

## A Patent Race for a Drastic Innovation with Large Set-Up Cost

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**Abstract** We analyze a situation in which a monopolist incumbent and a potential entrant compete for the patent of a new technology, the owner of which can monopolize the market. If the set-up cost for the new technology is so large that the incumbent would let the patent sleep, the incumbent's preemptive incentive is greater while its stand-alone incentive is smaller than the entrant's. We find that for a drastic innovation, the incumbent invests more in R&D than the entrant if the market is highly profitable under the current technology and the new technology incurs large set-up cost.

**Keywords** Patent race, Drastic innovation, Set-up cost

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## 1. INTRODUCTION

An innovation which is good enough to make the inventor a monopolist is called a drastic innovation. Whether the incumbent monopolist or potential entrants invest more in innovation is important because it explains how strong the persistence of the incumbency in monopoly is for the case of drastic innovations and how strong the persistence of monopoly is in case of non-drastic innovations. Since Arrow (1962) argued that for a drastic innovation, an incumbent monopolist has less incentive to invent than a potential entrant, there have been many researches on the effect of monopoly power to engage in innovation. Gilbert and Newbery (1982) use the auction model to examine this question when an incumbent monopolist and potential entrants compete for innovation and showed that the incumbent will win the auction. The incumbent wins the auction even when the new product or technology is inferior to its current technology for the preemptive purpose not to become a duopolist. In this case, the incumbent invests in a sleeping patent. Contrarily, in the stochastic racing model, Reinganum (1983) shows that for a drastic innovation, the incumbent invests less in innovation than the potential entrant.

To explain why the results derived in the stochastic racing model differ from those derived in the auction model, Reinganum (1989) used the concept of preemptive incentive and stand-alone incentive for innovation described by Katz and Shapiro (1987). The preemptive incentive represents the difference in the firm's profits when it innovates instead of the other firm, while the stand-alone incentive represents the difference in the firm's profits before and after the innovation. In the auction model, only the preemptive incentive matters in the firms' decisions and the incumbent's preemptive incentive for a non-drastic innovation is greater than a potential entrant's. But in the stochastic racing model, both the preemptive incentive and the stand-alone incentive matter in the firms' decisions. Moreover, for a drastic innovation, the preemptive incentive is the same for the two firms while the potential entrant's stand-alone incentive is greater than the incumbent's.

We consider a situation in which an incumbent monopolist and a potential entrant compete for the patent of a new technology as the result of a drastic innovation. If the set-up cost for the new technology is so large that the incumbent would let the patent sleep, the incumbent's preemptive incentive is greater while its stand-alone incentive is smaller than the entrant's. The larger the set-up cost, the more likely the incumbent's R&D investment is greater than the entrant's. We show that for a drastic innovation, the incumbent invests more in R&D than the entrant if the market is highly profitable and the new technology incurs large

Table 1: Preemptive Incentive (PI) and Stand-Alone Incentive (SI)

	(i) When $\pi^N > \pi^O$		(ii) When $\pi^N < \pi^O$	
	PI	SI	PI	SI
Incumbent	$\pi^N$	$\pi^N - \pi^O$	$\pi^O$	0
Entrant	$\pi^N$	$\pi^N$	$\pi^N$	$\pi^N$

set-up cost. Section 2 of this paper performs a formal analysis in the stochastic racing model and Section 3 concludes the paper.

## 2. DRASTIC INNOVATION WITH LARGE SET-UP COST

We consider a case in which an incumbent monopolist produces a product with current technology and earns the profit of  $\pi^O$ , where  $\pi^O > 0$ . The incumbent and a potential entrant which we will simply call the entrant simultaneously invest for an innovation to acquire a patent for a new technology. We assume that the innovation is drastic so that if any firm succeeds to obtain the patent, it can monopolize the market with the new technology and earn the profit of  $\pi^1$ , where  $\pi^1 > 0$ . We also assume that adopting the new technology incurs a set-up cost and let  $F$  denote the amortized set-up cost for the new technology. If  $\pi^1 - F < \pi^O$ , the incumbent would let the patent sleep when it acquires the patent. For a simpler notation, let  $\pi^N \equiv \pi^1 - F$ .

