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Recognizability of Medium of Exchange and Relative Prices

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Abstract

This paper provides a theoretical account for the fundamental defects of commodity money as an imperfectly-recognizable medium of exchange. We incorporate the recognizability of silver as a medium of exchange explicitly into the search-based model where silver can be either used as a medium of exchange or invested in the world market for a given rate of return. When the recognizability of silver becomes severe, both the real balance of silver as a medium of exchange and the quantity traded decrease substantially. The declining real balance of silver is due to either a decrease in the nominal balance of silver when silver is abundant or a decrease in the price of silver when silver is scarce. The variability of silver demand and price also affects relative prices as long as silver coin is used as an imperfectly-recognizable medium of exchange. An increase in the recognizability of silver improves welfare through its effects on extensive and intensive margins, net of opportunity cost of holding nominal balance of silver for a trade. This implies the superiority of fiat money which is almost perfectly recognizable.

Keywords Liquidity return, Medium of exchange, Relative prices

JEL Classification E40, E42, G12

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1. Introduction

Among the properties which money should possess as a medium of exchange is its recognizability on which Jevons (1875, chapter V) described as follows:

> By this name we may denote the capability of a substance for being easily recognized and distinguished from all other substances. As a medium of exchange, money has to be continually handed about, and it will occasion great trouble if every person receiving currency has to scrutinize, weigh, and test it. If it requires any skill to discriminate good money from bad, poor ignorant people are sure to be imposed upon. Hence, the medium of exchange should have certain distinct marks which nobody can mistake. Precious stones, even if in other respects good as money, could not be so used, because only a skilled lapidary can surely distinguish between true and imitation gems.

In the medieval Europe, despite the attempts to prevent counterfeiting and the fraudulent removal of silver from coin, it was not easy to distinguish between true and false coin because of illegal clipping and tampering with coin as well as legitimate wear and tear. Further, the recognizability problem varies across regions and over time. Jevons (1875, chapter XIII) noted properly that "the degrees of fineness employed in one country or another at different times are infinitely various. Silver has been coined of only 200 or even 150 parts in 1000, and gold of 750 or 700 parts; and coins exist of almost every fineness from these limits up to nearly pure metal."[1]

The goal of this paper is to provide a theoretical account for the fundamental defects of commodity money in general as an imperfectly-recognizable medium of exchange. This is executed using silver coin which was the most widely circulated commodity money in medieval Europe. Specifically, in the spirit of Jevons (1875), King and Plosser

[1] Jevons (1875, chapter VII) also noted the significance of this problem as follows :

> The use of money creates, as it were, an artificial crime of false coining, and so great is the temptation to engage in this illicit art that no penalty is sufficient to repress it, as the experience of two thousand years sufficiently proves. Thousands of persons have suffered death, and all the penalties of treason have been enforced without effect.

(1986), Williamson and Wright (1994), and Banerjee and Maskin (1996), we incorporate the recognizability of money explicitly into the model of Lagos and Wright (2005) to determine liquidity return of money as a medium of exchange and its effects on equilibrium relative price as well as welfare with different degrees of recognizability of money.

More specifically, we assume that there is a given stock of silver which is storable and perfectly divisible. The silver can be either carried into the decentralized market for a pairwise trade or invested in the world market for a given rate of return. In a pairwise meeting a seller can imperfectly recognize the quality of silver offered by a buyer in exchange for goods where recognizability is parameterized as a given probability with which a seller will identify the quality of silver. A buyer can costlessly produce counterfeit on the spot as in Lester et al. (2009) and hence, a seller who cannot discern the quality of silver will not accept it. The terms of trade in a pairwise meeting are determined by a buyer's take-it-or-leave-it offer.

We first show that, for a given recognizability of silver, its liquidity return is determined by the value of an additional unit of silver carried into the decentralized market for a bilateral trade. The comparison of equilibria for different degrees of recognizability shows that the liquidity return decreases with the recognizability of silver. As the recognizability problem becomes severe, liquidity return should increase in order to induce agents to carry silver to the decentralized market for a bilateral trade. This is consistent with the negative relationship between an asset's recognizability as a medium of exchange and its liquidity return as in Freeman (1985), Williamson and Wright (1994), Banerjee and Maskin (1996), Velde et al. (1999), Nosal and Wallace (2006), Li and Rocheteau (2008), Lester et al. (2009), Rocheteau (2009), and Lagos (2010).

The equilibrium real balance and quantity of output traded in exchange for silver in a pairwise trade increase with its recognizability. As the recognizability of silver increases, a seller who can discern the quality of silver is willing to trade more goods in exchange for silver. In particular, when the recognizability problem becomes severe, both the real balance of silver as a medium of exchange and the quantity traded decrease substantially. This is analogous to a currency shortage problem with an indivisible medium of exchange.[2] The declining real balance of silver as a medium of exchange is due to either a decrease in the nominal balance of silver or a decrease in the equilibrium price of silver. When the endowment of silver stock is sufficiently large, the price of silver is determined solely by its real return given in the world silver market and the silver "coins" carried into the decentralized market decrease with its recognizability problem. In an economy where the endowment of silver stock is scarce so that agents always carry all the silver into the decentralized market, the price of silver decreases with its recognizability problem.

Also, the variability of either silver demand or silver price affects relative prices across equilibria for different degrees of recognizability of silver. As long as silver coin is used as an imperfectly-recognizable medium of exchange, the equilibrium relative price varies with the recognizability of silver due to negative relationship between liquidity return of silver and its recognizability. This suggests that different degrees of recognizability of a commodity money across regions and over time in the medieval Europe must have affected relative prices and real allocations.

Finally, an increase in the recognizability of silver improves welfare through its effects on extensive margin in the informative single-coincidence meetings and intensive margin in the quantity traded, net of opportunity cost of holding nominal balance of silver for a bilateral trade. When the endowment of silver stock is sufficiently large, the positive effect of recognizability on quantity traded ends up offsetting its negative

[2] See, for instance, Wallace and Zhou (1997), Sargent and Velde (1999, 2002), Wallace (2003), and Kim and Lee (2010).

effect on the opportunity cost of silver holdings. When the endowment of silver stock is scarce so that the nominal balance of silver is determined by a given supply of silver, the recognizability of silver has no effect on the opportunity cost of silver holdings.

