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Economic Allocations with Permanent Heterogeneities: An Example

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Abstract Permanent heterogeneities in initial asset, preference for leisure, and individual productivity are introduced in a dynamic general equilibrium model and the functional relationship between the heterogeneities and economic allocations is studied. With the permanent heterogeneities, we show that a unique saddle path equilibrium is supported by an initial asset distribution. Depending on the heterogeneities, the distributions of consumption, income, asset and labor income may differ incredibly. However, the associated aggregate economies are not different very much.

Keywords Heterogeneities, Saddle Path Equilibria, Income Distribution, Aggregate Productivity

JEL Classification E13, E21, E25, O40

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1. INTRODUCTION

Until recently, distributional issues have not often been embedded in depth in dynamic stochastic general equilibrium framework. In an important contribution, Krusell and Smith (1998) introduce temporary individual productivity shocks together with a borrowing constraint to reproduce the wealth and income distribution in the U.S. economy.[1] They show that although it is not easy to reproduce in their model the upper tail shape of the U.S. economy, it is overall an exciting starting point. Castañeda, Díaz-Giménez and Ríos-Rull (2003) introduce a mix of the dynastic and lifecycle features in a model. They also introduce a borrowing constraint together with some of the quantitative properties of the U.S. social security system including progressive tax. The temporary employment shocks are individual specific and uninsurable. With the features, they are able to reproduce in their model the U.S. distributions of earnings and wealth almost exactly.

These contributions are very exciting. The urgent objective of building these models is to reproduce income and wealth distribution comparable to those in the U.S. economy. Temporary shocks together with borrowing constraint serve very well for that purpose.[2] However, if one would like to say something about consumption distribution, which is a more direct measure of welfare than income or wealth,[3] one has to rely on the guiding principle of consumption in modern macroeconomics like the life-cycle/permanent income hypothesis developed by Modigliani and Brumberg (1954) and Friedman (1957). According to the theory, only permanent changes in current income affect consumption and temporary changes have little effect. As far as income contains temporary component, it does not represent the lifetime resources very well and consumption or permanent income is a more accurate measure of lifetime resources and welfare.

[1]Javier Díaz-Giménez, Vincenzo Quadrini, and José-Victor Ríos-Rull (1997), and Santiago Budría, Javier Díaz-Giménez, Vincenzo Quadrini, and José-Victor Ríos-Rull (2002) find various interesting facts on the U.S. distributions of labour earnings, income, and wealth. Kopczuk and Saez (2004), Picketty and Saez (2005) use individual estate tax return data in U.S. to look at changes in top wealth shares from 1916 to 2000. Kennickell (2006) uses the 2004 wave of the Survey of Consumer Finances to study changes in the distribution of the wealth of U.S. families over the period of 1989-2004.

[2]Using an analogous setup, Chang and Kim (2006, 2007) study aggregation and labor market issues.

[3]Slesnick (1993, 1994, 2001) argues that consumption based welfare and inequality measures in U.S. tell a quite different story from the one read from the measures based on current income. He demonstrates that consumption based inequality indexes actually either have not chaneged very much or decreased over the post war period.

There is a plethora of evidences telling that all changes in current income are neither temporary nor permanent.[4] Moffitt and Gottschalk (2002)[5] show that transitory variance of male earnings in the United States rose strongly in 1980s. From 1974 to 1991, the transitory variance accounted for about seventy percent of the overall cross-sectional variance increase of male annual earnings. However, there has been a rapid reduction in the transitory variance after 1991. As far as all variations in current income are not permanent, consumption distribution may differ from that of income significantly.[6]

There are two conflicting lines of studies in the literature on the dynamics of consumption and income distribution. Krueger and Perri (2003, 2006) find that while the cross-sectional variation in earnings and disposable income has increased significantly, the overall dispersion in consumption has not changed very much. Over the period from 1972 and 1998 the standard deviation of the log of after tax labor income increased by twenty percents while that of log consumption increased by less than two percents. On the other hand, assessing the distribution of consumption in the 1960s, 1970s, and 1980s, Cutler and Katz (1991, 1992) and Johnson, Smeeding and Torrey (2005) find that changes in the distribution of consumption correspond closely to changes in the distribution of income and hence reach the conclusion that changes in family income appear to reflect primarily changes not in transitory income but in permanent income.

