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Sequential Quality and Price Competition in Hotelling Model*

Sung Hyun Kim†

Abstract We allow sellers in Hotelling model to choose "quality" (referring to the gross surplus of product) before choosing prices and examine welfare implications of introducing a horizontally differentiated product into a monopoly market. Under certain parameter assumptions, we show that entry results in lower prices, lower qualities, (typically) lower consumers' surplus and lower social welfare. Our findings suggest that we should not regard the number of sellers in a differentiated market to be a simple measure of competitiveness.

Keywords Horizontal differentiation, Vertical differentiation, Entry

JEL Classification D40, L10

*I thank three anonymous referees for very constructive comments. Remaining deficiencies are solely mine.

†Ewha Womans University, Department of Economics, 11-1 Daehyundong Seodaemun-gu, Seoul, 127-750, Republic of Korea, Phone/Fax: +82-2-3277-4063, E-mail: sungkim@ewha.ac.kr

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1. INTRODUCTION

Product differentiation is a prevalent feature of the modern economy, as forcefully championed by Chamberlin (1933). It is though arguable that we do not have a complete understanding of market outcomes when products are differentiated. In contrast, if products are assumed to be homogeneous, Cournot's (1838) analysis has established a unified and coherent set of results under fairly general and reasonable conditions.

This paper explores a well-known model of product differentiation from an unconventional angle and is another attempt in a series of recent work examining "counter-intuitive" effects of entry on market outcomes under the presence of product differentiation.

The number of sellers in an industry is often used as a convenient structural measure of competitiveness. It is almost common sense to suggest that as the number of sellers increases, the market will become more competitive and consumers will benefit from competition. However, several recent (and not so recent) papers have shown that equilibrium price may rise in response to entry, especially if products are horizontally differentiated. In this paper, we offer instances where entry may be harmful to consumers in a dimension other than price.

More specifically, we take Hotelling's (1929) classical linear city model, where standard treatments focus on locations (horizontal differentiation) or price but examine sellers' strategic choice of "quality". We show that entry of a horizontally differentiated product may lead to degradation of quality and reduction of consumers' surplus and social welfare.

It should be duly noted that our result is not completely general, being a curious feature exhibited in a simple model, but an important lesson is that we should not be content relying on our common sense intuitions as to how markets behave when products are differentiated.

The fact that a monopolist may not provide a socially efficient level of quality was pointed out by Spence (1975) in terms of the difference between marginal and average valuations of quality. (Also see Snyder and Nicholson, 2007, chapter 14 for a textbook treatment.) Ma and Burgess (1993)[1] extended this discussion to duopoly with both horizontal and vertical differentiation to examine whether equilibrium qualities are suboptimal. Their research questions clearly resonate with our inquiries and in interpreting our results later, we shall make references

[1] I am grateful to an anonymous referee who brought this work to my attention, which subsequently helped to clarify my own thinking about the issues of this paper.

to their arguments. However, one important difference is that Ma and Burgess (1993) rely on the assumption that higher qualities require higher fixed cost, while our model does not assume any fixed cost. Hence, this paper is not a mere re-iteration of results already known in the literature.

More generally, our contention that entry may not necessarily be socially efficient can be placed within a broader literature.[2] Classic papers such as Spence (1976) and Mankiw and Whinston (1986) have shown that entry may be socially inefficient. Inefficiency in those papers mainly arises from the presence of another kind of fixed cost—that of set-up or entry. This paper identifies a different source of inefficiency, as we do not assume any cost of entry. If we were to introduce entry cost into our analysis, it will make our result even stronger.

The debate on the complex relationship between the number of firms and efficiency can be traced even further back. The so-called Schumpeter's thesis that monopoly may be a necessary evil for a dynamically efficient economy has generated a huge literature. (Tirole, 1988, chapter 10; Kamien and Schwartz, 1982) One important element of the debate is related to the fact that R&D and innovation exhibit a feature of public goods due to the presence of high fixed cost in such activities. However, our analysis does not include any dynamic consideration hence does not directly contribute to this old debate.

The paper is organized as follows. After describing the basic ingredients of the model in Section 2, we characterize equilibria of a monopoly version and a duopoly version of the model in Section 3. In Section 4, we analyze the effects of entry.

2. MODEL

Consumers are uniformly distributed on the unit interval $[0,1]$. Each consumer decides whether to purchase one unit of a product. A consumer located at $x \in [0,1]$ contemplates purchasing from a seller located at $z \in [0,1]$ and faces three parameters s, p and t that determine her net utility $s - p - t|x-z|$. Each consumer's reservation utility is normalized to be zero, so she purchases a product only if $s - p - t|x-z| \geq 0$.

The parameter s represents consumer's gross surplus obtained from consuming the product and we refer to it as the product's "quality" hereafter. The parameter p is the mill price charged by the seller. The parameter t is the so-called transportation cost per distance $|x-z|$, where the distance may be interpreted in

[2] I thank another anonymous referee for encouraging me to include this discussion of broader literatures in this and the next paragraphs.

terms of geographical or any other dimensions of horizontal differentiation.[3]

In Hotelling's (1929) original analysis, s and t are exogenously given and the sellers choose z (location) and p (price) sequentially. Like many other writers, we maintain the assumption that t is given as it represents consumers' taste regarding horizontally differentiated products. On the other hand, we fix the locations of sellers to be the end points 0 and 1 of the interval and do not consider the issue of locational choice. (For a survey on the topic of locational choice, see Gabszewicz and Thisse, 1992.)

If we also fix s (as is usually done), then we obtain a pricing game between horizontally differentiated sellers, which leads to some familiar and not-so-familiar results. Many texts in microeconomic theory or industrial organization, such as Mas-Colell et al. (1995) or Tirole (1988), discuss the familiar pricing equilibria under "reasonable" parameter values of s and t. On the other hand, Wang (2006) and Kim (2006) explain less well-known characterizations of equilibria under broader ranges of parameter values.

An interesting finding reported in Kim (2006) is that post-entry duopoly equilibrium price may be higher than pre-entry monopoly price. In a similar vein, the fact that increasing number of sellers can lead to higher prices has been noted in such recent papers as Chen and Riordan (2008) and Roessler (2012).[4] One lesson from these papers is that we need to be careful about effects of price competition in differentiated product markets.

