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Under What Conditions Do the Poor Hold Cash in Pairwise-Trade Models?*

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Abstract Most existing pairwise-trade models of money predict that the poor may not hold any money in the presence of interest-bearing liquid assets. We resolve this seemingly implausible prediction by incorporating cash and interest-bearing checking account into a standard search-theoretic model. If the transaction cost of a debit card is neither too small nor too large, there is a coexistence equilibrium in which the poor hold cash for their small transactions, whereas the rich hold interest-bearing deposits and use a debit card for their large transactions.

Keywords Cash, Checking account, Debit card, Coexistence puzzle

JEL Classification E40, E41

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1. INTRODUCTION

Recently there has been a growing literature that explores the coexistence puzzle (or rate-of-return-dominance puzzle) with microfounded search-theoretic models of money. Among related studies are, for instance, Aiyagari, Wallace and Wright (1996), Shi (2005), Berentsen and Waller (2008), and Marchesiani and Senesi (2009). Essentially, these works introduce some frictions that make interest-bearing assets less liquid compared to non-interest-bearing money.

However, some historical anecdotes show that an interest-bearing asset that was almost a perfect substitute for money in daily trades failed to chase money from circulation. Gherity (1993) reports that several types of bonds issued during the U.S. Civil War circulated at face value without interest until shortly before maturity along with non-interest-bearing notes. Burdekin and Weidenmier (2008) also report that Arkansas' small-denomination, interest-bearing notes issued in 1861 circulated at a negligible discount alongside non-interest-bearing notes.

One of the recent studies that is consistent with the above observations is Zhu and Wallace (2007). They propose a two-stage trading protocol. In stage 1, a buyer makes a take-it-or-leave-it offer in the form of an offer of money for the good. In stage 2, a seller makes a take-it-or-leave-it offer in the form of the good for interest-bearing bonds. They show an existence of coexistence equilibrium in which sufficiently rich agents hold non-interest-bearing money. While this prediction is plausible in the sense that rich agents hold some money, it also has a seemingly implausible prediction in the sense that very poor agents may not hold any money.

Kim and Lee (2012) study the issue in a competitive-trade model by introducing an intermediary cost of interest-bearing asset. They show an existence of a coexistence equilibrium in which the rich hold both money and bond, whereas the poor hold money only. In a model of pairwise trade, however, their mechanism also generates a similar prediction to that in Zhu and Wallace (2007).

The goal of this note is to provide a search-theoretic model which has plausible predictions on the portfolio holdings and transaction patterns across the agents with different wealth. In particular, we focus on the environment in which monetary wealth can be either held in a form of non-interest-bearing cash or interest-bearing demand deposit. We do so because interest-bearing demand deposit is immediately available as a means of payment, which is consistent with historical instances discussed above. Furthermore, in modern economies, demand deposits are typically insured by the government. In these senses, as argued properly by Andolfatto (2006), the coexistence of money and interest-bearing demand deposits can be regarded as a present-day version of the coexistence puzzle.

More specifically, at the beginning of each period, each agent chooses a portfolio that consists of cash and checking-account deposit. With chosen portfolios, each agent is randomly matched with another agent. In a pairwise meeting, the terms of trade are determined by a buyer's take-it-or-leave-it offer. If a buyer in a pairwise trade uses a debit card to pay for her consumption purchase, it is withdrawn from her account and transferred to the seller immediately, where the seller incurs the associated transaction cost. In the real world, it is common that debit-card providers (usually banks) charge transaction fees to retailers (sellers) not to users of debit cards. If a buyer uses cash, there is no such cost. This is also consistent with an observation from the real world: i.e., for sellers, transaction cost of accepting debit card is much more expensive than cash (see, for example, Humphrey 2004). After pairwise meetings, the balance in checking account is redeemed to each agent with the promised return. There is no return for the early-withdrawn deposit for consumption purchases.

For the model, we characterize the conditions which ensure plausible predictions on the portfolio holdings and transaction patterns across the agents with different wealth. The poorest agent holds wealth in the form of cash and uses it for her small transaction if her willingness to transfer one unit of money as a buyer is not too small. Furthermore, the richest agent holds wealth in the form of interest-bearing deposit and use a debit card for her large transaction if her willingness to transfer one unit of money as a buyer is not too large. By numerical exercises, we then illustrate that such conditions are satisfied if the transaction cost of a debit card is neither too small nor too large. For a given return rate of checking account, the poorest agent is not willing to hold cash if the cost is low enough, whereas even the richest agent is willing to make a large transaction by cash if the cost is too high.

Several comments are in order about the salient features of our model compared to the previous ones. In Kim and Lee (2012), the transaction cost of interest-bearing asset is incurred at the time of portfolio choice instead of pairwise trade. This different assumption on the timing has somewhat different implication on the intensive margin. In Kim and Lee (2012), the terms of pairwise trades do not rely on the transaction cost of interest-bearing assets, whereas they depend on the transaction cost of a debit card in our model. Notice that in the real world, particularly in pairwise trades, more favorable terms of trade are sometimes offered to the buyers who use cash as a means of payment. There might be various incentives for sellers to do so such as tax evasion, saving credit-card (or debit-card) transaction fee and so on, although it is beyond the scope of this paper.