In sum, the evidence in the literature requires us to develope a model with both temporary and permanent differences in income. However, there are not many papers having permanent income differences in the dynamic general equilibrium literature.[7] This paper introduces permanent heterogeneities in a stochastic dynamic general equilibrium model. The heterogeneities are in inititial endowment, preference for leisure and individual productivity. The households are assumed to live infinitely. To have closed form solutions, we assume a stripped down version of the Long and Plosser (1983) economy.

With the setup, we ask what role permanent heterogeneities play for vari-

[4]See Attanasio (1999) for a survey.

[5]See Moffitt and Gottschalk (1995) for earlier covariance decomposition and Blundell and Preston (1998) for U.K. evidence.

[6]Of course, the extent that the pattern of consumption changes deviates from that of income variations depends on the existence of market and nonmarket devices of smoothing consumption across states and time. For example, Mace (1991) finds some evidence for complete consumption insurance. However, most of the studies on consumption support partial insurance. See, for example, Cochrane (1991), Townsend (1994), Nelson (1994), Attanasio and Davis (1996), and Blundell, Pistaferri and Preston (2003).

[7]Satyajit Chatterjee (1994) studies the transitional dynamics and the distribution of wealth in a neoclassical growth model with permanent personal differences in initial endowment.

ous distributions of an economy. Heterogeneities in the paper are the sources of difference in permanent income and hence affect consumption and other related distributions. Manipulating initial asset distribution, we can have very disperse equilibrium consumption distribution as well as a consumption which is equal across the agents. However, the equal consumption equilibrium requires the dispersion of the asset distribution which is implausibly large and many households have to have initial debt. Although the heterogeneities make the economies look incredibly different in terms of economic allocations, the associated aggregate economies are not different very much.

The paper consists of the following sections. In the next section, we will present the model with permanent heterogeneities and solve the model. We will prove in the section that the asset distribution relative to the aggregate counterpart is time invariant. In the third section, we will look at a steady state equilibrium and see the role of permanent heterogeneities. In the fourth section, we conclude.

2. LONG AND PLOSSER ECONOMY WITH PERMANENT HETEROGENEITIES

A stripped down version of Long and Plosser (1983) economy with heterogeneities is considered. We introduce heterogeneities in initial endowment, preference for leisure and labor productivity. However, the initial endowment in this section is not independent of other heterogeneities, which will be made clear later. We will let e denote labor productivity and b preference for leisure. We will call the two element vector $j = (e,b)$ the type. There can be many agents of the same type. They are born with initial capital stock, $k(j)_0$.[8]

Assume the following expected lifetime utility for an agent of type j.

$$U(j) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left[c\left(j\right)_t\right] + b \ln\left[1 - n\left(j\right)_t\right] \right\} \right\}, \tag{1}$$

where E_0 is the expectations operator conditional on the initial period information Ω_0, $c(j)_t$ is period t consumption and $n(j)_t$ is the amount of working hours. $0 < \beta < 1$ is the utility discounting factor.

We assume that the productivity and the preference parameters are independent of each other. $F(e)$ and $G(b)$ denote the distribution functions of e and b respectively. $F(e)$ and $G(b)$ are assumed to be time invariant and have positive

[8] All agents of type j are assumed to have the same initial asset.

supports. Each agent is endowed with one unit of time in each period. The type of an agent is a public information.

An agent of type j maximizes the expected lifetime utility subject to the following budget constraint.

$$c(j)_t + k(j)_{t+1} = w(e)_t n(j)_t + r_t k(j)_t, \tag{2}$$

where $w(e)_t$ and r_t are the wage rate for the agents with productivity e and the rental rate of capital respectively. To have a closed form solution, we assume one hundred percent depreciation of capital stock. Note that the agents with the same productivity face the same wage rate and all agents face the same rental price of capital. The first order conditions for the utility maximization can be obtained as follows.

$$\frac{b}{1-n(j)_t} = w(e)_t \frac{1}{c(j)_t} \tag{3}$$

$$\frac{1}{c(j)_t} = \beta E_t \left\{ r_{t+1} \frac{1}{c(j)_{t+1}} \right\} \tag{4}$$

and (2). The economic meaning of the conditions is straightforward. Each agent maximizes her utility by equating marginal cost and benefit of working hours and saving respectively.