This implies the superiority of fiat money as both an almost perfectly-recognizable medium of exchange and a numeraire in the sense that relative price is hardly affected by the medium-of-exchange role of fiat money. This is quite novel in the sense that the superiority comes from the inherent physical characteristics of commodity money such as the imperfect recognizability of silver. It depends on neither waste of resources of using a commodity as a medium of exchange such as Wallace (1980), Sargent and Wallace (1983), Kiyotaki and Wright (1989), Banerjee and Maskin (1996), Burdett et al. (2001), Lagos and Rocheteau (2008) nor the indivisibility of commodity money as in Kim and Lee (2010).

The paper is organized as follows. Section 2 describes the model economy, followed by the equilibrium characterization in Section 3. Section 4 discusses the role of recognizability of a medium of exchange in the determination of liquidity return and relative price. Section 5 summarizes the paper with a few concluding remarks.

2. Model

Consider a small open economy in the Lagos (2010) framework. There is a unit measure set of infinitely-lived agents and time is indexed by $t \in \mathbb{N}$, the set of positive integers. In each period t, there are two markets, the decentralized and the centralized markets that open sequentially, and two perishable and perfectly divisible consumption goods, fruit and general goods. Fruit is endowed and traded in the decentralized market, while general goods are produced and traded in the centralized market.

There is only one storable object across periods, silver, which is perfectly divisible with total endowment of stock $S > 0$. The silver can be either carried over to the decentralized market for a pairwise trade or invested in the world silver market at a given return rate γ in terms of general goods.[3] Hence, in contrast to the related literature such as Geromichalos et al. (2007), Lester et al. (2009), and Lagos (2010), there is an opportunity cost of carrying silver into the decentralized market. Our modeling stems from somewhat different interpretation of silver from "Lucas tree"; that is, transforming silver into a silver "coin" takes away its investment opportunity as a real asset.[4]

The rest of the model is best described following the sequence of events within a period. At the opening of the decentralized market, a half of the agents are endowed with $\varepsilon^h = (1+\varepsilon)A$ units of fruit and the remaining half with $\varepsilon^l = (1-\varepsilon)A$ units of fruit where $\varepsilon \in (0,1)$ and $A > 0$. We call the former as type-h agents and the latter as type-l agents. The realization of the stochastic individual endowments is i.i.d. across periods and agents. An agent gets utility $v(q)$ from consuming q units of fruit where $v''(q) < 0 < v'(q)$, $v(0) = 0$ and $v'(\varepsilon^l)$ is sufficiently large.

After the realization of endowment shock, each agent is randomly matched with another one. Trades can occur only in the pairwise meetings between a type-h agent and a type-l agent. Hereinafter, we refer to this type of a pairwise meeting as *single-coincidence* meeting. Agents cannot make any binding intertemporal commitments and their trading histories are private. Hence, all but *quid pro quo* trades in the both decentralized and centralized markets are ruled out.

A type-l agent as a potential buyer can freely access the technology of producing silver alloy by inserting valueless object into a silver bar at any time, while a type-h agent as a potential seller receives a common-knowledge signal regarding the quality

[3] Among the related models of commodity money are King and Plosser (1986), Velde and Weber (2000), and Sussman and Zeira (2003).

[4] If we interpret a real asset (e.g., silver) as Lucas tree, it is natural to follow the assumption made in existing relevant literature. As we will see in section 4, this difference induces quite dissimilar implications for trade outcomes.

of silver held by the buyer. With probability $\theta \in (0,1)$, the signal is informative and the quality of silver is revealed to the seller; with probability $1-\theta$, the signal is uninformative.[5] Then as in Lester et al. (2009), sellers who cannot discern the quality of silver refuse to accept it because buyers can costlessly produce a counterfeit on the spot. In a single-coincidence meeting in which a seller gets an informative signal, the terms of trade are determined by Nash bargaining where a buyer has all the bargaining power.[6]

In the centralized market, a given return from silver investment is realized in terms of γ units of general goods. An agent gets utility $u(y)$ from consuming y units of general goods where $u''(y) < 0 < u'(y)$, $u(0) = 0$ and $u'(0) = \infty$. Also, all agents can produce one unit of general goods using one unit of labor which incurs one unit of disutility. Agents can trade general goods and silver in this competitive market. Assuming that the model economy starts from the centralized market in period 1, the lifetime expected utility of an agent is given by

$$\mathbb{E}_1 \sum_{t=1}^{\infty} \beta^{t-1}[v(q_t) + u(y_t) - h_t] \tag{1}$$

where $\beta \in (0,1)$ is the discount factor between the centralized market and the next decentralized market, and h_t is labor supply in the centralized market.

3. Equilibrium

To facilitate the description of an equilibrium for a given degree of recognizability of silver, we first introduce some notations. Let ϕ denote the unit price of silver in terms of general goods. Let $V(\mathbf{s})$ be the value function for an agent who enters the decentralized

[5] Lester et al. (2009) endogenize θ by introducing the information cost of verifying quality of assets.

[6] It is not difficult to check that all the following main results hold for the generalized Nash bargaining as long as the bargaining power of a buyer in a pairwise meeting is sufficiently large.

market with a portfolio $\mathbf{s} = (s^d, s^c)$ and $W(\mathbf{s})$ be the value function when she enters the centralized market. Here, s^d and s^c denote respectively the silver carried into the decentralized market for a pairwise trade and the silver stock invested in the world silver market. In what follows, we will formulate an equilibrium in the recursive manner and work backward from the centralized market to the decentralized market. The way of analysis and equilibrium characterization are similar to those in the relevant literature such as Geromichalos et al. (2007), Lester et al. (2009), and Lagos (2010).

