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Contractual Matching of Finite Economy: Limits of Decentralization

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Abstract

This paper considers incentive-constrained efficient contractual matching of individuals in the presence of moral hazard. By considering an economy with finite number of individuals, this paper shows what were assumed away in continuum models. Contract arbitrageurs specializing in writing (randomized) contracts, insurers, a market for lotteries on contracts, Lindahl-like prices for the lotteries, and a public randomization device are required for the incentive-constrained efficiency. The public randomization device needs to coordinate the matchings of individuals, and each team's contract is dependent on other firms' outputs. Insurers are committed to avoid cream-skimming, and essentially cross-subsidize across teams. This unrealistic role of the insurers shows the impossibility of decentralization in a reasonable sense (even when economics agents are price takers). Applications in Labor economics and Theory of merger are discussed. Technical contribution is to formulate a finite economy by linear programming.

Keywords General equilibrium, Team, Finite economy, Duality of linear programming

JEL Classification C68, D50, D53, D86

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1 Introduction

This paper is a finite economy analogue of Song (2011) that considers incentive-constrained efficient contractual matching of continuum of individuals in the presence of moral hazard. By considering the economy of finite number of individuals, this paper shows what were assumed away in the continuum models. Additional to the requirement in Song (2011), (i) an insurer committed to taking care of the risk in the economy (i.e., committed to avoiding cream-skimming) is required, (ii) the role of a public randomization device to match individuals become more involved, and (iii) the contract of a team becomes dependent on other firms' outputs.

A finite number of individuals form a team/firm in order to produce stochastic output. Moral hazard problems exist within teams since the individuals' efforts are not verifiable. I characterize conditions for incentive-constrained efficiency by exploiting the duality in linear programming. Once a planner's problem is set as a linear programming problem, economic interpretation of the duality in linear programming characterizes an idealized market environment (institutions, technologies such as public randomization devices, and markets) for achieving efficiency.

A market for lotteries on matchings (with Lindahl-like prices), public randomization devices, contract arbitrageurs, and insurers are necessary for incentive-constrained efficiency.

Randomized matching improves efficiency by convexifying non-convex domain due to the indivisible structure of teams. For example, Rogerson (1988) showed that randomized assignment of labor can improve efficiency due to the indivisibility nature of labor. In order to implement randomized matching, a lottery market for matching and a public randomization device are required (for survey, see Prescott and Shell (2002)). Perfectly competitive contract arbitrageurs maximize profits by inventing exclusive randomized contracts that (i) depend on the states of all the teams formed in the economy and (ii) prohibit individuals from trading in the contingent claims market. Contract arbitrageurs and individuals trade lotteries on the randomized matching (I use "matching" and "contract" interchangeably). A contract arbitrageur could be active or inactive depending on whether his contract is chosen by the public randomization device or not.

Even though contract arbitrageurs are risk-neutral, they have to deliver non-money commodities promised by the contracts. Therefore, they purchase insurance to ensure the delivery of non-money commodities; hence, it is shown that access to the insurance market by a team promotes efficiency, while the access to the market by an individual is not desirable. The lastly required are perfectly competitive insurers who maximize profits and are committed to avoid cream-skimming. Markets for lotteries on matchings and insurance take place before the realization of matching. Contract arbitrageurs and insurers end up with zero profit since they are perfectly competitive.

The insurer's commitment to avoid cream-skimming means that the insurer cross-subsidizes; i.e., the insurance premium and the value of the expected payments do not coincide. However, contract arbitrageurs get zero expected profit since the subsidies are channeled to individuals. A combined Welfare theorem is proved, which states that the entire utility frontier could be decentralized; hence, there is no distinction between the first and the second Welfare theorems. Mathematically, I illustrate that the combined Welfare theorem is from two characteristics of the model: the matching structure and teams are essentially public goods, and the prices for contracts are non-linear. The uncertainty structure in the finite economy is endogenously determined by the matching structure of the economy; hence, my choice directly affects others' decisions. (On the other hand, the law of large number in a continuum model makes it possible for the consequence of one individual's action to be contained within a team, e.g., Song (2011) and Rahman (2005)). The prices of lotteries on a contract specifying effort and consumption cannot be linearly decomposed into prices of consumption and effort. (Note that the linear decomposition was possible in the continuum version of the model, see Song (2011)). These public good characteristics of matching and the non-linear pricing yield a large degree of freedom in choosing prices; therefore the entire utility frontier can be decentralized in the finite model.

The randomization device implementing randomized matching (or contract) needs to accommodate and signal enormous amounts of information in the sense that all the teams' randomized contracts are correlated. Insurers are committed not to engage in cream-skimming. Insurers committed to pooling the risks in this way are hardly realistic. Also, insurers need to know all the details of the economy to calculate the right prices. In other words, the current paper shows stronger result than that of Garatt (1995); not only a public randomization device is required for efficient decentralization, but also the unrealistic insurers are required for efficient decentralization. In short, the Welfare theorem in the current paper shows, in fact, the failure of the Welfare theorem in practice. Ellickson, Grodal, Scotchmer, and Zame (2001) show that in a finite team economy where lotteries are not used, exact decentralization is impossible by prices for private good and club membership. My finding suggests that even when lotteries are allowed, decentralization is impossible without these unrealistic players in the presence of the moral hazard problem.

The technical contribution is to formulate a team model by linear programming with finite number of individuals. The current paper, Rahman (2005), and Song (2011) are extensive exercise of the application of linear programming to general equilibrium theory. Once the planner's problem is set as a linear programming problem, the fundamental theorem of linear programming gives a direct interpretation of the Welfare theorems, and the definition of equilibria is obtained by investigating the dual linear programming problem.

The related team literatures can be classified into three categories. Firstly, Cole and Prescott (1997) consider a team economy of continuum of individuals with lottery trade. Song (2011) considers a similar economy, but extends their discussion by adding moral hazard problem into the model and by the usage of linear programming technique. The current paper is a finite economy analogue to Song (2011). Secondly, Ellickson, Grodal, Scotchmer, Zame (1999, 2001) consider team economies of continuum and finite number of individuals without lotteries. Zame (2007) extends the continuum model by incorporating adverse selection and moral hazard problem. Thirdly, Makowski and Ostroy (2003) and Rahman (2005) consider economies of continuum of individuals with or without moral hazard problem under the assumption of quasi-linear utilities. Other related team literatures include Bennardo and

Chiappori (2003), Prescott and Townsend (2000), and Jerez (2003, 2005).

In Section 2, I present a model of the finite economy. Section 3 is summary and conclusion. All the proofs are in appendices.

2 The model

The linear programming formulation of the planner's problem is proposed in Section 2.1. In Section 2.2 and 2.3, I derive the market environment required for the decentralization by the dual linear programming problem. Discussion on the Welfare theorem is in Section 2.4.

2.1 Planner's problem

Assignments of matching and efforts: There are finite number of individuals, N. A typical individual is denoted by $i \in N$. Let $\mathcal{E}$ be a finite set of efforts. When $T \subset N$ are matched, it is said that *team T is formed*. When they are matched, they exert efforts, which are denoted by $e_T := (e_i)_{i \in T} \in \mathcal{E}^{|T|}$.

Contractual matching function is

$$\begin{aligned} A \quad &: \quad N \to \{(T, e_T) | T \subset N, e_T \in \mathcal{E}^{|T|}\} \\ &s.t. \quad A(i) = (T, e_T) \to A(j) = (T, e_T), \forall j \in T \end{aligned}$$

For the simplicity of notation, I often write $(T, e_T) \in A$ if $A(i) = (T, e_T)$. I often write $T \in A$ when T is formed in matching A with a certain e_T. Define set $\mathcal{A}$ to be the set of all the contractual matching functions.

Note that the contractual matching function assigns not only teams but also the effort for the each assigned individual.