The firm produces output according to the Cobb-Douglas production function.

$$Y_t = A_t K_t^{\theta} N_t^{1-\theta}, \tag{5}$$

where A_t is the aggregate productivity shock, which is assumed to follow an AR(1) process.

$$\ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t, \;\; \varepsilon_t \sim i.i.d.\, N(0, \sigma_\varepsilon^2) \tag{6}$$

K_t is aggregate capital stock and N_t is aggregate working hours in efficiency unit.

$$N_t = \int_\Theta eN(e)_t dF(e), \tag{7}$$

where Θ is the support of the productivity distribution and $N(e)_t$ is the firm's demand for the hours of workers with efficiency e.

The firm maximizes the following profit function.

$$\Pi_t = A_t K_t^{\theta} \left(\int_\Theta eN(e)_t dF(e) \right)^{1-\theta} - \int_\Theta w(e)_t N(e)_t dF(e) - r_t K_t \tag{8}$$

The first order conditions for the profit maximization can be obtained as follows.

$$w(e)_t = (1-\theta)eA_tK_t^{\theta}N_t^{-\theta} \tag{9}$$

$$r_t = \theta A_t K_t^{\theta-1} N_t^{1-\theta} \tag{10}$$

The equilibrium for the economy can be defined as follows.

Definition. Given the distribution of initial capital stock $\{k(j)_0\}$ and the sequence of aggregate productivity shocks $\{A_t\}_{t=0}^{\infty}$, a *recursive competitive equilibrium* is defined by a sequence of individual choices:

$$\{c(j)_t, n(j)_t, k(j)_{t+1}\}_{t=0}^{\infty}, \tag{11}$$

a sequence of firm's choices:

$$\{Y_t, K_t, N(e)_t\}_{t=0}^{\infty}, \tag{12}$$

and a sequence of prices:

$$\{w(e)_t, r_t\}_{t=0}^{\infty} \tag{13}$$

such that

(1) the individual choices (11) solve the households' utility maximization,
(2) the firm's choices (12) solve the firm's profit maximization, and
(3) the prices (13) clear the markets in each period:

$$\int_{\Delta} n(j)_t dG(b) = N(e)_t \text{ for } \forall e$$

$$\int_{\Theta}\int_{\Delta} k(j)_t dG(b)dF(e) = K_t$$

$$\int_{\Theta}\int_{\Delta} c(j)_t dG(b)dF(e) + \int_{\Theta}\int_{\Delta} k(j)_{t+1} dG(b)dF(e) = Y_t,$$

where Δ is the support of the distribution function $G(b)$.

The competitive equilibrium can be obtained analytically as follows. First, we make a guess for the individual consumption as follows.[9]

[9] A referee has suggested an alternative solution process. First of all, use the following guess for the solution:

$$c(j)_t = \alpha_j C_t, \;\; C_t = (1-\beta\theta)Y_t.$$

Then the Euler equation for the saving implies the solution for the capital stock (saving).

$$K_{t+1} = \beta\theta Y_t,$$

$$c(j)_t = (1-\beta\theta)\left(\frac{z_1}{z_2}\right)A_t K_t^{\theta} N_t^{1-\theta}, \tag{14}$$

where z_1 is the following time invariant function of b, e and any vector of exogenous variables, x:

$$z_1 = \phi(j,x), \tag{15}$$

and z_2 is assumed to be the mean of z_1:

$$z_2 = E(z_1) = \int_{\Xi}\int_{\Theta}\int_{\Delta} z_1 dGdFdH, \tag{16}$$

where $H(x)$ is the distribution function of x, whose support is Ξ. To make the problem simple, we assume that $j = (b,e)$ and x are independent of each other.[10]

Aggregating (14), we can have the aggregate consumption and saving.

$$C_t = (1-\beta\theta)A_t K_t^{\theta} N_t^{1-\theta} \tag{17}$$

$$K_{t+1} = \beta\theta A_t K_t^{\theta} N_t^{1-\theta} \tag{18}$$

Note here that (10), (14), and (18) trivially satisfy the condition for individual saving (4).