3.1. Centralized Market

In the centralized market, agents produce, trade, and consume general goods, and trade silver. Hence, the problem for a representative agent entering the centralized market with a portfolio $\mathbf{s} = (s^d, s^c)$ is

$$W(\mathbf{s}) = \max_{(y,h,\hat{\mathbf{s}})} \{u(y) - h + \beta V(\hat{\mathbf{s}})\} \tag{2}$$

$$\text{s.t. } y + \phi\hat{\mathbf{s}} = h + (\phi + \gamma)s^c + \phi s^d \tag{3}$$

$$\mathbf{s} \geq 0,\ y \geq 0,\ h \in [0, \bar{h}]$$

where $\hat{\mathbf{s}} = (\hat{s}^d, \hat{s}^c)$ denotes the next-period value of $\mathbf{s} = (s^d, s^c)$ and $\bar{h}$ is an upper bound on h. We assume an interior solution for y and h.[7] Substituting h from the budget constraint (3), we have the following value function:

$$W(\mathbf{s}) = (\phi + \gamma)s^c + \phi s^d + \max_{(y,\hat{\mathbf{s}})} \{u(y) - y - \phi\hat{\mathbf{s}} + \beta V(\hat{\mathbf{s}})\}.$$

[7] An interior solution for y is guaranteed under the standard assumption on $u(y)$. It is also straightforward to characterize the conditions for $h \in (0, \bar{h})$ as in Lagos and Wright (2005).

The first order conditions with respect to y, $\hat{s}^d$, and $\hat{s}^c$ are as follows:

$$u'(y) = 1 \tag{4}$$

$$\phi \geq \beta \frac{\partial V(\hat{\mathbf{s}})}{\partial \hat{s}^d}, \ = \ \text{if } \hat{s}^d > 0 \tag{5}$$

$$\phi \geq \beta \frac{\partial V(\hat{\mathbf{s}})}{\partial \hat{s}^c}, \ = \ \text{if } \hat{s}^c > 0. \tag{6}$$

The envelope conditions are

$$\frac{\partial W(\mathbf{s})}{\partial s^d} = \phi \tag{7}$$

$$\frac{\partial W(\mathbf{s})}{\partial s^c} = \phi + \gamma. \tag{8}$$

The condition (4) implies that the consumption of general goods does not depend on the current portfolio holdings $\mathbf{s} = (s^d, s^c)$. The conditions (5) and (6) determine portfolio carried over the following period, $\hat{\mathbf{s}} = (\hat{s}^d, \hat{s}^c)$, which is also independent of $\mathbf{s} = (s^d, s^c)$. Further, as shown in Lagos and Wright (2005), a buyer's take-it-or-leave-it offer implies a unique solution of $\hat{\mathbf{s}} = (\hat{s}^d, \hat{s}^c)$, and hence portfolio distribution is degenerate at the beginning of each period. The envelope conditions (7) and (8) imply that $W(\mathbf{s})$ is linear.

3.2. Decentralized Market

For a given degree of recognizability of silver, the following two types of single-coincidence meetings arise in the decentralized market: one is a meeting where a seller receives an informative signal, and the other is a meeting where a seller receives an uninformative signal. Because a buyer can produce silver alloy as a counterfeit on the spot, a seller in the latter (uninformative) meeting will not accept silver bar and hence an exchange of fruit with silver does not occur.

In an informative single-coincidence meeting, a buyer hands over $p \in \mathbb{R}_+$ amount of silver to a seller in exchange for $q \in \mathbb{R}_+$ units of fruit. The terms of trade $(q,\, p)$ are determined by Nash bargaining in which a buyer has all the bargaining power. Let $v_b(q) \equiv v(\varepsilon^l+q)-v(\varepsilon^l)$ and $v_s(q) \equiv v(\varepsilon^h)-v(\varepsilon^h-q)$. Then, in a single-coincidence meeting where a buyer holds a portfolio $\mathbf{s} = (s^d, s^c)$ and a seller holds a portfolio $\tilde{\mathbf{s}} = (\tilde{s}^d, \tilde{s}^c)$, the terms of trade $(q,\, p)$ solve

$$\max_{(q\geq 0,\ p\leq s^d)} \left[v_b(q) + W(s^d - p, s^c) - W(\mathbf{s})\right]$$

subject to $v_s(q) \leq \left[W(\tilde{s}^d + p, \tilde{s}^c) - W(\tilde{\mathbf{s}})\right]$. By using the linear property of W with a tie-breaking rule by which a seller agrees to any offer that makes her indifferent between accepting and rejecting, we can simplify the above problem as follows:

$$\max_{(q\geq 0,\ p\leq s^d)} \left[v_b(q) - \phi p\right] \tag{9}$$

subject to $v_s(q) = \phi p$. The solution to (9) is

$$q = \begin{cases} \varepsilon A & \text{if } \phi s^d \geq v_s(\varepsilon A) \\ v_s^{-1}(\phi s^d) & \text{if } \phi s^d < v_s(\varepsilon A) \end{cases} \tag{10}$$

$$p = \begin{cases} v_s(\varepsilon A)/\phi & \text{if } \phi s^d \geq v_s(\varepsilon A) \\ s^d & \text{if } \phi s^d < v_s(\varepsilon A) \end{cases} . \tag{11}$$

Notice that εA represents the quantity of fruit traded that maximizes the buyer's surplus in a single-coincidence meeting, namely, $\varepsilon A = \arg\max\left[v_b(q) - v_s(q)\right]$. Hence, if a buyer has sufficiently large amount of the real balance of silver so that $\phi s^d \geq v_s(\varepsilon A)$, she gets εA units of fruit in exchange for the real balance $v_s(\varepsilon A)$. If $\phi s^d < v_s(\varepsilon A)$, however, a buyer spends all the real balance in exchange for q units of fruit which solves $v_s(q) = \phi s^d$. Notice also that, as in other variations of Lagos and Wright (2005), the terms of trade depend on the buyer's portfolio and not on the seller's portfolio.

Now, for a given degree of recognizability of silver $\theta \in (0,1)$, the bargaining solutions imply that the value function for a buyer (type-l agent) with a portfolio $\mathbf{s} = (s^d, s^c)$ satisfies

$$V_l(\mathbf{s}) = \frac{\theta}{2}\left\{v\left[\varepsilon^l + q(s^d)\right] + W(s^d - p(s^d), s^c)\right\} + \left(1 - \frac{\theta}{2}\right)\left[v(\varepsilon^l) + W(\mathbf{s})\right]. \qquad (12)$$

The expected utility of a buyer consists of the expected payoff from a single-coincidence meeting in which her trading partner gets an informative signal, and the expected payoff from all other cases where she just consumes her fruit endowment ε^l.

