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Optimal Feasible Tax Mechanism for a Heterogeneous Economy

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Abstract The main goal of this paper is to extend the model of optimal feasible tax mechanism developed in Rhee (2008) to a heterogeneous economy in which agents have different characteristics related to their income or wealth. We provide a full characterization of optimal feasible tax mechanism for such a heterogeneous economy and find that if the level of low endowment is relatively low, only the incentive compatibility constraint of a rich minority agent will be binding. We also present some interesting comparative statics analyses as to how the optimal mechanism will respond to a change in the primitives of the economy. These analyses explain how the incentive problem of a heterogeneous economy will be resolved efficiently under feasibility constraint.

Keywords Optimal taxation, Heterogeneity, Feasibility, Incentive compatibility, Informational rent

JEL Classification H21, D71, D82

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1. INTRODUCTION

Rhee (2008) characterizes an optimal feasible tax mechanism for a public good economy where the provision of public goods is to be financed by property taxes collected from individuals when a social planner is relatively uninformed of the properties of the individuals.[1] However, the model of Rhee (2008) considers only the economy consisting of *homogeneous* agents who are not distinguishable by any fundamental and public characteristics, which may be a restrictive assumption.

An individual in an economy has many characteristics that explain his/her economic status. Many of those characteristics are observable only privately, but some of them are publicly observable. Furthermore, one can easily recognize that some publicly observable characteristics of an individual are closely related to his/her income or wealth. For example, such characteristics include race (minority vs. majority), sex (female vs. male), class (blue-collar vs. white-collar) and so on.[2] We try to incorporate this reality.

The main goal of this paper is to extend the model of Rhee (2008) to a more general model with *heterogeneous* agents who have different characteristics that are publicly observable. We consider the case of one observable characteristic that can take two types, minority or majority; a majority agent has more advantageous probability distribution over endowment than a minority agent. It is assumed that their true initial endowment is still private information. As a result, an individual has two characteristics from the informational point of view; one is private information on his/her endowment and the other is publicly observable information on his/her race. Since a social planner is uninformed of the realization of individuals' wealth, she has to consider the *feasible* tax mechanism in the sense that a tax schedule should not impose more than what an individual really has.

Since the seminal model of Hurwicz, Maskin, and Postlewaite (1995), there have been some studies on the feasible implementation problem for a public good economy, such as Hong (1995), Tian (1993), Tian and Li (1995), and Dagan et al.

[1] Throughout this paper, we use the terms *property*, *income*, *wealth*, and *endowment* interchangeably according to context.

[2] The U.S. Census data indicate that there are notable income gaps by race and/or sex. For example, in 2008, the median income is $35,120 for white male, $20,950 for white female, $25,254 for black male, and $20,197 for black female. For further details, visit the U.S. Census Bureau website at http://www.census.gov/compendia/statab/2011edition.html. Also, based on a variety of socially observable individual characteristics, Schelling (1971) studies a dynamic equilibrium of segregation of a community using the model of neighborhood tipping.

(1999) for complete information cases, and Hong (1996, 1998) and Tian (1996, 1999) for incomplete information cases.[3] However, these studies have mainly focused on the implementability of a social choice rule, but not on the efficiency of an implementing mechanism. Rhee (2008) is the first attempt to deal with the efficiency aspect using optimal taxation scheme à la Mirrless (1971). Even though Rhee (2008) fully characterizes an optimal feasible tax mechanism which maximizes the sum of agents' expected utilities, it does not allow for agents' heterogeneity that is publicly recognized in many economies.

The vast majority of previous literature regarding heterogeneity in a public good economy has focused on experimental models.[4] Also, almost of those experimental researches mainly compare the effect of heterogeneity on the provision of public good.[5] Isaac and Walker (1988) studies the effect of communication in voluntary contribution mechanism experiments and concludes that heterogeneity as an asymmetric incomes decreases the level of contributions. Cardenas (2003) and Anderson et al. (2008) obtain the similar conclusion on the effect of asymmetric endowments. On the other hand, Chan et al. (1996, 1999) shows the experimental results that income inequality increases the aggregate contribution to public good. However, since most of experimental studies are based on voluntary contribution, to our best knowledge, there has been no study to find a socially optimal provision of public good under the probabilistic heterogeneity of individual endowments using Bayesian model, which is the main contribution of our model.

In this paper, we provide a social planner's problem for two heterogeneous agents and two types (rich and poor), and present the full characterization of optimal feasible tax mechanism. To begin with, we obtain the quite similar results as in Rhee (2008); (i) when the expected total endowment of the economy is relatively low or high enough, first best taxation is possible, (ii) the second best feasible tax mechanism is regressive, and (iii) the optimal feasible tax mechanism is increasing.

In addition, we find that for an optimal feasible tax schedule, (iv) if the level

[3]Some authors refer to this literature as "state-dependent implementation" or "endowment game."

[4]One of exceptions is the famous theoretical contribution by Bergstrom, et al. (1986). They provide a canonical model that deals with income heterogeneity using Nash equilibrium and show that an income inequality which increases the heterogeneity among players may increase the provision of public good, which is a different result from Warr (1983). See Ledyard (1995) for a survey of experimental researches on public good.

[5]A notable exception is Kaplow (2006), which studies the effect of public good provision on the income heterogeneity.

of low endowment is relatively low, then only the incentive compatibility (IC) constraint of a rich minority agent will be binding, while otherwise only the (IC) constraint of a rich majority agent will be binding. In other words, the social planner only need to worry about the incentive problem of a rich minority agent for the former case, and that of a rich majority agent for the latter case. Finally, for better understanding the optimal feasible tax mechanism, we conduct some comparative statics analyses when there is a change in the primitives of the economy. Although the optimal solution of the social planner's problem has many corner solutions by nature, we can employ a quite similar interpretation used in the corresponding homogeneous cases in Rhee (2008).

The remainder of this paper is organized as follows.[6] In Section 2, we present the model for a heterogeneous public good economy. In Section 3, we fully characterize the optimal feasible tax schedule for the economy with two agents and two possible types. Using the characterization results, in Section 4, we discuss the properties of the optimal mechanism and provide some comparative static analyses. In Section 5, we give concluding remarks.

2. THE MODEL

2.1. THE ECONOMY

Consider a public good economy with two agents, 1 and 2. There is one private good $x \in \mathbb{R}_+$ and one pure public good $y \in \mathbb{R}_+$, where the private good can be used to produce the public good according to a constant returns to scale technology. Without loss of generality, we normalize the production technology such that one unit of private good can be transformed into one unit of public good. Each agent $i = 1,2$ has the same quasilinear von Neumann-Morgenstern utility function u on $\mathbb{R}^2_+$,

$$u(x_i, y) = \log y + x_i,$$

where x_i is the consumption of private good by agent i. Initially, each agent i is endowed with private good $\omega_i \in \{\omega_L, \omega_H\}$ only, where $0 \leqq \omega_L < \omega_H < \infty$. Agent i is called *poor* when $\omega_i = \omega_L$ and *rich* when $\omega_i = \omega_H$. Let

$$\Omega = \{(\omega_L, \omega_H) \in \mathbb{R}^2_+ : \omega_L < \omega_H\}$$

denote the set of all possible pairs of initial endowments.

[6] We try to keep the same order of analysis as in Rhee (2008) for the purpose of comparison, but omit some detailed arguments to avoid repetition.

The information structure of this economy is as follows. The primitives of the economy are common knowledge, whereas each agent has private information about his own endowment. That is, agent i knows the realization of his own endowment ω_i and the initial probability distribution of the other agent's endowment, but does not know the realization of the other agent's endowment ω_j. The probabilities of agent 1 and 2 being poor are p and q respectively and independent, that is,

$$\Pr(\omega_1 = \omega_L) = p \in (0,1), \quad \Pr(\omega_2 = \omega_L) = q \in (0,1),$$

where $q > p$. We call agent 1 a p-type or *majority* and agent 2 a q-type or *minority*. An economic environment is equivalent to the realization of $\omega = (\omega_1, \omega_2)$.