Technology, state, and matching: The outcome of team T's production is denoted by $s_T \in S = \{1, \ldots, S\}$. For a given matching function A, the state of the economy is $\mathbf{s}_A = (s_T)_{T \in A} \in \mathbf{S}_A$ where $\mathbf{S}_A$ is the set of all possible state $\mathbf{s}_A$ for given matching A. $\mathbf{s}_A$ is realized with probability $\Pr(\mathbf{s}_A; A)$, i.e., the probability of state $\mathbf{s}_A$ is affected by the efforts specified in A. (Note that, if shocks for teams are independent, $\Pr(\mathbf{s}_A; A)$ is the product of the probabilities of individual shocks. For example, $\Pr(\mathbf{s}_A; A) = \prod_{T \in A} \varphi(s_T; e_T)$ where $\varphi(s_T; e_T)$ is the (independent) probability of s_T in team T implementing efforts e_T.)

The output of individual team T at state s_T is $q(s_T)$. There is a one-to-one relationship between s and q; however, I keep this notation to reduce any possible confusion. I often write $q_T(\mathbf{s}_A) := q(s_T)$ when s_T is an element in $\mathbf{s}_A$.

Commodities, allocation, and utility function: There are L non-money commodities and one money commodity. For each state of $\mathbf{s}_A$, $z_i^{\mathbf{s}_A} \in Z \subset \mathbb{R}^L$ is an allocation of commodities to individual i. Z is a finite subset of $\mathbb{R}^L$, but it can be extended to a continuum subset of $\mathbb{R}^L$.[1] Let $z_i^A := (z_i^{\mathbf{s}_A})_{\mathbf{s}_A \in \mathbf{S}_A}$ be an allocation defined for all states $\mathbf{s}_A$, for given matching A. (I leave superscript A for z_i^A because the dimension of vector s_A depends on A – the dimension is the number of teams in A. Thus, the dimension of z_i^A depends on A as well.)

Definition 1 (Assignment/Allocation) *Assignment/Allocation* $(A, (z_i^A)_{i \in N})$ *specifies how individuals are matched with which efforts, A, and what the consumption at each state of economy* $(A, \mathbf{s}_A)$ *are.*

The expected utility of i for given (A, z_i^A) is denoted by $\sum_{\mathbf{s}_A \in \mathbf{S}_A} v_i(z_i^{\mathbf{s}_A}) \Pr(\mathbf{s}_A; A) - C_i(e_i)$ where $C_i(e_i)$ represents the utility cost of effort e_i. $v_i(\cdot)$ is strictly increasing, strictly concave, and differentiable.

Probabilities: We define a few probabilities. $X(A)$ is the probability that A is realized. $X(A, (z_i^A)_{i \in N})$ is the probability that A is realized and consumption z_i^A are awarded to individuals. $X_T(A, (z_i^A)_{i \in T})$ is the probability that A is realized with team T formed, and $i \in T$ consumes z_i^A. $X_i(A, z_i^A)$ is the probability that A is realized, and i consumes z_i^A.

[1] If Z is a continuum set, the linear programming discussed in the current paper will be a double infinite linear programming (there are an infinite number of variables and infinite number of constraint). An extension to a continuum set would be possible in a similar way as Rahman (2005, proposition 2.6).

For individual i, randomized matching with efforts and consumption is described by probability $X_i(A, z_i^A)$. For example, if $X_i(A, z_i^A) = X_i(A, {z'}_i^A) = X_i(A', {z''}_i^{A'}) = 1/3$, individual i is situated in matching structure A with probability $2/3$, and in A' with probability $1/3$. In the case that A is realized, i consumes z_i^A or ${z'}_i^A$ with probability $1/2$. In the case that individual i is situated in A', individual i consumes ${z''}_i^{A'}$. $X_i(A, z_i^A)$ is a marginal probability of $X(A, (z_i^A)_{i\in N})$ since $X_i(A, z_i^A) = \sum_{(z_j^A)_{j\neq i}} X(A, (z_i^A)_{i\in N})$. Additionally we define $z_T^A := (z_i^A)_{i\in T}$ and $z_N^A := (z_i^A)_{i\in N}$.

Information structure: Suppose matching (T, e_T) was realized. The individuals in team (T, e_T) know that they are matched. However, one may or may not know how the other teams are formed. Two extreme assumptions on this information are the following.

Assumption 1 (*Local* Information Structure) *Once T is formed with (e_T), the members of T do NOT observe how others were matched.*

Assumption 2 (*Global* Information Structure) *Once T is formed with (e_T), the members of T DO observe how others were matched.*

There are many possible information structure in between. I only concentrate on the case of *global information structure.*

The incentive compatibility constraint (IC) is

$$DG_i(e_i'|A, z_i) := \left[\sum_{\mathbf{s}_A\in\mathbf{S}_A} v_i(z_i^{\mathbf{s}_A})\Pr(\mathbf{s}_A; A|e_i') - C_i(e_i')\right] - \left[\sum_{\mathbf{s}_A\in\mathbf{S}_A} v_i(z_i^{\mathbf{s}_A})\Pr(\mathbf{s}_A; A) - C_i(e_i)\right] \leq 0$$

where $\Pr(\mathbf{s}_A; A|e_i')$ is same as $\Pr(\mathbf{s}_A; A)$ with the only difference being $A(i) = (T, (e_i', (e_j)_{j\neq i}))$. Since $X_T(\cdot,\cdot) \geq 0$, I can write the incentive compatibility condition as the following.

$$DG_i(e_i'|A, z_i)X_T(A, z_T) \leq 0 \tag{1}$$

Note that I used unconditional probability $X_T(A, z_T)$ so that the constraints are linear in terms of $X_T(\cdot,\cdot)$.

Although I assumed that there is no assignment/allocation randomizing consumption and efforts in the incentive compatibility constraint, the framework is general enough to incorporate randomized contracts upon consumption and efforts. Extensions to the contracts randomizing efforts and consumption are illustrated in Rahman (2005) and Song (2006).

2.1.1 Linear Programming Formulation of Planner's Problem

I formulate the objective function of the planner as a linear function, and all the constraints as linear constraints.

Objective Function: The following is the expected utility for individual i.

$$U_i(X_i(\cdot,\cdot)) := \sum_A \sum_{z_i^A} \left[\sum_{\mathbf{s}_A \in \mathbf{S}_A} v_i(z_i^{\mathbf{s}_A}) \Pr(\mathbf{s}_A; A) - C_i(e_i) \right] X_i(A, z_i^A)$$

since $X_i(A, z_i^A)$ is the probability that the planner puts individual i in matching A with consumption z_i^A.

As each individual's weight changes in planner's objective, all the Pareto efficient allocation can be obtained. The objective function for the planner is

$$\sum_i \lambda_i U_i(X_i(\cdot,\cdot)) \tag{2}$$

where $\lambda := (\lambda_i)_{i \in N}$ is a weight profile such that $\lambda \gg 0$.

Probability constraints: Sum of probabilities $X_i(A, z_i^A)$ has to be equal to 1

$$\sum_A \sum_{z_i^A} X_i(A, z_i^A) = 1, \forall i \in N \tag{3}$$

Matching constraints: $X_i(A, z_i^A)$ cannot be arbitrary. Let $X_T(A, z_T^A)$ be the probability that T is formed in A with consumption $z_T^A := (z_i^A)_{i \in T}$. Then the following must hold.

$$\begin{aligned} X_T(A, z_T^A) &= \sum_{(z_i^A)_{i \notin T}} X(A, z_N^A) \text{ if } T \in A \\ X_i(A, z_i^A) &= \sum_{(z_j^A)_{j \in T \setminus \{i\}}} X_T(A, z_T^A), \forall A, z_i^A, i \in T \in A \end{aligned} \tag{4}$$

In other words, $X_i(A, z_i^A)$ is the marginal probability of $X(A, z_N^A)$.

Resource Constraint: For any realized matching A and state $\mathbf{s}_A$, the consumption cannot be larger than what was produced in the economy. Therefore,

$$\sum_{i\in N} z_i^{\mathbf{s}_A} \leq \sum_{T\in A} q(s_T), \forall \mathbf{s}_A \in \mathbf{S}_A, A$$

The constraint can be written as the following since $X(A, z_N^A) \geq 0$.

$$\left[\sum_{i\in N} z_i^{\mathbf{s}_A} - \sum_{T\in A} q_T(\mathbf{s})\right] X(A, z_N^A) \leq 0, \forall \mathbf{s}_A \in \mathbf{S}_A, A, z_N^A \quad (5)$$

Note that if $X(A, z_N^A) = 0$, inequality $\sum_{i\in N} z_i^{\mathbf{s}_A} \leq \sum_{T\in A} q(s_T)$ does not have to hold.

