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Liquidity and Selection in Asset Markets with Search Frictions*

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Abstract I construct a model of an asset market subject to search frictions, in an environment where both asset liquidity and market composition are determined endogenously. The analysis predicts that higher asset prices resulting from exogenously higher asset earnings imply: (i) a shorter search duration for sellers (higher liquidity), (ii) a shorter owner tenure before listing assets for resale (turnover), (iii) a higher stock of buyers, and (iv) a higher share of the asset stock traded (trade volume). Asset price-earnings ratios respond positively to earnings because liquidity premia respond to the size of earnings relative to the costs of search. I show that liquidity effects and selection effects reinforce each other in the presence of search frictions.

Keywords liquidity, selection, asset market, search

JEL Classification G1, D83, R31.

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1. INTRODUCTION

I construct a model of an asset market subject to search frictions. Search frictions introduce two dimensions to asset pricing. First, assets are not resalable instantaneously, and the arrival rate of matches with buyers determines the liquidity of the asset which affects its price. Second, the productivity or earnings of assets can vary among the population of sellers and buyers, such that the sales price depends on the match specific productivity of buyers and sellers. When the distribution of buyer and seller productivity varies, selection affects the *average* sale price of assets.

In the environment I consider, the earnings of assets consists of an aggregate component and an owner specific component. Both asset liquidity and market composition are determined endogenously. The analysis predicts that higher asset prices resulting from exogenously higher asset earnings (the aggregate component) imply: (i) a shorter search duration for sellers and a longer search duration for buyers (higher liquidity), (ii) a shorter owner tenure before listing assets for resale (turnover), (iii) a larger stock of buyers, and (iv) a higher share of the asset stock traded (trade volume).

Asset price-earnings ratios respond *positively* to earnings because liquidity premia respond to the size of earnings *relative* to the costs of search. Thus, an explicit characterization of liquidity premia can account for an excess volatility of asset prices in response to earnings. Furthermore, the analysis demonstrates how productivity shocks have amplified effects in the search environment via the trade volume of assets. In generating each of these outcomes, I show that liquidity effects and selection effects reinforce each other in the presence of search frictions.

These predictions are difficult to reconcile with the standard frictionless framework where assets are traded instantaneously, and there is a single asset price. By introducing search frictions, I demonstrate how these observations can be reconciled in a parsimonious way.

The conditions under which the decentralized outcomes under search frictions are socially efficient is discussed along with the robustness of the results to various modelling assumptions. The implications of the model are discussed in the context of the market for owner occupied housing. However, the insights of the model are applicable to other asset markets where search frictions are prevalent (other examples include cars, capital goods, and financial assets traded in over-the-counter markets). Numerical simulations of the model highlight the potential significance of search frictions in characterizing asset prices and asset markets.

The existing search-theoretic literature on financial markets includes Duffie, Gârleanu and Pedersen (2005), Gârleanu (2006), Lagos and Rocheteau (2009), Miao (2006), Rust and Hall (2003), Spulber (1996), and Weill (2007).[1] The existing search-theoretic literature specific to housing markets includes Arnott (1989), Krainer (2001), Krainer and LeRoy (2002), Ngai and Tenreyro (2008), Novy-Marx (2009), Wheaton (1990), and Williams (1995). Differentiating features of my analysis are the endogenous determination of liquidity and its interaction with the endogenous selection of buyers and sellers.[2]

The ingredients of the analysis borrow from the canonical search models of unemployment pioneered by Diamond-Mortensen-Pissarides.[3] Specially, I demonstrate how the structure of such models can be adapted to the analysis of asset markets. This is not obvious prima facie since in the analysis of unemployment, there are two distinct populations representing each side of the market (firms and workers). In the analysis of asset markets, agents switch from one side of the market to another (today's buyers are tomorrow's sellers). To facilitate comparison, I adopt their notation and language whenever it is deemed appropriate.

Section 2 introduces the benchmark model. Section 3 discusses extensions and alternative specifications. Section 4 relates predictions of the model to empirical features of the market for owner occupied housing, and section 5 conducts numerical simulations. The last section concludes.

2. MODEL

All agents are risk-neutral and infinitely lived, with time preferences determined by a constant discount rate $r > 0$. The productivity (or earnings) of an owner-asset match is $\pi x, x \in [0,1]$. In the case of owner occupied housing, this

[1]In the financial markets literature, the existence of bid-ask spreads of dealers who intermediate between buyers and sellers is generated in the context of random matching. In my framework this is ruled out since buyers can direct their search to sellers (and sellers can direct search to buyers) of different match productivity. However, in section 2.3.2, I discuss bid-ask spreads which are interpreted as the price differential between assets bought in spot markets and sold in friction markets by dealers with zero match productivity. Such bid-ask spreads are correlated with trade volume and speed of asset sales.

[2]More generally, I demonstrate insights obtained by using a production function approach to the match function inspired by the unemployment literature. Differences with the existing literature are discussed in the text.

[3]See Pissarides (2000) for a review of this literature. Wheaton (1990) also adopts the insights from this literature to the housing market, where he assumes buyers are also sellers to analyze changes in vacancy rates of the housing stock.

reflects the (unobserved) implicit owner utility from housing services. With Poisson arrival rate λ there is a new draw of match specific productivity x from a time invariant distribution $F(x)$ and density $f(x)$, where the expected x, $E(x) \equiv \bar{x}$.[4] $\pi > 0$ is the aggregate component of the match productivity which is common to all agents.

New matches have the productivity of the buyer-asset match which is constant at $\pi y = \pi$, i.e. for the new buyer, match specific productivity is 1. This assumption captures the idea that new buyers search for *ideal* matches with assets and match productivity can deteriorate after the match is formed.[5] Thus, the analysis will focus on the heterogeneity of sellers valuations of the asset, by keeping the ex ante buyer valuation constant at this level. As in Duffie, Gârleanu and Pedersen (2005), agents can trade only discrete units of the asset normalized to one.[6]

I consider outcomes when buyers can *direct* their search to sellers of different match productivity. Thus, there are different submarkets for every level of seller productivity x, although from the point of view of a buyer there is only one aggregate market (since buyers are ex ante indifferent between using assets owned by different sellers).[7] This last observation about the homogeneity of assets from the point of view of *buyers* motivates an analysis of liquidity and selection effects with reference to one aggregate asset market.

Following search models of unemployment, there is a constant returns to scale match function with the stock of buyers and sellers as arguments. In submarket x, the Poisson arrival rate of matches per buyer is $Aq(\theta(x))$, $q'(\theta(x)) \leq 0$, where $\theta(x)$ is the ratio of buyers to sellers or "market tightness", and A governs the search efficiency. From the assumption of constant returns to scale, the Poisson arrival rate of matches per seller is $Am(\theta(x)) \equiv \theta(x) Aq(\theta(x))$, $m'(\theta(x)) > 0$ with strict inequality. The elasticity of the match function $\eta(\theta(x)) \equiv -\frac{q'(\theta)\theta}{q(\theta)} \in [0,1)$, where the bounds are implied by the assumption of constant returns and $m'(\theta(x)) > 0$. The stock of assets over all submarkets is exogenous

[4]The special case where $F(x)$ is degenerate at $x = 0$ such that $F(0) = 1$, can be thought of as the case of exogenous match destruction.

[5]In the case of owner occupied housing, this reflects the fact that buyers are free to search in any neighborhood and type of housing when looking to form new matches.

[6]See Lagos and Rocheteau (2009) for a model where agents can adjust their marginal holdings of an asset. The consideration of selection and liquidity in this more general case would be interesting to consider.

[7]Later I consider outcomes with non-directed search in section 3.3. I demonstrate the main qualitative results do not depend on the assumption of directed search. The benchmark model is specified with directed search since this characterizes many asset markets better and keeps the analysis transparent.

and normalized at 1.[8]

Let $c > 0$ denote the exogenous flow cost of search for a buyer, and $\kappa c \geq 0$ denote the exogenous flow cost of search for a seller. In submarket x, let $V(x)$ denote the value of being a buyer, $J(x)$ the value of being a non-selling owner, $U(x)$ the value of being a seller (an owner who has listed his asset for sale) and $P(x)$ the asset price. There is a free entry of buyers such that in every period and in each submarket $V(x) = V = 0$.