A natural question then arises as to whether non-price competition can also exhibit a similar phenomenon. In our models, we address the question by allowing sellers to choose quality s, in a stage prior to price competition.[5]

This may be seen as a theoretical exercise of trying out different strategic dimensions in Hotelling model, but a number of real world markets correspond to our model settings. Many sellers do compete in both price and quality and how entry affects both price and quality is an important research question. (Courtemanche and Carden, 2011) In a model where sellers are able to set both prices

[3]The fact that the transportation cost is linear in distance is significant for the existence and the properties of equilibria and our later results may not be robust to other specifications. (d'Aspremont et al, 1979) But the linear specification is commonly employed in discussions of linear city models following the lead of Hotelling (1929).

[4]Older papers include Salop (1979), Satterthwaite (1979) and Rosenthal (1980). Surprisingly enough, such perverse effects of entry are also possible even in Cournot models where products are homogeneous. See Frank and Quandt (1963) and Frank (1963) where efforts are made towards removing such perverse effects and Amir and Lambson (2000) where the issue is treated in a more constructive manner.

[5]We could also consider a somewhat simpler case where quality is the only strategic aspect of competition with prices exogenously fixed, which is explored in a companion paper, Kim (2012).

and qualities, it is natural to treat price as a short-run variable compared to quality. (Tirole, 1988, p.205)

Before entry, a monopolist offers a product located at point 0. An entrant offers an identical product at location 1. Assume for simplicity that the per unit production cost is zero for both sellers, but the sellers need to incur cost $C(s)$ to provide quality s, with $C'(\cdot) > 0$ and $C''(\cdot) > 0$. For concreteness, let us impose a functional form for $C(s)$ as an assumption.

Assumption 1. $C(s) = \frac{1}{2}cs^2$, with $c > 0$

While we will use Assumption 1 to obtain closed-form expressions for results, key findings should be qualitatively similar with any convex cost function.

Sellers first choose quality s_i $(i = 0, 1)$, then choose price p_i $(i = 0, 1)$ for their products. The second stage of pricing corresponds to the second stage of standard Hotelling model where locations (horizontal differentiation) are fixed. In the case of monopoly, the sequential nature doesn't have to be imposed but is analytically more convenient and allows for an easier comparison with duopoly game. For the duopoly game, each seller chooses simultaneously in each stage and we look for subgame perfect equilibria.

3. EQUILIBRIA

3.1. PRE-ENTRY MONOPOLY

For an easy comparison between monopoly and duopoly cases, we shall use choice variables s and p without any subscripts for monopolist and put subscripts $i = 0, 1$ for duopolists.

Consider the second stage of sequential optimization. Some level s of quality has already been chosen and the seller now faces the problem of choosing price p to maximize px where $x = \max\left(\frac{1}{t}(s-p), 1\right)$, because the total demand is limited by the size of the city ($x \leq 1$) and if some consumers do not purchase ($x < 1$), then the location of the marginal consumer is $\frac{1}{t}(s-p)$. We can easily characterize profit-maximizing prices p as follows.

Lemma 3.1. *If $s \geq 2t$, then the monopolist chooses $p = s - t$. If $s \leq 2t$, then the monopolist chooses $p = \frac{1}{2}s$. (The pricing schedule is continuous at $s = 2t$.)*

Instead of a formal proof, we sketch an intuitive argument as follows. If s is sufficiently high, all consumers are willing to purchase and the seller faces inelastic demand, so the profit maximizing price is found by setting the net utility enjoyed by the farthest consumer to be zero, i.e. $s - p - t = 0$. On the other hand, if s is sufficiently low, then demand is $\frac{1}{t}(s-p)$ and profit is $\frac{1}{t}p(s-p)$, which is maximized at $p=\frac{1}{2}s$. At this price, we have $x=\frac{1}{2t}s \leq 1$. So the threshold value of s that determines which of the two cases occurs is $s=2t$.

Given Lemma 3.1, we can determine the optimal level of s that the monopolist chooses in the first stage. Let us introduce an assumption on the size of the "cost" parameters c and t, which will be in effect throughout the rest of the paper. This assumption ensures the viability of the market (positive profits for sellers in equilibrium).

Assumption 2. The cost parameters are sufficiently small, i.e. $ct < \frac{1}{2}$

If the seller chooses some $s < 2t$, then its profit will be $f(s) = px - \frac{1}{2}cs^2 = \frac{1}{2}s \cdot \frac{1}{2t}s - \frac{1}{2}cs^2 = \frac{1}{2}(\frac{1}{2t} - c)s^2 > 0$ (by Assumption 2). If the seller chooses some $s \geq 2t$, then its profit will be $g(s) = p - \frac{1}{2}cs^2 = s - t - \frac{1}{2}cs^2$. By plotting the profit as function of s, we can see that it is continuous at $s = 2t$ and the maximum is achieved at $s = \frac{1}{c} > 2t$. (See Figure 1.) These assertions are proved in the following lemma.

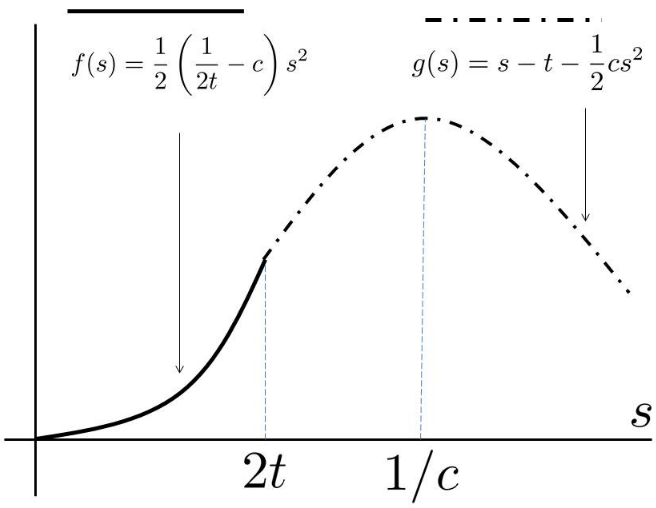

Figure 1: The maximum profit is achieved at $s = \frac{1}{c} > 2t$

Lemma 3.2. *The monopolist's profit is continuous as a function of s. The monopolist chooses $s = \frac{1}{c} > 2t$ and all consumers purchase the product, hence the* *monopoly price is* $p = s - t = \frac{1}{c} - t$.