In summary, the planner's problem with weight profile $(\lambda_i)_{i\in N}$ is

$$\max_{X_i(\cdot)\geq 0, X_T(\cdot)\geq 0, X(\cdot)\geq 0} (2) \text{ subject to } (1), (3), (4), (5)$$

If all the individuals become single-person teams, the economy becomes the classic exchange/production economy of Arrow-Debreu. Thus, a sufficient condition for non-empty feasible set is the condition for non-empty core of the classic exchange/production economy. Alternatively, a sufficient condition for non-empty feasible set is the possibility of autarky. Under one of the two assumptions (i.e., either under the possibility of the economy with *only* single person teams or under the possibility of the autarky), the feasible set is non-empty. Also there are finite number of variables (since Z and $\mathcal{A}$ are finite) and finite number of constraints. Thus, the maximum of the planner's problem always exists, and we derive the following proposition.

Proposition 1 *A solution for the planner's problem exists.*

Note that the solution of the planner's problem is incentive-constrained efficient by definition.

Definition 2 *A probabilistic assignment/allocation of teams and consumption,* $((X_i(\cdot))_{i\in N}$, $(X_T(\cdot))_{T\in A\in\mathcal{A}}, X(\cdot))$, *is* incentive-constrained efficient *if it solves the planner's problem.*

2.2 Decentralization of Efficient Assignment

The dual linear programming problem of the planner's problem specifies a market environment where the decentralized economy replicates the optimization of the planner. The market environment includes commodities, prices of the commodities, timing of relevant markets for the commodities, necessary commitment, technologies such as public randomization devices, and arbitrageurs of the commodities. Detailed derivation of the market environment is provided in Section 2.3, and I state the market environment here without derivation.

Contracts, markets: Contract $(T, e_T, (z_T^A)_{A \ni (T,e_T)})$ specifies who are matched, what payoffs are (depending on the realization of $A \in \mathcal{A}$ and $\mathbf{s}_A \in \mathbf{S}_A$), and what the efforts are. We can interpret the structure of team (T, e_T) as a public good: the implemented efforts enter the individuals' utilities. For the efficient allocation of the public good, it is known that Lindahl price is used; hence, the prices of contracts are Lindahl-like, denoted by $p_i(T, e_T, (z_i^A)_{A \ni (T,e_T)})$. Note that this is the price of *each probability unit* for contract $(T, e_T, (z_i^A)_{A \ni (T,e_T)})$ purchased by individual i.

Players of the economy: Besides the individuals in the planner's problem, perfectly competitive contract arbitrageurs and insurers are derived from the dual linear programming.

Contract arbitrageurs maximize expected profit by selling lotteries on matching to individuals. They purchase insurance from insurers to ensure the delivery of commodities to individuals in the team. The price of the insurance providing consumption z_T^A for team (T, e_T) when A has realized is $P_T(e_T, A, z_T^A)$. Note that the first argument e_T in $P_T(\cdot,\cdot,\cdot)$ is redundant since e_T is already described in A; however, I often use this notation to stress e_T, but I also sometimes use $P_T(A, z_T^A)$ – mainly in appendix to save space. Since they are perfectly competitive, they end up with zero profit in equilibrium.

Insurers maximize the expected profit by insuring all the risks in the economy. The insurers must be committed to taking care of all the risks faced by the teams in the economy. In other words, the insurers must meet all the demand from all the teams. For example, if the economy has two teams, T and T', and suppose that the insurer's profit from selling to

T' is negative. However, as long as the combined profit from T and T' is non-negative, the insurer is willing to be committed to selling insurance. In a competitive environment, the equilibrium expected payoff to the insurers would be zero.

Timing of markets: There are a market for lotteries on contracts and a market for insurance. The timing is described below.

The 1st stage: Individuals and contract arbitrageurs trade lotteries on contracts. Contract arbitrageurs and insurers trade insurance for allocation z_T^A for all $A \ni (T, e_T)$.

The 2nd stage: Matching A is realized.

The 3rd stage: Individuals choose efforts.

The 4th stage: The matching, A, and state of the economy, $\mathbf{s}_A$, are revealed. The contracts are exercised and individuals consume.

As certain prices in Lindahl equilibrium are negative, the (per-unit probability) price of a contract could be negative. For example, if one person is important in a specific team, we can imagine a team pays him to join the team (instead of him paying to join the team).

It is assumed that each individual has no private access to a contingent market. With the private access to a contingent market, an economic agent will try to smooth consumption over states; hence, the contracted effort cannot be enforced (see Tommasi and Weinschelbaum (2004)).

Public randomization device: A public randomization device is required since the outcome of the lotteries are correlated in general. First, the probability $Q_i(T, e_T, (z_i^A)_{A \ni (T,e_T)})$ that individual i purchase cannot be arbitrary: it has to be a marginal probability of $Q_T(A, z_T^A)$ that is the probability that a contract arbitrageur purchases, i.e., the following must hold.

$$Q_i(T, e_T, (z_i^A)_{A \ni (T,e_T)}) - \sum_{j \in T \setminus \{i\}, (z_j^A)_{A \ni (T,e_T)}} Q_T(e_T, (z_T^A)_{A \ni (T,e_T)}) = 0 \quad (6)$$

Second, $Q_T(A, z_T^A)$ cannot be arbitrary either: they have to be correlated through a probability, say $Q(A, z_N^A)$, i.e.,

$$Q_T(A, z_T^A) - \sum_{(z_j^A)_{j\notin T}} Q(A, z_N^A) = 0. \tag{7}$$

The first equality in equation (4) implies equation (6) and the second in equation (4) is identical to equation (7).

Formal definition of equilibrium is the following.

Definition 3 *A competitive equilibrium is a probabilistic assignment/allocation* $((Q_i)_{i\in N}$, $(Q_T)_{T\in A\in\mathcal{A}}, Q)$ *and a price system* $((p_i)_{i\in N}, (p_T)_{T\subset N})$ *that satisfy the following 1 – 5.*

1. Individual's Optimization: *Individuals trade lotteries over the set of contracts. After the randomization device chooses matching A, individuals join teams and choose efforts. Contracts award consumption bundles depending on the realization of the state of the economy* $(A, \mathbf{s}_A)$*. Formally,* $Q_i(\cdot)$ *solves*

$$\max_{\tilde{Q}_i(\cdot)} \sum_{T,e_T,(z_i^A)_{A\ni(T,e_T)}} \left\{ \sum_{A\ni(T,e_T),z_i} \max_{e_i'} \left[\sum_{\mathbf{s}} v_i(z_i^{\mathbf{s}}) \Pr(\mathbf{s}; A|e_i') - C_i(e_i') \right] \times \right.$$
$$\left. \bar{Q}_i(A, z_i^A|T, e_T, (z_i^{\tilde{A}})_{\tilde{A}\ni(T,e_T)}) \right\} \tilde{Q}_i(T, e_T, (\tilde{A}_i^A)_{\tilde{A}\ni(T,e_T)})$$
$$s.t. \sum_{T,e_T,(z_i^{\tilde{A}})_{\tilde{A}\ni(T,e_T)}} p_i(T, e_T, (z_i^{\tilde{A}})_{\tilde{A}\ni(T,e_T)}) \tilde{Q}_i(T, e_T, (z_i^{\tilde{A}})_{\tilde{A}\ni(T,e_T)}) = 0,$$
$$\sum_{T,e_T,(z_i^{\tilde{A}})_{\tilde{A}\ni(T,e_T)}} \tilde{Q}_i(T, e_T, (z_i^{\tilde{A}})_{\tilde{A}\ni(T,e_T)}) = 1$$