2.2. THE TAX MECHANISM

A tax mechanism consists of message spaces M_i for each agent $i = 1,2$, and an outcome function f which maps each message profile $m \in M \equiv M_1 \times M_2$ into agents' tax burdens $t(m) = (t_1(m), t_2(m)) \in \mathbb{R}^2_+$ and public good production y; $f : m \mapsto (t(m), y(m))$. The constant returns to scale technology implies that $y(m) \leqq t_1(m) + t_2(m)$ for all $m \in M$, but without loss of generality, we can assume that the equality always holds since no taxes will be wasted. Hence, we have the following simple definition.

Definition 2.1 (Tax Mechanism and Schedule). *A* tax mechanism Γ *is defined as* $\Gamma = \langle M, t\rangle$, *where* $t : M \to \mathbb{R}^2_+$ *is called a* tax schedule.

Given a tax mechanism $\Gamma = \langle M, t\rangle$, let $s_i : \{\omega_L, \omega_H\} \to M_i$ denote the strategy (report) of agent i. By the Revelation Principle, we are able to restrict our attention to a direct incentive compatible tax mechanism. Thus, we assume that $M_i = \{\omega_L, \omega_H\}$ for each $i = 1,2$. The expected utility of agent i when his endowment is ω_i and he reports s_i, assuming the other agent j is truthful, is

$$\begin{aligned} U_i(s_i|\omega_i, t) &= \mathbb{E}_{\omega_j}\left[u_i\Big(\omega_i - t_i(s_i, \omega_j), t_1(s_i, \omega_j) + t_2(s_i, \omega_j)\Big)\Big|\omega_i\right] \\ &= \mathbb{E}_{\omega_j}\left[\log\left(t_1(s_i, \omega_j) + t_2(s_i, \omega_j)\right) + \Big(\omega_i - t_i(s_i, \omega_j)\Big)\Big|\omega_i\right]. \end{aligned}$$

In this paper, we make two assumptions which a tax mechanism should satisfy.

Assumption 2.2 (No Exaggeration). *For each* $i = 1,2$, $s_i(\omega_i) \leqq \omega_i$.

This assumption implies that each agent is asked to *put* his report *on the table*, which, in fact, reduces informational disadvantage of the social planner. We employ another assumption that a tax schedule should not be affected by the change of agents' names. That is, two agent's tax payment must be the same if each reports the same endowment given the other agent's report. Suppose that the white people is majority and the black is minority. It is illegal in a democratic society to apply a different tax schedule between them even if they have different probability on their income. Thus we have

Assumption 2.3 (Anonymity)**.** *For all* $i, j = 1, 2$ *with* $i \neq j$*,*

$$s_i = s_j \implies t_i(s_i, s') = t_j(s', s_j)$$

for each $s' \in \{\omega_L, \omega_H\}$.

Under the anonymity assumption, a tax schedule t can be written as

$$(t_{LL}, t_{LH}, t_{HL}, t_{HH}),$$

where, for example, t_{LH} is the tax payment of an agent when he reports ω_L and the other agent reports ω_H. Since we are considering only direct mechanisms, we simply identify a (direct) tax mechanism $\Gamma = \langle M, t \rangle$ with a tax schedule t in this paper.

To state the social planner's problem, we need to look at three properties that a tax mechanism should satisfy. First, *feasibility* implies that no tax mechanism should impose more than the announced endowment.

Definition 2.4 (Feasibility)**.** *A tax mechanism t is* feasible *if for all* $k = L, H$*,*

$$0 \leqq t_{L,k} \leqq \omega_L \;\text{ and }\; 0 \leqq t_{H,k} \leqq \omega_H. \tag{1}$$

Throughout this paper, we require all tax mechanisms considered to be feasible. Second, by the Revelation Principle, we consider *incentive compatible* tax mechanisms only.

Definition 2.5 (Incentive Compatibility: IC)**.** *A tax mechanism t is* (Bayesian) incentive compatible *if for all* $i = 1, 2$*,*

$$U_i(\omega_H | \omega_H, t) \geqq U_i(\omega_L | \omega_H, t). \tag{2}$$

Note that due to the no exaggeration assumption, the incentive compatibility of a tax mechanism for this economy is just one-directional; the inequality

$U_i(\omega_L|\omega_L,t) \geqq U_i(\omega_H|\omega_L,t)$ is meaningless. Third, by making the same assumption for participation constraint as in the homogeneous case, we can ignore an individual rationality condition.

Finally, we add one more definition for a tax mechanism.

Definition 2.6 (Increasingness)**.** *A tax mechanism t is* increasing *if for all $k = L,H$,*

$$t_{L,k} \leqq t_{H,k}.$$

That is, a tax mechanism is increasing if an agent's tax payment is increasing with his endowment.

2.3. THE SOCIAL PLANNER'S PROBLEM

The social planner (or tax authority) who does not know the true realization of each agent's endowment, but knows its probability distribution, wants to find the optimal tax schedule $t^* = (t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH})$ under (1) and (2) which maximizes the expected sum of agents' utilities. Formally, given $(\omega_L, \omega_H) \in \mathbf{\Omega}$ and $p,q \in (0,1)$, the social planner's problem is:

$$\begin{aligned}
\max_t \quad W(t;p,q) = \; & pq[2\log(2t_{LL}) - 2t_{LL}] \\
& + \big[p(1-q)+(1-p)q\big][2\log(t_{LH}+t_{HL}) - (t_{LH}+t_{HL})] \\
& + (1-p)(1-q)[2\log(2t_{HH}) - 2t_{HH}] + (p+q)\omega_L + (2-(p+q))\omega_H
\end{aligned}$$

subject to

$$\text{(P)} \quad \begin{array}{ll}
(\text{IC}_1) & \begin{aligned} & q[\log(t_{LH}+t_{HL}) - t_{HL}] + (1-q)[\log(2t_{HH}) - t_{HH}] \\ \geqq \; & q[\log(2t_{LL}) - t_{LL}] + (1-q)[\log(t_{LH}+t_{HL}) - t_{LH}]; \end{aligned} \\[2ex]
(\text{IC}_2) & \begin{aligned} & p[\log(t_{LH}+t_{HL}) - t_{HL}] + (1-p)[\log(2t_{HH}) - t_{HH}] \\ \geqq \; & p[\log(2t_{LL}) - t_{LL}] + (1-p)[\log(t_{LH}+t_{HL}) - t_{LH}], \end{aligned} \\[2ex]
(\text{Feasibility}) & \begin{array}{ll} 0 \leqq t_{LL} \leqq \omega_L, & 0 \leqq t_{LH} \leqq \omega_L, \\ 0 \leqq t_{HL} \leqq \omega_H, & 0 \leqq t_{HH} \leqq \omega_H \end{array} .
\end{array}$$

For notational simplicity, given $\rho \in (0,1)$, define a function $\Delta : \mathbb{R}^4_+ \to \overline{\mathbb{R}} \equiv \mathbb{R} \cup \{-\infty, +\infty\}$, which represents incentive compatibility constraints, by

$$\Delta(t;\rho) = \rho\left[\log\frac{(t_{LH}+t_{HL})^2}{(2t_{LL})(2t_{HH})} - (t_{LH}+t_{HL}) + (t_{LL}+t_{HH})\right] - \left[\log\frac{t_{LH}+t_{HL}}{2t_{HH}} + (t_{HH}-t_{LH})\right].$$

Then, a tax schedule t satisfies (IC$_1$) (resp. (IC$_2$)) if $\Delta(t;q) \geqq 0$ (resp. $\Delta(t;p) \geqq 0$). We say t satisfies (IC) if it satisfies both (IC$_1$) and (IC$_2$).

