuations than the standard bargaining model does. This less volatility can lead to more volatile unemployment and vacancy as compared to the standard MP model. However, the magnitude of amplification seems not to be robust. The model can generate the observed magnitude of employment volatility only if the base wage, which is exogenously fixed here, is high enough to cover almost all the fraction of worker's reservation wage. When we investigate the source of real wage rigidity, we found that changes in worker's outside options seldom affect wage volatility. Limited bargaining power of worker is more crucial for explaining wage inertia. One of main findings is that moral hazard may not amplify wage fluctuations over cycle because of the weak incentive effect of adjusting the bonus.

There remain a lot of things for further research. Main theoretical limitation of this model is that we impose a exogenously fixed level of base wage on the model's contract arrangement, though it should be endogenously determined by firm's optimal choice. Since it is a critical factor affecting the model's amplification performance significantly, it is imperative to find theoretical justifications for firm's paying a substantial level of base wage in order to support the validity of the model's amplification. Second, although we put much emphasis on the important role of worker's limited bargaining power, how strong a worker's bargaining power is basically a empirical issue. Thus, it should be a natural next step to find some empirical evidences about the degree of worker's bargaining power. Finally, we need to find the more general conditions under which performance-based wage scheme incurred by moral hazard does not amplify wage fluctuations over cycles even under huge output variations. We explored only a parameterized version of those conditions here.

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Appendix : Mathematical Proofs

For the characterization of an optimal contract, the following general assumptions are needed:

- (A1) $g(y|e)$ is continuously twice differentiable in e.
- (A2) $\frac{g_e(y|e)}{g(y|e)}$ is increasing in y, $\forall e$.
- (A3) $G(y|e)$ is convex in e.
- (A4) $\exists \tilde{y},\ E_y(w^D(y)) - \phi(e^D) \geq z$

where $G(y|e)$ is a cumulative distribution function of gross productivity y conditioned on effort and $g(y|e)$ is its density. Assumption A1 and Assumption A3 are necessary for the existence and uniqueness of the optimal effort level. Assumption A2 represents the so-called "monotone likelihood-ratio property" which implies that higher output is more likely due to higher effort. Assumption A3 implies a stochastically diminishing return of effort and technically makes the first-order approach for worker's maximization valid. Assumption A4 states that there exist a certain range of threshold productivity under which a debt contract renders net positive surpluses over the reservation level for workers. It is crucial for the existence of an optimal debt contract.

Lemma 1. $$w^r = z \text{ } \mathit{if} \text{ } (1-\delta) > p(\theta)$$

Proof. When negotiation fails, the firm has three choices: destroying a job, posting a new vacancy, or doing a temporary layoff. The first two choices lead to zero expected profit by the free entry condition. When a firm lays off a worker temporarily, its value is,

$$J^L(a) = \beta(1-\delta)\int_{a'}\int_{x'} J(a', x')dH(x')dP(a'|a) > 0 \tag{5.1}$$

Thus, when negotiation fails, the firm chooses to lay the worker off temporarily rather than to destroy a job. When negotiation fails, workers prefer getting laid off to being fired, since the value of getting laid off W^L is higher than that of being fired U if $(1-\delta) > p(\theta)$.

$$W^L(a) = z + \beta[(1-\delta)\int_{a'}\int_{x'} W(a',x')dH(x')dP(a'|a) + \delta\int_{a'} U(a')dP(a'|a)] \quad (5.2)$$
$$> z + \beta[p(\theta)\int_{a'}\int_{x'} W(a',x')dH(x')dP(a'|a) + (1-p(\theta))\int_{a'} U(a')dP(a'|a)] = U(a)$$

so the reservation wage that the firm should guarantee is the same as z since
$W(a,x) = W^L(a)\ {}^\forall a, x$ only if $w(y) - \phi(e) = z,\ {}^\forall e, y$. Therefore $w^r = z$ ■

Before proceeding on, Lemma 2 below from Innes (1990) is helpful in characterizing the effort level induced by a debt contract.