Let $\widetilde{x}$ denote the draw of productivity from the distribution $F(x)$ following a draw shock with arrival rate λ. Steady state value equations are given by

$$\begin{aligned} rV &= -c + Aq(\theta(x))(J(1) - P(x)) = 0, \qquad (1)\\ rJ(x) &= \pi x + \lambda\left(E\max\{J(\widetilde{x}), U(\widetilde{x})\} - J(x)\right),\\ rU(x) &= \pi x + \lambda\left(E\max\{J(\widetilde{x}), U(\widetilde{x})\} - U(x)\right)\\ &\quad - \kappa c + Am(\theta(x))(P(x) - U(x)). \end{aligned}$$

The flow to a buyer rV consists of the per period search cost and the capital gain $(J(1) - P(x))$ resulting from a match with a seller which occurs at rate $Aq(\theta(x))$. The flow to a non-selling owner $rJ(x)$, consists of the per period productivity and the expected capital gain $(E\max\{J(\widetilde{x}), U(\widetilde{x})\} - J(x))$ resulting from a new productivity draw which occurs at rate λ. The flow to a selling owner $rU(x)$, consists of the flow to a non-selling owner minus the per period search cost plus the capital gain $(P(x) - U(x))$ resulting from a match with a buyer which occurs at rate $Am(\theta(x))$.

Here asset buyers enter the market, become owners following a match, and then become sellers following a λ shock. Upon realizing a sale, the seller exits the asset market. It is possible that some sellers become buyers upon the sale of their asset, but this is inconsequential under the assumption of free entry of buyers since the asset price of being a buyer is zero $V = 0$. In the Appendix, I consider a discussion and extension of the model where sellers become buyers upon selling their asset (or upon listing their current asset for sale).

Let $\beta \in (0,1)$ denote the bargaining share of sellers. A buyer-seller match in submarket x determines the sale price $P(x)$ as the outcome of Nash bargaining:

$$\max_{P(x)} (P(x) - U(x))^{\beta} (J(1) - P(x))^{1-\beta}.$$

[8] Here, sellers cannot direct their search to buyers with different match productivity y. Allowing them to do so results in only those buyers with $y = 1$ participating in the market given the free entry condition. This coincides with the assumption of the benchmark model. Later in section 3.2, I allow for heterogeneity in buyer productivity through a productivity draw which occurs *after* a buyer seller have been matched.

The Nash bargaining rule implies

$$(1-\beta)(P(x)-U(x)) = \beta(J(1)-P(x)). \tag{2}$$

The match surplus between a buyer and seller is $S(x) \equiv J(1)-U(x)$ which varies with x, the match productivity of the seller.

Let R denote the "listing margin": the endogenous level of match productivity *below* which owners decide to list their asset (become sellers). All assets are listed for resale, $R=1$ iff

$$J(1) \leq U(1) \tag{3}$$

Otherwise, at an interior solution for the listing margin $R<1$, sellers have no surplus from listing their asset. The seller listing margin is then given by

$$J(R) = U(R). \tag{4}$$

This system of equations solve for $\{V, J(x), U(x), P(x), \theta(x), R\}$ given $\{\pi, c, \kappa, r, \lambda\}$, the match function $Aq(\theta)$ and distribution $F(x)$.[9]

Figure 1 shows a typical path of match specific productivity x over ownership tenure. $x=1$ for new matches, and following a λ shock a draw of $x<R$ motivates the owner to list the asset for sale. A further shock and new draw of $x>R$, which occurs before a sale is realized, motivates the owner to withdraw his listing until another draw of $x<R$ is realized following a third λ shock after which a sale occurs terminating the tenure.

The bargaining rule (2) implies the asset price is given by

$$P(x) = U(x) + \beta S(x). \tag{5}$$

The sales price is the seller's outside option plus his bargaining share of the match surplus. Note the resalability of the asset affects its price through $U(x)$ via market tightness $\theta(x)$: this is a liquidity effect. The measure of matches in submarket x is given by $Am(\theta(x))s(x)$, where $s(x)$ denotes the measure of sellers. The average sales price over a time interval is given by the weighted average of sales prices across submarkets

$$\bar{P} \equiv \frac{\int_0^R P(x) Am(\theta(x)) s(x)\, dx}{\int_0^R Am(\theta(x)) s(x)\, dx}, \tag{6}$$

[9]Note that *unexpected* changes in exogenous variables $\{\pi, c, \kappa, r, \lambda, A\}$ lead to immediate changes in endogenous variables $\theta(x), R$ to their steady state levels due to the assumption of free entry of buyers. Thus, the analysis of steady state outcomes can accommodate the presence of random shocks.

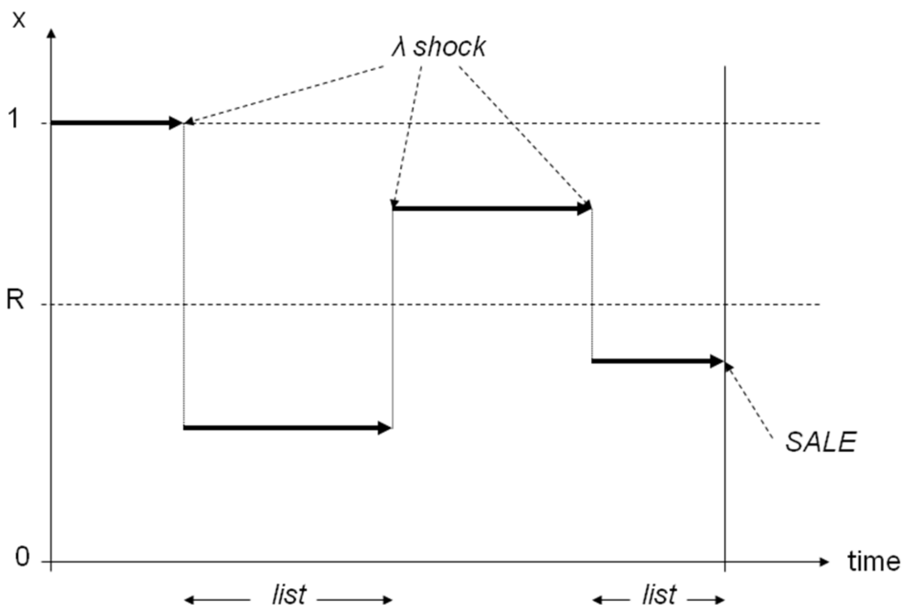

Figure 1: Match specific productivity x over ownership tenure.

which is rising in the listing margin R as long as $P(x)$ is rising in x (to be shown): this is a selection effect.[10]

2.1. EQUILIBRIUM

For any $x \leq R$, the equilibrium market tightness $\theta(x)$ is implicitly given by (derived in Appendix)

$$(r+\lambda) S(x) = \frac{r+\lambda}{1-\beta} \frac{c}{Aq(\theta(x))} = \pi(1-x) + \kappa c - \frac{\beta}{1-\beta} c\theta(x). \qquad (7)$$

Here, we can see that market tightness is falling in seller productivity $\theta'(x) < 0$. Suppose not and $\theta'(x) \geq 0$, then following an increase in x, the left hand side of

[10]The selection effect is given by

$$\frac{d\bar{P}}{dR} \equiv \frac{P(R) Am(\theta(R)) s(R) \left[1 - \frac{\int_0^R P(x)Am(\theta(x))s(x)dx}{P(R)\int_0^R Am(\theta(x))s(x)dx}\right]}{\int_0^R Am(\theta(x)) s(x) dx},$$

which is posiive if $P(R) \geq P(x)$.

this equation is weakly rising and the right hand side is strictly falling which is a contradiction. This uniquely solves for $\theta(x)$ where $x \leq R$.