3. OPTIMAL FEASIBLE TAX MECHANISM

3.1. POSSIBILITY OF FIRST BEST TAXATION

To begin with, we examine the possibility of the first best tax schedule which is the solution to (P) without (IC) constraint. If the social planner were to know the realization of each agent's endowment, she could easily find the first best tax schedule. However, she does not have such an information, so the question is when the (IC) constraint is not binding. First of all, to rule out the uninteresting cases, partition Ω (see Figure 1) into

$$\begin{aligned}\Omega_1 &= \{(\omega_L, \omega_H) \in \Omega : \omega_L \in [0,1)\}, \text{ and}\\ \Omega_2 &= \{(\omega_L, \omega_H) \in \Omega : \omega_L \in [1,\infty)\}.\end{aligned}$$

When $(\omega_L, \omega_H) \in \Omega_2$, the social planner can easily solve (P) by imposing a first best feasible tax schedule

$$t^F \in \{t \in B(\omega_L, \omega_H) : t_{LL} = t_{HH} = 1,\ t_{LH} + t_{HL} = 2, \text{ and } 1 \leqq t_{LH} \leqq \omega_L\},$$

since t^F satisfies the (IC) constraint; $\Delta(t^F;\rho) = -(1-t^F_{LH}) \geqq 0$. If the social planner insists that the tax schedule be increasing, then the unique solution to (P) is $t^F = (1,1,1,1)$. Therefore, in the following we just focus on the case of $(\omega_L, \omega_H) \in \Omega_1$. According to the welfare function $W(\cdot)$, it is easy to see that for $(\omega_L, \omega_H) \in \Omega_1$ the first best feasible tax schedule is

$$t^F = \left(t^F_{LL}, t^F_{LH}, t^F_{HL}, t^F_{HH}\right) = \left(\omega_L, \omega_L, \min\{\omega_L + \omega_H, 2\} - \omega_L, \min\{\omega_H, 1\}\right).$$

To find the conditions under which t^F is the solution to (P), consider the (IC) constraint at t^F:

$$\begin{aligned}\Delta(t^F;\rho) = \rho &\left[\log \frac{\min\{\omega_L+\omega_H,2\}^2}{(2\omega_L)(2\min\{\omega_H,1\})} - \min\{\omega_L+\omega_H,2\} + \left(\omega_L + \min\{\omega_H,1\}\right)\right]\\ &- \left[\log \frac{\min\{\omega_L+\omega_H,2\}}{2\min\{\omega_H,1\}} + \left(\min\{\omega_H,1\} - \omega_L\right)\right].\end{aligned}$$

Lemma 3.1. *For $(\omega_L, \omega_H) \in \Omega_1$, $\Delta(t^F;\rho)$ is strictly increasing in ρ.*

Proof. [7] Same as the proof of Lemma 3.1 in Rhee (2008) *mutatis mutandis*. □

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[7] To avoid repetition, we omit the proof of some lemmas and propositions below if it is similar to the corresponding proof in Rhee (2008).

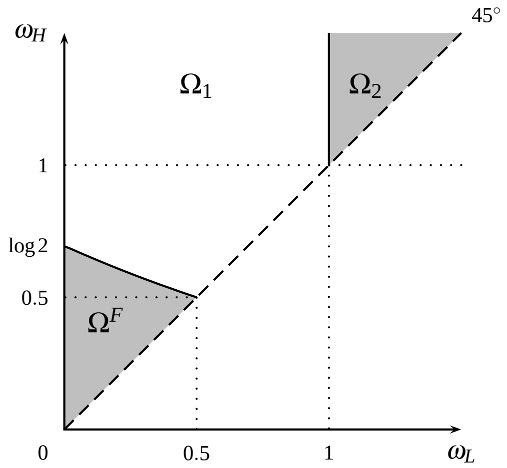


Figure 1: Possibility of First Best Taxation

For $(\omega_L, \omega_H) \in \Omega_1$, define $\overline{\rho} \in \overline{\mathbb{R}}$ by $\Delta(t^F; \overline{\rho}) = 0$, or equivalently,

$$\overline{\rho} = \frac{\log \frac{\min\{\omega_L+\omega_H,2\}}{2\min\{\omega_H,1\}} + \big(\min\{\omega_H,1\} - \omega_L\big)}{\log \frac{\min\{\omega_L+\omega_H,2\}^2}{(2\omega_L)(2\min\{\omega_H,1\})} - \min\{\omega_L+\omega_H,2\} + \big(\omega_L + \min\{\omega_H,1\}\big)},$$

and let $\widehat{\rho} = \min\{1, \overline{\rho}\}$. Define also

$$\Omega^F = \{(\omega_L, \omega_H) \in \Omega_1 : \lim_{\rho \to 0} \Delta(t^F; \rho) \geqq 0\}.$$

Proposition 3.2. *If $p \geqq \widehat{\rho}$, then the first best feasible tax schedule t^F is the unique solution to (P). In particular, if $(\omega_L, \omega_H) \in \Omega^F$, then t^F is the unique solution to (P) for all $p, q \in (0,1)$, $q > p$.*

Figure 1 depicts the possibility of first best feasible taxation.

3.2. SECOND BEST TAX SCHEDULE

Assume that $p < \widehat{\rho}$. To characterize the second best feasible tax schedule, we begin with three lemmas. The main purpose of these lemmas is to lower the dimension of the social planner's problem.

Lemma 3.3. *Suppose t^* is a solution to (P). Then,*

$$t^*_{HH} = \min\{\omega_H, 1\}.$$

Lemma 3.4. *Suppose t^* is a solution to (P). Then,*

$$t^*_{LH} + t^*_{HL} \leqq 2.$$

Lemma 3.5. *Suppose t^* is a solution to (P). Then,*

$$t^*_{LH} = \omega_L, \ \text{and} \ \ t^*_{HL} \geqq \omega_L.$$

By Lemmas 3.3–3.5, we found the two values of t^*_{LH} and t^*_{HH}, so all we have to find to solve the problem (P) is t^*_{LL} and t^*_{HL}. This implies that the dimension of (P) reduces from four to two. Let $T = t_{LH} + t_{HL}$. Lemmas 3.3–3.5 implies that we can restrict our attention to $(T, t_{LL}) \in [2\omega_L, \min\{\omega_L + \omega_H, 2\}] \times [0, \omega_L]$, which now can be called a *tax schedule*. Define *(IC)-function* $z(\cdot,\cdot;\rho) : [2\omega_L, \min\{\omega_L + \omega_H, 2\}] \times [0, \omega_L] \to \overline{\mathbb{R}}$, by

$$\begin{aligned} z(T, t_{LL}; \rho) &= \Delta(t_{LL}, t_{LH}, t_{HL}, t_{HH}; \rho)\big|_{t_{LH}=\omega_L, t_{HH}=\min\{\omega_H,1\}} \\ &= \rho \left[\log \frac{T^2}{(2t_{LL})(2\min\{\omega_H,1\})} - T + \big(t_{LL} + \min\{\omega_H, 1\}\big)\right] \\ &\quad - \left[\log \frac{T}{2\min\{\omega_H,1\}} + (\min\{\omega_H,1\} - \omega_L)\right]. \end{aligned}$$

Thus, a tax schedule (T, t_{LL}) is said to satisfy (IC_1) if $z(T, t_{LL}; q) \geqq 0$, and (IC_2) if $z(T, t_{LL}; p) \geqq 0$. We can call $z(T, t_{LL}; q)$ the majority agent's (IC)-function and $z(T, t_{LL}; p)$ the minority agent's (IC)-function.

Now, the social planner's problem (P) can be written as an equivalent but simplified version (P′): Given $(\omega_L, \omega_H) \in \Omega_1$ and $p, q \in (0,1)$,

$$\max_{(T,t_{LL})} \quad \widetilde{W}(T, t_{LL}; p, q) = pq[2\log(2t_{LL}) - 2t_{LL}] + [p(1-q) + (1-p)q][2\log T - T]$$

subject to

(P′) (IC_1) $\quad z(T, t_{LL}; q) \geqq 0$;

(IC_2) $\quad z(T, t_{LL}; p) \geqq 0$,

(Feasibility) $\quad (T, t_{LL}) \in [2\omega_L, \min\{\omega_L + \omega_H, 2\}] \times [0, \omega_L]$.