Kim and Lee (2010) introduce the carrying cost of cash which captures inconvenience of cash holding as well as interest on checking account. This cost is a kind of sunk cost in the sense that it is incurred at the portfolio-choice stage. Put in another way, an implicit interest on checking account is paid at the time of making deposit. Hence, the choice of a means of payment in a pairwise trade is irrelevant to the opportunity cost of cash holdings. In our model, interest on checking account is paid at the end of a period and there is no interest on the deposits that are used to purchase consumption goods. Hence, the choice of a means of payment in a pairwise trade relies upon interest on checking account.

2. MODEL

The background setup is the standard matching model of money studied in Zhu and Wallace (2007) and Kim and Lee (2010), which is in turn a version of Shi (1995) and Trejos and Wright (1995) augmented with distribution of wealth.

Time is discrete and continues forever. There is a $[0,1]$ continuum of each of $K > 2$ types of infinitely lived agents with K distinct types of specialized good and one general consumption good, where both type-specific and general goods are divisible and perishable. A type $k \in \{1,2,...,K\}$ agent produces only good k and consumes only good $k+1$ (modulo K). A General good is homogeneous and consumed by all agents regardless of specialization types. A type k agent enjoys per-period utility given by $u(q_{k+1}) - q_k + g$, where $q_{k+1} \in \mathbb{R}_+$ is consumption of good $k+1$, $q_k \in \mathbb{R}_+$ is production of good k, g is the consumption of general good, $u'' < 0 < u'$, $u(0) = 0$, $u'(\infty) = 0$, and $u'(0)$ is sufficiently large. Further, there is $q^* \in (0,\infty)$ that satisfies $u'(q^*) = 1$. Each agent maximizes expected discounted utility with a discount factor $\beta \in (0,1)$.

There exists indivisible money which is symmetrically distributed across the K specialized types. Let $\bar{z} > 0$ and $Z > 0$ denote the exogenous average wealth per each specialization type and the exogenous upper bound on individual wealth holdings, respectively. Let $\mathbf{Z} \subset \mathbb{Z}_+$ be the set of possible individual wealth holdings with the normalization of the smallest unit of money to be one. Each agent can hold monetary wealth in cash or checking-account deposits from which a debit card can be used as a means of payment. The intra-temporal information on accounts is kept by the government and by using checking-account deposits, she can produce general goods according to linear technology $G = \theta D$, where $\theta > 0$, G is the quantity of general good produced and D is the balance of deposits after the trades of specialized good.

The sequence of actions in a period is as follows. An agent entering a period with $z \in \mathbf{Z}$ chooses a portfolio $\omega = (c,d)$ subject to $c+d \leq z$, where c and d denote cash holdings and checking-account deposits, respectively. We assume that there is no cost in adjusting portfolios. After the portfolio is chosen, each person is randomly matched with another person. Trades occur only in single-coincidence meetings in which type-k agent meets type-$(k+1)$ agent. Agents in a meeting know each other's specialization type and portfolio. However, trading histories are private and agents cannot commit to their future actions, which makes a medium of exchange essential.

In a single-coincidence meeting, a buyer makes take-it-or-leave-it offer (q,p) to a seller where q denotes quantity of specialized good produced by a seller and p denotes the amount of wealth transferred by a buyer. If a buyer uses a debit card to pay p amount of money, it is withdrawn from her account and transferred to the seller immediately, where the seller incurs the associated fixed disutility cost $\phi > 0$.[1] The cost ϕ can be interpreted as a record-keeping cost which seems to be irrelevant to the amount of transaction. If a buyer uses cash, there is no such cost. After pairwise meetings, the balance in checking account is redeemed to each agent with the return of θ units of general good per unit of deposit. There is no return for the early-withdrawn deposit to purchase specialized good. Then intra-temporal transaction records of checking accounts are wiped out completely. Other than managing the accounts, there is no economic activity by the government so that its budget is always in balance. Finally, agents go on to the next period with the end-of-period money holdings.

3. STEADY STATE

We consider a steady-state allocation which is symmetric across specialization types. A symmetric steady state consists of a set of functions (v, π, λ) that satisfies the conditions described below. The function v and π map from $\mathbf{Z}$ to

[1] Kim and Lee (2012) assume a proportional transaction cost of interest-bearing assets. This is because they interpret the cost as a transaction fee charged by a security dealer which is typically proportional to the amount of transaction. Here we interpret the cost as a record-keeping cost of debit-card transaction and a flat fee per transaction and immediate clearance capture the exact features of a PIN-based debit card. According to the *Study of Consumer Payment Preferences* conducted by American Bankers Association and Dove Consulting shows that around 70% of debit-card holders use PIN-based debit cards to make in-store, internet, and bill payments. Meanwhile if we introduce a proportional cost, the result would be similar to that in Kim and Lee (2010): i.e., cash is used up to a certain amount of transaction and an extra amount of transaction exceeding the critical level will be paid by a debit card.

$\mathbb{R}$ and [0,1], respectively, where $v(z)$ and $\pi(z)$ denote the expected discounted value of having z and the fraction of each specialization type with z at the beginning of a period and prior to the choice of portfolios. The function λ maps from Ω to $[0,1]$, where $\lambda(\omega)$ denote the fraction of each specialization type with portfolio ω after the portfolio choice and before the pairwise meeting and $\Omega = \left\{\omega = (c,d) \in \mathbb{Z}_+^2 : c+d \leq Z\right\}$ is the set of feasible individual portfolios.