Lemma 2. *Suppose two wage contracts $w^1(y)$ and $w^2(y)$ are satisfying these two conditions,*

$$i)\ \ E_y(w^1(y) - w^2(y)) = 0 \quad (5.3)$$
$$ii)\ \ \exists y^0 \in (\underline{y}, \bar{y}),$$
$$w^1(y) \geq w^2(y),\ \ for\ \forall y \leq y^0\ and\ w^1(y) \leq w^2(y),\ \ for\ \forall y \geq y^0$$

then $e^1 < e^2$ where e^i is the optimal effort level under $w^i(y)$.

Proof. See the proof of Lemma 1 and Lemma 2 in Innes (1990) ■

Lemma 2 means that one contract, of which the payoff has the same expected value and crosses the other once from below to above, induces a higher effort level. With no loss of generality, from now on, We assume $\overline{w} = 0$ in the firm's problem (2.12).

Lemma 3. *If any solution to (2.12) exists, it is a debt contract.*

Proof. By definition, the optimal non-debt contract $(w^{ND}(y), e^{ND})$ satisfies all of the constraints of (2.12). Moreover, by definition of $(w^D(y), e^D)$, $(w^D(y), e^D)$ also satisfies all of the constraints of (2.12) except for (P.C.). By Lemma 1 and the definition of $(w^D(y), e^D)$

$$\begin{aligned} w^r = z \quad &\le \textstyle\int_y w^{ND}(y)dG(y|e^{ND}) - \phi(e^{ND}) \\ &= \textstyle\int_y w^D(y)dG(y|e^{ND}) - \phi(e^{ND}) \\ &\le \textstyle\int_y w^D(y)dG(y|e^D) - \phi(e^D) \end{aligned} \tag{5.4}$$

Thus, contract $(w^D(y), e^D)$ satisfies constraint (P.C.), too. Now then, firms prefer contract $(w^D(y), e^D)$ to $(w^{ND}(y), e^{ND})$, since, by definition of $w^D(y)$ and Lemma 2,

$$\begin{aligned} \int_y (y - w^{ND}(y))dG(y|e^{ND}) \quad &= \quad \int_y (y - w^D(y))dG(y|e^{ND}) \\ &< \quad \int_y (y - w^D(y))dG(y|e^D) \end{aligned} \tag{5.5}$$

The second inequality holds due to the fact that $\int_y (y - w^D(y))dG(y|e)$ is a increasing function of e. It is because the following can be shown by substituting $w^D(y)$.

$$\frac{\partial \int_y (y - w^D(y))dG(y|e)}{\partial e} = \frac{\partial E_y(y - w^D(y))}{\partial e} = -\int_{\underline{y}}^{\tilde{y}} G_e(y|e)dy > 0 \tag{5.6}$$

The last inequality holds due to the first order stochastic dominance (FOSD) property implied by Assumption (A2). Since firms prefer contract $(w^D(y), e^D)$ to $(w^{ND}(y), e^{ND})$, it is a contradiction. ■

Proposition 1. *Under the Assumption (A4), A solution to problem (2.12) exists and has the following properties:*

$$i) \;\; w(y) = w^D(y; \tilde{y}) \equiv y - \min(y, \tilde{y}), 0 < \tilde{y} < \bar{y} \tag{5.7}$$

$$ii) \;\; e^D < e^* \tag{5.8}$$

where e^* is the first-best effort level.

Proof. For i), by Lemma 3, the existence of an optimal debt contract (w^D, e^D) is a necessary and sufficient condition for (2.12) to have a solution, i.e., if and only if the following problem

$$Max_{\tilde{y}} \int_y (y - w^D(y;\tilde{y}))dG(y|e^D(\tilde{y})) \tag{5.9}$$
$$s.\ t.\ \int_y w^D(y;\tilde{y})dG(y|e^D(\tilde{y})) - \phi(e^D(\tilde{y})) \geq z$$

has a solution, then (2.12) has a solution. By definition of w^D, the choice set of $\tilde{y}$ is compact and, by (A4), nonempty. With the continuity of the objective function and $e^D(\tilde{y})$—it is straightforward to prove the continuity of $e^D(\tilde{y})$ by the Maximum Theorem, (5.9) has a solution by the Weierstrass Theorem.