Table 1 shows how the preemptive incentives and stand-alone incentives of the incumbent and entrant for drastic but unattractive innovations differ from those for drastic and attractive innovations. When innovation is drastic and the set-up cost is small so that  $\pi^N > \pi^O$ , the preemptive incentive is the same for the two firms, but the incumbent's stand-alone incentive is smaller than the entrant's and hence the incumbent invest less for the innovation than the entrant. When innovation is drastic but the set-up cost is large so that  $\pi^N < \pi^O$ , the incumbent has greater preemptive incentive and lower stand-alone incentive than the entrant. Therefore, it is not clear which firm invests more for the innovation. Reinganum (1983) analyzes the case in which  $\pi^N > \pi^O$  and concludes that for a drastic innovation, the incumbent invests less in innovation than the potential entrant. We will show that Reinganum's result can be overturned if the set-up cost is large so that  $\pi^N < \pi^O$ .

The incumbent does not necessarily invest more for the innovation than the

entrant when  $\pi^N$  is very small because the incumbent's preemptive incentive becomes less important for its decision as the entrant's R&D investment gets smaller. However, when  $\pi^O$  is large, we can show that the incumbent invests more for the innovation than the entrant.

From now on, we assume that  $\pi^N < \pi^O$ . To analyze the incentives of the incumbent and the entrant for drastic but unattractive innovation, we use the stochastic racing model developed in Reinganum (1983) which is based on Loury (1979) and Lee and Wilde (1980). Let  $x_I$  and  $x_E$  denote the R&D investment rates of the incumbent and the entrant, respectively.<sup>1</sup> Random success dates of the incumbent and the entrant for the innovation denoted by  $\tau_I(x_I)$  and  $\tau_E(x_E)$  respectively, are assumed to have probability distributions such that  $\Pr\{\tau_I(x_I) \leq t\} = 1 - e^{-h(x_I)t}$  and  $\Pr\{\tau_E(x_E) \leq t\} = 1 - e^{-h(x_E)t}$ . We assume that  $h(\cdot)$  is twice differentiable with  $h(0) = 0$ ,  $h'(x) > 0$ ,  $\lim_{x \rightarrow 0} h'(x) = \infty$ ,  $\lim_{x \rightarrow \infty} h'(x) = 0$  and  $h''(x) < 0$  for all  $x \in [0, \infty)$ . The race ends when a firm succeeds in the innovation.

For any pair of R&D investment rates  $(x_I, x_E)$ , we denote the present values of the expected profit over time to the incumbent and the entrant by  $V_I(x_I, x_E)$  and  $V_E(x_I, x_E)$ , respectively. The entrant has not yet succeeded and the incumbent succeeds at  $t$  with probability density  $h(x_I) e^{-(h(x_I)+h(x_E))t}$ , and the incumbent receives capitalized profit  $\pi^O/r$  at  $t$  in this case.<sup>2</sup> No firm has succeeded by  $t$  with probability  $e^{-(h(x_I)+h(x_E))t}$  and the incumbent receives flow profit  $\pi^O$  and pays flow cost  $x_I$  in this case. Thus,

$$\begin{aligned} V_I(x_I, x_E) &= \int_0^\infty e^{-rt} e^{-(h(x_I)+h(x_E))t} \left( h(x_I) \frac{\pi^O}{r} + \pi^O - x_I \right) dt \quad (1) \\ &= \frac{h(x_I) \frac{\pi^O}{r} + \pi^O - x_I}{r + h(x_I) + h(x_E)}. \end{aligned}$$

Analogously,

$$\begin{aligned} V_E(x_I, x_E) &= \int_0^\infty e^{-rt} e^{-(h(x_I)+h(x_E))t} \left( h(x_E) \frac{\pi^N}{r} - x_E \right) dt \quad (2) \\ &= \frac{h(x_E) \frac{\pi^N}{r} - x_E}{r + h(x_I) + h(x_E)}. \end{aligned}$$

<sup>1</sup>In the stochastic racing model, firms are assumed to choose stationary R&D investment rates. This assumption seems acceptable because the stochastic racing model is a stationary model.

<sup>2</sup>When the set-up cost for the new technology is low so that  $\pi^N > \pi^O$ , the incumbent receives  $\pi^N/r$  as in Reinganum (1983).

The patent race between the incumbent and the entrant is a game between the two firms whose strategies are  $x_I$  and  $x_E$  and whose payoffs are  $V_I(x_I, x_E)$  and  $V_E(x_I, x_E)$ , respectively. In our stationary patent race model, the game between the two firms is technically a static game and hence strategic interaction between the firms during the race cannot be reflected as in the non-stationary patent race model of Fudenberg *et al.* (1983).