Proof. Continuity of profit is easily checked by $f(2t) = g(2t) = t(1-2ct)$. Since $g(\cdot)$ is quadratic, $g'(\frac{1}{c}) = 0$ and $g''(\frac{1}{c}) = -c < 0$, it follows that $g(s)$ is maximized at $s = \frac{1}{c}$. Finally, $g(\frac{1}{c}) - f(2t) = \frac{1}{2c} - t - t(1-2ct) = \frac{(1-2ct)^2}{2c} > 0$, so $g(\frac{1}{c}) > f(2t)$. Therefore we conclude that the profit maximizing quality is $s = \frac{1}{c}$. The rest follows from Lemma 3.1. ■

3.2. POST-ENTRY DUOPOLY

We now let an entrant offer another product at point 1. The two sellers engage in a sequential game, timing of which can be described as follows. In the first stage, the two sellers simultaneously choose quality levels, denoted s_0 and s_1. In the second stage, after observing quality choices, the two sellers simultaneously choose prices, denoted p_0 and p_1.[6] We look for subgame perfect equilibria of this game.

3.2.1 The second stage (pricing)

Consider the second stage where s_0 and s_1 are given.

Lemma 3.3. *There are three kinds of Nash equilibria in the second stage game depending on the values of* s_0 *and* s_1 *chosen in the first stage.*

(1) **[duopoly]** *If* $s_0 + s_1 > 3t$*, then the unique Nash equilibrium prices are*

$$p_0 = t + \frac{s_0 - s_1}{3},\ p_1 = t - \frac{s_0 - s_1}{3}$$

with demand for the incumbent $y_0^H = \frac{1}{2} + \frac{1}{6t}(s_0 - s_1)$*. In this equilibrium, all consumers purchase the product from one of the sellers, i.e. demand for the entrant is* $y_1^H = 1 - y_0^H$.

(2) **[local monopolies]** *If* $s_0 + s_1 < 2t$*, then the unique Nash equilibrium prices are*

$$p_0 = \frac{1}{2}s_0,\ p_1 = \frac{1}{2}s_1$$

with demand for the incumbent $y_0^L = \frac{1}{2t}s_0$ *and for the entrant* $y_1^L = \frac{1}{2t}s_1$*. In this equilibrium, some consumers do not purchase the product from either of the sellers* ($y_0^L + y_1^L < 1$).

[6] Note that we do not consider the entry game itself. We simply assume that entry occurs and the incumbent accommodates it.

(3) **[adjacent sellers]** *If otherwise (i.e. $2t \leq s_0 + s_1 \leq 3t$), then there are infinitely many Nash equilibria where the equilibrium prices satisfy the relation*

$$p_0 + p_1 = s_0 + s_1 - t$$

with demand for the incumbent $y_0^M = 1 - \frac{1}{t}(s_1 - p_1)$. In this equilibrium, all consumers purchase the product from one of the sellers, i.e. demand for the entrant is $y_1^M = 1 - y_0^M$.

While proof is given in Appendix A.1, some remarks are in order. In Case (1), the condition $s_0 + s_1 \geq 3t$ ensures that net utility of the marginal consumer (who is indifferent between two sellers) is positive. It will be shown below that in the subgame perfect equilibrium corresponding to this stage game Nash equilibrium we have $s_0 = s_1 = s$, so the condition reduces to $s > \frac{3}{2}t$, which is weaker than $s \geq 2t$ (the condition introduced in Lemma 3.1 for monopolist covering the whole market). Case (1) describes a standard duopoly pricing equilibrium (easily found in textbook treatments), where the two sellers' markets overlap so they engage in a genuine competition and the marginal consumer enjoys a positive net surplus.

The fact that there are other Nash equilibria than is described by Case (1) was recently emphasized by Wang (2006). In Case (2), quality levels are set so low that some consumers do not bother to purchase from either of the sellers and each seller acts as a local monopolist. (Note that the described equilibrium is essentially identical to that given in Lemma 3.1 for the monopolist.) Each seller's market is isolated and each acts independently.

Case (3) poses some challenges for analysis in that there is a continuum of equilibria. These equilibria occur at a "kink" in the demand curve where local monopolies regime makes a transition to duopoly regime. (See Tirole 1988, p. 98, Figure 2.2) In any one of these equilibria, a seller's market is exactly adjacent to another's market. If a seller raises its price slightly, its own market shrinks and its profit also falls. If a seller lowers its price slightly, then the two markets overlap and the sellers need to engage in a full pricing competition but there is no equilibrium that ensures positive net surplus for the marginal consumer and non-negative profits for the sellers under the given moderate levels of quality. In the equilibrium, each seller is neither a monopolist nor a genuine duopolist, hence our term "adjacent sellers".

3.2.2 The first stage (quality setting)

Given Lemma 3.3's characterizations of Nash equilibria of the second stage game, we should now solve for subgame perfect equilibria of the whole game. A "complete" characterization of subgame perfect equilibria would proceed as follows: Seller i's profit function may be denoted $\pi_i(s_i, s_j; p_i(s_i, s_j), p_j(s_i, s_j))$ where it is noted that the second-stage choice of p_i depends on both s_i and s_j and similarly for the choice of p_j. If $\pi_i(\cdot)$ were a well-behaved function, then we would write down best responses s_i for arbitrary s_j (and vice versa) and look for the "intersections" of best responses.

But difficulty is caused by the adjacent sellers equilibria, Case (3) from Lemma 3.3. When the competitor's choice is $s_j < 3t$, the seller i can choose to enter into the adjacent sellers regime in the second stage, but the price equilibrium is indeterminate there and we cannot derive a unique "best" response s_i. In other words, a seller's best response s_i to a competitor's choice of s_j cannot be described as a standard function, but it must take the form of a set-valued function or correspondence.