Letting $\mathbf{V} : \Omega \to \mathbb{R}$ denote the expected discounted utility after the portfolio choice and before the pairwise meeting, the portfolio-choice problem for an agent with z can be expressed as

$$\mathbf{J}(z,\mathbf{V}) = \max_{\omega \in \Gamma(z)} \mathbf{V}(\omega) \tag{1}$$

where $\Gamma(z) = \{\omega = (c,d) \in \mathbb{Z}_+^2 : c+d \leq z\}$ is the set of feasible portfolios with z. Let $\mathbb{S}_1(z,\mathbf{V})$ denote the set of maximizers in (1). If $\mathbb{S}_1(z,\mathbf{V})$ contains multiple elements, we allow for all possible randomizations over them. This set of randomizations can be defined as $\Delta_1(z,\mathbf{V}) = \{\delta_z : \delta_z(\omega) = 0 \text{ if } \omega \notin \mathbb{S}_1(z,\mathbf{V})\}$. Then $\Lambda(\mathbf{V},\pi)$, the set of portfolio distributions on Ω, can be defined as

$$\Lambda(\mathbf{V},\pi) = \{\lambda : \lambda(\omega) = \sum_z \pi(z)\delta_z(\omega) \text{ for } \delta_z(\omega) \in \Delta_1(z,\mathbf{V})\}. \tag{2}$$

We next turn to a pairwise-trade stage. Consider a generic single-coincidence meeting between a buyer with $\omega = (c,d) \in \Omega$ and a seller with $\tilde{\omega} = (\tilde{c},\tilde{d}) \in \Omega$. Let $z_\omega = (c+d) \in \mathbf{Z}$ and $z_{\tilde{\omega}} = (\tilde{c}+\tilde{d}) \in \mathbf{Z}$ denote the total wealth implied by the portfolio ω and $\tilde{\omega}$, respectively. For the meeting, the set of feasible offers from the buyer to the seller can be defined as $\Gamma(\omega,\tilde{\omega}) = \{p : p \in \{0,1,...,\min\{z_\omega, Z - z_{\tilde{\omega}}\}\}\}$. With a tie-breaking rule by which the seller accepts all offers that leave her no worse off, the buyer's problem can be written as

$$\max_{p \in \Gamma(\omega,\tilde{\omega})} \left\{u\left[\beta v(z_{\tilde{\omega}}+p) - \beta v(z_{\tilde{\omega}}) - \phi 1_{\{p>c\}}\right] + \beta v(z_\omega - p) + \theta[d-(p-c)1_{\{p>c\}}]\right\} \tag{3}$$

where $1_{\{\chi\}} = 1$ if and only if χ is true. Let the set of maximizers in (3) be $\mathbb{S}_2(\omega,\tilde{\omega},v)$ and let the maximized value of (3) be $g(\omega,\tilde{\omega},v)$. Noting that the payoff with portfolio $\omega = (c,d)$ as a seller is $\beta v(z_\omega) + \theta d$, $\mathbf{V}(\omega)$, the expected payoff from holding $\omega = (c,d)$ before the pairwise meeting, should satisfy

$$\mathbf{V}(\omega) = \alpha \sum_{\tilde{\omega}} \lambda(\tilde{\omega}) g(\omega,\tilde{\omega},v) + (1-\alpha)\left[\beta v(z_\omega) + \theta d\right] \tag{4}$$

where $\alpha = 1/K$, the probability of a single-coincidence meeting as a buyer.

Now, we can describe the evolution of wealth distribution induced by pairwise trades. As in the portfolio-choice stage, we allow for all possible randomizations over the elements in $\mathbb{S}_2(\omega,\tilde{\omega},v)$. It is convenient to express the randomizations over the post-trade wealth holdings of the buyer as follows:

$$\Delta_2(\omega,\tilde{\omega},v) = \{\delta(\cdot;\omega,\tilde{\omega},v) : \delta(z;\omega,\tilde{\omega},v) = 0 \text{ if } z \notin \{z_\omega - p(\omega,\tilde{\omega},v)\} \text{ for } p(\omega,\tilde{\omega},v) \in \mathbb{S}_2(\omega,\tilde{\omega},v)\}. \quad (5)$$

Then, $\Pi(v,\lambda)$, the set of post-trade wealth distributions on $\mathbf{Z}$, can be defined as

$$\Pi(v,\lambda) = \left\{ \pi : \pi(z) = \alpha \sum_{(\omega,\tilde{\omega})} \lambda(\omega)\lambda(\tilde{\omega})[\delta(z;\cdot) + \delta(z_\omega + z_{\tilde{\omega}} - z;\cdot)] + (1-2\alpha)\sum_{\omega} \lambda(\omega) 1_{\{z_\omega = z\}} \text{ for } \delta \in \Delta_2(\omega,\tilde{\omega},v) \right\}. \quad (6)$$

The probability measure in the first line of the right-hand side in (6) corresponds to single-coincidence meetings, while the second line corresponds to all other cases. Since δ is defined over the post-trade wealth of the buyer, the buyer's post-trade wealth $(z_\omega + z_{\tilde{\omega}} - z)$ corresponds to the seller's post-trade wealth z.