Given the presupposed solution for the individual consumption, the hours of work of a type j agent can be obtained from the condition for the working hours.[11]

$$n(j)_t = \frac{(1-\theta)z_2 e + (1-\beta\theta)\left(z_3 e - bz_1 \int_{\Theta} edF\right)}{\left[(1-\theta)z_2 + (1-\beta\theta)z_3\right]e}, \tag{19}$$

where z_3 is defined as follows.

$$z_3 = \int_{\Xi}\int_{\Theta}\int_{\Delta} bz_1 dGdFdH. \tag{20}$$

where $r_{t+1} = \theta Y_{t+1}/K_{t+1}$ is used. Using these results in the first order condition for working hours and in the household budget constraint, we can have the solution for individual working hours and the law of motion of individual capital stock. The law of motion of individual capital stock can be used to have the initial asset distribution satisfying individual transversality condition.

However, it is not difficult to show that this alternative solution process produces the same equilibrium as in the text.

[10] In fact, independence among the three parameters is not important for the result. However, z_1 has to be exogenous and time invariant for the closed form solution. If, for example, z_1 is a function of individual choices like $n(j)_t$, we cannot in general solve the problem analytically.

[11] See the Appendix for the derivation.

By aggregating the individual hours, we can have the aggregate hours in efficiency unit as follows.

$$N_t = \frac{(1-\theta)z_2 \int_\Theta e dF}{(1-\theta)z_2 + (1-\beta\theta)z_3} \tag{21}$$

Hence the individual working hours and the aggregate hours are time invariant, which will be denoted as $n(j)$ and N without time subscript respectively.

Now we have to determine the initial asset distribution which supports the presupposed solution. In fact, we can determine the initial asset distribution by probing the transversality condition as follows.[12]

$$\begin{aligned} \frac{k(j)_t}{K_t} &= \frac{k(j)_0}{K_0} \\ &= \left(\frac{\beta}{1-\beta}\right)\left[\frac{(1-\beta\theta)z_1 N - (1-\theta)en(j)z_2}{\beta\theta z_2 N}\right] \end{aligned} \tag{22}$$

We have to note the following in the solution. First, the asset distribution relative to the aggregate asset is not changing over time. Second, a unique income, consumption, working hours and asset distribution are associated with an equilibrium. Third, any distribution of consumption satisfying (14) can be supported as a market equilibrium, which is associated with a unique asset distribution. Fourth, since factor inputs in production depend on asset distribution, output also depends on it.

3. A STEADY STATE EXAMPLE

The steady state can be obtained as follows.

$$\begin{aligned} \frac{K}{N} &= (\beta\theta A)^{\frac{1}{1-\theta}} \\ c(j) &= D(c)(1-\beta\theta)A(\beta\theta A)^{\frac{\theta}{1-\theta}}N, \\ k(j) &= D(k)K \\ y(j) &= D(y)A(\beta\theta A)^{\frac{\theta}{1-\theta}}N \\ y^w(j) &= D(w)A\,(\beta\theta A)^{\frac{\theta}{1-\theta}}\,N, \end{aligned}$$

where $D(c)$, $D(k)$, $D(y)$ and $D(w)$ are defined as follows.

$$D(c) = \frac{c(j)}{C} = \frac{z_1}{z_2} \tag{23}$$

[12]See the Appendix for the derivation.