The value function for a seller (type-h agent) with a portfolio $\mathbf{s} = (s^d, s^c)$ satisfies

$$\begin{aligned} V_h(\mathbf{s}) \quad &= \quad \frac{\theta}{2}\int\left\{v\left[\varepsilon^h - q(\mathfrak{s}^d)\right] + W(s^d + p(\mathfrak{s}^d), s^c)\right\} dF(\mathfrak{s}^d, \mathfrak{s}^c) \\ &\quad + \left(1 - \frac{\theta}{2}\right)\left[v(\varepsilon^h) + W(\mathbf{s})\right] \end{aligned} \qquad (13)$$

where $F(\mathfrak{s}^d, \mathfrak{s}^c)$ is a portfolio distribution across agents. The expected utility of a seller consists of the expected payoff from a single-coincidence meeting with an informative signal and the expected payoff from all other cases where a seller just consumes her fruit endowment ε^h.

From (12) and (13) with the linearity of W and degenerate distribution $F(\mathfrak{s}^d, \mathfrak{s}^c)$, the expected utility of an agent entering the decentralized market with portfolio $\mathbf{s} = (s^d, s^c)$, before knowing the endowment shock, can be written as

$$V(\mathbf{s}) = \frac{\theta}{4}[v(\varepsilon^h - q) + v(\varepsilon^l + q)] + \frac{2-\theta}{4}[v(\varepsilon^h) + v(\varepsilon^l)] + W(\mathbf{s}). \qquad (14)$$

Now, from (14), the first derivatives of V for $q \in [0, \varepsilon A]$ become[8]:

$$\frac{\partial V(\mathbf{s})}{\partial s^d} = \phi \left\{ 1 + \frac{\theta}{4} \left[\frac{v_b'(q)}{v_s'(q)} - 1 \right] \right\} \tag{15}$$

$$\frac{\partial V(\mathbf{s})}{\partial s^c} = \phi \left(1 + \frac{\gamma}{\phi} \right). \tag{16}$$

By substituting (15) and (16) into (5) and (6), respectively, we have the conditions determining portfolio demands $\hat{\mathbf{s}} = (\hat{s}^d, \hat{s}^c)$:

$$\phi \geq \beta \hat{\phi} \left\{ 1 + \frac{\theta}{4} \left[\frac{v_b'(q)}{v_s'(q)} - 1 \right] \right\} = \text{if } \hat{s}^d > 0 \tag{17}$$

$$\phi \geq \beta \hat{\phi} \left(1 + \frac{\gamma}{\hat{\phi}} \right) \qquad\qquad = \text{if } \hat{s}^c > 0 \tag{18}$$

where $\{1 + (\theta/4)[(v_b'(q)/v_s'(q)) - 1]\}$ represents the expected marginal benefit from the liquidity of silver carried into the decentralized market for a bilateral trade, whereas $(1 + \gamma/\hat{\phi})$ is the real return from the silver stock invested in the world market.

For a given degree of recognizability θ, an equilibrium can be defined as follows.

Definition 1 *For a given* θ *and an initial portfolio* $\mathbf{s}_1 = (s_1^d, s_1^c)$*, an equilibrium is the sequence of allocations, price, and terms of trade such that (i) for a given* ϕ_t*, the allocations* $\{y_t, h_t, \mathbf{s}_{t+1}\}$ *for each* $t \in \mathbb{N}$ *are solutions to the optimization problem of a representative agent in the centralized market as summarized by (3), (4), (17) and (18); (ii) the bilateral terms of trade* $\{q_t, p_t\}$ *for each* $t > 1$ *are determined by a buyer's take-it-or-leave-it offer as summarized by (10) and (11); and (iii) the price* ϕ_t *clears the centralized market:* $s_{t+1}^d + s_{t+1}^c = S$ *for all* $t \in \mathbb{N}$*.*

[8]The slope of V with respect to s^d at $s^d = v_s(\varepsilon A)/\phi$ which corresponds to $q = \varepsilon A$ is the limiting case from below.

4. Recognizability and Relative Price

We now compare the properties of equilibria for different degrees of recognizability of silver. This comparison should provide some implications of the variable recognizability for different relative prices across regions and over time in the medieval Europe as noted by Jevons (1875).

We first show that silver of a given recognizability is essential as a medium of exchange ($\hat{s}^d > 0$), and quantity of fruit traded in the decentralized market is strictly less than the first-best one ($q < \varepsilon A$). Notice that if there is no opportunity cost of holding silver for a pairwise trade as in Geromichalos et al. (2007), Lester et al. (2009), and Lagos (2010), then we have $q \leq \varepsilon A$.[9]

Lemma 1 *In an equilibrium for a given θ, $\hat{s}^d > 0$. Further, the quantity of fruit traded in the decentralized market is strictly less than εA.*

Proof. See Appendix. ■

For a given θ, let $\mathcal{L}(q_\theta) \equiv [v_b'(q_\theta) - v_s'(q_\theta)]/v_s'(q_\theta)$ denote the liquidity return which captures the value of an additional unit of silver carried into the decentralized market for a bilateral trade. Now, consider two different equilibria for θ_1 and θ_2, respectively, where $\theta_1 > \theta_2$.

Proposition 1 *$\mathcal{L}(q^*_{\theta_1}) < \mathcal{L}(q^*_{\theta_2})$ and $q^*_{\theta_1} > q^*_{\theta_2}$ for $\theta_1 > \theta_2$ where q^*_θ is a solution to $(1-\beta)/\beta = (\theta/4)\mathcal{L}(q_\theta)$.*

Proof. See Appendix. ■

[9] As we will see in section 4.1, no opportunity cost of holding silver in an economy where the stock of silver is sufficiently large implies $q = \varepsilon A$ regardless of θ. Then, Proposition 1∼2 and Corollary 1∼2 do not hold in general. This is the case for no-fiat-money version of Lester et al. (2009) in which fiat money is introduced to discuss the monetary policy.