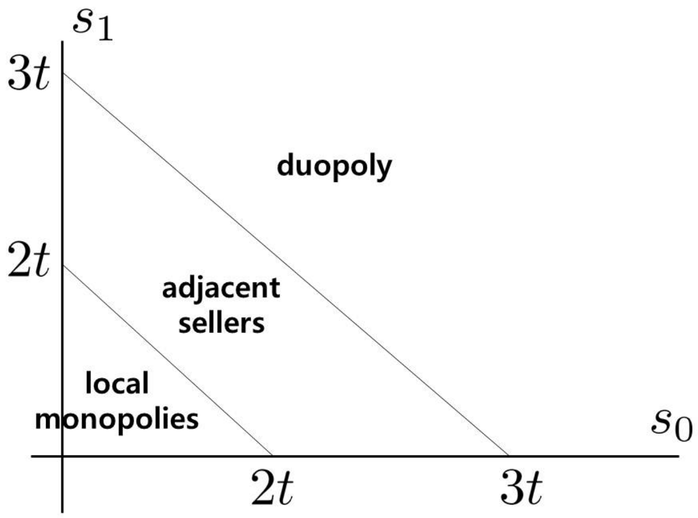

Figure 2: Different regimes for the second-stage

Moreover, even if we impose some *ad hoc* restrictions on the price equilibrium (as we will shortly do), the optimal choice depends on the parameter values and a complete analysis requires consideration of a great number of cases.

Therefore, a tractable alternative is to develop a few interesting scenarios and focus on them. Instead of deriving complete best responses, we will consider 3 regimes (see Figure 2) separately. As different levels of quality s_0 and s_1 are chosen in the first stage, we are led into different regimes in the second stage. The choice of quality is then governed by values of parameters c and t.

Suppose $s_0 + s_1 > 3t$ so that Case (1) of Lemma 3.3, the duopoly equilibrium, realizes in the second stage. Then sellers would choose s_i in the first stage in order to maximize

$$\Pi_0(s_0, s_1) = p_0 y_0^H - \frac{1}{2}cs_0^2 = \left(t + \frac{s_0 - s_1}{3}\right)\left(\frac{1}{2} + \frac{1}{6t}(s_0 - s_1)\right) - \frac{1}{2}cs_0^2$$

and

$$\Pi_1(s_0, s_1) = p_1(1 - y_0^H) - \frac{1}{2}cs_1^2 = \left(t - \frac{s_0 - s_1}{3}\right)\left(\frac{1}{2} - \frac{1}{6t}(s_0 - s_1)\right) - \frac{1}{2}cs_1^2$$

respectively. The first order conditions $\frac{\partial \Pi_0}{\partial s_0} = \frac{\partial \Pi_1}{\partial s_1} = 0$ reveal the interior equilibrium to be $s_0 = s_1 = \frac{1}{3c}$. For this equilibrium to make sense, we must have $s_0 + s_1 = \frac{2}{3c} > 3t$ or $ct < \frac{2}{9}$. In other words, if cost parameters are sufficiently low (i.e. producing quality is sufficiently cheap and consumers do not dislike a non-ideal product too much), then sellers can choose qualities high enough to induce standard pricing competition. In this equilibrium, each seller's profit is $\frac{1}{18c}(9ct - 1)$. Hence, we also need $ct \geq \frac{1}{9}$ for this to be a viable equilibrium for sellers. Hence, we have found an interior subgame perfect equilibrium for $\frac{1}{9} \leq ct < \frac{2}{9}$.

On the other hand, suppose $s_0 + s_1 < 2t$ so that Case (2), the local monopolies equilibrium, realizes in the second stage. (Note that this regime is uninteresting from our perspective because the two sellers never interact and it is obvious that entry is socially desirable in this case.) Then $p_i = \frac{1}{2}s_i$ and $y_i^L = \frac{1}{2t}s_i$ so that profit is a single-variable function of s_i as follows:

$$\Pi_i(s_i) = \frac{1}{2}s_i \frac{1}{2t}s_i - \frac{1}{2}cs_i^2 = \frac{1}{2}\left(\frac{1}{2t} - c\right)s_i^2$$

By Assumption 2 ($ct < \frac{1}{2}$), this profit is strictly increasing in s_i. If we assume symmetry between the sellers, the highest profit *within local monopolies regime* is achieved at $s_i = t$ with profit amounting to

$$\Pi^L = \frac{1}{4}t(1 - 2ct).$$

We will keep this value Π^L as a benchmark.

Finally, suppose chosen quality levels are intermediate so that Case (3), the adjacent sellers equilibrium, realizes in the second stage. All equilibria in this case satisfy the relation

$$p_0 + p_1 = s_0 + s_1 - t$$

From the range of s_i's allowed, we can see that $t \leq p_0 + p_1 \leq 2t$.

In principle, sellers may choose very divergent prices and qualities as long as those choices satisfy the above formula. For a tractable analysis, let us impose an *ad hoc* (but not unreasonable) requirement that their choices do not differ significantly (perhaps because of pressures from consumer groups or by government regulations). Specifically, we shall introduce the following pricing rule.

Quality-adjusted pricing rule: $p_0 - p_1 = s_0 - s_1$

Under this pricing rule, sellers (via some coordination mechanism) adjust their prices so as to reflect differences in their qualities. This can be achieved if we set $p_i = s_i - \frac{1}{2}t$. With these prices, two sellers divide the market equally. The seller i's profit is $\Pi_i(s_i) = \frac{1}{2}s_i - \frac{1}{4}t - \frac{1}{2}cs_i^2$. The optimal qualities are $s_0 = s_1 = \frac{1}{2c}$. These values meet Case (3) conditions if $\frac{1}{3} \leq ct < \frac{1}{2}$.

In this equilibrium, since qualities are identical, prices are also eventually identical. Each seller earns profit $\Pi^M = \frac{1}{8c}(1-2ct)$, which is always higher than $\Pi^L = \frac{1}{4}t(1-2ct)$ derived earlier (by Assumption 2).