Definition 1. *For given (ϕ,θ), a symmetric steady state is (v,π,λ) such that (i) $v(z) = \mathbf{J}(z,\mathbf{V})$ where $\mathbf{J}(z,\mathbf{V})$ is given by (1) and $\mathbf{V} : \Omega \to \mathbb{R}$ is given by (4); (ii) $\pi \in \Pi(v,\lambda)$ where $\Pi(v,\lambda)$ is given by (6); (iii) $\lambda \in \Lambda(\mathbf{V},\pi)$ where $\Lambda(\mathbf{V},\pi)$ is given by (2).*

The existence of a steady state for some parameters is a straightforward extension of the results in Lee, Wallace and Zhu (2005) and Zhu and Wallace (2007): if $\bar{z}$ and $Z/\bar{z}$ are large enough, and (ϕ,θ) are sufficiently close to zero, then there exists a steady state with strictly increasing and strictly concave v, and π having full support.

4. PORTFOLIO CHOICES OF HETEROGENEOUS AGENTS

We now characterize the conditions that imply plausible predictions on the portfolio holdings and transaction patterns across the agents with different wealth.

We first consider the agent with $z = 1$, the poorest one among the agents with $z \in \mathbf{Z}\backslash\{0\}$. Notice that the feasible set of offers for her as a buyer is $\{0,1\}$ and her offer decreases with her trading partner's wealth (see Kim and Lee 2010). Hence we can define $\bar{z}_1 \in \mathbf{Z}$ such that $p(\omega, z_{\tilde{\omega}}, v) = 1$ if $z_{\tilde{\omega}} \leq \bar{z}_1$ and $p(\omega, z_{\tilde{\omega}}, v) = 0$ otherwise, where $p(\omega, z_{\tilde{\omega}}, v)$ denotes the optimal offer in a single-coincidence meeting with a seller having wealth $z_{\tilde{\omega}} \in \mathbf{Z}$.[2] Let $\Theta_1 \equiv \sum_{z_{\tilde{\omega}} \leq \tilde{z}_1} \pi(z_{\tilde{\omega}})$, the fraction of single-coincidence meetings transferring one unit of money to a seller. Then the following proposition says that if Θ_1 is not too small, the poorest agent is willing to hold her wealth in the form of cash rather than deposit it into interest-bearing checking account. It is worth stressing here that in Zhu and Wallace (2007), the richest agent rather than the poorest one is more willing to hold cash.

Proposition 1. *An agent with $z = 1$ chooses the portfolio $\omega = (1,0)$ if*

$$\Theta_1 \geq \frac{\theta}{\alpha[\phi u'(\beta Q^*) + \theta]} \tag{7}$$

where $Q^* \equiv [\alpha(u(q^*) - q^*) + (1-\alpha)\theta]/(1-\beta)$.

Proof. See Appendix. □

Intuitively, the magnitude of consumption for the poorest agent is relatively small and hence the utility loss implied by the transaction cost of a debit card (ϕ) would be substantial. This then suggests that such an utility loss dominates an expected return from holding interest-bearing deposit if it is most likely to be withdrawn for her consumption purchase. In sum, Proposition 1 suggests that in order to have a plausible prediction on the portfolio holdings in a pairwise-trade model, some ingredients that promote the willingness of the poor to trade are indispensable.

Consider now the agent with $z = Z$, the richest one among the agents with $z \in \mathbf{Z}\backslash\{0\}$. Notice that by Lemma 6 in Zhu (2003), $p(\omega, z_{\tilde{\omega}}, v) \geq 1$ for $z_\omega = Z$ unless $z_{\tilde{\omega}} = Z$: i.e., the richest agent is almost always willing to transfer at least one unit of money in a single-coincidence meeting as a buyer. Let $\hat{p} \equiv p(\omega, 0, v)$ for $z_\omega = Z$, the largest possible offer for the agent with Z. Since $p(\cdot)$ decreases with a seller's wealth ($z_{\tilde{\omega}}$), we can define $\hat{z}_i$ for $i \in \{1, 2, ..., \hat{p} - 1\}$ such that $p(\omega, z_{\tilde{\omega}}, v) = i$ if $\hat{z}_i < z_{\tilde{\omega}} \leq \hat{z}_{i-1}$ with $\hat{z}_0 = Z - 1$ and $p(\omega, z_{\tilde{\omega}}, v) = \hat{p}$ if $z_{\tilde{\omega}} \leq \hat{z}_{\hat{p}-1}$. As in Proposition 1, let $\Theta_Z^i \equiv \sum_{\hat{z}_i < z_{\tilde{\omega}} \leq \hat{z}_{i-1}} \pi(z_{\tilde{\omega}})$, the frequency of single-coincidence meetings transferring i units of wealth to a seller. Then the following proposition says that if Θ_Z^1 is not large enough, the richest agent is willing to hold her wealth in the form of interest-bearing liquid deposits and use a debit card for her consumption purchase.

[2] We simply express $p(\cdot)$ as a function of $(\omega, z_{\tilde{\omega}}, v)$ because, as we can see in the definition of $\Gamma(\omega^b, \omega^s)$ and (3), seller's total wealth implied by $\tilde{\omega}$ does matter for the terms of trade in the pairwise meeting.