Proposition 1 means that the liquidity return of silver, $\mathcal{L}(q_\theta)$, is negatively related to its recognizability, while the quantity of fruit traded in a bilateral meeting is positively related to the recognizability of silver. Intuitively, as the recognizability problem becomes more severe (i.e., as θ decreases), liquidity return should increase in order to induce silver holdings carried over to the decentralized market as a medium of exchange. Further, as the recognizability of silver increases, a seller in an informative single-coincidence meeting will find it worthwhile to trade more fruit in exchange for silver. This is consistent with the negative relationship between an asset's recognizability as a medium of exchange and its liquidity return as in Freeman (1985), Williamson and Wright (1994), Banerjee and Maskin (1996), Velde et al. (1999), Nosal and Wallace (2006), Li and Rocheteau (2008), Lester et al. (2009), Rocheteau (2009), and Lagos (2010). This also implies that the equilibrium real balance carried into the decentralized market for a bilateral trade increases with the recognizability of silver.

Corollary 1 $v_s(q^*_{\theta_1}) > v_s(q^*_{\theta_2})$ *for* $\theta_1 > \theta_2$.

Proof. See Appendix. ■

This, together with Proposition 1, implies that when the recognizability problem becomes severe, both the real balance of silver as a medium of exchange and the quantity traded decrease substantially. This is analogous to a currency shortage problem with an indivisible medium of exchange as in Wallace and Zhou (1997), Sargent and Velde (1999, 2002), Wallace (2003), and Kim and Lee (2010).

Noting that the real balance of silver is given by $v_s(q) = \phi s^d$, a decrease in the real balance with recognizability problem is due to either a decrease in the nominal balance of silver (s^d) carried into the decentralized market or a decrease in the equilibrium price of silver (ϕ). In general, this will depend on the endowment of silver stock.

4.1. High Endowment of Silver Stock

When the endowment of silver stock is sufficiently large so that agents not only carry silver into the decentralized market but also invest a positive amount in the world silver market, the equilibrium price of silver is constant regardless of θ. Hence, the decline in the real balance $v_s(q) = \phi s^d$ with the recognizability problem will come from a decrease in the nominal balance of silver (s^d).

Specifically, when $\hat{s}^d > 0$ and $\hat{s}^c > 0$, (17) and (18) hold with equality which yield a constant equilibrium price of silver given by $\phi = \beta\gamma/(1-\beta)$. Now let $\bar{S}_\theta \equiv [v_s(q_\theta^*)(1-\beta)]/\beta\gamma$, stock of silver required to obtain q_θ^* at the silver price $\phi = \beta\gamma/(1-\beta)$. Then agents holding silver greater than $\bar{S}_\theta$ would never carry silver into the decentralized market more than $\bar{S}_\theta$. This critical level $\bar{S}_\theta$ becomes larger as θ increases because q_θ^* increases with θ from Proposition 1. This can be formalized by the following set of results.

Lemma 2 *In an equilibrium for a given* θ*, if* $S > \bar{S}_\theta \equiv [v_s(q_\theta^*)(1-\beta)]/\beta\gamma$*, then* $\phi = \beta\gamma/(1-\beta)$*,* $\hat{s}^d = \bar{S}_\theta$ *and* $\hat{s}^c = S - \bar{S}_\theta$*.*

Proof. See Appendix. ■

Intuitively, if the given supply of silver is sufficient, the equilibrium silver price is determined solely by its real return given in the world silver market. Hence, as shown below, nominal balance of silver increases with the recognizability of silver as a medium of exchange.

Proposition 2 *Suppose* $S = \bar{S}_{\theta_1} = [v_s(q_{\theta_1}^*)(1-\beta)]/\beta\gamma$ *and* $\theta_1 > \theta_2 > \theta_3$*. Then* $\hat{s}_{\theta_1}^d > \hat{s}_{\theta_2}^d > \hat{s}_{\theta_3}^d$*.*

Proof. See Appendix. ■

That is, if θ decreases as the recognizability problem becomes severe, the nominal balance of silver decreases. With the constant equilibrium silver price irrespective of the recognizability problem, therefore, the real balance of silver carried into the decentralized market also decreases.

This implies that equilibrium relative price varies with the recognizability of silver. From Lemma 2, the price of general goods in terms of silver is $1/\phi$ regardless of θ. And the price of fruit in terms of silver with $\theta = \theta_1$ is $s^d_{\theta_1}/q^*_{\theta_1} = v_s(q^*_{\theta_1})/(\phi q^*_{\theta_1})$, while that with $\theta = \theta_2$ is $s^d_{\theta_2}/q^*_{\theta_2} = v_s(q^*_{\theta_2})/(\phi q^*_{\theta_2})$. Let $\mathbf{P}_\theta$ denote the relative price of fruit to general goods in terms of silver for a given θ. Then $\mathbf{P}_\theta = [v_s(q^*_\theta)/(\phi q^*_\theta)]/(1/\phi) = [v_s(q^*_\theta)/q^*_\theta]$.

Corollary 2 $\mathbf{P}_{\theta_1} > \mathbf{P}_{\theta_2}$ *for* $\theta_1 > \theta_2$.

Proof. See Appendix. ■

Corollary 2 says that if silver as an imperfectly-recognizable medium of exchange is adopted as a numeraire, the equilibrium price of fruit relative to general goods will change with the recognizability of silver. That is, $\mathbf{P}_\theta$ can be interpreted as the seller's willingness to transfer fruit in the equilibrium, which on average increases with the recognizability of silver. This suggests that different degrees of recognizability of a commodity money across regions and over time in the medieval Europe must have affected relative prices and real allocations.

4.2. Low Endowment of Silver Stock

When the endowment of silver stock is relatively scarce so that it is all carried into the decentralized market for a pairwise trade (i.e., $\hat{s}^c = 0$), the nominal balance of silver as a medium of exchange will be equal to the given supply of silver. Hence, the decline in the real balance will come from a decrease in the equilibrium price of silver.

Specifically, when $\hat{s}^d > 0$ and $\hat{s}^c = 0$, substituting $\phi = v_s(q^*_\theta)/S$ from the market-clearing condition $v_s(q^*_\theta) = \phi \hat{s}^d = \phi S$ in (18) yields $S \leq [v_s(q^*_\theta)(1-\beta)]/\beta\gamma \equiv \bar{S}_\theta$. This can be summarized by the following lemma:

Lemma 3 *In an equilibrium for a given* θ*, if* $S \leq \bar{S}_\theta$*, then* $\hat{s}^d = S$*,* $\hat{s}^c = 0$*, and* $\phi = v_s(q^*_\theta)/S$*.*

Proof. See Appendix. ■

Intuitively, for a given θ, when the silver stock is relatively scarce so that $S \leq \bar{S}_\theta$, agents carry all the silver into the decentralized market for a bilateral trade. Since the real balance of silver increases with its recognizability by Corollary 1, the equilibrium silver price will also increase with recognizability.