2. Contract-arbitrageur's Optimization: *Perfectly competitive contract-arbitrageurs trade lotteries with individuals. Also, contract-arbitrageurs insure themselves through an insurer. Formally,* Q_T *solves*

$$\max_{\tilde{Q}_T(e_T,(z_T^A)_{A\ni(T,e_T)})} \left[\sum_{i\in T} p_i(T, e_T, (z_i^A)_{A\ni(T,e_T)}) - \sum_{A\ni(T,e_T)} P_T(e_T, A, z_T^A) \right] \tilde{Q}_T(e_T, (z_T^A)_{A\ni(T,e_T)})$$
$$s.t.\ (T, e_T, z_T^A) \text{ is incentive compatible for each } A \ni (T, e_T).$$

3. Insurer's Optimization: *Perfectly competitive insurers try to maximize the expected payoff. Formally,*

$$\max_{\tilde{Q}_T(\cdot,\cdot)} \sum_{T,e_T} \sum_{A \ni (T,e_T)} \Big[\sum_{z_T^A} P_T(e_T, A, z_T^A) \Big] \tilde{Q}_T(e_T, (z_T^A)_{A \ni (T,e_T)}) \;\; s.t. \;\; \sum_{T \in A} \Big[\sum_{i \in T} z_i^{\mathbf{s}_A} - q_T(\mathbf{s}_A) \Big] \leq 0$$

4. Clearance of Commodity Market: *The commodity market clears at each $(A, \mathbf{s}_A)$.*

$$\sum_{i \in N} z_i^{\mathbf{s}_A} \leq \sum_{T \in A} q_T(\mathbf{s}_A), \forall \mathbf{s}_A \in \mathbf{S}_A, A$$

5. Matching Market Clearance: *The matching market clears in the sense that lottery purchases are consistent across the population. In summary, the public randomization device is also required to satisfy the following.*

$$\begin{aligned} & Q_i(A, z_i^A | T, e_T, (z_i^{\tilde{A}})_{\tilde{A} \ni (T,e_T)}) Q_i(T, e_T, (z_i^{\tilde{A}})_{\tilde{A} \ni (T,e_T)}) \\ = & \sum_{(z_j^{\tilde{A}})_{\tilde{A} \ni (T,e_T)}, j \in T \setminus \{i\}} Q_T(A, z_T^A | e_T, (z_T^{\tilde{A}})_{\tilde{A} \ni (T,e_T)}) Q_T(e_T, (z_T^{\tilde{A}})_{\tilde{A} \ni (T,e_T)}), \\ & Q_T(A, z_T^A | e_T, (z_T^{\tilde{A}})_{\tilde{A} \ni (T,e_T)}) Q_T(e_T, (z_T^{\tilde{A}})_{\tilde{A} \ni (T,e_T)}) = \sum_{(z_j^A)_{j \notin T}} Q(A, z_T^A). \end{aligned}$$

Remark: In the same way as markets clear (i.e., supply equals to demand) in the classical exchange economy by the "right" prices, the purchases of probabilities by individuals, contract arbitrageurs and insurers end up with being exactly correlated by the "right" prices of contracts. In the individual's optimization, $\bar{Q}_i(A, z_i^A | T, e_T, (z_i^{\tilde{A}})_{\tilde{A} \ni (T,e_T)})$ is the probability distribution on matching A and consumption z_i^A, conditional that contract $(T, e_T, (z_i^{\tilde{A}})_{\tilde{A} \ni (T,e_T)})$ is chosen. This conditional probability implements contract $(T, e_T, (z_i^{\tilde{A}})_{\tilde{A} \ni (T,e_T)})$, which awards consumption bundle depending on the realization of A. In other words, the randomization device simply implements $Q_i(T, e_T, (z_i^A)_{A \ni (T,e_T)})$ that is constructed from the individuals' purchase of probabilities.

Note that the model assumes that individuals are price takers, i.e., I do ignore the strategic interactions between market players. In this papers, the only information problem I consider is the moral hazard problem inside teams.

Also note that individuals are assumed to have no initial endowment; hence, the right hand side of the budget constraint is zero. ∎

Theorem 1 *(i) A competitive equilibrium is incentive-constrained efficient. (ii) Any incentive-constrained efficient probabilistic assignment/allocation can be decentralized with any weight profile* λ.

Proof. See Section 2.3. ∎

2.3 Characterization of Equilibria and Proof of Theorem 1

Dual linear programming problem of the planner's linear programming problem is derived. From the dual constraints (the constraints of the dual linear programming probelm), a proper definition of price-taking equilibrium is derived.

2.3.1 Dual Linear Programming

Let the dual variables corresponding to each constraint of the planner's linear program be $y_i, p_i(A, z_i^A)$, $\bar{P}_T(A, e_T, z_T^A)$, $\alpha_i(e_i'|e_T, (z_j^A)_{j\in T\setminus\{i\}})$, and $\phi(A, \mathbf{s}_A, z_N^A)$. Then the following is the dual linear program.

$$
\begin{aligned}
(D)\min \quad & \sum_{i\in N} y_i \\
s.t. \quad & y_i \geq \lambda_i \left[\sum_{\mathbf{s}_A} v_i(z_i^{\mathbf{s}_A})\Pr(\mathbf{s}_A; A) - C_i(e_i)\right] - p_i(A, z_i^A) \\
& 0 \geq \sum_{i\in T} p_i(A, z_i^A) - \bar{P}_T(A, e_T, z_T^A) - \sum_{i\in T}\sum_{e_i'} \alpha_i(e_i'|e_T, (z_j^A)_{j\in T\setminus\{i\}}) DG_i(e_i'|A, z_i^A) \\
& 0 \geq \sum_{(T,e_T)\in A} \bar{P}_T(A, e_T, z_T^A) + \left[\sum_{T\in A} q_T(\mathbf{s}_A) - \sum_{i\in N} z_i^{\mathbf{s}_A}\right] \phi(A, \mathbf{s}_A, z_N^A) \\
& \phi(A, \mathbf{s}_A, z_N^A) \geq 0
\end{aligned}
$$

Proposition 2 (Fundamental Theorem of Linear Programming) *(1) There exists a solution for each of the primal linear programs. (2) There exists a solution for the dual linear programs. (3) The values of the primal and dual programming are same.*

Proposition 3 (Complementary Slackness)

$$X_i(A, z_i^A) > 0 \;\Rightarrow\; y_i = \lambda_i \left[\sum_{\mathbf{s}_A} v_i(z_i^{\mathbf{s}_A}) \Pr(\mathbf{s}_A; A) - C_i(e_i)\right] - p_i(A, z_i^A)$$

$$X_i(A, z_i^A) = 0 \;\Leftarrow\; y_i > \lambda_i \left[\sum_{\mathbf{s}_A} v_i(z_i^{\mathbf{s}_A}) \Pr(\mathbf{s}_A; A) - C_i(e_i)\right] - p_i(A, z_i^A)$$

Similarly,

$X_T(A, z_T^A) > 0 \;\Rightarrow\;$ *Equality of the relevant second dual constraint*

$X_T(A, z_T^A) = 0 \;\Leftarrow\;$ *Strict inequality of the relevant second dual constraint*

$X(A, z_N^A) > 0 \;\Rightarrow\;$ *Equality of the third dual constraint*

$X(A, z_N^A) = 0 \;\Leftarrow\;$ *Strict inequality of the third dual constraint*

Before analyzing and interpreting the meaning of the dual constraints, I prove two lemmas that can be shown by a direct application of the fundamental theorem of linear programming.