To find the second best tax schedule, first consider the shape of the *(IC)-curve* $z(T, t_{LL}; \rho) = 0$. In fact, we can find a point that satisfies $z(T, t_{LL}; \rho) = 0$ for all $\rho \in (0,1)$. For $(\omega_L, \omega_H) \in \Omega_1 \setminus \Omega_F$, let

$$\widetilde{T} = 2\min\{\omega_H, 1\}\exp\big\{-\big(\min\{\omega_H,1\} - \omega_L\big)\big\}, \text{ and}$$

$$\widetilde{t}_{LL} = -W_0\left(-\exp\left\{\log\frac{\widetilde{T}}{2} - \widetilde{T} + \omega_L\right\}\right),$$

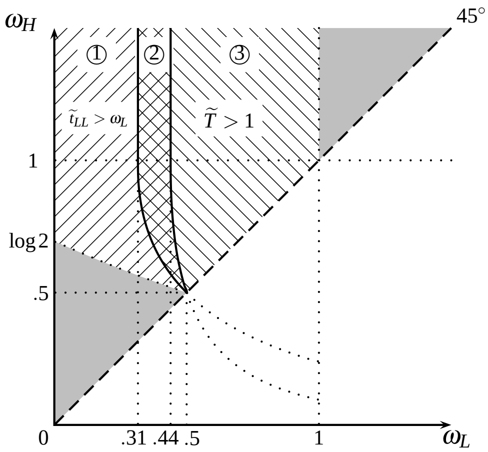


Figure 2: Relative size of $(\widetilde{T}, \widetilde{t}_{LL})$ on Ω

where W_0 is the principal branch of Lambert W function. By the definition of $\widetilde{T}$, we can rewrite the (IC)-curve as

$$z(T, t_{LL}; \rho) = \rho \left[\log \frac{T^2}{(2t_{LL})(\widetilde{T})} - T + \left(t_{LL} + \omega_L\right) \right] - \left[\log \frac{T}{\widetilde{\widetilde{T}}} \right], \qquad (3)$$

so, it is clear that $z(\widetilde{T}, \widetilde{t}_{LL}; \rho) = 0$ for all $\rho \in (0, \widehat{\rho})$. That is, the (IC)-curve $z(T, t_{LL}; \rho) = 0$ always goes through the pivotal point $(\widetilde{T}, \widetilde{t}_{LL})$ regardless of ρ value. Furthermore,

Lemma 3.6. *(a)* $(\widetilde{T}, \widetilde{t}_{LL}) \in (2\omega_L, \min\{\omega_L + \omega_H, 2\}) \times [0, 1)$.
(b) If $\widetilde{T} \leqq 1$, *then* $\widetilde{t}_{LL} > \omega_L$.

This lemma tells that if $\widetilde{T} \leqq 1$, the pivotal point $(\widetilde{T}, \widetilde{t}_{LL})$ is above the feasible set $[2\omega_L, \min\{\omega_L + \omega_H, 2\}] \times [0, \omega_L]$. Another property of the (IC)-curve is that it turns around the pivotal point $(\widetilde{T}, \widetilde{t}_{LL})$ counterclockwise as ρ increases.

Lemma 3.7. *For all* $p, q \in (0, 1)$ *such that* $q > p$, *if* $z(T, t_{LL}; p) = 0$, *then*

$$z(T, t_{LL}; q) = \begin{cases} < 0 & \text{if } T < \widetilde{T} \\ \geqq 0 & \text{if } T \geqq \widetilde{T} \end{cases}.$$

Figure 2 depicts the subsets of Ω that satisfy $\widetilde{t}_{LL} \geqq \omega_L$ and $\widetilde{T} > 1$.

Now, consider the slopes of *(IC)-curve* $z(T,t_{LL};\rho)=0$ and *welfare-curve* $\widetilde{W}(T,t_{LL};p,q)=\bar{w}$, where $\bar{w}$ is a constant. By the same argument in Rhee (2008), it follows that for $(T,t_{LL})\in[0,2]\times[0,1)$,

$$\left.\frac{dt_{LL}}{dT}\right|_{z(T,t_{LL};p)=0}=-\left.\frac{\frac{2p-1}{T}-p}{p\left(1-\frac{1}{t_{LL}}\right)}\right|_{z(T,t_{LL};p)=0}. \tag{4}$$

and

$$\left.\frac{dt_{LL}}{dT}\right|_{\widetilde{W}(T,t_{LL};p,q)=\bar{w}}=-\left.\frac{[p(1-q)+(1-p)q]\left(\frac{2}{T}-1\right)}{2pq\left(\frac{1}{t_{LL}}-1\right)}\right|_{\widetilde{W}(T,t_{LL};p,q)=\bar{w}}<0. \tag{5}$$

To describe the second best feasible tax schedule, we need some definitions. First, for $(\omega_L,\omega_H)\in\Omega_1$ such that $\omega_L+\omega_H\leqq 1$ and $\rho\in(0,\widehat{\rho})$, define $\underline{t}^{\rho}_{LL}\in(0,\omega_L)$ by $z(\omega_L+\omega_H,\underline{t}^{\rho}_{LL};\rho)=0$. Second, for $\rho\in(0,\widehat{\rho})$, define $\underline{T}^{\rho}\in(2\omega_L,\min\{\omega_L+\omega_H,2\})$ by $z(\underline{T}^{\rho},\omega_L;\rho)=0$. Third, define simply $T^{\rho o}=\frac{2\rho}{p+q}$. Finally, for $\widetilde{T}<T^{\rho o}$ and $\underline{T}^{\rho}<T^{\rho o}$, define $t^{\rho o}_{LL}\in(0,\omega_L)$ by $z(T^{\rho o},t^{\rho o}_{LL};\rho)=0$.

Now, we can state the main result of this paper. The main difference of this result for the heterogeneous case compared to that for the homogeneous case of Rhee (2008) is that since we have to consider two (IC) constraints, the cases of corner solutions have increased.

Proposition 3.8. *For $p<\widehat{\rho}$, the solution to (P) is: If $\widetilde{t}_{LL}\geqq\omega_L$, then*

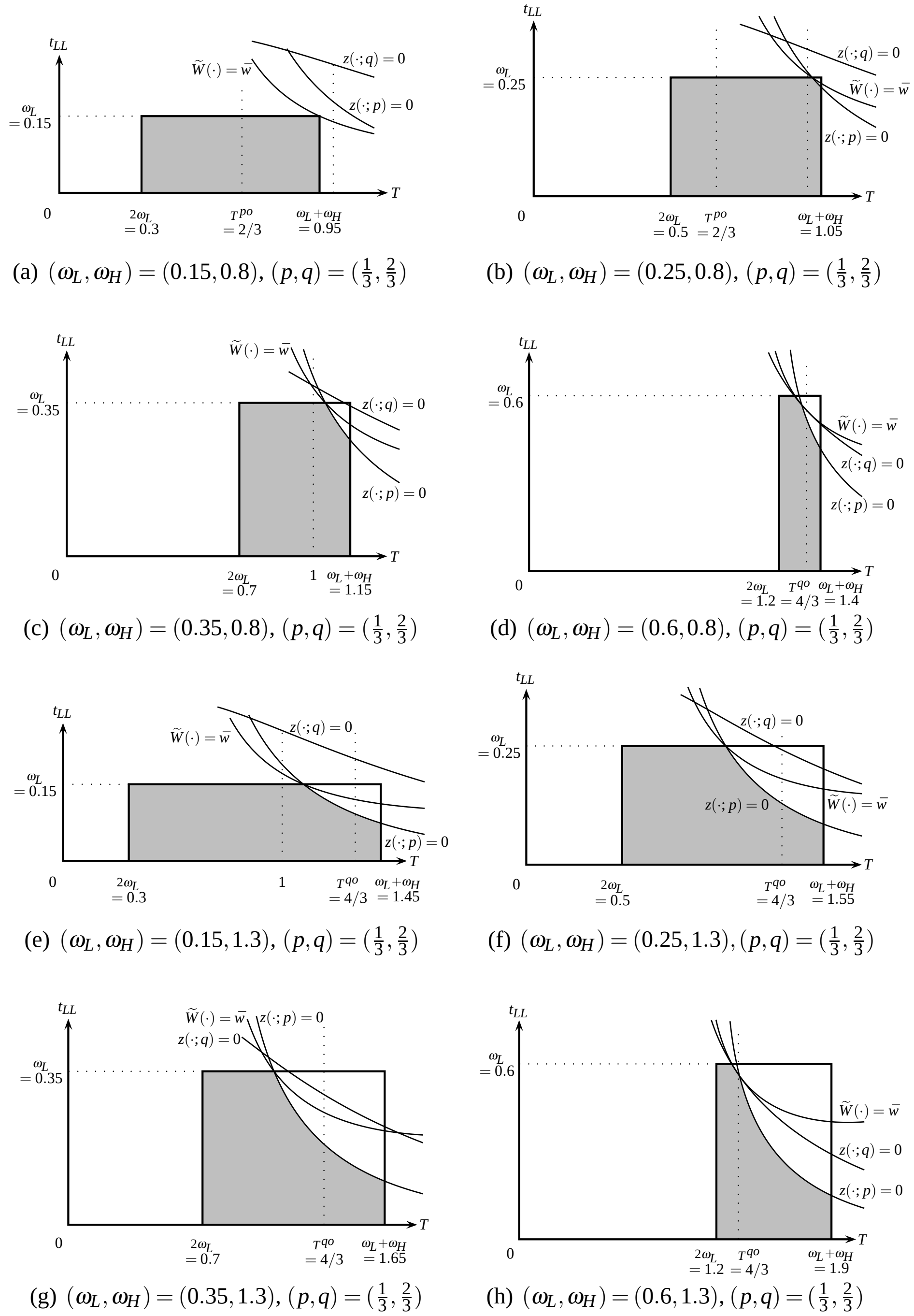
$$t^*=\begin{cases}(\underline{t}^{p}_{LL},\omega_L,\omega_H,\omega_H) & \text{if } \omega_L+\omega_H\leqq T^{po}\\ (t^{po}_{LL},\omega_L,T^{po}-\omega_L,\min\{\omega_H,1\}) & \text{if } \underline{T}^{p}\leqq T^{po}<\omega_L+\omega_H\\ (\omega_L,\omega_L,\underline{T}^{p}-\omega_L,\min\{\omega_H,1\}) & \text{if } T^{po}<\underline{T}^{p}\end{cases}.$$