So it becomes apparent that local monopolies regime of Case (2) can be ruled out for some parameter values. Whether Case (1) or Case (3) obtains as an optimum depends on the value of ct. When ct is sufficiently low (but not too low), Case (1) obtains and if ct is sufficiently high, then Case (3) with quality-adjusted pricing rule is applicable. Therefore, while not a complete characterization, we can summarize our analysis so far as the following lemma.

Lemma 3.4. (1) *if $\frac{1}{9} \leq ct < \frac{2}{9}$, then the unique subgame perfect equilibrium outcome is*

$$s_0 = s_1 = \frac{1}{3c},\ p_0 = p_1 = t$$

(2) *if $\frac{1}{3} \leq ct < \frac{1}{2}$, then there is a subgame perfect equilibrium (under quality-adjusted pricing rule) where*

$$s_0 = s_1 = \frac{1}{2c},\ p_0 = p_1 = \frac{1}{2c} - \frac{1}{2}t$$

4. EFFECTS OF ENTRY

From the results summarized in 4 lemmata in the previous section, we can now compare market outcomes before and after entry, hence determine the effects of entry.

4.1. EFFECTS ON QUALITY LEVELS

Let us first compare quality levels chosen before and after entry.

Proposition 1. *Suppose Assumptions 1 and 2 hold for our models of pre-entry monopoly and post-entry duopoly. Both qualities and prices decrease by entry if cost parameters are sufficiently low ($\frac{1}{9} \leq ct < \frac{2}{9}$) or sufficiently high ($\frac{1}{3} \leq ct < \frac{1}{2}$).*

Proof. We have $s = \frac{1}{c}$ and $p = \frac{1}{c} - t$ before entry. (Lemma 3.2)

(1) When $\frac{1}{9} \leq ct < \frac{2}{9}$, post-entry values are $s_0 = s_1 = \frac{1}{3c}$ and $p_0 = p_1 = t$. [Lemma 3.4 (1)] So $s_0 = s_1 = \frac{1}{3}s < s$. We also have $p - p_0 = \frac{1}{c} - t - t = \frac{1}{c} - 2t = \frac{1-2ct}{c} > 0$.

(2) When $\frac{1}{3} \leq ct < \frac{1}{2}$, post-entry values are $s_0 = s_1 = \frac{1}{2c}$ and $p_0 = p_1 = \frac{1}{2c} - \frac{1}{2}t$. [Lemma 3.4 (2)] So $s_0 = s_1 = \frac{1}{2}s < s$. We also have $p_0 = \frac{1}{2}p < p$. ■

We could further strengthen the proposition by extending it to include other intermediate values of ct, but we still can draw some implications from this limited version.

First, the fact that prices fall by entry is standard and is not inconsistent with "counter-intuitive" results referred to earlier in the paper. Equilibrium price may rise due to entry when the incumbent monopolist was not serving the whole potential market. (Kim, 2006, Proposition 1) This happens when quality is sufficiently low so that the incumbent does not find it profitable to serve consumers on the other side of the linear city.

But in our model, since the monopolist is allowed to choose quality before setting price, it will choose a sufficiently high quality so that the whole potential market is covered. When entry occurs, the monopolist is forced to share the market with the entrant. Pricing competition leads to a lower price. Furthermore, the finding that qualities will fall by entry is not surprising either. Since cost of producing quality is convex, it is natural that equilibrium quality falls as the size of individual market decreases.

Ma and Burgess (1993), though in a different model setting, derived a similar result and interpreted it in terms of two opposing effects—that of price undercutting and of higher marginal cost. One seller's higher quality leads to the competitor's price undercutting in the subsequent stage. Anticipating this, sellers try not to raise qualities too high in the first stage. This is the price undercutting effect. On the other hand, a higher quality means a higher marginal cost (under convex cost) and this may offset some of the price undercutting effect. In fact, in Ma and Burgess (1993), when quality production does not involve any fixed cost, the two effects cancel each other out. But in our model, only the price undercutting effect seems to be in play. This is probably due to the special simplifying nature of Hotelling model.

Even though we have concluded that qualities tend to fall by entry in our model, what is not so clear is whether this degradation of quality will result in reduction of consumers' overall welfare—this is the central question of this paper. This question is addressed in the next subsection.

For ease of reference, Table 1 tabulates market outcomes. (Computations are given in Appendix A.2.)

Table 1: Market Outcomes

	Monopoly	Duopoly ($\frac{1}{9} \le ct < \frac{2}{9}$)	Adjacent Sellers ($\frac{1}{3} \le ct < \frac{1}{2}$)
Quality	$s = \frac{1}{c}$	$s_0 = s_1 = \frac{1}{3c}$	$s_0 = s_1 = \frac{1}{2c}$
Price	$p = \frac{1}{c} - t$	$p_0 = p_1 = t$	$p_0 = p_1 = \frac{1}{2c} - \frac{1}{2}t$
Consumers' surplus	$\frac{1}{2}t$	$\frac{1}{3c} - \frac{5}{4}t$	$\frac{1}{4}t$
Profits	$\frac{1}{2c} - t$	$t - \frac{1}{9c}$	$\frac{1}{4c} - \frac{1}{2}t$
Social Welfare	$\frac{1}{2c} - \frac{1}{2}t$	$\frac{2}{9c} - \frac{1}{4}t$	$\frac{1}{4c} - \frac{1}{4}t$

4.2. EFFECTS ON CONSUMERS' SURPLUS

As we saw in the previous subsection, entry will result in lower qualities (lower surplus) and lower prices (higher surplus), hence the net effect on consumers' surplus is ambiguous.

From Table 1, it is easy to see that the following results obtain.