Let  $\phi_I(x_E)$  and  $\phi_E(x_I)$  denote best response functions of the incumbent and the entrant, respectively. That is,  $V_I(\phi_I(x_E), x_E) \geq V_I(x_I, x_E)$  and  $V_E(x_I, \phi_E(x_I)) \geq V_E(x_I, x_E)$  for all  $x_I$  and  $x_E$ . Then, a strategy profile  $(x_I^*, x_E^*)$  is a Nash equilibrium if  $x_I^* = \phi_I(x_E^*)$  and  $x_E^* = \phi_E(x_I^*)$ . Existence and differentiability of the best response functions and existence of Nash equilibrium, as well as the continuity of  $\phi_I(x_E)$ ,  $\phi_E(x_I)$ ,  $x_I^*$  and  $x_E^*$  in the parameter  $\pi^O$  are proved in Proposition 1 of Reinganum (1983).

The first-order conditions which implicitly define  $\phi_I(x_E)$  for all  $x_E > 0$  and  $\phi_E(x_I)$  for all  $x_I \geq 0$  are

$$\begin{aligned} \frac{\partial V_I(\phi_I, x_E)}{\partial x_I} &\propto (r + h(\phi_I) + h(x_E)) \left( h'(\phi_I) \frac{\pi^O}{r} - 1 \right) \\ &- \left( h(\phi_I) \frac{\pi^O}{r} + \pi^O - \phi_I \right) h'(\phi_I) = 0, \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial V_E(x_I, \phi_E)}{\partial x_E} &\propto (r + h(x_I) + h(\phi_E)) \left( h'(\phi_E) \frac{\pi^N}{r} - 1 \right) \\ &- \left( h(\phi_E) \frac{\pi^N}{r} - \phi_E \right) h'(\phi_E) = 0.^3 \end{aligned} \quad (4)$$

Now, Lemma 1 characterizes best response functions of the two firms and the Proposition 1 gives our main result.

**Lemma 1.** (i)  $\phi_I(0) = 0$ . For all  $x_E \geq 0$ ,  $\phi_I'(x_E) > 0$ ,  $\frac{\partial \phi_I}{\partial \pi^N} = 0$ . For all  $x_E > 0$ ,  $\frac{\partial \phi_I}{\partial \pi^O} > 0$ .

(ii)  $\phi_E(0) > 0$ . For all  $x_I \geq 0$ ,  $\phi_E'(x_I) \geq 0$ ,  $\frac{\partial \phi_E}{\partial \pi^N} > 0$ ,  $\frac{\partial \phi_E}{\partial \pi^O} = 0$ .

(iii) For all  $x > 0$ ,  $\lim_{\pi^O \rightarrow \pi^N} \phi_I(x) < \phi_E(x) < \lim_{\pi^O \rightarrow \infty} \phi_I(x) = \infty$ .

*Proof.* See Appendix A. □

**Proposition 1.** There exist  $\bar{\pi}^O$  such that  $\bar{\pi}^O > \pi^N$  and  $x_I^* > x_E^*$  if  $\pi^O > \bar{\pi}^O$ .

*Proof.* See Appendix B. □

<sup>3</sup>The first order condition (3) cannot define  $\phi_I(0)$  because  $\partial V_I(x_I, 0)/\partial x_I < 0$  for all  $x_I \geq 0$ .

When  $\pi^N = \pi^O$ , the preemptive incentive is  $\pi^N$  for the incumbent and the entrant but the stand-alone incentive is zero for the incumbent while it is  $\pi^N$  for the entrant. Thus, the entrant has greater total incentive than the incumbent. If  $\pi^N$  decreases while  $\pi^O$  keeps constant, the incumbent's incentives do not change while the entrant's incentives decrease. If  $\pi^O$  increases while  $\pi^N$  remains constant, the entrant's incentives and the incumbent's stand-alone incentive do not change while the incumbent's preemptive incentive increases. Because the incumbent's preemptive incentive increases infinitely when  $\pi^O$  increases infinitely, the incumbent has greater total incentive than the entrant if the market is highly profitable and the new technology incurs large set-up cost. Both firms' equilibrium R&D investments increase in  $\pi^O$ , but the entrant's equilibrium R&D investment is bounded while the incumbent's equilibrium R&D investment increases infinitely as  $\pi^O$  increase infinitely. Therefore, the incumbent invests more in R&D than the entrant when  $\pi^O$  is much larger than  $\pi^N$ .

### 3. CONCLUSION

We have analyzed a situation in which a monopolist incumbent and a potential entrant compete for the patent of a new technology, the owner of which can monopolize the market. If the set-up cost for the new technology is so large that the incumbent would let the patent sleep, the incumbent's preemptive incentive is greater while its stand-alone incentive is smaller than the entrant's. We found that for a drastic innovation, the incumbent invests more in R&D than the entrant if the market is highly profitable under the current technology and the new technology incurs large set-up cost.