For ii), the first order condition for effort level can be rewritten,
$\frac{\partial E_y(w^D(y)-\phi(e^D))}{\partial e} = \frac{\partial E_y(y-\phi(e^D)-z)}{\partial e} - \frac{\partial E_y(y-w^D(y))}{\partial e} = 0$
Since $\frac{\partial E_y(y-w^D(y))}{\partial e} > 0$ as shown in the inequality (5.6), $\frac{\partial E_y(y-\phi(e^D)-z)}{\partial e} > 0$, which means first-best effort level e^* is higher than e^D. ■

Corollary 1. *The solution $\tilde{x}$ to (3.4),(3.5) satisfies $\frac{h(\tilde{x})}{2(1-H(\tilde{x}))} \leq a(1-H(\tilde{x}))$. Under this condition, $\tilde{x}$ is nondecreasing function of both a and $\overline{w}$. If the condition holds as a strict inequality, $\tilde{x}$ is strictly increasing in them.*

Proofs. From the first order condition of problem (3.4),

$$\frac{h(\tilde{x})}{(1-H(\tilde{x}))^2} = 2a \tag{5.10}$$

Note that if the right hand side of this condition is strictly larger than the left hand side for all $\tilde{x}$ satisfying (3.5), it implies that (3.5) is binding.

First, suppose that (3.5) is not binding, so that (5.10) holds as an equality. The left-hand side of (5.10) is a non-decreasing function of $\tilde{x}$ as long as Assumptions (A1)-(A3) are satisfied[12]. Thus, $\tilde{x}$ is increasing in a and not affected by $\overline{w}$.

Next, Suppose that (3.5) is binding. As mentioned, it is binding if and only if $\frac{h(\tilde{x})}{(1-H(\tilde{x}))^2} <$

[12]It is straightforward to show that $\frac{1}{ea}\frac{h(\tilde{x})}{(1-H(\tilde{x}))} = -\frac{G_e(\tilde{y}|e)}{1-G(\tilde{y}|e)}$ Then, we can show that $-\frac{G_e(\tilde{y}|e)}{(1-G(\tilde{y}|e))}$ is a non-decreasing function of $\tilde{y}$ (or $\tilde{x}$) if Assumption A1-A3 is satisfied. See the proof of Lemma 1 in Kim (1997).

$2a$ for all $\tilde{x}$ satisfying (3.5). As an equality, totally differentiating (3.5),

$$\frac{d\tilde{x}}{da} = \frac{\tilde{x}(1 - H(\tilde{x})) - \int_{\tilde{x}}^{\bar{x}} xh(x)dx}{(1 - H(\tilde{x}))(\frac{1}{2}\frac{h(\tilde{x})}{1-H(\tilde{x})} - a)} \tag{5.11}$$

$$\frac{d\tilde{x}}{d\overline{w}} = -\frac{1}{(1 - H(\tilde{x}))(\frac{1}{2}\frac{h(\tilde{x})}{1-H(\tilde{x})} - a)} \tag{5.12}$$

The numerator of the right-hand side of (5.11) is negative irrespective of the distributional property, but the sign of the denominator depends on it. If $\frac{h(\tilde{x})}{1-H(\tilde{x})} < 2a$, $\frac{d\tilde{x}}{da} > 0$ and $\frac{d\tilde{x}}{d\overline{w}} > 0$ hold. Note that this condition is automatically satisfied as long as (3.5) is binding implying $\frac{h(\tilde{x})}{(1-H(\tilde{x}))^2} < 2a$. So when (3.5) is binding, $\tilde{x}$ is a strictly increasing function of a and $\overline{w}$.