Lemma 1 *At an optimal solution of linear programs,*

$$y_i = \lambda_i \sum_A \sum_{z_i^A} \left[\sum_{\mathbf{s}_A \in \mathbf{S}_A} v_i(z_i^{\mathbf{s}_A}) \Pr(\mathbf{s}_A; A) - C_i(e_i)\right] X_i(A, z_i^A) - \sum_A \sum_{z_i^A} p_i(A, z_i^A) X_i(A, z_i^A)$$

$$0 = \sum_{A \ni (T, e_T)} \sum_{i \in T} \sum_{z_i^A} p_i(A, z_i^A) X_i(A, z_i^A) - \sum_{A \ni (T, e_T)} \sum_{z_T^A} \bar{P}_T(A, e_T, z_T^A) X_T(A, z_T^A)$$

$$0 = \sum_A \sum_{z_N^A} \bar{P}_T(A, e_T, z_T^A) X(A, z_N^A)$$

Lemma 2 *For any weight profile λ, there exists a dual solution, $(y_i, p_i(A, z_i^A))$ such that*

$$\sum_A \sum_{z_i^A} p_i(A, z_i^A) X_i(A, z_i^A) = 0.$$

2.3.2 Characterization of Decentralization

The planner's problem can be interpreted as a revenue maximization problem. The inputs for the planner's problem are individuals. All other constraints can be interpreted as technological constraints. The dual variables of the individual probability constraint measure the value of the individual, which has the direct interpretation of the individual's utility. The dual value of the second primal constraint measures the value of assignment (A, z_i^A) to individual i, which is the price of assignment (A, z_i^A) to individual i. The dual value of the third primal constraint measures the value of assignment (A, z_T^A) to team T, which is the price of assignment (A, z_T^A) to team T. The dual variable of the fourth primal constraint measures the value of the resource. Note that the notion of a market has to be introduced in order to consider these values as prices. Lastly, the prices of the incentive compatibility constraints are derived from the last primal constraints, which will be discussed later.

Note that the prices of the second and the third primal constraints measure the value of assigning individuals and team. Since the constraints are indexed by i and T, the natural definition of prices would be Lindahl-like.

2.3.3 Individual Choice: the first dual constraint

The dual variable of the first primal linear program, y_i, is the value of individual i to the planner. The first dual constraint is interpreted as individuals' maximization.

Define prices using the optimal value of the dual linear program.

$$p_i(T, e_T, (z_i^A)_{A \ni (T,e_T)}) := \sum_{A \ni (T,e_T)} \sum_{z_i^A} p_i(A, z_i^A) Q_i(A, z_i^A | T, e_T, (z_i^A)_{A \ni (T,e_T)})$$

$p_i(T, e_T, (z_i^A)_{A \ni (T,e_T)})$ is the price of a randomized contract that (i) realizes matching (T, e_T) with probability $\sum_{A \ni (T,e_T)} \frac{\sum_{z_i} Q_i(A,z_i)}{\sum_A \sum_{z_i} Q_i(A,z_i)}$ and that (ii) gives consumption $z_i^{\mathbf{s}_A}$ at state $(A \ni (T, e_T), \mathbf{s}_A)$ with probability $\sum_{\tilde{z}_i^{\mathbf{s}} : \tilde{z}_i = z_i^{\mathbf{s}}} \frac{Q_i(A,\tilde{z}_i)}{\sum_{\tilde{z}_i} Q_i(A,\tilde{z}_i)}$.

By summing up the first dual constraints with arbitrary probability $Q_i(A, z_i^A)$, I derive

$$y_i \geq \lambda_i \sum_{A,z_i^A} \left[\sum_{\mathbf{s}_A \in \mathbf{S}_A} v_i(z_i^{\mathbf{s}_A}) \Pr(\mathbf{s}_A; A) - C_i(e_i) \right] Q_i(A, z_i^A) - \sum_{A,z_i^A} p_i(A, z_i^A) Q_i(A, z_i^A) \quad (8)$$

$$\Leftrightarrow \; y_i/\lambda_i \geq \sum_{T,e_T,(z_i^A)_{A\ni(T,e_T)}} \left\{ \sum_{A\ni(T,e_T),z_i^A} \left[\sum_{\mathbf{s}_A} v_i(z_i^{\mathbf{s}_A}) \Pr(\mathbf{s}_A; A) - C_i(e_i) \right] \times Q_i(A, z_i^A | T, e_T, (z_i^A)_{A\ni(T,e_T)}) \right\} Q_i(T, e_T, (z_i^A)_{A\ni(T,e_T)}) - \frac{1}{\lambda_i} \sum_{T,e_T,(z_i^A)_{A\ni(T,e_T)}} p_i(T, e_T, (z_i^A)_{A\ni(T,e_T)}) Q_i(T, e_T, (z_i^A)_{A\ni(T,e_T)})$$

If $Q_i(A, z_i^A) = X_i(A, z_i^A)$, then the inequalities are equalities by Complementary Slackness.

From Lemma 2, pick optimal dual variables such that $\sum_A \sum_{z_i} p_i(A, z_i) X_i(A, z_i) = 0$. Then, I derive

$$\sum_{T,e_T,(z_i^A)_{A\ni(T,e_T)}} p_i(T, e_T, (z_i^A)_{A\ni(T,e_T)}) X_i(T, e_T, (z_i^A)_{A\ni(T,e_T)}) = 0.$$

The meaning of the equality in Lemma 2 is that money expenditure on the lottery purchase is zero if the purchase is the same as that of the planner's solution. The purchased contracts are incentive compatible by both Complementary Slackness and the primal constraints of the incentive compatibility constraints. It will be shown that non-incentive compatible contracts are never sold by contract arbitrageurs.

Therefore, the above inequality (8) summarizes individual i's optimization: if individual i has chosen a different probability than that of the planner's, the purchase of the different probability would be infeasible or suboptimal. Also, y_i/λ_i is interpreted as the *ex-ante* utility of i before the realization of $(A, \mathbf{s}_A)$.

2.3.4 Contract Arbitrageurs' optimization: the second dual constraints

The second dual constraint is the contract arbitrageurs' optimization. There is no probability constraint for contract arbitrageurs, i.e. arbitrageurs are a freely available input to the planner. The price of a freely available input must be zero, so arbitrageurs get zero profit unlike individuals. They specialize in writing contracts for team T. The only way for them to gain profit is to innovate a contract for team T, and to sell a lottery on that contract.

The contract arbitrageur pays non-money commodity z_i^A to $i \in T$, which are not necessarily same to $q_T(\mathbf{s})$. Therefore, they insure themselves through an insurer to deliver z_i to each individual in team T.

In the equilibrium, there are two kinds of arbitrageurs: *ex-ante active* arbitrageurs and *ex-ante inactive* arbitrageurs. *Ex-ante active* arbitrageur sells a positive amount of lotteries on the job assignment, while *ex-ante inactive* arbitrageur sells zero amount of lotteries because her contract is not profitable. Not all ex-ante active arbitrageurs are active *ex-post*. Some are *ex-post active*, and others are *ex-post inactive* depending on the realization of A.

Define the insurance premium using the optimal value of the dual linear programming problem.

$$P_T(e_T, A, z_T^A) := \bar{P}_T(e_T, A, z_T^A) Q_T(A|e_T, (z_i^{\tilde{A}})_{\tilde{A} \ni (T,e_T)})$$

$P_T(e_T, A, z_T^A)$ is the price of insurance that pays non-money commodity $\sum_{i \in T} z_i^{\mathbf{s}_A} - q_T(\mathbf{s}_A)$ at the realization of $(A \ni (T, e_T), \mathbf{s}_A)$ with probability $\sum_{(\tilde{z}_i)_{i\in T}: \tilde{z}_i^{\mathbf{s}} = z_i^{\mathbf{s}}, \forall i \in T} \frac{Q_T(A,(\tilde{z}_i)_{i\in T})}{\sum_{A \ni (T,e_T)} \sum_{(\tilde{z}_i)_{i\in T}} Q_T(A,(\tilde{z}_i)_{i\in T})}$.

By summing up the second dual constraints with arbitrary $Q_T(A, z_T)$, I get

$$\begin{aligned} 0 \geq & \left[\sum_{i \in T} p_i(T, e_T, (z_i^A)_{A \ni (T,e_T)}) - \sum_{A \ni (T,e_T)} P_T(e_T, A, z_T^A) \right] Q_T(e_T, (z_i^A)_{A \ni (T,e_T)}) \\ & - \sum_{i \in T} \sum_{A \ni (T,e_T)} \sum_{z_T^A} \sum_{e_i'} \alpha_i(e_i'|T, e_T^A, z_j) DG_i(e_i'|A, z_i^A) Q_T(A, z_T^A) \end{aligned} \tag{9}$$

If $Q_T(A, z_T^A) = X_T(A, z_T^A)$, the inequality becomes an equality. Moreover, the second line is zero.