Combining (1) and (3), the seller "listing margin" $R = 1$, if the market tightness implied by (7) implies seller search costs are very low

$$\kappa < \frac{\beta}{1-\beta}\theta(1).$$

Otherwise, from (4) the market tightness $\theta(R)$ in submarket $x = R$ is given by

$$(r+\lambda)S(R) = \frac{r+\lambda}{1-\beta}\frac{c}{Aq(\theta(R))} = \pi(1-R), \tag{8}$$

where the seller listing margin R implied by (4) is implicitly given by

$$\kappa = \frac{\beta}{1-\beta}\theta(R). \tag{9}$$

Market tightness at the interior listing margin $\theta(R)$ is constant. Substituting this into (8), uniquely solves for R. We can state the following:

Proposition 1 (Equilibrium with trade)**.** *There exists a unique equilibrium set* $\{\theta(x), R\}$. *Assets trade in equilibrium iff seller search costs are not too high*

$$\kappa < \frac{\beta}{1-\beta}q^{-1}\left(\frac{1}{A\frac{\pi}{c}\frac{(1-\beta)}{r+\lambda}}\right).$$

This condition combines (8) and (9) after setting $R = 0$. For the rest of the analysis, I assume this condition holds throughout. From (7) the creation margin, $\theta(x)$ is increasing in $\frac{\pi}{c}, A$ and decreasing in $(r+\lambda)$. From (9) the listing margin, and (8), R is increasing in $\frac{\pi}{c}, A$ and decreasing in $(r+\lambda)$. Thus, we can state the following:

Proposition 2 (Liquidity and selection)**.** *Equilibrium market tightness* $\theta(x)$ *and listing margin* R *are (i) increasing in productivity* π, *and search efficiency* A, *and (ii) decreasing in search cost* c, *interest rate* r, *and arrival rate of shocks* λ.

Changes to $\theta(x)$ capture changes in liquidity in market x. Changes in R capture selection effects in the aggregate market. The rate at which a seller meets a buyer in market x is $Am(\theta(x))$, and the average rate at which an owner lists an asset for resale (asset turnover) is $\lambda F(R)$. These are both increasing in π, A and decreasing in c, r, λ. Comparative statics with respect to the seller bargaining power β are discussed in the context of efficiency below.

2.2. COMPOSITION AND TRADE VOLUME

The evolution of selling owners $s(x)$ for submarket $x \leq R$ is

$$\dot{s}(x) = \lambda f(x) - \lambda s(x) - Am(\theta(x))\, s(x). \tag{10}$$

New sellers arrive at rate $\lambda f(x)$, and sellers leave the submarket following shocks at rate λ and matches at rate $Am(\theta(x))$. In steady states, the stock of sellers in submarket x is $s(x) = \frac{\lambda f(x)}{\lambda + Am(\theta(x))}$. The stock of buyers in submarket x is $\theta(x)s(x) = \frac{\theta(x)\lambda f(x)}{\lambda + Am(\theta(x))}$, which moves in the opposite direction to the stock of sellers given changes in $\theta(x)$. The inverse relationship between the stock of sellers and buyers for different levels of tightness $\theta(x)$ maps out a "Beveridge curve" for the asset market.

The total stock of sellers and buyers in the aggregate market is

$$\begin{aligned} \int_0^R s(x)\, dx &= \int_0^R \frac{\lambda f(x)}{\lambda + Am(\theta(x))} dx \\ \int_0^R \theta(x)\, s(x)\, dx &= \int_0^R \frac{\theta(x)\lambda f(x)}{\lambda + Am(\theta(x))} dx. \end{aligned}$$

For the total stock of sellers, liquidity (higher $\theta(x)$) and selection effects (higher R) work in opposite directions. For the total stock of buyers, these effects work in the same direction. Combining these arguments with Proposition 2 implies the following.

Proposition 3 (Composition). *(i) In submarket x, the stock of sellers (buyers) is decreasing (increasing) in the productivity π, search efficiency A and increasing (decreasing) in search costs c, interest rate r. The stock of sellers is increasing in the arrival rate of shocks λ. (ii) In the aggregate market, the total stock of sellers can increase or decrease in response to these variables, and the total stock of buyers is increasing in the productivity π, search efficiency A, and decreasing in search costs c, interest rate r.*

In the absence of selection effects (R constant), the total stock of sellers will move inversely with the total stock of buyers in response to π, A, c, r.[11] I denote trade volume as the per period share of the asset stock traded. The trade volume is given by

$$\int_0^R Am(\theta(x))\, s(x)\, dx = \int_0^R \frac{Am(\theta(x))\, \lambda f(x)}{\lambda + Am(\theta(x))} dx. \tag{11}$$

[11] Selection effects are absent when seller search costs low$\Rightarrow R = 1$, or when the distribution $F(x)$ is degenerate at $x = 0$, the case of exogenous match destruction.

This is rising in $\theta(x)$ and R. Combining with Proposition 2 implies the following Proposition.

Proposition 4 (Trade volume)**.** *The share of assets traded (trade volume) is (i) increasing in the productivity π, search efficiency A and (ii) decreasing in search costs c, interest rate r.*

Both liquidity and selection effects work in the same direction to affect trade volume in response to a change in these variables. Meanwhile, the overall effect of λ on trade volume is ambiguous. The direct effect on trade volume works in the opposite direction of the indirect effects through $\theta(x), R$.

The average arrival rate of buyers to sellers is the trade volume divided by the total stock of sellers. The average arrival rate of sellers to buyers is the trade volume divided by the total stock of buyers. These are given by

$$\begin{aligned} Em(\theta(\widetilde{x} \mid \widetilde{x} \leq R)) &= \frac{\int_0^R Am(\theta(x))\, s(x)\, dx}{\int_0^R s(x)\, dx}, \\ Eq(\theta(\widetilde{x} \mid \widetilde{x} \leq R)) &= \frac{\int_0^R Am(\theta(x))\, s(x)\, dx}{\int_0^R \theta(x)\, s(x)\, dx}. \end{aligned}$$

The average arrival rate of buyers to sellers can increase or decrease in response to changes in productivity π, search efficiency A and search costs c, interest rate r. In contrast, given the elasticity $\eta \leq 1$ from the assumption of constant returns to scale, the average arrival rate of sellers to buyers always moves inversely to the total stock of buyers following a change in these variables.

2.3. ASSET PRICES

Combining (1), (2) and (5), the asset price in submarket x is given by (derived in Appendix)

$$\begin{aligned} P(x) &= U(x) + \beta S(x) \qquad (12) \\ &= \frac{1}{(r+\lambda)}\left[\pi x - \kappa c + \frac{\beta}{1-\beta} c\theta(x)\right] \\ &\quad + \frac{\lambda}{r}\frac{1}{(r+\lambda)}\left[\pi\bar{x} + E\max\left\{0, -\kappa c + \frac{\beta}{1-\beta} c\theta(\tilde{x})\right\}\right] \\ &\quad + \frac{\beta}{1-\beta}\frac{c}{Aq(\theta(x))}. \end{aligned}$$

This equation implies price $P(x)$ is rising in π through the direct effect and the liquidity effect (acting through higher $\theta(x)$). Meanwhile, $P(x)$ is rising in x since the buyer match surplus $(1-\beta)S(x)=\frac{c}{Aq(\theta(x))}=J(1)-P(x)$ and $\theta'(x)<0$. Recall that $m'(\theta(x))>0$. From the chain rule we can ignore the effect of R on $P(x)$. These imply the following.

Proposition 5 (Price heterogeneity)**.** *Sales prices $P(x)$ and listing durations $\frac{1}{Am(\theta(x))}$ are rising in the seller productivity x.*

Thus, more distressed sellers (lower x) will meet buyers faster and charge a lower price. This Proposition implies the average sales price $\bar{P}$ given by (6) will be rising in R, which is the selection effect. Thus, through the direct effect, liquidity effect and selection effect, the average sales price $\bar{P}$ is rising in the earnings of the asset π.

The following implications of our equilibrium equations (7)-(9) (proved in the Appendix) are useful in the analysis to follow:

Lemma 1. *In each submarket $x\leq R$,*
(i) the normalized match surplus $\frac{S(x)}{\pi}$ is falling in $\frac{\pi}{c}$, A and rising in r,λ,
(ii) the limit as $A\to\infty$ or $\frac{\pi}{c}\to\infty$ implies $\frac{S(x)}{\pi}\to 0$, and
(iii) the limit as $A\to\infty$ or $\frac{\pi}{c}\to\infty$ implies $R\to 1$.