Let $\tilde{x}^B$ the threshold level satisfying (3.5) as the equality, conditional on aggregate productivity a and base wage $\overline{w}$; implying $\tilde{x}^B(a, \overline{w})$. Similarly, suppose a threshold level satisfying (5.10) as the equality. That is also a function of a and I refer to it as $\tilde{x}^{NB}(a)$. The optimal threshold level $\tilde{x}$ is determined by the following relationship.

$$\tilde{x}(a, \overline{w}) = \min\{\tilde{x}^B(a, \overline{w}), \tilde{x}^{NB}(a)\} \tag{5.13}$$

Both $\tilde{x}^B(a, \overline{w})$ and $\tilde{x}^{NB}(a)$ are continuous and nondecreasing functions of a and $\overline{w}$. Thus, $\tilde{x}(a, \overline{w})$ is also a continuous and nondecreasing function of a and $\overline{w}$. ■

Table 1. Parameter Calibration

Parameter	Interpretation	Value
$\frac{z}{y}$	Ratio of non-market utility to average output per worker (standard Nash bargaining model)	0.4
δ	Exogenous separation rate	0.1
β	Discount factor	0.99
η	Bargaining power parameter	0.6
α	Elasticity of the matched to vacancy	0.4
ρ_a	Persistence parameter of aggregate shock	0.95
σ_a	SD of innovation to aggregate shock process	0.008
μ_x	Mean of logged idiosyncratic shock	0.93
σ_x	SD of logged idiosyncratic shock	0.58
c	flow cost of vacancy posting	1
B	Effort disutility	0.5

Table 2. Summary Statistics (Quarterly U.S. data (1964 - 2003))

		a	θ	u	v
Standard deviation		0.019	0.371	0.180	0.202
Quarterly autocorrelation		0.881	0.952	0.956	0.946
Correlation matrix	a	1	0.304	-0.319	0.276
	θ	0	1	-0.969	0.976
	u	0	0	1	-0.892
	v	0	0	0	1

Table 3. Summary Statistics (standard Nash bargaining)

		a	θ	u	v
Standard deviation		0.0192	0.0329	0.0116	0.0235
Quarterly autocorrelation		0.866	0.866	0.886	0.707
Correlation matrix	a	1	0.999	-0.866	0.970
	θ	0	1	-0.866	0.970
	u	0	0	1	-0.719
	v	0	0	0	1

Table 4. Summary Statistics (optimal contract)

	$\gamma = 0.9$		$\gamma = 0.95$		$\gamma = 0.985$	
	SD	Autocorrelation	SD	Autocorrelation	SD	Autocorrelation
a	0.0192	0.866	0.0192	0.865	0.0192	0.865
θ	0.0750	0.866	0.1104	0.866	0.6630	0.857
u	0.0263	0.886	0.0384	0.887	0.1777	0.913
v	0.0537	0.708	0.0787	0.708	0.5130	0.740

Table 5. Share of Variable Pay in Total Payroll

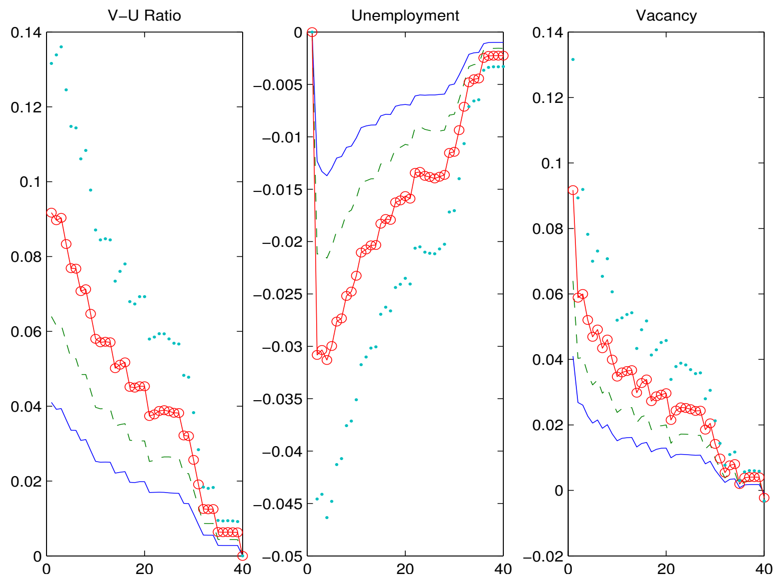
	Data(1994)	Data(2003)	Model			
γ			0.6	0.9	0.95	0.985
Share(%)	6.4	8.8	49.9	15.3	8.3	3.1