The last term is the shadow value of the incentive compatibility constraints. In other words, the seller's profit *internalizes* the shadow value of the incentive compatibility constraints. If $X_T(A, z_T)$ is the same as the planner's solution, the arbitrageur would get zero profit. If the arbitrageur sells a different amount of the lotteries than the planner's solution, the profit (including the incentive cost represented by the shadow value) would be smaller than zero. Of course, the shadow values of the incentive compatibility constraints are imaginary. A realistic story is that arbitrageurs cannot observe the efforts of the team members, thus they will offer only incentive compatible contracts.

Lemma 3 *If* $(T, e_T, (z_T^A)_{A\ni(T,e_T)})$ *is incentive compatible for all* $i \in T$*, then*

$$0 \geq \left[\sum_{i\in T} p_i(T, e_T, (z_i^A)_{A\ni(T,e_T)}) - \sum_{A\ni(T,e_T)} P_T(e_T, A, z_T^A)\right] Q_T(e_T, (z_T^A)_{A\ni(T,e_T)})$$

Therefore, inequality (9) summarizes contract arbitrageur's optimization, since she would not gain more even if she chose a different probability than that of the planner.

The *ex-ante active* contract arbitrageurs spend all non-money commodities by paying non-money wage $\sum_{i\in T} z_i$, by receiving $\sum_{i\in T} z_i - q_T(\mathbf{s})$, and by production $q_T(\mathbf{s})$.

2.3.5 Insurer's Choice: the third dual constraints

The third constraint is

$$0 \geq \sum_{(T,e_T)\in A} \bar{P}_T(A, e_T, z_T^A) + \phi(A, \mathbf{s}_A, (z_N^A)) \left[\sum_{T\in A} q_T(\mathbf{s}_A) - \sum_i z_i^{\mathbf{s}_A}\right]. \tag{10}$$

Summing up the inequalities with an arbitrary weight $Q(A, z_N^A)$, we derive

$$0 \geq \sum_{A,z_N^A} \sum_{(T,e_T)\in A} \bar{P}_T(e_T, A, z_T^A) Q(A, z_N^A) + \sum_{A,z_N^A} \phi(A, \mathbf{s}_A, (z_N^A)) \left[\sum_{T\in A} q_T(\mathbf{s}_A) - \sum_i z_i^{\mathbf{s}_A}\right] Q(A, z_N^A) \tag{11}$$

Note that the first part of the above equations's right-hand side is

$$\begin{aligned}
\sum_{T,e_T} \sum_{A\ni(T,e_T)} \sum_{z_T^A} \bar{P}_T(e_T, A, z_T^A) \sum_{z_{-T}^A} Q(A, z_N^A) &= \sum_{T,e_T} \sum_{A\ni(T,e_T)} \sum_{z_T^A} \bar{P}_T(e_T, A, z_T^A) Q_T(A, z_T^A) \\
&= \sum_{T,e_T} \sum_{A\ni(T,e_T)} \sum_{z_T^A} \bar{P}_T(e_T, A, z_T^A) Q_T(A, e_T, z_T^A | (z_N^{\tilde{A}})_{\tilde{A}\ni(T,e_T)}) Q_T(e_T, (z_N^{\tilde{A}})_{\tilde{A}\ni(T,e_T)}) \\
&= \sum_{T,e_T} \sum_{A\ni(T,e_T)} \left[\sum_{z_T^A} P_T(e_T, A, z_T^A)\right] Q_T(e_T, (z_N^{\tilde{A}})_{\tilde{A}\ni(T,e_T)})
\end{aligned}$$

The last term in equation (11) is the shadow value of the resource constraint. In other words, the insurer's profit *internalizes* the shadow value of the resource constraint. If $X(A, z_N)$ is the same as the planner's solution, then the insurer gets zero profit. If the insurer sells insurance that is different from the planner's solution, then the profit (including the shadow cost) would be smaller than zero. Of course, the shadow value of the resource constraints are imaginary. A justifying story is that, if the insurer sells insurance that does not satisfy the resource constraint (hence, the insurer cannot deliver what is promised in the insurance contract), the disutility of the insurer is infinite. Even if the insurer sells insurance satisfying the resource constraint, but different from that of the planner's solution, the profit would not go up by the following proposition.

Lemma 4 *If $Q(A, (z_i))$ is the probability that resource constraints are satisfied for all the realizations, then*

$$0 \geq \sum_{A} \sum_{z_N^A} \left[\sum_{T \in A} P_T(A, z_T^A) \right] Q(A, z_N^A)$$

Therefore, inequality (10) summarizes the insurer's optimization, since she would not gain more even if she chose a different probability than that of the planner. However, the insurer behaves differently from ordinary insurers. Insurers are committed to taking care of all the risk involved in the economy. Without commitment, they could have sold more insurance to some teams if the insurance premium exceeds the expected payment to them, and none to the others if not. The following example illustrates the point.

Example 1 *Suppose there are only two possible teams, (T, e_T) and $(T', e'_{T'})$. In general,*

$$P_T(e_T, (z_i^A)_{A \ni (T, e_T)}) X_T(e_T, (z_i^A)_{A \ni (T, e_T)})$$

$$\neq \sum_{A \ni (T, e_T), z_T^A} \sum_{\mathbf{s}_A} \phi(A, \mathbf{s}_A, z_N) \left[\sum_{i \in T} z_i^{\mathbf{s}_A} - q_T(\mathbf{s}_A) \right] X_T(A, z_T^A)$$

$$P_{T'}(e_{T'}, (z_i'^A)_{A \ni (T, e_T)}) X_{T'}(e_{T'}, (z_i'^A)_{A \ni (T', e_{T'})})$$

$$\neq \sum_{A \ni (T, e_T), z_T^A} \sum_{\mathbf{s}_A} \phi(A, \mathbf{s}_A, z_N^A) \left[\sum_{i \in T'} z_i^{\mathbf{s}_A} - q_{T'}(\mathbf{s}_A) \right] X_{T'}(A, z_{T'})$$

In other words, the insurance premium $T_i(e_T, (z_T^A)_{A \ni (T, e_T)})$ does not correctly reflect the expected value of the payment from the insurer to the insured. Suppose

$$T_i(e_T, (z_T^A)_{A \ni (T, e_T)}) > \sum_{A \ni (T, e_T), z_T^A} \sum_{\mathbf{s}_A} \phi(A, \mathbf{s}_A, z_N^A)[\sum_{i \in T} z_i^{\mathbf{s}_A} - q_T(\mathbf{s}_A)] X_T(A, z_T^A).$$

If the insurer has access to a contingent claims market where she can purchase contingent claims $\sum_{i \in T} z_i^{\mathbf{s}_A} - q_T(\mathbf{s}_A)$ at price $\sum_{A \ni (T, e_T)} \sum_{z_T^A} \sum_{\mathbf{s}_A} \phi(A, \mathbf{s}, z_N^A)[\sum_{i \in T} z_i^{\mathbf{s}_A} - q_T(\mathbf{s}_A)] X_T(A, z_T^A)$, she would not sell insurance to T', but only to T. In other words, unless the risk premium correctly reflects the expected value of future payments, commitment of the insurer is critical in proving the Welfare theorems. ∎

One might wonder if it would be possible to pick a weight profile λ so that the insurance premium reflects the expected value of the payment from insurance. However, the number of possible matching structures is in general larger than that of individuals, so it is not possible to adjust λ to make all the insurance premiums reflect the true values of the insurance. For example, suppose $N = I \cup J$ where $I = \{i, i'\}$, $J = \{j, j'\}$. Suppose also that matching is not possible between i and i' or j and j', and that there are five ways of assignments A_1, A_2, A_3, A_4, A_5 with positive probability of realization for each of them. Note that matching specifies efforts too. Then, the effective number of the equalities that has to be satisfied is 4, while the number of the control variables is effectively 3. Therefore, the equalities would be impossible in general.

Discussion on the equality of insurance premium and the expected value of future payment is revisited in Song (2006) where finite and continuum economies are compared in terms of the convergence of the former to the latter.