A sufficient condition for Lemma 1(i) is that the elasticity of the match function $\eta\leq 1$, which is implied by the assumption of constant returns. Lemma 1 (ii) and (iii) consider outcomes as search frictions disappear which occurs either when the search efficiency becomes very large $A\to\infty$, or when the match productivity becomes very large relative to the search costs $\frac{\pi}{c}\to\infty$.

Combining (7) and (12), the price-earnings ratio can be expressed as

$$\frac{P(x)}{\pi}=\frac{1}{r}-(1-\beta)\frac{S(x)}{\pi}+\frac{\lambda}{r}E\max\left\{\begin{array}{c}\frac{\bar{x}-1}{(r+\lambda)},\\ \frac{\bar{x}-\tilde{x}}{(r+\lambda)}-\frac{S(\tilde{x})}{\pi}\end{array}\right\}. \tag{13}$$

From this equation we can determine outcomes as search frictions disappear, $A\to\infty$ or $\frac{\pi}{c}\to\infty$. In either case, $R\to 1$ from Lemma 1$\Rightarrow E(\bar{x}-\tilde{x})\to 0$ and $E\max\left\{\frac{\bar{x}-1}{(r+\lambda)},\frac{\bar{x}-\tilde{x}}{(r+\lambda)}-\frac{S(\tilde{x})}{\pi}\right\}=E\max\left\{-\frac{S(\tilde{x})}{\pi}\right\}$. From Lemma 1, $\frac{S(x)}{\pi}\to 0$. These imply $\frac{P(x)}{\pi}\to\frac{1}{r}$. Thus, in the absence of search frictions, there is a single asset price which would be $\frac{\pi}{r}$, and assets are sold instantly as soon as a shock λ arrives, i.e. for any $x<1$.[12]

[12]This would correspond to the Lucas asset pricing equation in the absence of interest rate, dividend and discount rate uncertainty as assumed in the analysis, since there is no rematch uncertainty or search friction.

In the absence of search frictions, the price-earnings ratio is invariant to earnings, $\frac{d\frac{P(x)}{\pi}}{d\pi} = 0$. I refer to outcomes where this ratio is increasing in earnings levels $\frac{d\frac{P(x)}{\pi}}{d\pi} > 0$ as exhibiting "excess volatility". From equation (13), the price-earnings ratio $\frac{P(x)}{\pi}$ is falling in the match surplus normalized by the match productivity $\frac{S(x)}{\pi}$. Lemma 1 and some calculations (in the Appendix) imply the following.

Proposition 6 (Excess volatility)**.** *With search frictions, there is an excess volatility of asset prices* $\frac{d}{d\pi}\frac{P(x)}{\pi} > 0$, *and* $\frac{d}{d\pi}\frac{P(x)}{\pi}$ *is falling in the elasticity of the match function* $\eta\left(\theta\left(x\right)\right)$.

Importantly, for a given change in π, such excess volatility is further amplified on the *average* price by selection effects through changes in the listing margin R from (6).

2.3.1 Liquidity premia

In the riskless stationary environment there is no risk premia, or expected price appreciation, while asset depreciation rate has been set to zero. I define the difference between the earnings yield and the interest rate $\frac{\pi}{P(x)} - r$, as the *liquidity premium*. This premium is equal to zero in the absence of search frictions. An alternative interpretation of Proposition 6 is as follows: in the presence of search frictions, assets exhibit liquidity premia which move inversely with the ratio of asset earnings to search costs $\frac{\pi}{c}$. We can consider other determinants of the liquidity premium. From Lemma 1, following an improvement in search efficiency A, the normalized match surplus $\frac{S(x)}{\pi}$ is lower. These results are summarized in the following Proposition.

Proposition 7 (Liquidity premia)**.** *Asset liquidity premia are (i) decreasing in the productivity* π, *search efficiency* A, *and (ii) increasing in search costs* c.

Meanwhile, the effect of λ on the liquidity premium is more subtle. Recall from (5) that the asset price is increasing in the seller value $U(x)$ and match surplus $S(x)$. The value equation for sellers can be rewritten as

$$rU\left(x\right) = \pi x - \kappa c + \beta A m\left(\theta\left(x\right)\right) S\left(x\right) + \lambda\left(E\max\left\{J\left(\widetilde{x}\right), U\left(\widetilde{x}\right)\right\} - U\left(x\right)\right).$$

From Proposition 2, liquidity $\theta\left(x\right)$ and the surplus of match with a buyer $S\left(x\right)$ are falling in λ, which cause the asset price to fall and liquidity premium to rise. The last term for $rU\left(x\right)$ represents the option value of the seller to keep the asset. Depending on whether the expected capital gain is positive or negative following a shock λ, the option value responds in different ways to a change in the frequency of shocks. In particular, for sellers with low $U(x)$, the option value effect could possibly outweigh the effects on $\theta(x)$ and $S(x)$ such that prices would rise with λ, implying a fall in the liquidity premium.

In the standard frictionless framework, the only source of differences in price-earnings ratios are interest rates, risk premia, asset depreciation and expected price appreciation. By introducing search frictions we can consider several more dimensions by which asset price-earnings ratios vary in systematic ways through the liquidity premium. From Proposition 2, these effects on price-earnings ratios are further amplified on the *average* price by selection effects through changes in the listing margin R from (6).

Combining with Proposition 4, assets exhibit larger trade volumes when they have higher price-earnings ratios and lower liquidity premia resulting from higher earnings π, and search efficiency A or lower search costs c.

2.3.2 Fire sales and bid-ask spread

$P(x)$ denotes the sales price of a seller with match productivity πx to an end buyer with match productivity π. When the seller has no use for the asset, the sales price is $P(0)$. In this case, the seller may not need to search for an end buyer with positive match productivity, and instead sell the asset in a spot market without search frictions. Following Lagos and Rocheteau (2007), if we assume there exists a free entry of potential non-end buyers or "dealers" with zero match productivity, such a seller can conduct a "fire-sale" of the asset to such dealers in a spot market where the fire-sale price is given by $U(0)$. The fire-sale discount rate (one minus the ratio of the fire sale price to sale price) is

$$1 - \frac{U(0)}{P(0)} = \beta \frac{S(0)}{P(0)}, \tag{14}$$

which is rising in ratio of the the surplus of a match with an end-buyer over the end buyer sale price $\frac{S(0)}{P(0)}$.[13] Note that when the seller has no bargaining power $\beta = 0$, the discount rate is zero.

An alternative interpretation of this discount rate is the bid-ask spread, a standard measure of liquidity in the finance literature ($U(0)$ is the ask price and

[13]When the lower bound for x is zero, the presence of dealers plays no role in affecting equilibrium outcomes. However, allowing for this lower bound to be below zero justifies their presence in equilibrium.

$P(0)$ is the bid price).[14] This spread captures the percentage price difference by which dealers buy assets in spot markets and sell in markets subject to search frictions.

Recall from our discussion of liquidity premia that the price-earnings level $\frac{P(0)}{\pi}$ is falling in the match surplus normalized by the aggregate productivity $\frac{S(0)}{\pi}$. Thus, combining with Lemma 1 yields the following Proposition.

Proposition 8 (Bid-ask spread)**.** *The bid-ask spread is (i) decreasing in the productivity π, search efficiency A, and (ii) increasing in search costs c.*

Combining with Proposition 4, the bid-ask spread negatively co-moves with transaction volume following changes to π, A, c. Combining with Proposition 7, the bid-ask spread positively co-moves with liquidity premia following changes to π, A, c. Thus, in many applications, the bid-ask spread, liquidity premium and trade volume are interchangeable measures of liquidity.