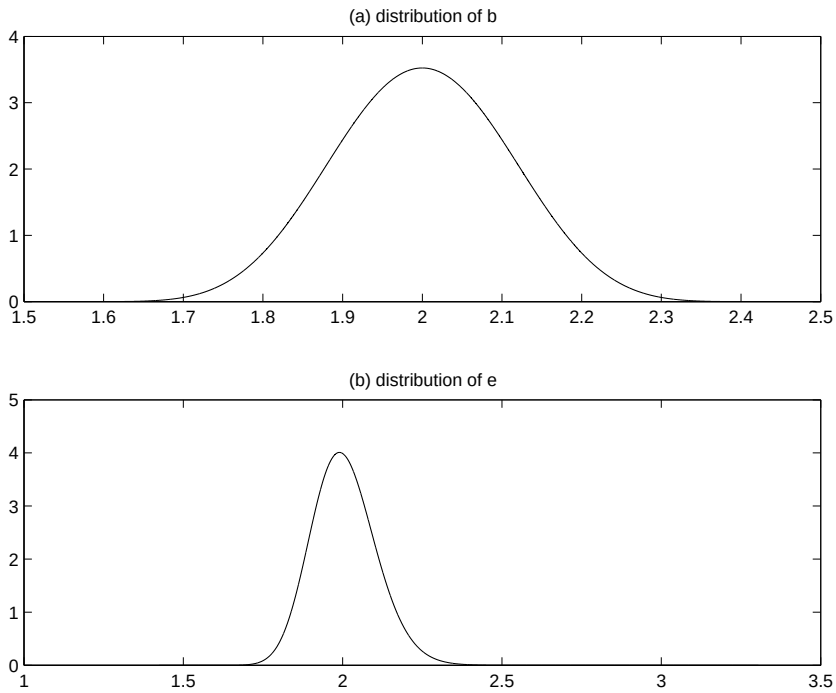

Figure 1: Underlying Distribution

$$D(k) = \frac{k(j)}{K} = \left(\frac{\beta}{1-\beta}\right)\left\{\frac{(1-\beta\theta)z_1N-(1-\theta)en(j)z_2}{\beta\theta z_2N}\right\} \tag{24}$$

$$D(y) = \frac{y(j)}{Y} = (1-\theta)\frac{en(j)}{N} + \theta ND(k) \tag{25}$$

$$D(w) = \frac{y^w(j)}{Y^w} = (1-\theta)\left(\frac{en(j)}{N}\right), \tag{26}$$

where Y^w and $y^w(j)$ are aggregate and individual wage income. Define also $D(n) = n(j)/N$ for working hours.

To look into the economic allocations in the steady state, we calibrate the parameter values as follows.

$$\beta = 0.99,\ \theta = 0.36,\ A = 1$$

These numbers have been used in the literature numerous times.[13] However, we do rarely have any references for the distributions of b and e and thus we tena-

[13]See, for example, Prescott(1986) and Cooley(1995).

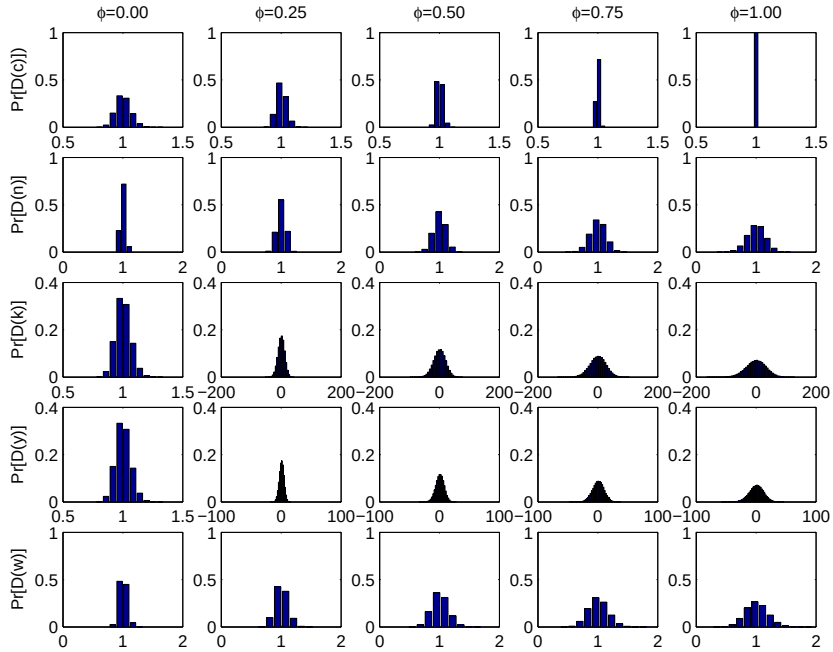

Figure 2: Permanent Heterogeneities and Distribution

tively use the following distributions. Since working hours are strictly bounded, $(b-1.5)$ is assumed to have the beta distribution with parameters $\alpha=\gamma=10$. Hence b is distributed symmetrically[14] and its support is the unit interval between 1.5 and 2.5.[15] On the other hand, we assume that $e-1$ has a log normal distribution with mean 1 and standard deviation 0.75. Hence the mean of e is 2 and its support is $(1,\infty)$. The distributions of b and e are depicted in Figure 1.

As can be seen in the steady state expressions, the key to understanding distributions of the economy is the distribution factors in (23)–(26) and $D(n)$,

[14]The mean and variance of $b-1.5$ is the following.

$$E(b)=\frac{\alpha}{\alpha+\gamma},\ var(b)=\frac{\alpha\gamma}{(\alpha+\gamma+1)(\alpha+\gamma)^2}$$

See Mood, Graybill and Boes(1974) for detailed derivation.