Proposition 2. *Suppose Assumptions 1 and 2 hold for our models of pre-entry monopoly and post-entry duopoly. Before entry, the incumbent monopolist serves the whole market.*

(1) *If* $\frac{1}{9} \leq ct < \frac{2}{9}$*, then entry leads to a duopoly equilibrium and consumers' surplus may increase or decrease by entry. More specifically,*

- *If* $\frac{1}{9} \leq ct < \frac{4}{21}$*, then consumers' surplus increases.*
- *If* $\frac{4}{21} < ct < \frac{2}{9}$*, then consumers' surplus decreases.*

(2) *If* $\frac{1}{3} \leq ct < \frac{1}{2}$*, then there is a subgame perfect equilibrium under quality-adjusted pricing rule where entry leads to an adjacent sellers equilibrium and consumers' surplus decreases by entry.*

Proof. (1) From Table 1, when $\frac{1}{9} \leq ct < \frac{4}{21}$, the change in consumers' surplus is $\frac{1}{3c} - \frac{5}{4}t - \frac{1}{2}t = \frac{1}{3c} - \frac{7}{4}t = \frac{4-21ct}{12} \gtrless 0 \Longleftrightarrow ct \lessgtr \frac{4}{21}$.

(2) Similarly from Table 1, when $\frac{1}{3} \leq ct < \frac{1}{2}$, the change in consumers' surplus is $\frac{1}{4}t - \frac{1}{2}t = -\frac{1}{4}t < 0$. ■

In the duopoly equilibrium (1), a lower c implies a higher level of qualities and a lower t implies a lower level of prices. So only when ct is sufficiently low, entry leads to a sufficiently high qualities (even after degradation) and a sufficiently low prices so that consumers benefit from entry.

On the other hand, in the adjacent sellers equilibrium (2) consumers are worse off than under monopoly. This can be understood as follows. The key characteristic of the adjacent sellers equilibrium is that the marginal consumer is indifferent between purchasing (from any of the sellers) and not purchasing at all because her net utility is zero. This is, in fact, identical to a situation where the incumbent monopolist (located at point 0) decides to offer another product at point 1 and become a multiproduct monopolist. (Alternatively, we can imagine that the incumbent merges with the entrant to form the multiproduct monopolist.) Obviously, such a move by the monopolist would only harm consumers. (See Tirole, 1988, p. 105, Exercise 2.3)

4.3. EFFECTS ON SOCIAL WELFARE

Finally, we examine the total social welfare, defined as the sum of consumers' surplus and sellers' profits (which include the cost of producing qualities, but do *not* include entry costs). While we observed in the previous subsection that consumers' surplus may increase by entry if ct is sufficiently low, it turns out that social welfare *always* decreases by entry. In other words, even when consumers benefit from entry, the benefit is more than offset by losses in sellers' profits.

Proposition 3. *Suppose Assumptions 1 and 2 hold for our models of pre-entry monopoly and post-entry duopoly. Social welfare decreases by entry if* $\frac{1}{9} \le ct < \frac{2}{9}$ *or if* $\frac{1}{3} \le ct < \frac{1}{2}$.

Proof. Again the relevant quantities can be taken from Table 1 above.

(1) When $\frac{1}{9} \le ct < \frac{4}{21}$, the change in social welfare is $\frac{2}{9c} - \frac{1}{4}t - \left(\frac{1}{2c} - \frac{1}{2}t\right) = -\frac{5}{18c} + \frac{1}{4}t = \frac{-10+9ct}{36c} < 0$ by Assumption 2.

(2) When $\frac{1}{3} \le ct < \frac{1}{2}$, the change in social welfare is $\frac{1}{4c} - \frac{1}{4}t - \left(\frac{1}{2c} - \frac{1}{2}t\right) = -\frac{1}{4c} + \frac{1}{4}t = \frac{-1+ct}{4c} < 0$ by Assumption 2. ■

5. CONCLUSION

In this paper, we took the well-known Hotelling model of horizontal differentiation and examined what happens when we allow sellers to choose "quality" (gross-surplus enjoyed by consumers from the product, normally assumed fixed). Retrospectively, our findings are not shocking—as long as the cost of producing quality is convex, entry forces the incumbent monopolist (who needs to share the market with the entrant) to lower both quality levels and prices.

However, somewhat surprising and possibly counter-intuitive implications (at least for the raw common sense intuition unaided by our models) are found for the effects on consumer's surplus and social welfare. Unless the relevant parameters (on quality-producing cost c and consumers' distaste for horizontal differentiation t) are sufficiently small, price reduction is not enough to offset quality degradation hence consumers may be harmed by entry. Moreover, even when consumers benefit from entry, such a beneficial effect is again offset by profit losses so that entry is always socially undesirable.

Our findings are of course limited because we have focused on very simple models and a limited range of parameter values. But we cannot ignore even such limited results when we set out to consider real-world markets where both vertical and horizontal differentiations are prevalent. If we simplify our model even further by assuming sellers compete exclusively in quality-setting (with exogenously fixed prices), we can obtain a cleaner set of results (reported in a separate paper, Kim, 2012) but with a similar implication that consumers may be harmed by entry.

APPENDIX

A.1 Proof of Lemma 3.3

This proof closely follows Wang's (2006) arguments.

Case (1): Let us first consider the case of a standard duopoly equilibrium. If sellers choose p_0 and p_1, then the location y_0^H of the marginal consumer who is indifferent between the two sellers is determined by

$$s_0 - p_0 - ty_0^H = s_1 - p_1 - t(1-y_0^H) \quad \Longrightarrow \quad y_0^H = \frac{1}{2} + \frac{(s_0-s_1)-(p_0-p_1)}{2t}$$

Profits are

$$\pi_0(p_0,p_1) = p_0 y_0^H = p_0\left(\frac{1}{2} + \frac{(s_0-s_1)-(p_0-p_1)}{2t}\right)$$

$$\pi_1(p_0,p_1) = p_1(1-y_0^H) = p_1\left(\frac{1}{2} - \frac{(s_0-s_1)-(p_0-p_1)}{2t}\right)$$

The (interior) Nash equilibrium of this stage game can be found by solving the first order conditions

$$\frac{\partial \pi_0}{\partial p_0} = \frac{1}{2} + \frac{(s_0-s_1)-(p_0-p_1)}{2t} + p_0 \times \left(-\frac{1}{2t}\right) = \frac{1}{2} + \frac{s_0-s_1}{2t} - \frac{1}{t}p_0 + \frac{1}{2t}p_1 = 0$$

$$\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} - \frac{(s_0-s_1)-(p_0-p_1)}{2t} + p_1 \times \left(-\frac{1}{2t}\right) = \frac{1}{2} - \frac{s_0-s_1}{2t} + \frac{1}{2t}p_0 - \frac{1}{t}p_1 = 0$$