2.4. ASSET PRODUCTIVITY

The evolution of average asset productivity w is

$$\dot{w} = \pi \int_0^R (1-x) Am(\theta(x)) s(x) dx + \lambda (\pi \bar{x} - w). \tag{15}$$

The first term is the increase in asset productivity from new matches, and the second term is the change in asset productivity from new shocks. So in steady states, substituting the equation for $s(x)$, the average productivity *normalized* by aggregate productivity parameter π is

$$\frac{w}{\pi} = \bar{x} + \int_0^R (1-x) \frac{Am(\theta(x)) f(x)}{\lambda + Am(\theta(x))} dx, \tag{16}$$

which is rising in $\theta(x)$ and R. Thus, through the liquidity (higher $\theta(x)$) and selection effects (higher R), the elasticity of average asset productivity w with

[14] See references in the introduction. This formulation of dealers differs somewhat from that in the existing literature on financial markets where the arrival rate of sellers to dealers and buyers to dealers is assumed to be the same and exogenous, in the random matching environments they consider. In that case, setting the arrival rate of dealers to sellers as infinite (as assumed here with spot markets) eliminates bid-ask spreads (e.g. Duffie, Gârleanu, and Pedersen (2005)). My formulation of bid-ask spreads is robust to the introduction of such spot markets because the spread arises from the dichotomous nature of the markets which dealers trade in when they buy and sell. Note that since the sell rate $m(\theta(x))$ is falling in x, dealers who sell at rate $m(\theta(0))$, will be observed to sell assets at faster rates than regular sellers.

respect to aggregate productivity π exceeds 1. In this way, the environment we have considered can generate a *amplification* of aggregate productivity shocks. Note that as search frictions disappear, $A \to \infty$ or $\frac{\pi}{c} \to \infty$, then $R \to 1$ and $\frac{Am(\theta(x))f(x)}{\lambda+Am(\theta(x))} \to f(x)$ which implies $\frac{w}{\pi} \to 1$. In this case, amplification effects are not present.

Following an unexpected increase in π, $\theta(x)$ and R jump to their new steady state levels, but $s(x)$ adjusts gradually downwards from equation (10). Thus, the steady state level of average asset productivity is reached gradually from (15). This speaks to a *propagation* of aggregate productivity shocks.

2.5. EFFICIENCY

The source of inefficiencies are the congestion externalities which arise from listing assets for sale and taking them off the market when matches occur. In each submarket $x \leq R$, the level of the seller bargaining power $\beta(x)$ which generates socially optimal outcomes is given by

$$\max_{\beta(x)} U(x) \text{ s.t. } (7). \tag{17}$$

This optimization problem is related to the standard social planners problem in the Appendix. Given the solution to this optimization in each submarket x, the listing margin implied by (9) is socially efficient. $\max_{\beta(x)} U(x)$ is equivalent to $\max_{\beta(x)} \frac{\beta}{1-\beta}\theta(x)$.[15] Solving this subject to (7) implies the social optimum is achieved under the Hosios condition: $\beta(x) = \eta(\theta(x))$.

The creation margin (7) implies that $\theta(x)$ is falling in $\beta(x)$. Recall the listing margin is determined by $\kappa = \frac{\beta}{1-\beta}\theta(R)$. Since the Hosios condition maximizes the right hand side for a given R, any $\beta \neq \eta(\theta(R))$ implies lower R since $\theta'(R) < 0$. These results are summarized in the following Proposition.

Proposition 9 (Efficiency)**.** *(i) The decentralized outcome with search frictions is efficient subject to the Hosios condition: the seller bargaining share equals the elasticity of the match function,* $\beta(x) = \eta(\theta(x))$.

(ii) When $\beta(x) > \eta(\theta(x))$, *market tightness* $\theta(x)$ *is lower, and when* $\beta(x) < \eta(\theta(x))$, *market tightness* $\theta(x)$ *is higher than the social optimum in each sub-*

[15]The expression for the value of sellers is

$$rU(x) = \pi x + \lambda\left(E\max\{J(\tilde{x}), U(\tilde{x})\} - U(x)\right) - \kappa c + \frac{\beta}{1-\beta}c\theta(x).$$

market $x \leq R$. The listing margin R is lower than the social optimum for $\beta \neq \eta(\theta(R))$.

Under the Hosios condition, the positive externality of buyers and the negative externality of sellers (to the existing stock of sellers) cancel out. A high seller bargaining power is associated with excessively low asset liquidity (and vice versa) relative to the social optimum. Asset turnover (at rate $\lambda F(R)$) is always excessively low when outcomes deviate from the social optimum. These insights on efficiency are analogous to results in search models of unemployment (see Pissarides (2000)).

3. EXTENSIONS

3.1. ENDOGENOUS BUYING

In this section, I consider the margin by which buyers decide to purchase an asset which they have been matched with. Given a match with seller productivity x, buyers draw from the same distribution $F(y)$ and determine a margin $Z(x)$ such that only when $y \geq Z(x)$ do they purchase. The draw is common knowledge to both buyer and seller. If the margin $Z(x) < R$, this gives rise to the phenomenon of "flipping" (listing for resale immediately after purchase). Steady state value equations are now given by

$$\begin{aligned} rV &= -c + Aq(\theta(x)) E_{\tilde{y}}(\max(0, \max\{J(\tilde{y}), U(\tilde{y})\}) - P(x,\tilde{y})) = 0, \qquad (18) \\ rJ(x) &= \pi x + \lambda(E_{\tilde{x}} \max\{J(\tilde{x}), U(\tilde{x})\} - J(x)), \\ rU(x) &= \pi x + \lambda(E_{\tilde{x}} \max\{J(\tilde{x}), U(\tilde{x})\} - U(x)) - \kappa c \\ &\quad + Am(\theta(x)) E_{\tilde{y}}(\max\{U(x), P(x,\tilde{y})\} - U(x)). \end{aligned}$$

where $E_{\tilde{y}}$ denotes expectations taken over the buyer productivity y, and $E_{\tilde{x}}$ that taken over the seller productivity x. The Nash bargaining rule and destruction margin are now given by

$$\begin{aligned} (1-\beta)(P(x,y) - U(x)) &= \beta(\max\{J(y), U(y)\} - P(x,y)), \\ J(R) &= U(R). \end{aligned}$$

The match surplus $(J(y) - U(x))$ is increasing in y and decreasing in x. The purchase decision rule is given by

$$\max\{J(Z(x)), U(Z(x))\} = U(x) \Rightarrow Z(x) = x.$$

since $\max\{J(Z(x)), U(Z(x))\} = U(Z(x))$ for $Z(x) \leq R$. Intuitively, sales occur whenever buyers have a higher productivity than sellers $y \geq x$.

For a given match, in submarket x, the incidence of flipping is given by $F(R) - F(x)$ which is rising in R. Flippers can be interpreted as intermediaries whose match productivity is not too low or not too high, $x \leq y(x) \leq R$. They facilitate exchange by transferring assets from low valuation buyers to high valuation buyers. Note that flippers differ from dealers analyzed above in the context of fire-sales, in the sense that they derive positive productivity from the asset $y \geq 0$.

3.2. NON-DIRECTED SEARCH

In this section, I consider outcomes when buyers cannot direct their search to sellers of different productivity. There is a common search market for all sellers and $\theta(x) = \theta$ for $x \leq R$. I again assume all buyers have match productivity $\pi y = \pi$. Steady state value equations are now given by

$$\begin{aligned} rV &= -c + Aq(\theta)(J(1) - EP(\widetilde{x} \mid \widetilde{x} < R)) = 0, \\ rJ(x) &= \pi x + \lambda(E\max\{J(\widetilde{x}), U(\widetilde{x})\} - J(x)), \\ rU(x) &= \pi x + \lambda(E\max\{J(\widetilde{x}), U(\widetilde{x})\} - U(x)) - \kappa c + Am(\theta)(P(x) - U(x)). \end{aligned} \tag{19}$$

Nash bargaining outcomes and the determination of R are as in the benchmark model, (2) and (4).

The equilibrium market tightness is given by (the derivation is analogous to (7))

$$\frac{r+\lambda}{1-\beta}\frac{1}{Aq(\theta)} = \frac{\pi}{c}(1 - E(\widetilde{x} \mid \widetilde{x} < R)) + \kappa - \frac{\beta}{1-\beta}\theta. \tag{20}$$

Since $E(\widetilde{x} \mid \widetilde{x} < R)$ is rising in R, this specifies an inverse relationship between θ, R. The listing margin $R = 1$, if given the market tightness implied by (20) implies search costs are very low

$$\kappa < \frac{\beta}{1-\beta}\theta(R = 1). \tag{21}$$

Otherwise, the listing margin is given by an interior $R < 1$[16]

$$\frac{r+\lambda}{\beta}\frac{\kappa}{Am(\theta)} = \frac{\pi}{c}(1-R). \tag{22}$$

This specifies a positive relationship between θ, R. These two schedules cross at a unique pair $\{\theta, R\}$.