Proposition 3 *Suppose* $S = \bar{S}_{\theta_2} = [v_s(q^*_{\theta_2})(1-\beta)]/\beta\gamma$ *and* $\theta_2 < \theta_1$*. Then* $\phi_{\theta_2} < \phi_{\theta_1}$*.*

Proof. See Appendix. ■

Finally, as in the case of a relatively high endowment of silver stock, equilibrium relative price varies with θ. For a given θ, the price of general goods in terms of silver is $(1/\phi_\theta) = S/v_s(q^*_\theta)$, while the price of fruit in terms of silver is $s^d_\theta/q^*_\theta = S/q^*_\theta$. Therefore, the relative price of fruit to general goods in terms of silver with $\theta = \theta_1$ and $\theta = \theta_2$ becomes, respectively, $\mathbf{P}_{\theta_1} = [S/q^*_{\theta_1}]/[S/v_s(q^*_{\theta_1})] = [v_s(q^*_{\theta_1})/q^*_{\theta_1}]$ and $\mathbf{P}_{\theta_2} = [S/q^*_{\theta_2}]/[S/v_s(q^*_{\theta_2})] = [v_s(q^*_{\theta_2})/q^*_{\theta_2}]$, which are identical to those in Corollary 2.

4.3. Superiority of fiat money

In an equilibrium with a unique solution for $\hat{\mathbf{s}} = (\hat{s}^d, \hat{s}^c) = (s^d, s^c) = \mathbf{s}$, the expected utility of a representative agent (14) for a given θ can be rewritten as

$$V(\mathbf{s};\theta) = \frac{\beta}{(1-\beta)} \left\{ \frac{\theta}{4} [v_b(q) - v_s(q)] + \gamma(S - s^d) + \kappa \right\} + \bar{u}.$$

Here, noting that the model economy starts from the centralized market in period 1, $\bar{u} \equiv u(y^*) - y^*$ is the first period utility and $\kappa \equiv (1/2)\left[v(\varepsilon^h) + v(\varepsilon^l)\right] + \bar{u}$ is constant regardless of the recognizability of silver.

Then, the recognizability of silver $\theta \in (0,1)$ affects V as follows:

$$\frac{\partial V}{\partial \theta} = \frac{\beta}{(1-\beta)}\left\{\frac{1}{4}\left[v_b(q) - v_s(q)\right] + \frac{\theta}{4}\left[v_b'(q) - v_s'(q)\right]\frac{\partial q}{\partial \theta} - \gamma\frac{\partial s^d}{\partial \theta}\right\}. \tag{19}$$

The right-hand side of (19) shows that a change in θ affects welfare through its respective effect on "extensive margin" in the informative single-coincidence meetings, "intensive margin" in the quantity traded, and opportunity cost of holding silver for a bilateral trade in the decentralized market.

Alternatively, we can interpret the first term in the brace of (19) as the effect of recognizability of silver as a medium of exchange in the sense that recognizability facilitates trades across anonymous agents. This effect increases with θ because $v_b(q) - v_s(q) = [v(\varepsilon^l + q) + v(\varepsilon^h - q)] - [v(\varepsilon^l) + v(\varepsilon^h)] > 0$. The second term together with the third term can be interpreted as the effect of recognizability of silver as a numeraire in the sense that recognizability affects relative price and real allocations. Notice that the second term is positive because $v_b'(q) - v_s'(q) = v'(\varepsilon^l + q) - v'(\varepsilon^h - q) > 0$ for $q \in (0, \varepsilon A)$ by the concavity of v and $\partial q/\partial\theta > 0$ by Proposition 1. However, the overall effect as a numeraire depends on the endowment of silver stock in general.

When the endowment of silver stock is sufficiently large so that $S > \bar{S}_\theta = [v_s(q_\theta^*)(1-\beta)]/\beta\gamma$, the nominal balance of silver increases with recognizability ($\partial s^d/\partial\theta > 0$) from Proposition 2. Since, by Lemma 2, (17) and (18) hold with equality and ϕ is constant regardless of θ, we have $(\theta/4)\{[v_b'(q) - v_s'(q)]/v_s'(q)\} = (\theta/4)\mathcal{L}(q_\theta) = (\gamma/\phi)$ and $(\partial q/\partial\theta) = (\partial q/\partial s^d)(\partial s^d/\partial\theta)$ where $\partial q/\partial s^d = [\phi/v_s'(q)]$. Therefore, the expected utility of a representative agent (19) can be shown to increase with θ as follows:

$$\begin{aligned}\frac{\partial V}{\partial \theta} &= \frac{\beta}{(1-\beta)}\left\{\frac{1}{4}\left[v_b(q)-v_s(q)\right]+\frac{\theta\phi}{4}\mathcal{L}(q_\theta)\frac{\partial s^d}{\partial \theta}-\gamma\frac{\partial s^d}{\partial \theta}\right\} \\ &= \frac{\beta}{(1-\beta)}\left\{\frac{1}{4}\left[v_b(q)-v_s(q)\right]+\gamma\frac{\partial s^d}{\partial \theta}-\gamma\frac{\partial s^d}{\partial \theta}\right\}>0\end{aligned} \tag{20}$$

where the intensive-margin effect of recognizability ends up offsetting its effect on the opportunity cost of silver holdings. Hence, higher welfare implied by higher recognizability eventually comes from the enhanced role of silver as a medium of exchange.