If $\widetilde{t}_{LL}<\omega_L$, then

$$t^*=\begin{cases}(\widetilde{t}_{LL},\omega_L,\widetilde{T}-\omega_L,\min\{\omega_H,1\}) & \text{if } \widetilde{T}\leqq T^{qo}\\ (t^{qo}_{LL},\omega_L,T^{qo}-\omega_L,\min\{\omega_H,1\}) & \text{if } \underline{T}^{q}\leqq T^{qo}<\widetilde{T}\\ (\omega_L,\omega_L,\underline{T}^{q}-\omega_L,\min\{\omega_H,1\}) & \text{if } T^{qo}<\underline{T}^{q}\end{cases}.$$

Proof. From (4) and (5), it follows that $z(T,t_{LL};q)=0$ and $z(T,t_{LL};p)=0$ are tangent to the welfare-curve $\widetilde{W}(T,t_{LL};p,q)=\bar{w}$ at (T^{po},t^{po}_{LL}) and $T^{qo}.t^{qo}_{LL}$, respectively. According to Lemma 3.7, the proof of Proposition 3.9 in Rhee (2008) is valid *mutatis mutandis*. □

Table 1 summarize the optimal feasible tax schedules and their relative size for each possible case.



(a) $(\omega_L, \omega_H) = (0.15, 0.8)$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

(b) $(\omega_L, \omega_H) = (0.25, 0.8)$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

(c) $(\omega_L, \omega_H) = (0.35, 0.8)$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

(d) $(\omega_L, \omega_H) = (0.6, 0.8)$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

(e) $(\omega_L, \omega_H) = (0.15, 1.3)$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

(f) $(\omega_L, \omega_H) = (0.25, 1.3)$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

(g) $(\omega_L, \omega_H) = (0.35, 1.3)$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

(h) $(\omega_L, \omega_H) = (0.6, 1.3)$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

Figure 3: Examples of second best taxation

Table 1: Optimal feasible tax schedules for a heterogeneous economy

Cases			t^*_{LL}	t^*_{LH}	t^*_{HL}		t^*_{HH}
$p \geqq \widehat{\rho}$ (including Ω^F)			$\omega_L =$	$\omega_L <$	$\min\{\omega_L+\omega_H, 2\} - \omega_L$	$\geqq$	$\min\{\omega_H, 1\}$
$p < \widehat{\rho}$	$\widetilde{t}_{LL} \geqq \omega_L$	$\omega_L + \omega_H \leqq \frac{2p}{p+q}$	$\underline{t}^p_{LL} \leqq$	$\omega_L <$	ω_H	$=$	ω_H
		$\underline{T}^p \leqq \frac{2p}{p+q} < \omega_L + \omega_H$	$t^{po}_{LL} \leqq$	$\omega_L <$	$\frac{2p}{p+q} - \omega_L$	$<$	$\min\{\omega_H, 1\}$
		$\frac{2p}{p+q} < \underline{T}^p$	$\omega_L =$	$\omega_L <$	$\underline{T}^p - \omega_L$	$\lesseqqgtr$	$\min\{\omega_H, 1\}$
	$\widetilde{t}_{LL} < \omega_L$	$\widetilde{T} \leqq \frac{2q}{p+q}$	$\widetilde{t}_{LL} \leqq$	$\omega_L <$	$\widetilde{T} - \omega_L$	$\lesseqqgtr$	$\min\{\omega_H, 1\}$
		$\underline{T}^q \leqq \frac{2q}{p+q} < \widetilde{T}$	$t^{qo}_{LL} \leqq$	$\omega_L <$	$\frac{2q}{p+q} - \omega_L$	$\lesseqqgtr$	$\min\{\omega_H, 1\}$
		$\frac{2q}{p+q} < \underline{T}^q$	$\omega_L =$	$\omega_L <$	$\underline{T}^q - \omega_L$	$\lesseqqgtr$	$\min\{\omega_H, 1\}$

3.3. SIMULATED EXAMPLES

In this section, we illustrate some examples that show the specific optimal feasible tax schedules for different parameter values. Due to the low dimensionality of the social planner's problem, we can draw the results graphically. We consider the case of $(p, q) = (\frac{1}{3}, \frac{2}{3})$.

i. Suppose that $\omega_H = 0.8$.

(a) If $\omega_L = 0.15$, then we have $\widehat{\rho} \approx 0.20$. Since $p \geqq \widehat{\rho}$, the first best tax schedule $(t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH}) = (0.15, 0.15, 0.8, 0.8)$ is obtained (Figure 3(a)).