The solutions for the two equations are

$$p_0 = t + \frac{1}{3}(s_0-s_1), \quad p_1 = t - \frac{1}{3}(s_0-s_1)$$

With these prices, the location of the marginal consumer is

$$y_0^H = \frac{1}{2} + \frac{s_0-s_1}{2t} - \frac{p_0-p_1}{2t} = \frac{1}{2} + \frac{s_0-s_1}{2t} - \frac{\frac{2}{3}(s_0-s_1)}{2t} = \frac{1}{2} + \frac{1}{6t}(s_0-s_1)$$

However, for this to be actually an equilibrium, the marginal consumer must enjoy a positive net utility. The net utility of consumer at y_0^H is $s_0 - p_0 - ty_0^H = \frac{1}{2}(s_0+s_1) - \frac{3}{2}t > 0$. Therefore, we need to have $s_0 + s_1 > 3t$ for this case to obtain.

Case (2): Let us now consider the case of local monopolies. The two sellers need not act strategically because their respective markets do not overlap, or there is a middle region of consumers who do not purchase from either of them.

The second part of Lemma 3.1 tells us that if the incumbent acts as a monopolist, it will choose $p_0 = \frac{1}{2}s_0$ and its demand is $\frac{1}{t}(s_0-p_0) = \frac{1}{2t}s_0$. Hence, $y_0^L = \frac{1}{2t}s_0$. Similarly, if the entrant acts as a monopolist, it will choose $p_1 = \frac{1}{2}s_1$

and its demand will be $\frac{1}{2t}s_1$. Because the entrant is located at point 1, the location of the marginal consumer for the entrant will be $y_1^L = 1 - \frac{1}{2t}s_1$.

For this to make sense, we must have $y_0^L < y_1^L$, which yields the condition $s_0 + s_1 < 2t$.

Case (3): The above characterizations leave a range of parameter values $2t \leq s_0 + s_1 \leq 3t$. Since the interior first order conditions in Case (1) do not yield an equilibrium in this case, we need to consider individual seller's best response, given an arbitrary choice by its competitor. We shall focus on the position of the incumbent. (The entrant's position can be argued along similar lines.) Its optimal choices depend on the entrant's choices of p_1.

We can quickly restrict the possible ranges of p_1 as follows. If $p_1 > s_1$, then no consumer finds it worthwhile to purchase from the entrant, so the incumbent can safely act as a monopolist [Case (2)]. On the other hand, if $p_1 < s_1 - t$, then the entrant's price is so low that all consumers would consider purchasing from the entrant, hence the incumbent must engage in duopoly competition [Case (1)].

Therefore, suppose $s_1 - t \leq p_1 \leq s_1$. Then the location y_1^M of the marginal consumer for the entrant is found by $s_1 - p_1 - t(1 - y_1^M) = 0$ or $y_1^M = 1 - \frac{1}{t}(s_1 - p_1)$. Now, the maximum price $\overline{p}$ that the *incumbent* can charge to the entrant's marginal consumer is found by setting the consumer's net utility from the incumbent to be zero, or $s_0 - \overline{p} - ty_1^M = 0$, which leads to $\overline{p} = s_0 - ty_1^M = s_0 + s_1 - t - p_1$. Notice that here we have obtained the relation between the two prices described in the Lemma: $p_0 + p_1 = s_0 + s_1 - t$. We are going to argue that the equilibria obtain at this price $\overline{p}$. At $p_0 = \overline{p}$, the incumbent serves the exact remainder of the entrant's market, so the incumbent's demand is exactly $y_0^M = 1 - y_1^M = \frac{1}{t}(s_1 - p_1)$.

The incumbent faces a demand schedule with a "kink". If it charges a price $p_0 < \overline{p}$, then all consumers purchase from one of the sellers. The incumbent's demand is (just as in the duopoly case) $y_0^M = \frac{1}{2} + \frac{(s_0 - s_1) - (p_0 - p_1)}{2t}$. On the other hand, if $p_0 > \overline{p}$, then some consumers do not purchase at all. The incumbent's demand is (just as in the local monopoly case) $y_0^M = \frac{1}{t}(s_0 - p_0)$. Each demand is linear, with the slope changing at $p_0 = \overline{p}$. (See Figure 3 below.)

Since the demand schedule is piecewise linear, we can easily derive marginal revenue functions. If we denote the upper and the lower parts of the demand with letters U and L, we can easily check that marginal revenue at the kink is such that $MR^U(\overline{y}) > MR^L(\overline{y})$.

Since production marginal cost is zero, the standard interior optimum is found by setting marginal revenue to be zero. But since there are two marginal revenue functions to deal with, we can divide our consideration into three subcases.

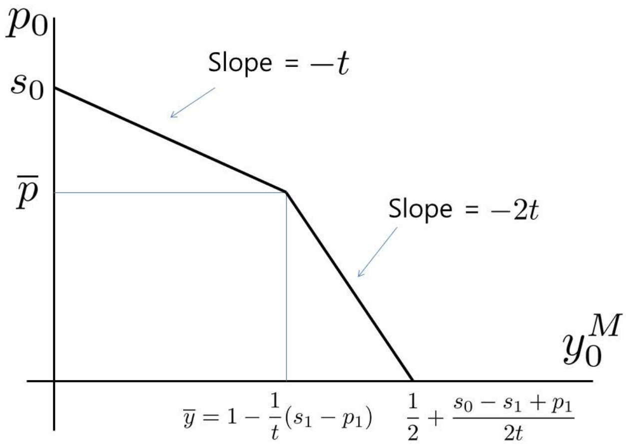

Figure 3: Two-part demand schedule faced by the incumbent

(A) $MR^U(\bar{y}) < 0$: Marginal revenue at the kink is negative, so the profit maximizing quantity is less. The incumbent act as a monopolist choosing $y = \frac{1}{2t}s_0 < \bar{y}$ (by choosing $p_0 = \frac{1}{2}s_0$). The case assumption states that $MR^U(\bar{y}) = s_0 - 2t\left(1 - \frac{1}{t}(s_1 - p_1)\right) < 0$, which leads to $p_1 > \frac{s_0+2s_1}{2} - t$. So this happens when the entrant charges a high price claiming a small market share and it is not profitable for the incumbent to try to capture the rest. This case properly belongs to the local monopolies regime. [Case (2)]