When the endowment of silver stock is relatively scarce so that $S \leq \bar{S}_\theta$, the equilibrium price of silver increases with recognizability ($\partial\phi/\partial\theta > 0$) from Proposition 3. Since (17) holds with equality and $\partial s^d/\partial\theta = 0$ due to $\hat{s}^d = S > 0$ from Lemma 3, we have $[(1-\beta)/\beta] = (\theta/4)\{[v_b'(q)-v_s'(q)]/v_s'(q)\}$ and $(\partial q/\partial\theta) = (\partial q/\partial\phi)(\partial\phi/\partial\theta)$ where $\partial q/\partial\phi = [S/v_s'(q)]$ from $\phi = v_s(q)/S$. Also, $\partial s^d/\partial\theta = 0$ implies that the recognizability of silver has no effect on the opportunity cost of silver holdings. Therefore, the expected utility of a representative agent (19) increases with θ:

$$\frac{\partial V}{\partial \theta} = \frac{\beta}{(1-\beta)}\left\{\frac{1}{4}\left[v_b(q)-v_s(q)\right]+\frac{(1-\beta)S}{\beta}\frac{\partial\phi}{\partial\theta}\right\}>0 \tag{21}$$

where the right-hand side represents respectively positive effects on the informative single-coincidence meetings and the quantity traded through silver price. That is, higher welfare implied by higher recognizability of silver comes from the roles of silver as both a medium of exchange and numeraire.

In sum, if silver as an imperfectly-recognizable medium of exchange is adopted as a numeraire, welfare is improved as the recognizability of silver increases. A higher θ enhances the role of silver as a medium of exchange when the endowment of silver stock is sufficiently large, while it improves the welfare via both channels of medium of exchange and numeraire when the endowment of silver stock is scarce.

These results essentially suggest the superiority of fiat money as an almost perfectly recognizable medium of exchange ($\theta \approx 1$).[10] This is quite novel in the sense that superiority of fiat money comes from the physical characteristics of money such as the imperfect recognizability of silver. It depends on neither the waste of resources of using a commodity or an asset as a medium of exchange such as Wallace (1980), Sargent and Wallace (1983), Kiyotaki and Wright (1989), Banerjee and Maskin (1996), Burdett et al. (2001), and Lagos and Rocheteau (2008) nor the indivisibility of a commodity money as in Kim and Lee (2010).

5. Concluding Remarks

In this paper, we have explored the fundamental demerits of commodity money (e.g., silver coin) as an imperfectly-recognizable medium of exchange and its consequence on relative prices. When the recognizability problem becomes severe, both the real balance of silver as a medium of exchange and the quantity traded decrease substantially. The declining real balance of silver is due to either a decrease in the nominal balance of silver when silver is abundant or a decrease in the price of silver when silver is scarce. Moreover, as long as silver coin is used as an imperfectly-recognizable medium of exchange, the equilibrium relative price varies with the recognizability of silver due to negative relationship between liquidity return of silver and its recognizability.

An increase in the recognizability of silver improves welfare through its positive effects on the informative single-coincidence meetings, the quantity traded, net of the opportunity cost of holding silver for a bilateral trade in the decentralized market. This implies the superiority of fiat money which is almost perfectly recognizable. This is

[10] U.S. Department of the Treasury reported that "the likelihood that a counterfeit note will be found in a batch of otherwise genuine overseas notes, is generally quite small, on the order of 1 or 2 counterfeits in 10,000 notes, about the same ratio as is found inside the United States."(www.ustreas.gov/press/releases/reports/2003.pdf.)

quite novel in the sense that the welfare-enhancing role of fiat money comes from its recognizability in addition to its divisibility and the saving of resources associated with using a commodity as a medium of exchange.

These results are robust to a change in silver supply over time. For instance, when the silver supply varies over time according to $\hat{S} = \mu S$, the only differences are the equation characterizing q, $[(\mu - \beta)/\beta] = \Phi(q;\theta)$, and the equilibrium price of silver in Lemma 2, $\phi = \beta\gamma[\mu/(\mu - \beta)]$. Finally, it should be noted that our main results come from the comparison across equilibria for different degrees of recognizability of commodity money. We do not deal with the transition from one equilibrium to another, which could be undertaken with an approximation of dynamic equilibrium paths.

6. Appendix: Proofs

Proof of Lemma 1: Since $v'(\varepsilon^l)$ is sufficiently large, $\beta\hat{\phi}\{1+(\theta/4)[(v'(\varepsilon^l)/v'(\varepsilon^h))-1]\} > \beta(\hat{\phi}+\gamma)$. This inequality implies $\hat{s}^d > 0$. Since $\hat{s}^d > 0$, (17) holds with equality and hence q should solve $(1-\beta)/\beta = \Phi(q;\theta)$ where $\Phi(q;\theta) = (\theta/4)[v_b'(q) - v_s'(q)]/v_s'(q)$ and $\phi = \hat{\phi}$ with a fixed (S, θ, γ). Since for $q \in [0, \varepsilon A]$, $v_b'(q) = v'(\varepsilon^l + q) > 0$, $v_b''(q) = v''(\varepsilon^l + q) < 0$, $v_s'(q) = v'(\varepsilon^h - q) > 0$, and $v_s''(q) = -v''(\varepsilon^h - q) > 0$,

$$\Phi'(q;\theta) = \frac{\theta}{4}\left\{\frac{v_b''(q)v_s'(q) - v_s''(q)v_b'(q)}{[v_s'(q)]^2}\right\} < 0.$$

Further, $\Phi(0;\theta) = (\theta/4)\{[v'(\varepsilon^l)/v'(\varepsilon^h)] - 1\} > (1-\beta)/\beta$ because $v'(\varepsilon^l)$ is sufficiently large, and $\Phi(\varepsilon A;\theta) = (\theta/4)\{[v'(\varepsilon^l + \varepsilon A)/v'(\varepsilon^h - \varepsilon A)] - 1\} < (1-\beta)/\beta$ because $v'(\varepsilon^l + \varepsilon A) = v'(A) = v'(\varepsilon^h - \varepsilon A)$. Therefore, we have $\Phi(0;\theta) > (1-\beta)/\beta > \Phi(\varepsilon A;\theta)$, and hence q which solves $(1-\beta)/\beta = \Phi(q;\theta)$ is strictly less than εA.