2.3.6 Public randomization device: How to Exercise Randomized Contracts

Lastly, a public randomization device is required in order to fully decentralize the planner's solution. The first matching market clearance condition changes into the following.

$$Q_i(A, z_i^A, T, e_T, (z_i^{\tilde{A}})_{\tilde{A}\ni(T,e_T)}) = \sum_{(z_j^A)_{j\in T\setminus\{i\}}} Q_T(A, z_T^A, e_T, (z_T^{\tilde{A}})_{\tilde{A}\ni(T,e_T)})$$

The left-hand side is in fact identical to $Q_i(A, z_i^A)$ since (T, e_T) is already contained in A and z_i^A was realized among $(z_i^{\tilde{A}})_{\tilde{A}\neq A}$. The right-hand side is identical to $\sum_{(z_j^A)_{j\in T\setminus\{i\}}} Q_T(A, z_T^A)$ by the same reasons. Thus, this condition is identical to the first constraint in equation (4).

The second matching market clearance constraint changes into

$$Q_T(A, z_T, e_T, (z_T^A)_{A\ni(T,e_T)}) = \sum_{(z_j)_{j\notin T}} Q(A, z_N^A).$$

Again, the left-hand side is $X_T(A, z_T)$ by the same reason above. Thus, this condition is identical to the second constraint in equation (4).

If it is assumed that a reputable institution announces the outcome of the public randomization device, the story of public randomization can be justified. However, the information that the institution has to process is enormous. So the story might not conform to the true spirit of Welfare theorems or Arrow-Debreu, in which small economic entities observe a very small set of information (e.g., prices) compared to the size of the economy (e.g., allocations).

Song (2011) considers a continuum model and Song (2006) shows convergence of the finite model in this paper to the continuum model in Song (2011), in which the economy-wide public randomization device can be "decentralized" into team-specific randomization devices.

2.4 Comment on Combined Welfare theorems

In the classical private good exchange economy, only one point on the contract curve can be typically decentralized without money transfers among individuals. However, Theorem

1 shows that all the incentive-constrained efficient allocations can be decentralized. The difference comes from the fact that commodity price $\phi(A, \mathbf{s}_A, z_N^A)$ is non-linear in the sense that it is a function of z_N^A as shown in Section 2.3. In the classical exchange economy, the expenditure is a linear function of consumption z_i, e.g., $p \cdot z_i$.

The non-linearity of the price is illustrated by the linear programming formulation. Makowski and Ostroy (1996) consider the classical exchange economy and write down the resource constraint as

$$\sum_i \sum_z z x_i(z) = 0$$

$x_i(z)$ is the individual measure that is in the objective function in the planner's problem and in the resource constraint at the same time. Therefore, $x_i(z)$ directly connects the utility function and the resource constraint; hence, the formulation enabled them to reflect the concavity of the utility function in price, i.e. linear price. On the other hand, in the finite model of this paper, the analogues of the resource constraint of the classical exchange economy are the probability constraints and the resource constraint, which are

$$X_i(A, z_i^A) - \sum_{(z_j^A)_{j \in T \setminus \{i\}}} X_T(A, z_T^A) = 0, \quad X_T(A, z_T^A) - \sum_{(z_i^A)_{i \notin T}} X(A, z_N^A) = 0$$

$$\left[\sum_i z_i^{\mathbf{s}_A} - \sum_{T \in A} q_T(\mathbf{s}_A)\right] X(A, z_N^A) = 0, \forall \mathbf{s}_A \in \mathbf{S}_A, A, z_N^A$$

It is impossible to write down the resource constraint using measures on individuals' or active contract arbitrageurs' behavior. Therefore, an economy-wide measure $X(A, z_N^A)$ is used. Since the dual value of the resource constraints represent the price of the commodities in the linear programming formulation, the commodity price becomes a function of allocation – non-linear price. Also, the existence of the economy-wide measure means that the corresponding dual constraint has to be interpreted as an economic agent's maximization problem. The fact that $X(A, z_N^A)$ describes the details of allocations in the economy is the main reason that the insurer has to take care of all the risks in the economy.

Through measure $X(A, z_N^A)$, unlike Makowski and Ostroy (1996), individual constraints here become redundant. Formally,

$$\sum_A \sum_{z_i^A} X_i(A, z_i^A) = 1 \quad \text{implies} \quad \sum_A \sum_{z_j^A} X_j(A, z_j^A) = 1$$

since

$$\sum_A \sum_{z_i^A} X_i(A, z_i^A) = \sum_A \sum_{z_i^A} \sum_{(z_j^A)_{j \in N \setminus \{i\}}} X(A, z_N^A) = \sum_A \sum_{z_j^A} \sum_{(z_i^A)_{i \in N \setminus \{j\}}} X(A, z_N^A) = \sum_A \sum_{z_j^A} X_j(A, z_j^A).$$

It is well known that redundant constraints in linear program make it possible to give arbitrary dual values to the constraints as long as the sum of the dual values is a certain constant. Thus, the linear programming can assign arbitrary dual values of the individual probability constraints, which are individuals' utilities. Therefore, the planner's problem with arbitrary weight can be decentralized. In summary, the existence of the economy-wide measure, non-linearity of commodity prices, and the redundancy of individuals' probability constraints are equivalent. (Note that none of these are required for the continuum model in Song (2011).)

Considering that the insurer's role is essentially to cross-subsidize (because the insurer has to take care of all the risks in the economy), the combined Welfare theorem is not surprising. The role of cross-subsidization by insurer (or the commitment of no cream-skimming) is hardly a competitive behavior. In short, the Welfare theorem shows, in fact, the failure of the Welfare theorem since an unrealistic player is required.

3 Conclusion

I study contractual matching of finite number of individuals with moral hazard. I characterize incentive-constrained efficient randomized assignments of teams, efforts, and consumption. By exploiting the duality of linear programming, necessary conditions for the incentive-constrained efficient decentralization are derived.

Contractual arbitrageurs, insurers pooling risks in the economy, and public randomization devices implementing matching and contracts are found to be necessary. Insurers need to know the details of the economy in order to set the right insurance premiums, and the public randomization device needs to accommodate and signal enormous amount of information for the allocation of resources. No cream-skimming by insurers is hardly a competitive behavior. Therefore, the unrealistic role of the insurer shows the failure of the Welfare theorems in practice.

The implication from the study is the following. In the finite economy, it is well-known that individuals typically do not behave as price-takers since they perceive the possibility of influencing prices. Even when they behave as price takers and trades of lotteries exist, it is less likely that efficiency would be achieved because of the large informational role of insurers and the existence of the public randomization device implementing randomized contracts. Therefore, I argue that government intervention could be desirable in a small economy. Even in the continuum model (see Song (2011)), it is shown that the lottery market and the common randomization device implementing randomized assignment of individuals to teams is required for efficiency. Considering that such a market and the public randomization device are not observed in the real world, a kind of government intervention on the job matching would be desirable.

Labor economics is an application of the model. For efficiency, the lottery trade is required.[2] At the lack of lottery trade, randomized assignment of individuals by governing institution would be required. On top of that, it was shown that cross-subsidies across teams by insurers were found to be required for efficiency. At the lack of such insurer, the government intervention would be required for efficiency. In South Korean economy from the sixties to the late eightieth, the cross-subsidies by government was extensively practiced. This paper does not argue that the practice was innocent or the only source of South Korean economic success, but tries to give one possible explanation for the success.

[2]In many European countries, government recommends jobs to the unemployed. The unemployed can reject the offers a certain number of times. However, if the unemployed rejects the final offer, the unemployment payment drops drastically. Considering that an available job arrives randomly, the role of the government can be interpreted to complement the lack of lottery trade.

Theory of merger is another application of the model when teams are considered as subsidiary-firms of a big firm. Even without the complementarity of subsidiary-firms, the indivisibility of the team structure implies that a merger with randomized assignment of individuals improves efficiency. Moreover, the model predicts that cross transfers across teams is unavoidable for efficiency. Until the late nineties, Samsung Group actively cross-transfered across subsidiary firms. These features were typical in many other South Korean conglomerate, which made the typical image of South Korean conglomerate of not respecting shareholders. Song (2011) shows that the randomized assignment is required, but the cross-subsidy is not required in the continuum economy. Considering that the size of Samsung Group has exploded during the late nineties, it could be approximated as a continuum model as of now. The reason that Samsung Group stopped practicing cross-subsidization during the late nineties could be because of the expansion of its size.