### APPENDIX A

*Proof of Lemma 1.* (i)  $\phi_I(0) = 0$  because  $V_I(x_I, 0) = \frac{\pi^O}{r} - \frac{x_I}{r+h(x_I)}$  which is maximized when  $x_I = 0$ .

From the first order condition  $\frac{\partial V_I(\phi_I, x_E)}{\partial x_I} = 0$ ,  $\frac{\partial \phi_I}{\partial x_I} = -\frac{\partial^2 V_I(\phi_I, x_E)/\partial x_E \partial x_I}{\partial^2 V_I(\phi_I, x_E)/\partial x_I^2}$  by the implicit function theorem.  $\partial^2 V_I(\phi_I, x_E)/\partial x_I^2 < 0$  all  $x_E \geq 0$  by the second order condition. From (3),  $\partial^2 V_I(\phi_I, x_E)/\partial x_E \partial x_I \propto h'(x_E) \left( h'(\phi_I) \frac{\pi^O}{r} - 1 \right) = h'(x_E) h'(\phi_I) V_I(\phi_I, x_E) > 0$  for all  $x_E \geq 0$ , where the equality holds because  $V_I(\phi_I, x_E) = \frac{h'(\phi_I) \frac{\pi^O}{r} - 1}{h'(\phi_I)}$  by (1) and (3). Therefore,  $\phi_I'(x_E) > 0$  for all  $x_E \geq 0$ .

From (3), by the implicit function theorem,  $\frac{\partial \phi_I}{\partial \pi^N} = 0$  for all  $x_E \geq 0$  and  $\frac{\partial \phi_I}{\partial \pi^O} =$

$$-\frac{h(x_E)h'(\phi_I)}{(h(x_E)\pi^O+r\phi_I)h''(\phi_I)} > 0 \text{ for all } x_E > 0.$$

$$(ii) \phi_E(0) > 0 \text{ because } \frac{\partial V^E(0,0)}{\partial x_E} \propto r \left( h'(0) \frac{\pi^N}{r} - 1 \right) > 0.$$

By the same reason why  $\phi_I'(x_E) > 0$  for all  $x_E \geq 0$  except that  $V_I(\phi_I, x_E) > 0$  all  $x_E \geq 0$  but  $V_E(x_I, \phi_E) \geq 0$  all  $x_I \geq 0$ ,  $\phi_E'(x_I) \geq 0$  for all  $x_I \geq 0$ . From (4), by the implicit function theorem,  $\frac{\partial \phi_E}{\partial \pi^N} = -\frac{(r+h(x_I))h'(\phi_E)}{r\pi^N+h(x_I)\pi^N+r\phi_E}h''(\phi_E) > 0$  and  $\frac{\partial \phi_E}{\partial \pi^O} = 0$  for all  $x_I \geq 0$ .

(iii) For all  $x$ ,  $\lim_{\pi^O \rightarrow \pi^N} \phi_I(x) < \phi_E(x)$  because continuity of  $\phi_I(x_E)$  in  $\pi^O$  and the fact that  $\phi_I(x) < \phi_E(x)$  when  $\pi^O = \pi^N$  are proved in Proposition 1 in Reinganum (1983). Moreover,  $\lim_{\pi^O \rightarrow \infty} \phi_I(x_E) = \infty$  for all  $x_E > 0$  because (3) shows that  $\lim_{\pi^O \rightarrow \infty} h'(\phi_I) = 0$  if  $x_E > 0$ .  $\square$

### APPENDIX B

*Proof of Proposition 1.* Because  $\lim_{\pi^O \rightarrow \infty} \phi_I(x_E) = \infty$  for all  $x_E > 0$  and  $\phi_E(x_I) > 0$  for all  $x_I \geq 0$ ,  $\lim_{\pi^O \rightarrow \infty} x_I^* = \infty$ . Let  $H$  denote an upper bound of  $h(x)$ , which exists due to the assumption that  $\lim_{x \rightarrow \infty} h'(x) = 0$ . If  $x_E \geq H\pi^N/r$ , then  $V_E(x_I, x_E) < 0$  for all  $x_I \geq 0$ . Therefore,  $x_E^* < H\pi^N/r$  for all  $\pi^O$ . Finally, continuity of  $x_I^*$  in  $\pi^O$  proves the proposition.  $\square$

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