Proof of Proposition 1: Notice that $q^*_{\theta_1}$ solves $(1-\beta)/\beta = (\theta_1/4)[v_b'(q) -$

$v_s'(q)]/v_s'(q)$, while $q_{\theta_2}^*$ solves $(1-\beta)/\beta = (\theta_2/4)[v_b'(q) - v_s'(q)]/v_s'(q)$. Since the left hand side of both equations has the same constant, $(\theta_1/4)[v_b'(q_{\theta_1}^*) - v_s'(q_{\theta_1}^*)]/v_s'(q_{\theta_1}^*) = (\theta_2/4)[v_b'(q_{\theta_2}^*) - v_s'(q_{\theta_2}^*)]/v_s'(q_{\theta_2}^*)$. Then $\theta_1 > \theta_2$ implies $\mathcal{L}(q_{\theta_1}^*) \equiv [v_b'(q_{\theta_1}^*) - v_s'(q_{\theta_1}^*)]/v_s'(q_{\theta_1}^*) < [v_b'(q_{\theta_2}^*) - v_s'(q_{\theta_2}^*)]/v_s'(q_{\theta_2}^*) \equiv \mathcal{L}(q_{\theta_2}^*)$. Finally, $q_{\theta_1}^* > q_{\theta_2}^*$ for $\theta_1 > \theta_2$ is obtained from $\mathcal{L}'(q_\theta) = [v_b''(q_\theta)v_s'(q_\theta) - v_s''(q_\theta)v_b'(q_\theta)]/[v_s'(q_\theta)]^2 < 0$.

Proof of Corollary 1: From Proposition 1 and Lemma 1, we have $q_{\theta_2}^* < q_{\theta_1}^* < \varepsilon A$. Then, $v_s'(q) = v'(\varepsilon^h - q) > 0$ for $q \in (0, \varepsilon A)$ implies the result.

Proof of Lemma 2: Since $\hat{s}^d > 0$ by Lemma 1, (17) holds with equality. Hence, $[(1-\beta)/\beta] = (\theta/4)[v_b'(q_\theta^*) - v_s'(q_\theta^*)]/v_s'(q_\theta^*)$. Now, suppose $\hat{s}^c = 0$ and $\hat{s}^d = S$. Since $S > \bar{S}_\theta = [v_s(q_\theta^*)(1-\beta)]/\beta\gamma$ by assumption, we have $[\beta\gamma/(1-\beta)]S > v_s(q_\theta^*)$. This inequality and $\phi > \beta\gamma/(1-\beta)$ with $\hat{s}^c = 0$ imply that $\hat{s}^d = S$ is not optimal. Therefore, (18) should hold with equality, which implies $\phi = \beta\gamma/(1-\beta)$. Finally, from $s^d\phi = v_s(q_\theta^*)$, we have $\hat{s}^d = [v_s(q_\theta^*)(1-\beta)]/\beta\gamma = \bar{S}_\theta$ and $\hat{s}^c = S - \bar{S}_\theta > 0$.

Proof of Proposition 2: Since $q_{\theta_3}^* < q_{\theta_2}^* < q_{\theta_1}^* < \varepsilon A$ by Proposition 1 and Lemma 1, and $v_s'(q) = v'(\varepsilon^h - q) > 0$ for $q \in (0, \varepsilon A)$, we have $S = \bar{S}_{\theta_1} = [v_s(q_{\theta_1}^*)(1-\beta)]/\beta\gamma > [v_s(q_{\theta_2}^*)(1-\beta)]/\beta\gamma = \bar{S}_{\theta_2} > [v_s(q_{\theta_3}^*)(1-\beta)]/\beta\gamma = \bar{S}_{\theta_3}$. Then, Lemma 2 immediately implies the result.

Proof of Corollary 2: Notice first the relative price of fruit to general goods in terms of silver with $\theta = \theta_1$ is $\mathbf{P}_{\theta_1} = (s_{\theta_1}^d/q_{\theta_1}^*)/(1/\phi) = [v_s(q_{\theta_1}^*)/\phi q_{\theta_1}^*]/(1/\phi) = v_s(q_{\theta_1}^*)/q_{\theta_1}^*$, while that with $\theta = \theta_2$ is $\mathbf{P}_{\theta_2} = (s_{\theta_2}^d/q_{\theta_2}^*)/(1/\phi) = [v_s(q_{\theta_2}^*)/\phi q_{\theta_2}^*]/(1/\phi) = v_s(q_{\theta_2}^*)/q_{\theta_2}^*$. Then $q_{\theta_2}^* < q_{\theta_1}^* < \varepsilon A$, $v_s(q_{\theta_1}^*) > v_s(q_{\theta_2}^*)$ by Corollary 1, and $v_s''(q) = -v''(\varepsilon^h - q) > 0$ for $q \in (0, \varepsilon A)$ yield $\mathbf{P}_{\theta_1} > \mathbf{P}_{\theta_2}$.

Proof of Lemma 3: Suppose $\hat{s}^c > 0$. Then (18) holds with equality and hence $\phi = \beta\gamma/(1-\beta)$ and $\bar{S}_\theta = [v_s(q_\theta^*)(1-\beta)]/\beta\gamma = v_s(q_\theta^*)/\phi$. Since $S \leq \bar{S}_\theta$, $\phi\hat{s}^d =$

$\phi(S - \hat{s}^c) < \phi\bar{S}_\theta = v_s(q^*_\theta)$ which is a contradiction to the market-clearing condition. Therefore, $(\hat{s}^d, \hat{s}^c) = (S, 0)$. From $v_s(q) = \phi s^d$, we have $\phi = v_s(q^*_\theta)/S$.

Proof of Proposition 3: Since $q^*_{\theta_2} < q^*_{\theta_1} < \varepsilon A$ and $v'_s(q) = v'(\varepsilon^h - q) > 0$ for $q \in (0, \varepsilon A)$, we have $S = \bar{S}_{\theta_2} = [v_s(q^*_{\theta_2})(1-\beta)]/\beta\gamma < [v_s(q^*_{\theta_1})(1-\beta)]/\beta\gamma = \bar{S}_{\theta_1}$. Therefore, $\hat{s}^d_{\theta_1} = \hat{s}^d_{\theta_2} = S$ by Lemma 3. Now $\phi_{\theta_1} > \phi_{\theta_2}$ for $\theta_1 > \theta_2$ is followed by $v_s(q^*_{\theta_1}) = \phi_{\theta_1} S$ and $v_s(q^*_{\theta_2}) = \phi_{\theta_2} S$ with $v_s(q^*_{\theta_1}) > v_s(q^*_{\theta_2})$ from Corollary 1.

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