[15]If the setup is of the representative agent, b is usually set to be 2, which implies the steady state working hour to be one third of time endowment.

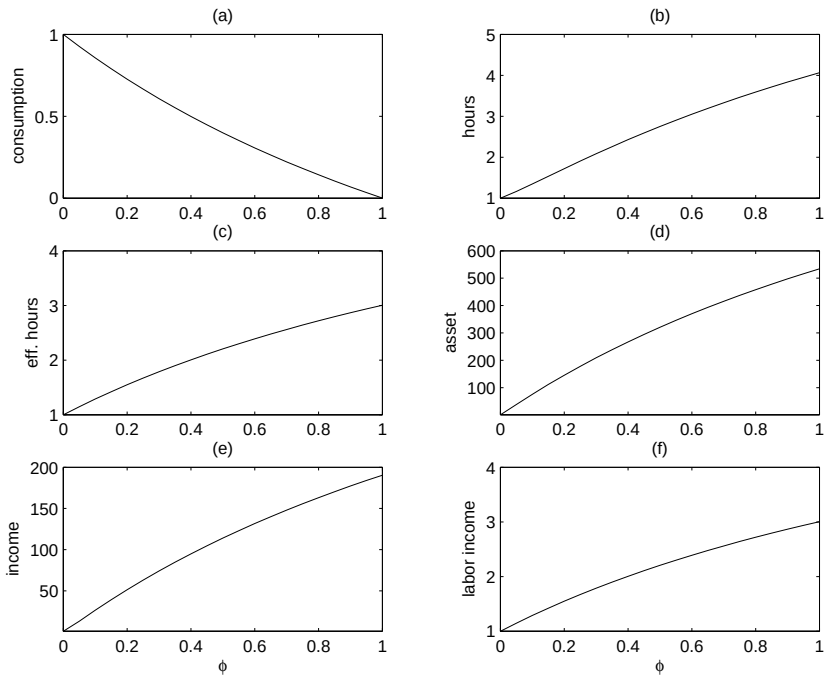

Figure 3: Relative Distribution

which depend on the definition of z_1. We define z_1 as follows.

$$z_1 = \phi + (1-\phi)\left[\frac{(1-\theta)e}{1-\theta+(1-\beta\theta)b}\right], \quad 0 \le \phi \le 1 \tag{27}$$

If $\phi = 1$, (27) returns to the case of equal consumption across agents and if $\phi = 0$, it returns to a case of unequal consumption. Hence ϕ determines the extent of consumption difference across agents. As ϕ approaches 0, consumption gets more proportional to labor efficiency and working hours.

From the assumed distributions, a set of one hundread thousand pairs of random numbers for b and e is drawn and the distribution factors are obtained. Figure 2 depicts the cases with different values for ϕ. We can clearly see the fact that the larger the value of ϕ gets and hence the smaller the dispersion of consumption distribution becomes, the more disperse are working hours, income and asset distribution. Achieving the complete equality in consumption requires a colossal dispersion of asset and income distribution.

Reducing consumption inequality to a small extent does not mean that there will not be large increase in asset and income dispersion. Figure 3 describes