Proposition 2. *An agent with $z = Z$ chooses the portfolio $\omega = (0, Z)$ if*

$$\Theta_Z^1 \leq \frac{\theta[\alpha\pi(Z) + (1-\alpha)]}{\phi\alpha u'(q_0)} \tag{8}$$

where $q_0 \equiv \beta[v(Z) - v(Z-1)] - \phi$, the smallest possible quantity of good traded over single-coincidence meetings.

Proof. See Appendix. □

Notice that small enough Θ_Z^1 essentially implies that an expected magnitude of consumption for the agent with Z is large enough. Hence the utility loss implied by the transaction cost of a debit card (ϕ) would be relatively small. This then suggests that such an utility loss is dominated by an expected marginal return from depositing her entire wealth into an interest-bearing checking account.

5. NUMERICAL EXAMPLE

At this point, it is natural to ask what conditions are necessary so that (7) and (8) are satisfied. Noting that (7) and (8) are associated with an endogenous non-degenerate wealth distribution π, further analytical result is almost ruled out. Hence we here explore such conditions via numerical exercises. In order to solve the model numerically, we need to specify $(u, K, \beta, Z, \bar{z}, \theta, \phi)$. Since it is not easy to calibrate the model as DSGE ones, we take reasonable and simple parameter values that satisfy our assumptions.[3] Hence, as hardly needs to be mentioned, our numerical example is literally an illustration.

Specifically, we parameterize the background matching model as follows. We let $u(q) = q^{1-\eta}/1-\eta$ with $\eta = 1/2$ which simply implies $q^* = 1$. We set $K = 3$, the smallest number of specialization types eliminating the possibility of double-coincidence of wants in pairwise meetings. We let $\beta = 0.96$, a standard annual discount factor. We choose $\bar{z} = 20$ which, as we will see, makes the indivisibility of money not-too severe. According to Zhu (2003), we then choose $Z = 4\bar{z}$ that is one of the necessary conditions for the existence result. Finally, we set $\theta = 0.03\%$ that is approximately consistent with an annual real return rate of *MZM* deposits reported in Šustek (2010).

With the parameterized version of model, we compute steady states for a wide range of ϕ to find an equilibrium that implies the predictions of Proposition

[3]For the parameterizations of similar random matching models of money, see Lee and Wallace (2006), Kim and Lee (2009, 2010), and Lee (2010).

1 and 2. Table 1 reports summary statistics of steady states with different values of ϕ. In the table, $\mathbb{E}(q)$, $\mathbb{E}(p)$ and $\mathbb{E}(c/z)$ denote respectively average consumption over single-coincidence meetings $[\mathbb{E}(q) = \sum_z \sum_{z'} \pi(z)\pi(z')q(z,z')]$, average monetary offer over single-coincidence meetings $[\mathbb{E}(p) = \sum_z \sum_{z'} \pi(z)\pi(z')p(z,z)]$, and average cash-holding ratio $[\mathbb{E}(c/z) = \sum_z \pi(z)(c_z/z)]$. And $\mathscr{W}$ denotes the welfare that is measured by the expected lifetime utility of a representative agent prior to the assignment of wealth according to π, $\mathscr{W} = [\alpha/(1-\beta)](\pi\mathbf{U}\pi') + [(1-\alpha)/(1-\beta)][\theta\pi\omega(:,2)]$ where the element in row $z \in \mathbf{Z}$ and column $z' \in \mathbf{Z}$ of the matrix $\mathbf{U}$ is $u[q(z,z')] - \tilde{q}(z,z') + \theta[d_z - (p(z,z') - c_z)1_{\{p(z,z')>c_z\}}]$ with (c_z, d_z) denoting cash and checking-account deposits of the agent with z, respectively, and $\tilde{q}(z,z') = q(z,z') + \phi 1_{\{p(z,z')>c_z\}}$.

Table 1: Summary statistics of steady states

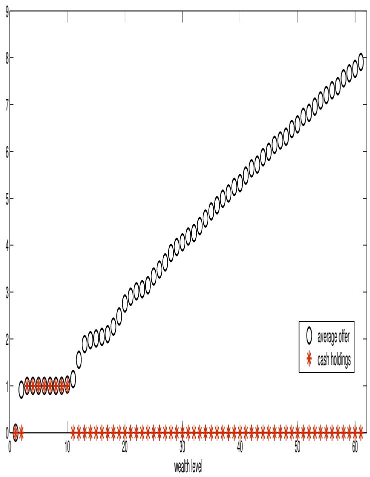
	$\phi \times 10^3$			
	0.36	1.80	4.50	7.50
$\mathbb{E}(q)$	0.7700	0.7684	0.7656	0.7652
$\mathbb{E}(p)$	2.7971	2.7959	2.7874	2.7869
$\mathbb{E}(c/z)$	0.0005	0.0863	0.1484	0.1501
$\mathscr{W}$	8.1089	8.1000	8.0929	8.0927

First of all, for all values of ϕ, $\mathbb{E}(q)$ is less than $q^* = 1$ and $\mathbb{E}(p)$ is far from one, both of which together imply that the indivisibility of money is not too severe. (See, for instance, Berentsen, Molico and Wright, 2002.) Not surprisingly, as ϕ increases, $\mathbb{E}(c/z)$ increases, whereas $\mathbb{E}(q)$ decreases due to the negative intensive margin of ϕ. Consequently the welfare $\mathscr{W}$ declines with ϕ.