The other contribution of the paper is the usage of linear programming to model a finite economy. Once a planner's problem is set without the consideration of decentralized economy, dual linear program of the planner's problem characterizes an idealized environment for efficiency of the decentralized economy. Under the characterized environment, the Welfare theorems are automatically derived.

A Proofs

A.1 Derivation of dual

From the following equality, the result follows.

$$\sum_{i\in N} y_i + \sum_{i}\sum_{A,z_i}\Big(\lambda_i\Big[\sum_{\mathbf{s}_A} v_i(z_i^{\mathbf{s}_A})\Pr(\mathbf{s}_A;A) - C_i(e_i)\Big] - p_i(A,z_i^A) - y_i\Big)X_i(A,z_i)$$
$$+\sum_{T,e_T}\sum_{A\ni(T,e_T),z_T}\Big(\sum_{i\in T} p_i(A,z_i^A) - \bar{P}_T(A,e_T,z_T^A) - \sum_{i\in T}\sum_{e_i'}\alpha_i(e_i'|e_T,(z_j^A)_{j\in T\setminus\{i\}})DG_i(e_i'|A,z_i^A)\Big)X_T(A,z_T)$$
$$+\sum_{A,z_N}\Big(\sum_{(T,e_T)\in A}\bar{P}_T(A,e_T,z_T^A) + \Big[\sum_{T\in A} q_T(\mathbf{s}_A) - \sum_{i\in N} z_i^{\mathbf{s}_A}\Big]\phi(A,\mathbf{s}_A,z_N^A)\Big)X(A,z_N)$$

$$
\begin{aligned}
=&\sum_i \sum_{A,z_i^A} \lambda_i \left[\sum_{\mathbf{s}_A} v_i(z_i^{\mathbf{s}_A})\Pr(\mathbf{s}_A;A) - C_i(e_i)\right] X_i(A,z_i^A) + \sum_i y_i \left(1 - \sum_{A,z_i^A} X_i(A,z_i^A)\right) \\
&- \sum_i \sum_{A,z_i^A} p_i(A,z_i^A)(X_i(A,z_i^A) - \sum_{(z_j^A)_{j\in T\setminus\{i\}}} X_T(A,z_T^A)) \\
&- \sum_{T,e_T} \sum_{A\ni(T,e_T)} P_T(A,z_T^A)(X_T(A,z_T^A) - \sum_{(z_j^A)_{j\notin T}} X(A,z_N^A)) \\
&+ \sum_A \sum_{\mathbf{s}_A} \phi(A,\mathbf{s}_A,z_N^A) \left[\sum_i z_i^A - \sum_{(T,e_T)\in A} q_T(\mathbf{s}_A)\right] X(A,z_N^A) \\
&- \sum_i \sum_{T\ni i} \sum_{e_T} \sum_{A\ni(T,e_T)} \sum_{e_i'} \sum_{z_T^A} \alpha_i(e_i'|A,z_T^A), DG_i(e_i'|A,z_i^A) X_T(A,z_T)
\end{aligned}
$$

A.2 Other Proofs

From now on, I use z_i, z_T, and z_N instead of z_i^A, z_T^A, and z_N^A when there is no possibility of confusion.

A.2.1 Proof of Proposition 3 (Complementary Slackness)

The results are direct application of Complementary Slackness of linear programming.

A.2.2 Proof of Lemma 1

Let $X_i(\cdot)$, $X_T(\cdot)$, and $X(A,z_N)$ be an optimal solution of the planner's problem. By complementary slackness of linear programming, I get

$$
\left[y_i - \sum_{\mathbf{s}_A\in\mathbf{S}_A} v_i(z_i^{\mathbf{s}_A}) \Pr(\mathbf{s}_A;A) - C_i(e_i) + p_i(A,z_i)\right] X_i(A,z_i) = 0
$$

Summing it over A and z_i^A proves the equality for the first line.

From complementary slackness of linear programming,

$$
\left[\sum_{i\in T} p_i(A,z_i) - \bar{P}_T(A,z_T) - \sum_{i\in T}\sum_{e_i'} \alpha_i(e_i'|A,z_{T\setminus\{i\}}) DG_i(e_i'|A,z_i)\right] X_T(A,z_T) = 0
$$

Summing them over (z_T),

$$
\sum_{i\in T}\sum_{z_i} p_i(A,z_i)X_i(A,z_i) - \sum_{z_T} \bar{P}_T(A,z_T)X_T(A,z_T) - \sum_{i\in T}\sum_{z_T}\sum_{e_i'} \alpha_i(e_i'|A,z_{T\setminus\{i\}}) DG_i(e_i'|A,z_i) X_T(A,z_T) = 0
$$

It is left to show the last term is zero. The last term can be written

$$\sum_{i\in T}\sum_{z_{T\setminus\{i\}}}\sum_{e'_i}\alpha_i(e'_i|A,z_{T\setminus\{i\}})\sum_{z_i}DG_i(e'_i|A,z_i)X_T(A,z_T)=0$$

From duality of linear programming, $\alpha_i(e'_i|A,z_{T\setminus\{i\}})\sum_{z_i}DG_i(e'_i|A,z_i)X_T(A,z_T)=0$. Therefore, the desired result is shown.

For the last statement, the result follows directly from Proposition 3 since

$$[\sum_{T\in A}q_T(\mathbf{s})-\sum_i z_i^{\mathbf{s}_A}]\phi(A,\mathbf{s},z_N)X(A,z_N)=0$$

A.2.3 Proof of Lemma 2

Suppose not, i.e. $\sum_A\sum_{z_i}p_i(A,z_i)X_i(A,z_i)\neq 0$ for some i. Define new variables

$$\begin{aligned}\hat{y}_k &:= y_i+\sum_{\tilde{A}}\sum_{\tilde{z}_k}p_i(\tilde{A},\tilde{z}_k)X_i(A,z_i)\\ \hat{p}_k(A,z_i) &:= p_i(A,z_i)-\sum_{\tilde{A}}\sum_{\tilde{z}_k}p_i(\tilde{A},\tilde{z}_k)X_i(A,z_i)\\ \hat{P}_T(A,z_T) &:= \bar{P}_T(A,z_T)-\sum_{i\in T}\sum_{\tilde{A}}\sum_{\tilde{z}_i}p_i(\tilde{A},\tilde{z}_i)X_i(A,z_i)\end{aligned}$$

Then the first and the second dual constraints hold trivially. The following equalities prove that the third constraint holds.

$$\begin{aligned}&\sum_{(T,e_T)\in A}\hat{P}_T(A,z_T)=\sum_{(T,e_T)\in A}\bar{P}_T(A,z_T)\\ \iff&\sum_i\sum_A\sum_{z_i}p_i(A,z_i)X_i(A,z_i)=0\\ \iff&\sum_A\sum_{(z_i)}\sum_{(T,e_T)\in A}\bar{P}_T(A,z_T)X(A,z_N)=0\text{ by lemma 2}\end{aligned}$$

Therefore, the result follows. ∎

A.2.4 Proof of Lemma 3

It is enough to show

$$\sum_{i\in T}\sum_{A\ni(T,e_T)}\sum_{z_T}\sum_{e'_i}\alpha_i(e'_i|A,z_{T\setminus\{i\}})DG_i(e'_i|A,z_i)q_T(A,z_T)\leq 0,$$

which is true since

$$\sum_{A\ni(T,e_T)}\sum_{z_i}DG_i(e'_i|A,z_i)q_T(A,z_T)\leq 0$$

from the incentive compatibility constraints. ∎

A.2.5 Proof of Lemma 4

Proof. For $Q(A,(z_i))$ such that $X(A,(z_K)) > 0$, it is trivial from proposition 2. For $Q(A,(z_i))$ such that $X(A,z_N) = 0$, it is enough to show

$$\phi(A,\mathbf{s},(z_i))\left[\sum_{T\in A} q_T(\mathbf{s}) - \sum_i z_i^{\mathbf{s}}\right] \geq 0,$$

which is trivially true from the complementary slackness of linear programming. ∎

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