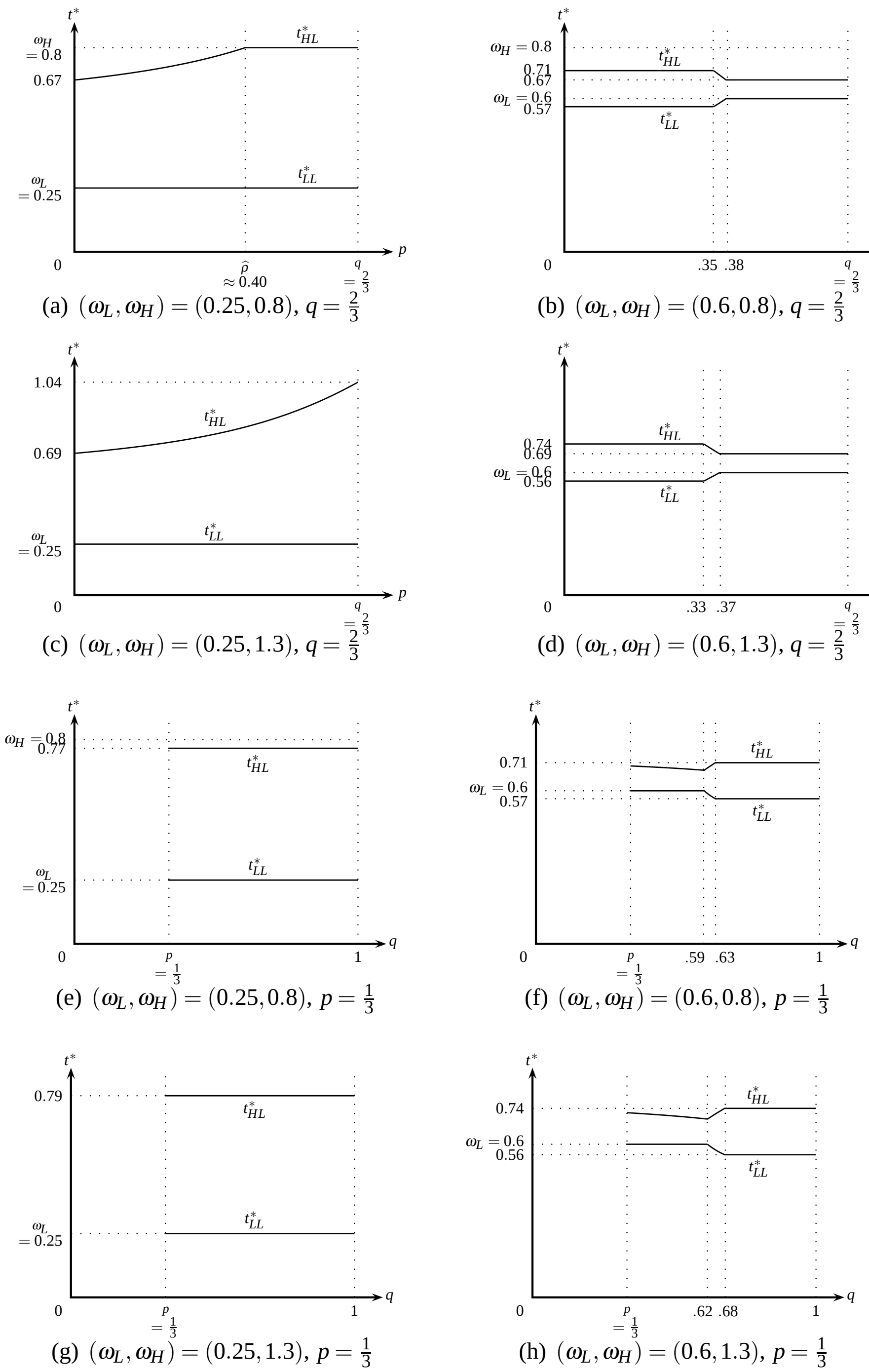
(b) If $\omega_L = 0.25$, then $\widetilde{t}_{LL} \approx 0.33 > \omega_L$ and $\widehat{\rho} \approx 0.40$. Since $T^{po} = \frac{2}{3} < 1.02 \approx \underline{T}^p$, by Proposition 3.8, the second best tax schedule $t^* = (\omega_L, \omega_L, \underline{T}^p - \omega_L, \omega_H)$ is obtained. Figure 3(b) illustrates this case in which the optimal tax schedule is $(t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH}) = (0.25, 0.25, 0.77, 0.8)$.

(c) If $\omega_L = 0.35$, then we have $\widetilde{t}_{LL} \approx 0.38 > \omega_L$, $\widehat{\rho} \approx 0.72$ and $T^{po} = \frac{2}{3} < 1.05 \approx \underline{T}^p$, thus the second best tax schedule $t^* = (\omega_L, \omega_L, \underline{T}^p - \omega_L, \omega_H)$ is obtained, too. Figure 3(b) illustrates this case in which the optimal tax schedule is $(t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH}) = (0.35, 0.35, 0.70, 0.8)$.

(d) If $\omega_L = 0.6$, then $\widetilde{t}_{LL} \approx 0.57 < \omega_L$ and $\widetilde{T} \approx 1.31 < \frac{4}{3} = T^{qo}$. Thus, the second best tax schedule $t^* = (\widetilde{t}_{LL}, \omega_L, \widetilde{T} - \omega_L, \omega_H)$ is obtained. Figure 3(d) illustrates this case in which the optimal tax schedule is $(t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH}) = (0.57, 0.6, 0.71, 0.8)$.

ii. Suppose that $\omega_H = 1.3$.

(e) If $\omega_L = 0.15$, then $\widetilde{t}_{LL} \approx 0.28 > \omega_L$ and $\widehat{\rho} \approx 0.55$. Since $T^{po} = \frac{2}{3} < 1.10 \approx \underline{T}^p$, by Proposition 3.8, the second best tax schedule $t^* = (\omega_L, \omega_L, \underline{T}^p - \omega_L, \omega_H)$ is obtained. Figure 3(e) illustrates this case in which the optimal tax schedule is $(t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH}) = (0.15, 0.15, 0.95, 1)$.



(a) $(\omega_L, \omega_H) = (0.25, 0.8)$, $q = \frac{2}{3}$

(b) $(\omega_L, \omega_H) = (0.6, 0.8)$, $q = \frac{2}{3}$

(c) $(\omega_L, \omega_H) = (0.25, 1.3)$, $q = \frac{2}{3}$

(d) $(\omega_L, \omega_H) = (0.6, 1.3)$, $q = \frac{2}{3}$

(e) $(\omega_L, \omega_H) = (0.25, 0.8)$, $p = \frac{1}{3}$

(f) $(\omega_L, \omega_H) = (0.6, 0.8)$, $p = \frac{1}{3}$

(g) $(\omega_L, \omega_H) = (0.25, 1.3)$, $p = \frac{1}{3}$

(h) $(\omega_L, \omega_H) = (0.6, 1.3)$, $p = \frac{1}{3}$

Figure 4: Responses of t^*_{LL} and t^*_{HL} to p and q

(f) If $\omega_L = 0.25$, then $\widetilde{t}_{LL} \approx 0.33 > \omega_L$ and $\widehat{\rho} \approx 0.86$. Since $T^{po} = \frac{2}{3} < 1.04 \approx \underline{T}^p$, by Proposition 3.8, the second best tax schedule $t^* = (\omega_L, \omega_L, \underline{T}^p - \omega_L, \omega_H)$ is obtained. Figure 3(f) illustrates this case in which the optimal tax schedule is $(t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH}) = (0.25, 0.25, 0.79, 1)$.

(g) If $\omega_L = 0.35$, then $\widetilde{t}_{LL} \approx 0.38 > \omega_L$ and $\widehat{\rho} = 1$. Since $T^{po} = \frac{2}{3} < 1.07 \approx \underline{T}^p$, by Proposition 3.8, the second best tax schedule $t^* = (\omega_L, \omega_L, \underline{T}^p - \omega_L, \omega_H)$ is obtained again. Figure 3(g) illustrates this case in which the optimal tax schedule is $(t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH}) = (0.35, 0.35, 0.72, 1)$.

(h) If $\omega_L = 0.6$, then $\widetilde{t}_{LL} \approx 0.56 < \omega_L$ and $\widetilde{T} \approx 1.34 > \frac{4}{3} = T^{qo}$. Thus, by Proposition 3.8, the second best tax schedule $t^* = (t^{qo}_{LL}, \omega_L, T^{qo} - \omega_L, \omega_H)$ is obtained. Figure 3(h) illustrates this case in which the optimal tax schedule is $(t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH}) = (0.56, 0.6, 0.73, 1)$

4. PROPERTIES AND COMPARATIVE STATICS

4.1. PROPERTIES OF OPTIMAL FEASIBLE TAX SCHEDULES

The results and intuitions of the comparative statics analysis in Rhee (2008) are also valid to this heterogeneous economy. That is, (i) when the expected total endowments of the economy is relatively low or high enough, then first best taxation is possible; (ii) the second best feasible tax mechanism is regressive; and (iii) it is increasing.[8]

In addition, we have another interesting result about the incentive compatibility for the heterogeneous case.

Corollary 4.1. *If the initial endowment (ω_L, ω_H) satisfies $\widetilde{t}_{LL} \geqq \omega_L$, then only the incentive compatibility constraint of the rich minority agent (IC$_2$) is binding at t^*. Otherwise, only (IC$_1$) is binding.*

Proof. By Lemma 3.7 and Proposition 3.8, the result is straightforward. □

According to Figure 2, the subset of Ω_1 that satisfies $\widetilde{t}_{LL} \geqq \omega_L$ represents the case in which the amount of low endowment ω_L is relatively low. Thus, this corollary can be interpreted as follows: If the endowment level of a poor agent is indeed low, a rich agent is reluctant to pretend to be poor since the size of public good provision could be too small in case the other agent was poor (and reporting truthfully). This reluctancy is greater for a rich majority agent than for a rich minority agent because by definition a minority is more likely to be poor than a majority. Thus, the misreporting incentive of a rich

[8]We do not repeat the same explanations of the results for simplicity.

Table 2: Responses of t^* to p and q.