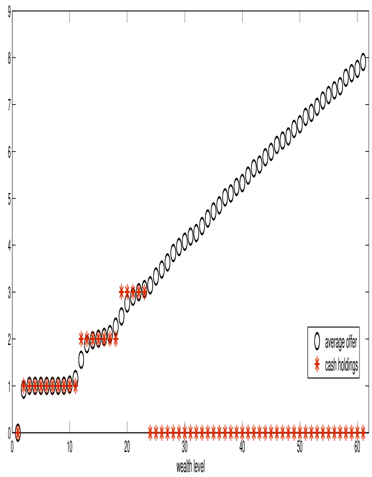
Figure 1 shows the average offers and cash holdings as a function of z. The average offers across single-coincidence meetings increase with the buyer's wealth. Further, most agents are not willing to hold cash with a low enough ϕ such as $\phi = 0.36 \times 10^{-3}$, whereas almost all the agents are willing to hold sufficient amount of cash for trades with a high enough ϕ such as $\phi = 7.5 \times 10^{-3}$.

Finally, we find that in the steady state with $\phi \in (0.36 \times 10^{-3}, 7.50 \times 10^{-3})$, the poorest agent chooses the portfolio consisting of cash only, whereas the richest agent chooses the one consisting of checking-account deposit only. Aiyagari, Braun and Eckstein (1998) report that the cost incurred by the U.S. banks in providing checkable deposits is around $0.2 \sim 0.4\%$ of GDP. Notice that in our model, $\mathbb{E}(q)$ can be regarded as GDP, and in the steady state with $\phi = 0.36 \times 10^{-3}$, the ratio of $[\phi/\mathbb{E}(q)]$ is 0.05% and it is 0.98% in the steady state

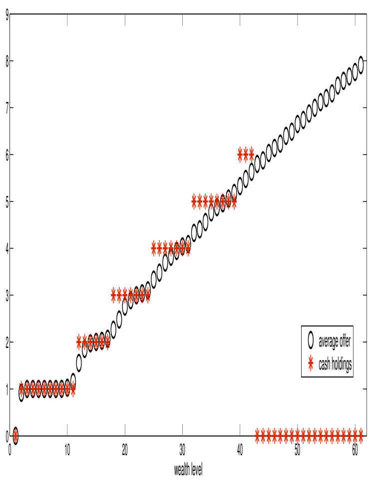
Figure 1: Average offers and cash holdings as a function of z



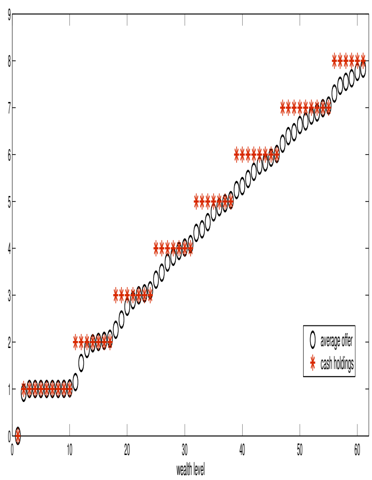
(a) $\phi = 0.36 \times 10^{-3}$



(b) $\phi = 1.80 \times 10^{-3}$



(c) $\phi = 4.50 \times 10^{-3}$



(d) $\phi = 7.50 \times 10^{-3}$

with $\phi = 7.50 \times 10^{-3}$. That is, ϕ reported in Aiyagari, Braun and Eckstein (1998) is in the range of ϕ implying the predictions of Proposition 1 and 2 in our model. This suggests that although we parameterize the background matching model somewhat arbitrarily, it is not out of the ordinary at all.

6. CONCLUDING REMARKS

In this note, we provide a search-theoretic model that has more plausible predictions on the portfolio holdings and transaction patterns compared to the previous models. As buyers, if the poor are sufficiently willing to transfer their money to sellers and the rich are willing to spend large enough amount of money, there is an equilibrium wherein the poor hold cash and use it for their small consumption purchases, and the rich hold interest-bearing deposits and use a debit card for their large consumption purchases. Furthermore, the conditions mentioned above are satisfied if the transaction cost of a debit card is neither too small nor too large.

It is worth noting that the conditions required to have the plausible predictions seem to be quite natural. In the real world, it is readily observable that the marginal propensity to consume for the poor is very close to one and the expenditure size of the rich is sufficiently large. However, we cannot characterize analytically the condition which ensures the existence of such an equilibrium. It does not seem at all easy in our framework due to an endogenous wealth distribution but that is indispensable to capture the feature of portfolio holdings across heterogeneous agents. In order to avoid this difficulty, we might take the model of Lagos and Wright (2005) wherein distribution is nondegenerate within a period but degenerate across periods.

With our model as a base, the welfare cost of inflation with interest-bearing liquid assets would be explored. Most existing literature on the cost of inflation assumes that all assets in M1 are non-interest bearing. But M1 is consisted of cash and interest-bearing demand deposits. To the best of our knowledge, no one has studied the cost of inflation by taking into account interest-bearing liquid assets explicitly in a search-theoretic model. Given a sufficiently plausible specification, numerical simulations can be carried out to study the cost of inflation in the presence of interest-baring liquid assets.