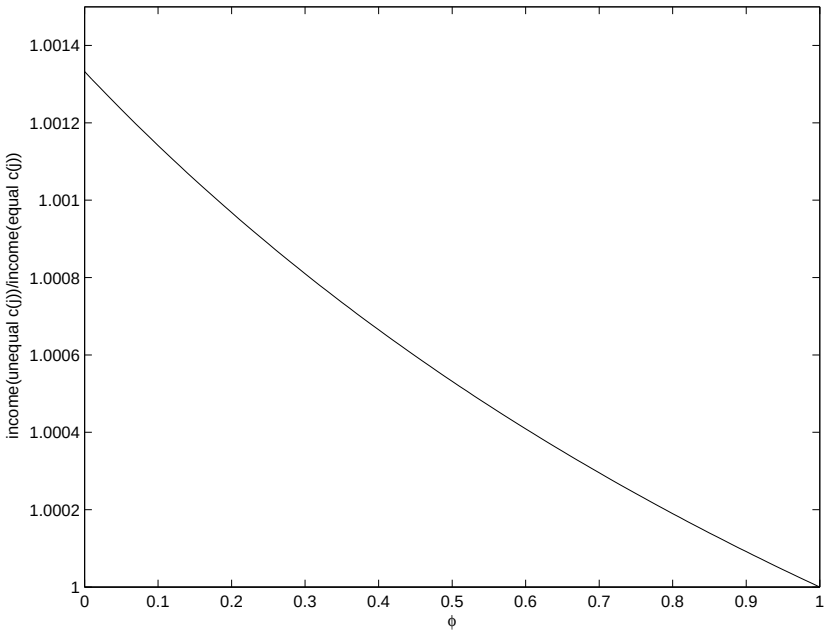

Figure 4: Distributions and Aggregate Economies

the increase in standard deviations when the value of ϕ increases. The vertical axis of the panels in Figure 3 measures the ratio of standard deviation relative to that in the case that $\phi = 0$. The figure shows that increasing the value of ϕ by twenty percent requires the dispersion of working hours in physical and efficiency unit increasing about two times and one and half times respectively. The same increase in the value of ϕ increases asset dispersion by about two hundred times and income dispersion by about fifty times. Hence reducing consumption inequlity by redistributing asset holdings involves huge cost in terms of asset and income dispersion.

Note here that since the aggregate hours (21) depend on ϕ, so does aggregate output. To see how the aggregate economy depends on ϕ, we obtain the ratio of aggregate output when ϕ is less than one relative to that when ϕ is one and depict the result in Figure 4. We can see in Figure 4 that aggregate economies are not that different. The figure shows that aggregate income when $0 \leq \phi < 1$ is always greater than that when $\phi = 1$ and that the difference between the two incomes is shrinking as the value of ϕ increases. The ratio of the two income when $\phi = 0$

is 1.0014, which means that aggregate income when $\phi = 0$ is 0.14% higher than that when $\phi = 1$ and hence aggregate economies are not much variant with the value of ϕ. If we consider the extent of the increases in the dispersion of asset and income distribution with the value of ϕ, the result is striking.

4. CONCLUSION

Atkinson (1997) observes that the subject of income distribution has been much out in the cold for much of the last century. In fact, massive changes in the labor market and redistributive programs in the postwar period did not alter earnings and income distributions very much up to early 1980s and hence distributional issues did not attract much attention in the profession. The lack of interest in distributional issues changed in the early 1980s since there appeared notable changes in the dynamics of earnings, income and wealth distributions.

To reproduce the income and wealth distribution in U.S., they have introduced individual specific temporary income shocks and borrowing constraint in dynamic general equilibrium models. With some fiscal arrangements, temporary shocks and borrowing constraint have served very well for that purpose. On the contrary, permanent heterogeneities are not considered seriously in the literarure. The reason is that the income and wealth distributions under permanent heterogeneities are implausible. However, the empirical literature on income and wealth distribition suggests that it is better to combine temporay heterogeneities with permanent ones in a model to better understand the evolution of those distributions.

In this paper, permanent heterogeneities in initial asset, preference for leisure, and individual productivity are introduced in a dynamic general equilibrium model and the functional relationship between the heterogeneities and economic allocations is studied. To have closed form solutions, we have used the Long and Plosser(1983) economy. With the permanent heterogeneities, we show that a unique saddle path equilibrium is supported by an initial asset distribution, which means that a functional relationship exists between the heterogeneities and economic allocations. In fact, individual asset position relative to the aggregate counterpart is not changing over time. Depending on the heterogeneities, the distributions of consumption, income, asset and labor income may differ incredibly. However, the associated aggregate economies are not different very much.