Cases			given q		given p	
			$\frac{dt^*_{LL}}{dp}$	$\frac{t^*_{HL}}{dp}$	$\frac{t^*_{LL}}{dq}$	$\frac{t^*_{HL}}{dq}$
$p \geqq \widehat{\rho}$ (including Ω^F)			0	0	0	0
$p < \widehat{\rho}$	$\widetilde{t}_{LL} \geqq \omega_L$	$\omega_L + \omega_H \leqq \frac{2p}{p+q}$	$+$	0	0	0
		$\underline{T}^p \leqq \frac{2p}{p+q} < \omega_L + \omega_H$	$-$	$+$	$+$	$-$
		$\frac{2p}{p+q} < \underline{T}^p$	0	$+$	0	0
	$\widetilde{t}_{LL} < \omega_L$	$\widetilde{T} \leqq \frac{2q}{p+q}$	0	0	0	0
		$\underline{T}^q \leqq \frac{2q}{p+q} < \widetilde{T}$	$+$	$-$	$-$	$+$
		$\frac{2q}{p+q} < \underline{T}^q$	0	0	0	$-$

minority agent is greater than that of a rich majority. Together with the no-exaggeration assumption defined below, it follows that once the (IC) constraint of a rich minority agent has been satisfied, then that of a rich majority agent is obviously satisfied. On the contrary, if the level of low endowment is relatively high, the (IC) constraints of rich minority and majority are both binding, but in fact we find that only the rich majority's (IC) constraint is binding at a solution.

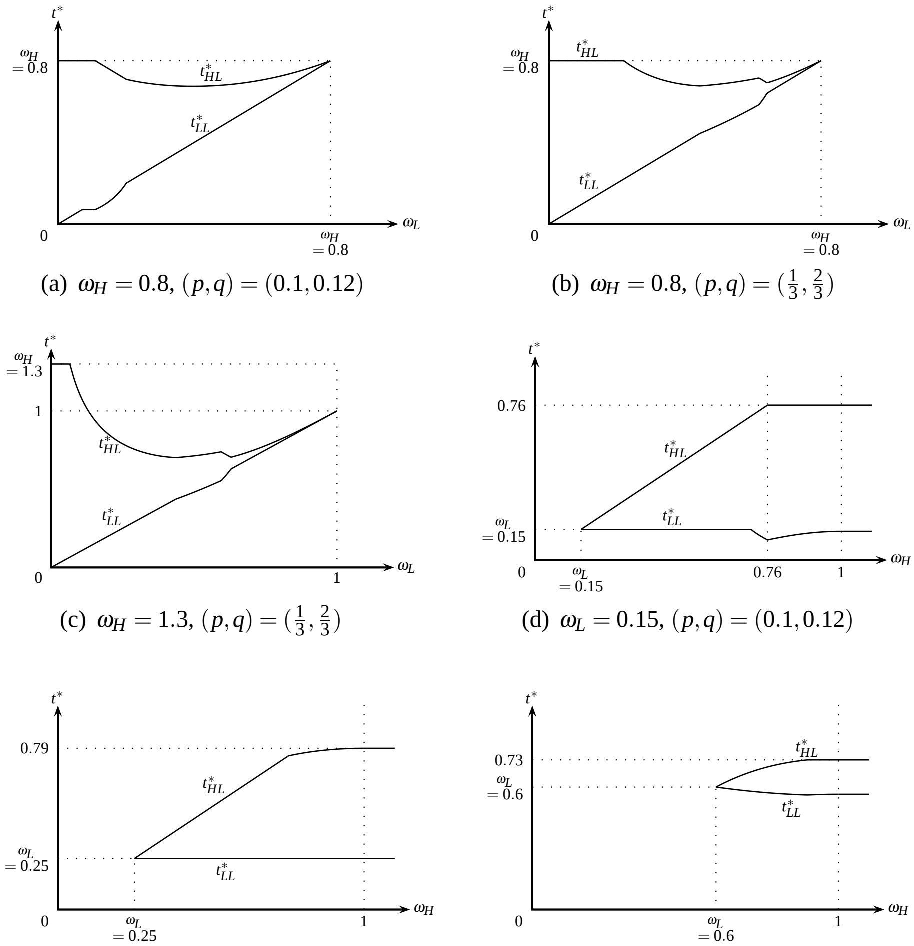
4.2. COMPARATIVE STATICS

In this section, we first study the responses of t^* to p and q analytically, and then show some examples of the responses of t^* to ω_L and ω_H by simulation approach. In the following, we exclude the trivial case Ω_2 in which first best taxation is always possible.

4.2.1 Responses of t^* to p and q

Since both t^*_{LH} and t^*_{HH} are independent of p and q, it suffices to analyze the responses of t^*_{LL} and t^*_{HL}. Given $(\omega_L, \omega_H) \in \Omega_1$, if $p \geqq \widehat{\rho}$, then t^* is independent of the change in p and q. Thus, suppose $p < \widehat{\rho}$. By applying a similar analysis in Rhee (2008), we can obtain the responses of t^* to p and q. Table 2 summarizes the responses of t^* to p and q and Figure 4 provides some examples for different parameter values.

The economic intuition is as follows. First, consider the change in p. Suppose first that the initial low endowment is small enough such that $\widetilde{t}_{LL} \geqq \omega_L$ (the



(a) $\omega_H = 0.8$, $(p,q) = (0.1, 0.12)$

(b) $\omega_H = 0.8$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

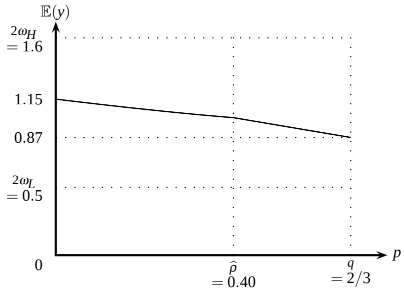
(c) $\omega_H = 1.3$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

(d) $\omega_L = 0.15$, $(p,q) = (0.1, 0.12)$

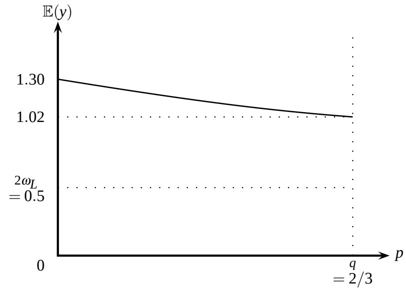
(e) $\omega_L = 0.25$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

(f) $\omega_L = 0.6$, $(p,q) = (\frac{1}{3}, \frac{2}{3})$

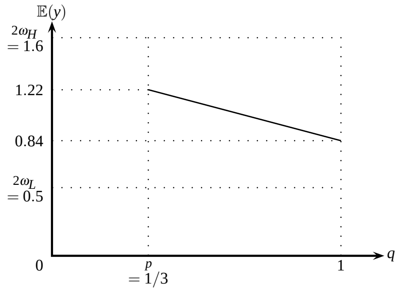
Figure 5: Responses of t^*_{LL} and t^*_{HL} to ω_L and ω_H



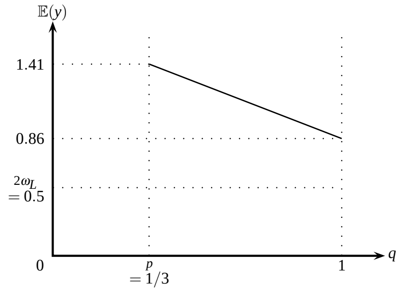
(a) $(\omega_L, \omega_H) = (0.25, 0.8)$ and $q = \frac{2}{3}$



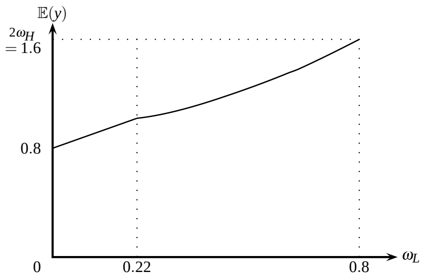
(b) $(\omega_L, \omega_H) = (0.25, 1.3)$ and $q = \frac{2}{3}$



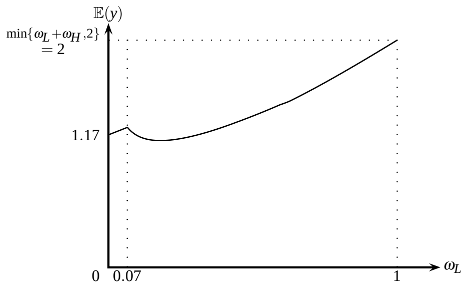
(c) $(\omega_L, \omega_H) = (0.25, 0.8)$ and $p = \frac{1}{3}$