(B) $MR^U(\bar{y}) > 0 > MR^L(\bar{y})$: The first order condition cannot hold as marginal cost (zero) falls in a discontinuous region of the marginal revenues. The best response for the incumbent is simply to choose $\bar{y}$, meaning $p_0 = s_0 + s_1 - t - p_1$. From the case assumption $MR^U(\bar{y}) > 0$ we have $p_1 < \frac{s_0+2s_1}{2} - t$, while from $MR^L(\bar{y}) = t + s_0 - s_1 + p_1 - 4t\bar{y} < 0$ we have $p_1 > \frac{s_0+3s_1}{3} - t$. Therefore, we need to have $\frac{s_0+3s_1}{3} - t < p_1 < \frac{s_0+2s_1}{2} - t$ for this case.

(C) $MR^L(\bar{y}) > 0$: Obviously this case obtains when $p_1 < \frac{s_0+3s_1}{3} - t$. The incumbent will seek to have $MR^L(y) = 0$ where $y > \bar{y}$. The best-response price is $p_0 = \frac{1}{2}(t + s_0 - s_1 + p_1)$.

In conclusion, we have delineated in the sub-case (B) the region of parameter values where neither local monopoly nor genuine duopoly prevails. If $s_0 + s_1$

have intermediate values (between $2t$ and $3t$) and if p_1 have intermediate values, then the equilibrium requires $p_0 + p_1 = s_0 + s_1 - t$.

A similar reasoning will provide us with best-response prices for the entrant. A very tedious, hence omitted (see Wang, 2006), computations of several different ways that these two best-response curves can intersect will determine that for our parameter values $2t \leq s_0 + s_1 \leq 3t$, the equilibrium is such that $p_0 + p_1 = s_0 + s_1 - t$. ■

A.2 Derivations of quantities in Table 1

Qualities (s, s_0, s_1) and prices (p, p_0, p_1) have been derived in Lemma 3.1 through 3.4. We can use these in deriving the remaining quantities.

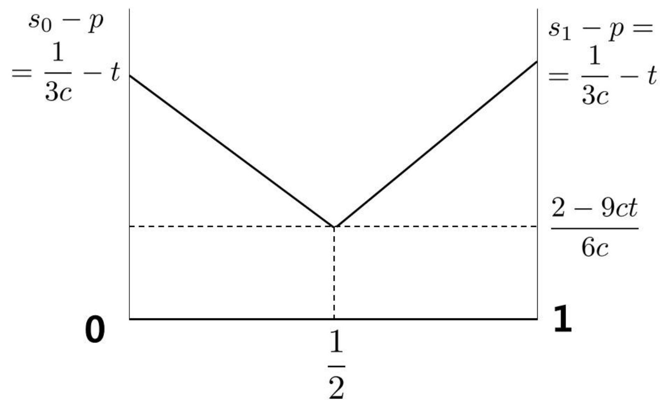

Figure 4: Consumers' surplus for duopoly

Consumers' surplus: For monopoly, we have observed in Lemma 3.2 that the monopolist covers the market and extracts the marginal consumer's surplus completely so that consumers' surplus is represented by a right triangle with two legs of lengths $s - p = t$ and 1, respectively. Hence, it is $\frac{1}{2}t$. For duopoly, two sellers divide the market equally so the marginal consumer is located at $\frac{1}{2}$. Moreover, the marginal consumer enjoys a positive net utility of $s_i - p_i - \frac{1}{2}t = \frac{1}{3c} - t - \frac{1}{2}t = \frac{1}{3c} - \frac{3}{2}t = \frac{2-9ct}{6c} > 0$. (See Figure 4.) Hence we need to compute the areas for two right triangles and a rectangle, which yields the answer $\frac{1}{3c} - \frac{5}{4}t$. Details are omitted. Finally, for adjacent sellers, again two sellers divide the

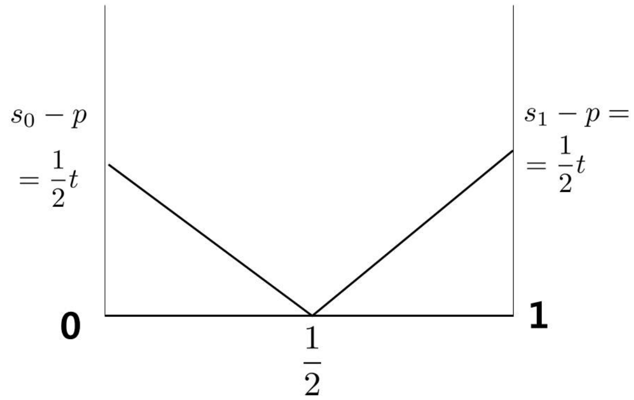

Figure 5: Consumers' surplus for adjacent sellers

market equally, but the difference is that the marginal consumer earns zero net utility. Hence, the situation is as depicted in Figure 5. It is straightforward to check that CS is $\frac{1}{4}t$.

Profits: Profit for monopolist is easily obtained from Lemma 3.2. For duopoly, one seller's profit is $\frac{1}{2}p_i - \frac{1}{2}cs_i^2 = \frac{1}{2}t - \frac{1}{2}c\frac{1}{9c^2} = \frac{1}{2}(t - \frac{1}{9c})$, hence the sum of profits is $t - \frac{1}{9c}$. For adjacent sellers, again one seller's profit is $\frac{1}{2}p_i - \frac{1}{2}cs_i^2 = \frac{1}{2}\left(\frac{1}{2c} - \frac{1}{2}t\right) - \frac{1}{2}c\frac{1}{4c^2} = \frac{1}{2}\left(\frac{1}{4c} - \frac{1}{2}t\right)$, hence the sum of profits is $\frac{1}{4c} - \frac{1}{2}t$.

Social welfare computations are straightforward. ■

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