APPENDIX: PROOFS

Proof of Proposition 1. Since our existence result is an extension of Zhu (2003) and Lee, Wallace and Zhu (2005), we adopt their assumptions: (A.1) $u'(0) > [2/(\kappa\beta)]^2$ where $\kappa \equiv [K-(K-1)\beta]^{-1}$; (A.2) $Z/\bar{z} \geq 4$; (A.3) $\bar{z} \geq \beta W/D$ where D is the solution of $u'(D) = [2/(\kappa\beta)]^2$ and W is the unique solution of $K(1-\beta)W = u(\beta W) + (K-1)\theta Z$. First, we claim that $v(1) \leq Q^*$. Let $\omega_1^i = (i, 1-i)$ for $i \in \{0,1\}$. Then by definition, $\mathbf{V}(\omega_1^i) \leq \alpha \sum_{\tilde{\omega}} \lambda(\tilde{\omega}) g(\omega, \tilde{\omega}, v) + (1-\alpha)[\beta v(1) + \theta] \leq \alpha u[\beta v(1)] + (1-\alpha)[\beta v(1) + \theta] = \alpha\{u[\beta v(1)] - \beta v(1)\} + \beta v(1) + (1-\alpha)\theta \leq \alpha[u(q^*) - q^*] + \beta v(1) + (1-\alpha)\theta$. Therefore, we have $v(1) \leq \alpha[u(q^*) - q^*] + \beta v(1) + (1-\alpha)\theta$, which can be rearranged as

$$v(1) \leq [1/(1-\beta)]\{\alpha[u(q^*) - q^*] + (1-\alpha)\theta\} \equiv Q^*.$$

Second, it is straightforward to show that if $p(\omega, z_{\tilde{\omega}}, v) = 1 \in \mathbb{S}_2(\omega_1^0, \tilde{\omega}, v)$, $p(\omega, z_{\tilde{\omega}}, v) = 1 \in \mathbb{S}_2(\omega_1^1, \tilde{\omega}, v)$. Hence,

$$\begin{aligned}
\mathbf{V}(\omega_1^1) - \mathbf{V}(\omega_1^0) &= \alpha \sum_{z_{\tilde{\omega}} \leq \bar{z}_1} \pi(z_{\tilde{\omega}})[g(\omega_1^1, \tilde{\omega}, v) - g(\omega_1^0, \tilde{\omega}, v)] \\
&\quad + \alpha \sum_{z_{\tilde{\omega}} > \bar{z}_1} \pi(z_{\tilde{\omega}})[g(\omega_1^1, \tilde{\omega}, v) - g(\omega_1^0, \tilde{\omega}, v)] - (1-\alpha)\theta \\
&= \alpha \sum_{z_{\tilde{\omega}} \leq \bar{z}_1} \pi(z_{\tilde{\omega}}) \left\{u[q_1^h(z_{\tilde{\omega}})] - u[q_1^l(z_{\tilde{\omega}})]\right\} \\
&\quad - \alpha\theta \sum_{z_{\tilde{\omega}} > \bar{z}_1} \pi(z_{\tilde{\omega}}) - (1-\alpha)\theta \\
&= \alpha \sum_{z_{\tilde{\omega}} \leq \bar{z}_1} \pi(z_{\tilde{\omega}}) u'[q_1^*(z_{\tilde{\omega}})]\phi - \alpha\theta \sum_{z_{\tilde{\omega}} > \bar{z}_1} \pi(z_{\tilde{\omega}}) - (1-\alpha)\theta
\end{aligned}$$

where $q_1^l(z_{\tilde{\omega}}) = \beta v(z_{\tilde{\omega}}+1) - \beta v(z_{\tilde{\omega}}) - \phi$, $q_1^h(z_{\tilde{\omega}}) = \beta v(z_{\tilde{\omega}}+1) - \beta v(z_{\tilde{\omega}})$, and the third equality follows from Mean Value Theorem with $q_1^*(z_{\tilde{\omega}}) \in [\beta v(z_{\tilde{\omega}}+1) - \beta v(z_{\tilde{\omega}}) - \phi, \beta v(z_{\tilde{\omega}}+1) - \beta v(z_{\tilde{\omega}})]$ for each $z_{\tilde{\omega}}$. Since $u'[\beta v(1)]$ is the minimum value of $u'[q_1^*(z_{\tilde{\omega}})]$ and $v(1) \leq Q^*$, $\alpha\phi\Theta_1 u'(\beta Q^*) \geq [\alpha(1-\Theta_1) + (1-\alpha)]\theta$ implies $\mathbf{V}(\omega_1^1) - \mathbf{V}(\omega_1^0) > 0$. □