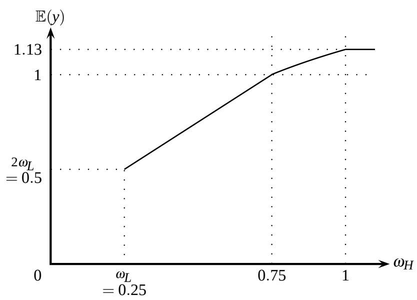
(d) $(\omega_L, \omega_H) = (0.25, 1.3)$ and $p = \frac{1}{3}$



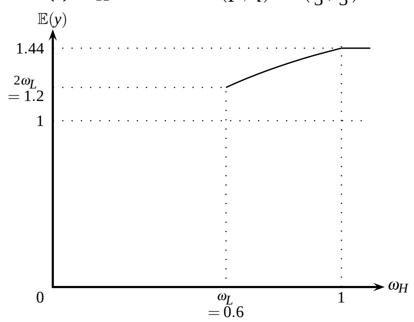
(e) $\omega_H = 0.8$ and $(p, q) = (\frac{1}{3}, \frac{2}{3})$



(f) $\omega_H = 1.3$ and $(p, q) = (\frac{1}{3}, \frac{2}{3})$



(g) $\omega_L = 0.25$ and $(p, q) = (\frac{1}{3}, \frac{2}{3})$



(h) $\omega_L = 0.6$ and $(p, q) = (\frac{1}{3}, \frac{2}{3})$

Figure 6: Expected Total Provision of Public Good: $\mathbb{E}(y)$

areas of ① and ② in Figure 2). In this case, by Corollary 4.1, only (IC_2) is binding. Thus, the increase in p makes the set of incentive compatible and feasible tax schedules larger, which implies that the social planner can increase t^*_{LL} or t^*_{HL} as long as the feasibility constraint is binding. Figure 4(a) and (c) depict this case. Given $q = \frac{2}{3}$, if $p < \widehat{\rho}$, a corner solution like Figure 3(b) is obtained, so t^*_{LL} stays at it maximum ω_L but t^*_{HL} increases as p increases. Suppose instead that (ω_L, ω_H) satisfies $\widetilde{t}_{LL} < \omega_L$. Then, by Corollary 4.1, only (IC_1) is binding. Since $T^{qo} = \frac{2q}{p+q}$ is decreasing in p, a corner solution like Figure 3(d) is obtained for a small p. Thus, t^*_{LL} and t^*_{HL} stay at $\widetilde{t}_{LL}$ and $\widetilde{T} - \omega_L$ respectively. As p increases more, an interior solution like Figure 3(h) takes place. In this case, t^*_{LL} increases but t^*_{HL} decreases. For a large p, we have a corner solution in which t^*_{LL} is at its maximum ω_L and t^*_{HL} stay the same at $\underline{T}^q - \omega_L$.

Next, consider the change in q. If (ω_L, ω_H) is such that $\widetilde{t}_{LL} \geqq \omega_L$, then only ($IC_2$) is binding. In this case, the change of q affects t^*_{LL} and t^*_{HL} through the change of T^{po} only when an interior solution occurs. Figure 4(e) and (g) show the cases where no interior solution is possible, so that t^*_{LL} and t^*_{HL} stays the same. On the other hand, if $\widetilde{t}_{LL} \geqq \omega_L$, then only ($IC_1$) is binding. Since $T^{qo} = \frac{2q}{p+q}$ is increasing in q, the effects of q on t^*_{LL} and t^*_{HL} are the inverse to those of p. That is, as q increases, t^*_{LL} stays initially at its maximum, and then decreases, and finally ends up at $\widetilde{t}_{LL}$ whereas t^*_{HL} decreases initially, and then increases, and finally ends up at $\widetilde{T} - \omega_L$. Refer to Figure 4(f) and (h).

4.2.2 Responses of t^* to ω_L or ω_H

We provide some simulated examples for the responses of t^*_{LL} and t^*_{HL} to ω_L or ω_H in Figure 5. The similar interpretations given in the homogeneous case also apply here. Figure 5(a)–(c) show the responses of t^* to ω_L for some cases. For a given value of ω_H, As ω_L increases, t^*_{LL} increases, but t^*_{HL} weakly decreases for lower level of ω_L and increases for higher level of ω_L. This results reflect whether the optimal tax schedule occurs at a corner of an interior. Note that an increase of ω_L may decrease the tax burden of a rich agent since the social planner should allow more information rent to the rich agent. However, we want to emphasize the role of (IC_1) and (IC_2) depending on the relative size of ω_L.

Figure 5(d)–(f) depict the responses of t^* to ω_H, and can be interpreted by the similar way above.

4.2.3 Expected Total Provision of Public Good

Finally, we show how much public goods will be provided as p, q, ω_L, or ω_H varies (Figure 5(a)–(h) respectively). The expected total provision of public good is expressed as

$$\mathbb{E}(y) = pq(2t^*_{LL}) + [p(1-q) + (1-p)q](t^*_{LH} + t^*_{HL}) + (1-p)(1-q)(2t^*_{HH}).$$

The quite similar interpretations given in Rhee (2008) apply here, too. That is, the expected public good provision increases as the given level of endowments ω_L or ω_H increases, and the probability of being poor p or q decreases. Exceptionally, the increases in ω_L may reduce the expected provision $\mathbb{E}(y)$ for large ω_H because it may decease t^*_{HL} so much.

It is meaningful to compare the levels of the expected provision of public good between homogeneous and heterogeneous economies. Although the analytical comparison is nearly impossible due to the feature that the solution has many corner solutions, a computational comparison indicates that the expected public good provision of heterogeneous economy is less than that of homogeneous economy.[9] This results reflect that in a heterogeneous economy the social planner has to care more about the incentive for a rich majority to pretend to be poor than the incentive for a rich agent in a homogeneous economy. This implies that a larger informational rent should be given to a rich at the cost of lower level of public good in a heterogeneous economy than in a homogeneous economy.

5. CONCLUDING REMARKS

In this paper, we consider the feasible taxation problem of a heterogeneous public good economy from an efficiency point of view. The heterogeneity adopted here is the assumption that each agent has a publicly observable characteristic. First, using a Bayesian mechanism design approach, we fully characterize the optimal feasible tax mechanism for an economy with two agents. The features of the obtained optimal feasible tax mechanism are quite similar to those of Rhee (2008), namely, (i) if the overall level of endowment of the economy is low enough or high enough, then the optimal mechanism is first best, (ii) when the optimal mechanism is second best, it is regressive, and (iii) the optimal tax schedule is increasing. In particular, the result of (ii) is mainly due to the feature of the model that levying a tax on a poor agent does not involve any incentive problem to misreport so that the social planner, who does not mind which agent

[9] The simulated results of comparison are available upon request from the author.

pays how much tax as long as the total amount of taxes stays the same, prefers to impose as much tax as possible on the poor agent rather than the rich agent who may request some informational rent.

Second, we conduct some comparative statics analyses of the mechanism when there are changes in the given parameters (ω_L, ω_H) and (p, q). The simulated results show how the optimal tax mechanism deals with the agents' incentives to misreport or free-ride and the individual feasibility constraint simultaneously in the heterogeneous economy. Specifically, in addition to the similar results and interpretations of the homogeneous case in Rhee (2008), the heterogeneous case indicates that if the level of ω_L is low enough, the free-riding incentive of a rich minority only matters, while otherwise the free-riding incentive of rich majority only matters. This implication results from the fact that if ω_L is low enough a rich minority who is more probable to be poor has more incentive to misreport his endowment than a rich majority who is less probable to be poor, and if ω_L is high enough, *vice versa*.

As a future research agenda, we need to study the performance comparison of the optimal feasible tax mechanisms between homogeneous and heterogeneous economies from the perspectives of social welfare. In other words, it would be an interesting research to compare which economy gives the higher social welfare, an economy consisting of homogeneous communities or an economy consisting of heterogeneous communities.

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