APPENDIX: DERIVATION OF THE EQUILIBRIUM IN SECTION 2

Given the presupposed solution for the individual consumption, the hours of work of a type j agent can be obtained from the condition for the working hour.

$$\begin{aligned}\frac{b}{1-n(j)_t} &= \frac{w(j)_t}{c(j)_t} \\ &= \frac{(1-\theta)ez_2A_tK_t^{\theta}N_t^{-\theta}}{(1-\beta\theta)z_1A_tK_t^{\theta}N_t^{1-\theta}} \\ &= \frac{(1-\theta)ez_2}{(1-\beta\theta)z_1N_t}\end{aligned} \tag{A.1}$$

Hence we can have the following individual working hours.

$$n(j)_t = 1 - \frac{(1-\beta\theta)bz_1N_t}{(1-\theta)ez_2} \tag{A.2}$$

By aggregating individual hours in efficiency unit, we can have the aggregate counterpart.

$$N_t = \frac{(1-\theta)z_2\int_{\Theta}edF}{(1-\theta)z_2+(1-\beta\theta)z_3} = N, \tag{A.3}$$

where z_3 is defined as:

$$z_3 = \int_{\Xi}\int_{\Theta}\int_{\Delta} bz_1dGdFdH. \tag{A.4}$$

Aggregate hours are time invariant, which are denoted as N without time subscript. Susbtituting (A.3) in (A.2), we can rewrite the individual hours.

$$\begin{aligned}n(j)_t =& 1 - \frac{(1-\beta\theta)bz_1\int_{\Theta}edF}{[(1-\theta)z_2+(1-\beta\theta)z_3]\,e} \\ =& \frac{(1-\theta)z_2e+(1-\beta\theta)\left(z_3e-bz_1\int_{\Theta}edF\right)}{[(1-\theta)z_2+(1-\beta\theta)z_3]\,e} = n(j)\end{aligned} \tag{A.5}$$

The individual specific working hours are also time invariant, which are denoted as $n(j)$.

Probing the transversality condition, we can determine the initial asset distribution which supports the presupposed solution. Using wage rate, rental price of capital and consumption in the budget constraint, individual saving can be

obtained as follows.

$$\begin{aligned} k(j)_{t+1} &= (1-\theta)eA_tK_t^{\theta}N^{-\theta}n(j) + \theta A_tK_t^{\theta}N^{1-\theta}k(j)_t - c(j)_t \\ &= -z_4A_tK_t^{\theta}N^{-\theta} + \theta A_tK_t^{\theta-1}N^{1-\theta}k(j)_t, \\ z_4 &= -(1-\theta)en(j) + (1-\beta\theta)\left(\frac{z_1}{z_2}\right)N \end{aligned} \tag{A.6}$$

Now, we use (A.6) in the transversality condition to have the the initial asset distribution supporting the presupposed solution as an equilibrium.

The transversality condition for j can be written as follows.

$$\lim_{t\to\infty}\beta^t E_0\left\{\frac{1}{c(j)_t}k(j)_{t+1}\right\} \to 0 \tag{A.7}$$

If we use (14) and (A.6) in (A.7), we can have the following expression.

$$\beta^t\frac{1}{c(j)_t}k(j)_{t+1} = \beta^t\left[-\frac{z_2z_4}{(1-\beta\theta)z_1N} + \frac{\theta z_2}{(1-\beta\theta)z_1}\left(\frac{k(j)_t}{K_t}\right)\right], \tag{A.8}$$

Moreover, dividing (A.6) by (18) gives us the following difference equation.

$$\frac{k(j)_{t+1}}{K_{t+1}} = z_5 + \left(\frac{1}{\beta}\right)\left(\frac{k(j)_t}{K_t}\right), \quad z_5 = -\frac{z_4}{\beta\theta N} \tag{A.9}$$

Substituting repeatedly, we have the solution as follows.

$$\begin{aligned} \frac{k(j)_{t+1}}{K_{t+1}} =& z_5\sum_{j=0}^{t}\left(\frac{1}{\beta}\right)^j + \left(\frac{1}{\beta}\right)^{t+1}\left(\frac{k(j)_0}{K_0}\right) \\ =& \frac{z_5}{1-1/\beta} + \left(\frac{1}{\beta}\right)^{t+1}\left(\frac{k(j)_0}{K_0} - \frac{z_5}{1-1/\beta}\right) \end{aligned} \tag{A.10}$$

Hence the following has to be satisfied for the transversality condition to hold.

$$\begin{aligned} \frac{k(j)_0}{K_0} =& \frac{k(j)_t}{K_t} = \frac{z_5}{1-1/\beta} \\ =& \left(\frac{\beta}{1-\beta}\right)\left\{\frac{(1-\beta\theta)z_1N - (1-\theta)en(j)z_2}{\beta\theta z_2N}\right\} \end{aligned} \tag{A.11}$$

That is, the asset distribution relative to the aggregate counterpart is time invariant.

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