Proposition 10 (Non-directed search)**.** *There exists a unique equilibrium pair* $\{\theta, R\}$ *with non-directed search.*

An increase in π, A or a decrease in c, r, λ, raises R on the buyer creation margin and raises R on the seller listing margin so R increases unambiguously. Following a change in $\frac{\pi}{c}, A$ or $(r+\lambda)$ higher $R \Leftrightarrow$ higher θ when the following condition is satisfied (derived in Appendix)

$$\frac{d}{dR}\frac{E(\widetilde{x} \mid \widetilde{x} < R)}{R} < 0. \tag{23}$$

This is a somewhat mild restriction which I assume is satisfied.[17] Thus, altering the model specification with non-directed search does not overturn the main implications of the benchmark model.

In steady states, the stock of sellers is $s = \frac{\lambda F(R)}{\lambda + Am(\theta)}$. The trade volume is given by

$$Am(\theta)s = \frac{Am(\theta)\lambda F(R)}{\lambda + Am(\theta)},$$

which is increasing in π, A (decreasing in c, r) since both θ, R are increasing in π, A (decreasing in c, r).

4. DISCUSSION

In this section, I discuss predictions of the benchmark model with some stylized facts from the market for owner occupied housing where search frictions are prevalent and there exist a rich set of empirical results. Since the asset is

[16]Solving

$$\begin{aligned} \kappa c &= Am(\theta)\beta[J(1) - U(R)] \\ (r+\lambda)[J(1) - U(R)] &= \pi(1-R) \end{aligned}$$

[17]This assumption is alternaltively given by $\frac{d}{dR}\frac{1}{F(R)}\int_0^R x dF(x) < 1$.

immobile, buyers must physically go and visit the asset. Also, the assumption of $\{0,1\}$ holdings of assets seems particularly appropriate for the owner occupied housing market. I proceed by listing the stylized facts and relating them to results of the paper. From the analysis, an exogenous change in aggregate asset productivity π is associated with an increase in the average sales price.[18] Other associated implications are as follows:

Main implications

- [Proposition 2] In time series, Stein (1995) documents there are ”sellers” markets (house sales occur quickly for sellers) when prices are higher relative to trend and vice versa. In cross sections, Zahirovic-Herbert and Turnbull (2007) document that school quality positively affects house price and negatively affects time to sale, while Ong and Koh (2000) report that in high rise condos, lower level units (which are otherwise identical) are priced lower and take longer to sell.
- [Proposition 2] Genesove and Mayer (1997) and Chan (2001) document that owner mobility is lower when house prices are lower.
- [Proposition 4] Stein (1995) and Ortalo-Magne and Rady (2006) among others document a positive correlation between prices and transactions volumes.
- [Proposition 5] Glower, Haurin and Hendershott (1998) document the correlation between sales prices and time on market is explained by factors related to seller motivation (stronger motivation results in shorter time on market). I address their conclusion that “theoretical models of the housing search process should be recast to allow for heterogeneous sellers”. Merlo and Ortalo-Magne (2004) document that initial list prices are correlated with time on market using UK data. Hendel, Nevo and Ortalo-Magne (2008) find that homeowners who do not use agents, sell homes at higher prices and take longer to sell. Finally, Anglin, Rutherford and Springer (2003) document that the list prices of houses which are withdrawn before sale have a higher mean. This suggests that properties with higher seller productivity are being withdrawn following a change in market conditions as predicted by the model.[19]

[18] Alternatively we may consider an increase in search efficiency A, or a decrease in search cost c.

[19] They also document no systemic difference in the ratio of list price to sales prices.

- [Proposition 6] Stein (1995) and Ortalo-Magne and Rady (2006) among others document the excess volatility of owner occupied house prices. Recent discussions have focused on the movements in the price-earnings ratio of houses (proxied by price-income or price-rent ratios) and its correlation with measures of earnings (income or rents), e.g. Case and Shiller (2003) and Campbell, Davis, Gallin, and Martin (2007).

Other implications

- [Proposition 2] Wheaton (1990) documents an inverse correlation between house prices and vacancy rates. In my model, a productivity shock which takes match productivity to $x = 0$ can be interpreted as an incidence of vacancy. The vacancy share is given by $\frac{\lambda f(0)}{\lambda + Am(\theta(0))}$ which is falling in $\theta(0)$. Thus, vacancy rates are inversely linked with house prices when price changes arise due to changes in any of our key exogenous variables: π, A, c, r, λ.
- Case and Shiller (1989) document a serial correlation in house price movements. Such price changes can be generated by the current analysis through changes in the gradual selection of house sales (in terms of seller productivity x) following an unexpected shock (such as to π, A, c, r, λ) as suggested by (15).

Wheaton (1990) considers search frictions to study the determinants of the vacancy rate and the intensity of search. More related to the current paper, Krainer (2001) analyzes the correlation of prices, liquidity and trade volumes with search frictions. I target these correlations, and also predict an excess volatility of prices. I also target selection effects in the search environment. Stein (1995), Genesove and Mayer (1997) and Ortalo-Magne and Rady (2006) link trade volume, house prices and excess volatility in environments with downpayment constraints. I complement their work by generating these correlations in the absence of downpayment constraints.[20]

5. NUMERICAL SIMULATION

To further investigate these movements (and movements in other financial markets) I consider a numerical simulation of the model. The numerical example

[20]In addition, Genesove and Mayer (2001) and Engelhardt (2003) link time to sale, house prices and trade volume in a model of seller loss aversion.

is constructed as follows. I assume the match function is $q(\theta) = \theta^{-\eta}$. Benchmark parameters are picked to represent parameters in the market for owner occupied housing. I consider the simplest version of the model where the distribution of x is degenerate at $x = 0$, $F(0) = 1$: the case of exogenous match destruction. In this case, there is only one submarket with trade where $x = 0$.

The bargaining share β is set at 50%, and $\eta = \beta$ to satisfy the efficiency condition (Hosios condition). r is set to match a 5% effective annual interest rate (taking into account depreciation, taxes and risk premia), $\lambda = 0.1$ is set to generate the productivity shock every 10 years, $\frac{\pi}{c} = 1$ to match a monthly benefit of homeownership, which is 1 times the monthly search cost of home buyers (which includes realtor fees), the relative seller search cost is set to $\kappa = 1$. Given these parameter choices, A is set to generate an average buyer arrival duration of 6 months in the submarket $x = 0$, that is $2 = A\theta(0)^{1-\eta}$. This corresponds to average time to sale for the US, reported by the National Association of Realtors. Table 1 summarizes the choice of benchmark parameters.

Table 1: Benchmark parameters

Parameter	β_0	η_0	r_0	λ_0	$\frac{\pi_0}{c_0}$	κ_0	A_0
Value	0.5	0.5	0.05	0.1	1	1	1.52

For convenience, I rewrite the equations of interest from the analysis. Under the assumption of exogenous match destruction these are simplified from (1), (11), (13), (14) and (16) as follows

$$\begin{aligned}
Surplus &= \frac{S(0)}{\pi} = \frac{1}{(1-\beta)A\theta(0)^{-\eta}}\frac{c}{\pi}, \\
P/E\ ratio &= \frac{P(0)}{\pi} = \frac{1}{r} - \left[(1-\beta) + \frac{\lambda}{r}\right]\frac{S(0)}{\pi}, \\
Liquidity\ Premium &= \frac{\pi}{P(0)} - r, \\
Bid-ask\ spread &= \beta\frac{S(0)}{P(0)} = \frac{r\beta}{r\beta + \frac{\pi}{S(0)} - (r+\lambda)}, \\
Volume &= \lambda\frac{A\theta(0)^{1-\eta}}{\lambda + A\theta(0)^{1-\eta}}, \\
Productivity &= \frac{w}{\pi} = \frac{A\theta(0)^{1-\eta}}{\lambda + A\theta(0)^{1-\eta}}.
\end{aligned}$$

The endogenous level of market tightness $\theta(0)$ is picked up from equation (7).

Results

I first report results for the statistics of interest under the benchmark parameters, and after varying β, η by $\pm 50\%$ of their initial values (and re-calibrating A). These are summarized in Table 2.