Proof of Proposition 2. Let $\omega_Z^i = (i, Z-i)$. We first show that for an agent with Z, $\omega_Z^0 = (0, Z)$ is preferred to $\omega_Z^1 = (1, Z-1)$. Notice that there is $\hat{z}_1$ such that $u[q_{Z,2}^l(\hat{z}_1)] - u[q_{Z,1}^l(\hat{z}_1)] = \beta v(Z-1) - \beta v(Z-2) + \theta$ where $q_{Z,2}^l(z_{\tilde{\omega}}) = \beta v(z_{\tilde{\omega}}+2) - \beta v(z_{\tilde{\omega}}) - \phi$ and $q_{Z,1}^l(z_{\tilde{\omega}}) = \beta v(z_{\tilde{\omega}}+1) - \beta v(z_{\tilde{\omega}}) - \phi$. This equality means that if a buyer with Z meets a seller with $\hat{z}_1$, she is indifferent to offer

$p(\omega, \hat{z}_1, v) = 1$ and $p(\omega, \hat{z}_1, v) = 2$. If $\min\{\mathbb{S}_2(\omega_Z^0, \tilde{\omega}, v)\} \geq 1$, $\mathbb{S}_2(\omega_Z^0, \tilde{\omega}, v) = \mathbb{S}_2(\omega_Z^1, \tilde{\omega}, v)$ and hence

$$\begin{aligned}
\mathbf{V}(\omega_Z^0) - \mathbf{V}(\omega_Z^1) &= \alpha\pi(Z)\theta + \alpha \sum_{Z-1 \geq z_{\tilde{\omega}} > \hat{z}_1} \pi(z_{\tilde{\omega}})[g(\omega_Z^0, \tilde{\omega}, v) - g(\omega_M^1, \tilde{\omega}, v)] \\
&\quad + \alpha \sum_{z_{\tilde{\omega}} \leq \hat{z}_1} \pi(z_{\tilde{\omega}})[g(\omega_Z^0, \tilde{\omega}, v) - g(\omega_Z^1, \tilde{\omega}, v)] + (1-\alpha)\theta \\
&= \alpha\pi(Z)\theta - \sum_{Z-1 \geq z_{\tilde{\omega}} > \hat{z}_1} \alpha\pi(z_{\tilde{\omega}}) \left\{ u[q_{Z,1}^h(z_{\tilde{\omega}})] - u[q_{Z,1}^l(z_{\tilde{\omega}})] \right\} \\
&\quad + (1-\alpha)\theta \\
&= \alpha\pi(Z)\theta - \sum_{Z-1 \geq z_{\tilde{\omega}} > \hat{z}_1} \alpha\pi(z_{\tilde{\omega}}) u'[q_Z^*(z_{\tilde{\omega}})]\phi + (1-\alpha)\theta
\end{aligned}$$

where $q_{Z,1}^h(z_{\tilde{\omega}}) = \beta v(z_{\tilde{\omega}} + 1) - \beta v(z_{\tilde{\omega}})$ and the third equality follows from Mean Value Theorem with $q_Z^*(z_{\tilde{\omega}}) \in [\beta v(z_{\tilde{\omega}} + 1) - \beta v(z_{\tilde{\omega}}) - \phi, \beta v(z_{\tilde{\omega}} + 1) - \beta v(z_{\tilde{\omega}})]$ for each $z_{\tilde{\omega}}$. Since $u'[\beta v(Z) - \beta v(Z-1) - \phi] = u'(q_0)$ is the maximum value of $u'[q_Z^*(z_{\tilde{\omega}})]$, $\alpha\Theta_Z^1 u'(q_0)\phi \leq [\alpha\pi(Z) + (1-\alpha)]\theta$ implies $V(\omega_Z^0) - V(\omega_Z^1) > 0$. Now let $q_0^i \equiv \beta[v(\hat{z}_{i-1} + i) - v(\hat{z}_{i-1})] - \phi$ for $i \in \{2, \ldots, \hat{p}\}$. Then for any $i \in \{2, \ldots, \hat{p}\}$, $\alpha\Theta_Z^i u'(q_0^i)\phi \leq [\alpha\tilde{\Theta}_Z^{i-1} + (1-\alpha)]\theta$ implies $V(\omega_Z^{i-1}) - V(\omega_Z^i) > 0$ by the same argument, where $\tilde{\Theta}_Z^i = \sum_{\hat{z}_i < z_{\tilde{\omega}}} \pi(z_{\tilde{\omega}})$ denotes the fraction of single-coincidence meetings as a buyer to offer less than $i+1$ units of wealth to a seller. Since v is strictly concave, $\beta[v(Z) - v(Z-1)] - \phi = q_0 \approx 0$ if Z is sufficiently large. Therefore we have

$$\min_{1 \leq i \leq \hat{p}} \left\{ \frac{\alpha\tilde{\Theta}_Z^{i-1} + (1-\alpha)}{\alpha\Theta_Z^i u'[\beta v(\hat{z}_{i-1} + i) - \beta v(\hat{z}_{i-1}) - \phi]} \right\} = \left\{ \frac{\alpha\pi(Z) + (1-\alpha)}{\alpha\Theta_Z^1 u'(q_0)} \right\}.$$

This again implies that if $\alpha\Theta_Z^1 u'(q_0)\phi \leq [\alpha\pi(Z) + (1-\alpha)]\theta$, then $\alpha\Theta_Z^i u'(q_0^i)\phi \leq [\alpha\tilde{\Theta}_Z^{i-1} + (1-\alpha)]\theta$ for all $i \in \{2, \ldots, \hat{p}\}$ which completes the proof. □

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