Note: Data is from Hewitt Associates (2007).

Table 6. Comparison between Model's Real Wage and Real Wage Data

Real Wage		U. S. Data	NB	Optimal Contract
SD		0.0220	0.020	0.0021
Autocorrelation		0.958	0.882	0.636
Correlation with	a	0.425	0.999	0.610
	θ	0.180	0.999	0.612
	u	-0.164	-0.884	-0.918
	v	0.186	0.960	0.405

Figure 1. The Response of Unemployment, Vacancy and V-U Ratio to 1 SD Productivity Shock

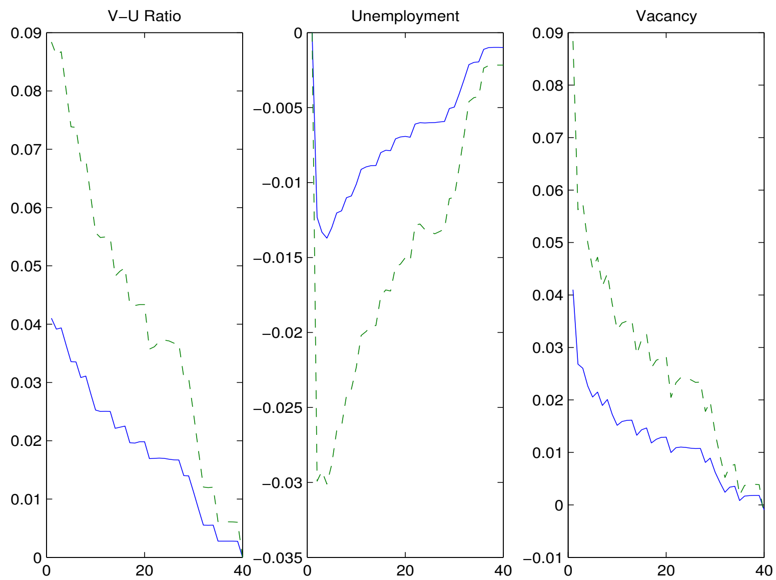


Note: The solid line, the dashed line, the marked line, and the dotted line respectively denote the impulse response of the standard model, the optimal contract model with $\gamma = 0.6$, the optimal contract model with $\gamma = 0.9$, and the optimal contract model with $\gamma = 0.95$

Figure 2. Real Wage and Labor Market Quantities (standard and optimal contract model)

Note: The solid line denotes the real wage (blue line) and labor-market quantities (green line) of the standard model. The dashed line denotes the real wage (red line) and labor-market quantities (cyan line) of the optimal contract model($\gamma = 0.9$).

Figure 3. The Response of Unemployment, Vacancy and V-U Ratio to 1 SD Productivity Shock (optimal contract model with a time-varying reservation wage)



Note: The solid line denotes the impulse response of the standard model and the dashed line denotes that of the optimal contract model with a time-varying reservation wage($\gamma = 0.9$)

Figure 4. Real Wage and Labor Market Quantities (optimal contract model with a time-varying reservation wage)

Note: The solid line denotes the real wage (blue line) and labor-market quantities (green line) of the standard model. The dashed line denotes the real wage (red line) and labor-market quantities (cyan line) of the optimal contract model($\gamma = 0.9$).