In the benchmark specification, the price-earnings ratio is 15.7 versus that of 20 in a frictionless setting. The liquidity premium is substantial at 1.4%. The trade volume and average productivity (relative to the frictionless case) are 9.5% and 95.2% respectively. Under the frictionless case these are 10% and 100% respectively. The bid-ask spread is also quite substantial at 5.6%.

A lower seller bargaining power β results in large reductions in the price-earnings ratio and large increases in liquidity premia and the bid-ask spread. The opposite is true for a higher seller bargaining power. Changing the match elasticity η affects the calibrated level of search efficiency A, but does not affect other statistics given our strategy for calibrating $A\theta(0)^{1-\eta} = 2$. For the same reason, trade volume and average productivity are unaffected by changing β.

Next, I report results for the endogenous variables of interest after varying $\left\{r, \lambda, \frac{\pi}{c}, \kappa, A\right\}$ to double their initial values and holding all other variables constant (including A). These are summarized in Table 3.

Doubling the interest rate r or the arrival rate of shocks λ substantially lowers price-earnings ratios, more than doubles liquidity premia and substantially increases bid-ask spreads. Doubling the relative seller search cost κ has a similar effect. Meanwhile, doubling the ratio of match productivity to search costs $\frac{\pi}{c}$ or search efficiency A raises price-earnings ratios, and halves liquidity premia

Table 2: Varying β, η

β	β_0	0.25	0.75	β_0	β_0
η	η_0	η_0	η_0	0.25	0.75
A	A_0	1.0	2.7	1.4	1.8
P/E	15.7	11.5	17.3	15.7	15.7
Liq prem	1.4%	3.7%	0.8%	1.4%	1.4%
Bid ask	5.6%	6.7%	5.3%	5.6%	5.6%
Volume	9.5%	9.5%	9.5%	9.5%	9.5%
Prod.	95.2%	95.2%	95.2%	95.2%	95.2%

Table 3: Varying $r, \lambda, \frac{\pi}{c}, \kappa, A$

	Bench	$r = 2r_0$	$\lambda = 2\lambda_0$	$\frac{\pi}{c} = 2\frac{\pi_0}{c_0}$	$\kappa = 2\kappa_0$	$A = 2A_0$
P/E	15.7	7.5	12.5	17.3	14.6	17.8
Liq prem	1.4%	3.4%	3.0%	0.8%	1.8%	0.6%
Bid ask	5.6%	11.4%	6.6%	3.1%	7.4%	2.4%
Volume	9.5%	9.5%	18.1%	9.6%	9.6%	9.8%
Prod	95.2%	95.1%	90.5%	96.1%	96.1%	97.6%

and bid-ask spreads. In particular, doubling the level of earnings results in prices increasing by a factor of 2.2. Both trade volume and average productivity respond strongly to a doubling of the arrival rate of shocks in opposite directions (increase in volume to 18.1% and decrease in productivity to 90.5%). Other changes in variables have smaller effects on trade volume and average productivity. Overall, these results highlight the potential quantitative significance of search frictions in asset pricing.

6. CONCLUSION

This paper considered the interaction of endogenous liquidity and selection in asset markets with search frictions. I characterized the response of various measures of liquidity (time to sale, stock of sellers and buyers, trade volume, liquidity premia, bid-ask spreads) in response to exogenous changes in asset productivity, search costs, interest rates, and arrival rate of shocks. The analysis revealed that many of these changes are reinforced by endogenous changes in the selection of sellers in the market. The predictions of the model correspond well with some stylized facts documented from the market for owner occupied housing, and the numerical simulations suggest a quantitative role for the forces analyzed. However, further work should look at mapping the implications of the model to empirical features of various asset markets in a more structural way to see whether the forces analyzed play a substantive role in the workings of asset markets.

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APPENDIX 1. PROOFS

Derivation of (7). Combining the free entry condition and value equations (1) and substituting in the Nash Bargaining outcome (2)

$$\frac{c}{Aq(\theta(x))} = J(1) - P(x) = (1-\beta)(J(1) - U(x))$$
$$(r+\lambda)(J(1) - U(x)) = \pi(1-x) + \kappa c - \frac{\beta}{1-\beta} c\theta(x).$$

Derivation of (12). The value equations are

$$(J(1) - U(x)) = \frac{1}{1-\beta}\frac{c}{Aq(\theta(x))}$$
$$(r+\lambda)U(x) = \pi x + \lambda E \max\{J(\tilde{x}), U(\tilde{x})\} - \kappa c + \frac{\beta}{1-\beta} c\theta(x)$$
$$rE\max\{J(\tilde{x}), U(\tilde{x})\} = \pi\bar{x} + E\max\left\{0, -\kappa c + \frac{\beta}{1-\beta} c\theta(\tilde{x})\right\}$$

Combining these expressions and simplifying yields the price formula.

Proof of Lemma 1. (i) From the creation margin (7),

$$(r+\lambda)\frac{S(x)}{\pi} \equiv X = \frac{r+\lambda}{1-\beta}\frac{1}{Aq(\theta(x))}\frac{c}{\pi} = (1-x) + \kappa\frac{c}{\pi} - \frac{\beta}{1-\beta}\frac{c}{\pi}\theta(x) \quad (24)$$

the differential **[This sentence is strange]**

$$\frac{dX}{d\frac{\pi}{c}} = -X\frac{c}{\pi} + \frac{\eta\left(\theta\left(x\right)\right)}{\theta\left(x\right)}X\frac{d\theta\left(x\right)}{d\frac{\pi}{c}} \tag{25}$$

$$= -\kappa\frac{c}{\pi}\frac{c}{\pi} + \frac{\beta}{1-\beta}\frac{c}{\pi}\theta\left(x\right)\frac{c}{\pi} - \frac{\beta}{1-\beta}\frac{c}{\pi}\frac{d\theta\left(x\right)}{d\frac{\pi}{c}}$$

$$\left(1-x\right) = \left[\eta\left(\theta\left(x\right)\right)X + \frac{\beta}{1-\beta}\frac{c}{\pi}\theta\left(x\right)\right]\frac{d\theta\left(x\right)}{d\frac{\pi}{c}}\frac{\frac{\pi}{c}}{\theta\left(x\right)}$$

$$\frac{dX}{d\frac{\pi}{c}}\frac{\frac{\pi}{c}}{X} = -1 + \eta\left(\theta\left(x\right)\right)\frac{d\theta\left(x\right)}{d\frac{\pi}{c}}\frac{\frac{\pi}{c}}{\theta\left(x\right)} \tag{26}$$

$$= -1 + \frac{\eta\left(\theta\left(x\right)\right)\left(1-x\right)}{\left[\eta\left(\theta\left(x\right)\right)X + \frac{\beta}{1-\beta}\frac{c}{\pi}\theta\left(x\right)\right]}$$

$$= -1 + \frac{\eta\left(\theta\left(x\right)\right)\left(1-x\right)}{\left[\eta\left(\theta\left(x\right)\right)\left[\left(1-x\right)+\kappa\frac{c}{\pi}\right] + \left(1-\eta\left(\theta\left(x\right)\right)\right)\frac{\beta}{1-\beta}\frac{c}{\pi}\theta\left(x\right)\right]} \tag{27}$$

$$< 0$$

A sufficient condition for this is that $\eta\left(\theta\left(x\right)\right) < 1$ which is implied by the assumption of the constant returns to scale match function. From Proposition 2, $\theta\left(x\right)$ is rising in A, and falling in r, λ which implies the results from the equation (24).

(ii) As $\frac{c}{\pi} \to 0, \theta\left(x\right)$ potentially has no upper limit, $\theta\left(x\right) \to \infty$. Under the constraint that $\frac{S(x)}{\pi} \geq 0$, the upper limit for $\frac{c}{\pi}\theta\left(x\right)$ as $\frac{c}{\pi} \to 0$ is given by

$$\left(1-x\right) = \frac{\beta}{1-\beta}\frac{c}{\pi}\theta\left(x\right).$$

Imposing this upper limit

$$\frac{S\left(x\right)}{\pi} = \frac{1}{1-\beta}\frac{1}{\theta\left(x\right)Aq\left(\theta\left(x\right)\right)}\frac{c}{\pi}\theta\left(x\right) = \frac{1}{\beta}\frac{\left(1-x\right)}{Am\left(\theta\left(x\right)\right)},$$

which converges to zero as $\theta\left(x\right) \to \infty$, given our assumption that $m'\left(\theta\left(x\right)\right) > 0$. Next, when $\frac{1}{A} \to 0, \theta\left(x\right)$ has a finite upper limit from (7) under the constraint that $\frac{S(x)}{\pi} \geq 0$, which implies $\frac{1}{Aq(\theta(x))} \to 0$.

(iii) Finally, substituting (9) into (8), $R \to 1$ as $\frac{c}{A\pi} \to 0$.

Derivation of (13). The creation margin (7) and the price equation (12) can be rewritten as

$$-\kappa\frac{c}{\pi}+\frac{\beta}{1-\beta}\frac{c}{\pi}\theta(x)=1-x-\frac{r+\lambda}{1-\beta}\frac{1}{Aq(\theta(x))}\frac{c}{\pi}$$

$$\begin{aligned}\frac{P(x)}{\pi}&=\frac{1}{(r+\lambda)}\left(\frac{\lambda}{r}\bar{x}+1\right)-\frac{1}{Aq(\theta(x))}\frac{c}{\pi}\\&\quad+\frac{\lambda}{r}E\max\left\{\begin{array}{c}0,\\ \frac{1}{(r+\lambda)}(1-\tilde{x})-\frac{1}{1-\beta}\frac{1}{Aq(\theta(\tilde{x}))}\frac{c}{\pi}\end{array}\right\}\\&=\frac{1}{r}-\frac{1}{Aq(\theta(x))}\frac{c}{\pi}\\&\quad+\frac{\lambda}{r}E\max\left\{\begin{array}{c}\frac{\bar{x}-1}{(r+\lambda)},\\ \frac{\bar{x}-\tilde{x}}{(r+\lambda)}-\frac{1}{1-\beta}\frac{1}{Aq(\theta(\tilde{x}))}\frac{c}{\pi}\end{array}\right\}.\end{aligned}$$

Proof of Proposition 6. From the derivations for Lemma 1,

$$\frac{dX}{d\frac{\pi}{c}}=-X\frac{c}{\pi}\left[-1+\eta(\theta(x))\frac{d\theta(x)}{d\frac{\pi}{c}}\frac{\frac{\pi}{c}}{\theta(x)}\right]$$

$$\eta(\theta(x))\frac{d\theta(x)}{d\frac{\pi}{c}}\frac{\frac{\pi}{c}}{\theta(x)}=\frac{\eta(\theta(x))(1-x)}{\left[\eta(\theta(x))X+\frac{\beta}{1-\beta}\frac{c}{\pi}\theta(x)\right]}$$

which is rising in $\eta(\theta(x))$ which implies the excess volatility is falling in $\eta(\theta(x))$.

Proof of Proposition 9: Social planner's problem. The social planner's problem is given by

$$\max_{w,s(x),\theta(x),R}\int_0^\infty e^{-rt}\left[w-\int_0^R[s(x)\theta(x)c+s(x)\kappa c]\,dx\right]dt \text{ s.t. (10) and (15).}$$

The implied Euler conditions (where μ_1,μ_2 are co-state variables) are

$$\begin{aligned}w&:e^{-rt}-\lambda\mu_1+\dot{\mu}_1=0,\\s(x)&:-e^{-rt}(\theta(x)c+\kappa c)+\mu_1\pi(1-x)Am(\theta(x))-\mu_2(\lambda+Am(\theta(x)))\\&\qquad+\dot{\mu}_2=0,\\\theta(x)&:-e^{-rt}s(x)c+\mu_1\pi(1-x)s(x)Am'(\theta(x))-\mu_2 s(x)Am'(\theta(x))=0,\\R&:-e^{-rt}[s(R)\theta(R)c+s(R)\kappa c]+\mu_1\pi(1-R)Am(\theta(R))s(R)=0.\end{aligned}$$

Solving for the co-state variables

$$\begin{aligned}\mu_1 &= \frac{e^{-rt}}{r+\lambda},\\ \mu_2 &= e^{-rt}\left(\frac{\pi(1-x)}{r+\lambda}-\frac{c}{Aq(\theta(x))(1-\eta)}\right).\end{aligned}$$

Substituting into the Euler condition for $s(x)$

$$\frac{c}{Aq(\theta(x))(1-\eta)}(r+\lambda+Am(\theta(x)))=\pi(1-x)+\theta(x)c+\kappa c,$$

which is equivalent to (7) under the Hosios condition. In particular, this holds for $x=R$ so that

$$\frac{1}{1-\eta}\frac{c}{Aq(\theta(R))}=\frac{\pi(1-R)}{r+\lambda}+\frac{1}{r+\lambda}\left[\kappa c-\frac{\eta}{1-\eta}c\theta(R)\right].$$

Next, substituting in this expression and the solution for μ_1 into the first order condition for R implies

$$\frac{r+\lambda}{Am(\theta(R))}\left[\frac{\eta}{1-\eta}\theta(R)-\kappa\right]=\left[\kappa-\frac{\eta}{1-\eta}\theta(R)\right],$$

which is equivalent to (9) under the Hosios condition. These results confirm that the optimization problem in (17) coincides with the solution to the planner's problem.

Derivation of (23). Using (20) and (22) and substituting in for $\frac{\pi}{c}$

$$\frac{\frac{(r+\lambda)}{1-\beta}\frac{1}{Aq(\theta)}-\kappa+\frac{\beta}{1-\beta}\theta}{\frac{(r+\lambda)}{\beta}\frac{\kappa}{Am(\theta)}}=\frac{1-E(\widetilde{x}\mid\widetilde{x}<R)}{1-R}.$$

Using (20) and (22) and substituting in for $(r+\lambda)$ and rearranging

$$\begin{aligned}\frac{\pi}{c}(1-R)\frac{\beta}{1-\beta}\frac{\theta}{\kappa}-\left(\frac{\kappa}{(1+\delta)}-\frac{\beta}{1-\beta}\theta\right)&=\frac{\pi}{c}(1-E(\widetilde{x}\mid\widetilde{x}<R)),\\ \frac{\beta}{1-\beta}\frac{\theta}{\kappa}&=\frac{\kappa+\frac{\pi}{c}(1-E(\widetilde{x}\mid\widetilde{x}<R))}{\kappa+\frac{\pi}{c}(1-R)}.\end{aligned}$$

Then (23) is a sufficient condition for the claim.

APPENDIX 2. MODEL WHERE SELLERS BECOME POTENTIAL BUYERS

The main analysis can be extended to accommodate behavioral assumptions where asset sellers become potential asset buyers (which maybe appropriate in the context of the housing market). In this section, I explicitly refer to the asset as an owner occupied home which delivers housing services. The key ingredient is that there is a rental market for housing services which is distinct from the owner occupied housing market. Such services are assumed to be traded in spot markets which are not subject to search frictions. The asset value of being a renter is normalized to zero (this would actually be the case when the cost and benefit of housing services through the rental market are equal). To keep the analysis simple, I consider the case of degenerate distribution of λ shocks where $x = 0$ following every shock, so there is only one submarket.

Let N denote the population of agents or households. At any point in time, the sum of home buyers v and owners is assumed to be strictly smaller than this population, $v + 1 < N$, which implies there is always a "surplus" measure of agents who are renting and not looking to buy a home for ownership (in the housing market about 40% of households are renters). Under this assumption, the option value of being a home buyer is driven to zero, $V = 0$. Moreover, in terms of our value equations in (1), imposing that (i) a home seller become a buyer upon the sale of his home or (ii) a home owner become a buyer upon the listing of his home, is *inconsequential* for the equilibrium analysis in the main text, since we would simply be adding a zero term. This implies that the decision to buy and sell assets can be analyzed distinctly as assumed in the main text.

Thus, the analysis can accommodate the assumption of asset sellers becoming buyers in a simple way under the assumption that agents may become renters in between episodes of home ownership (in the real world this is clearly often the case, most obviously when agents are relocating across cities). Note, in the case where we assume owners become potential buyers upon listing their existing home, we may in practice observe many agents actually facing *no rental spells* as they transition between ownership of different homes. Also, in this case, a sufficient condition for $V = 0$ is $v + 1 - v/\theta < N$, where v/θ is the measure of home sellers.