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Speculation under Bounded Rationality

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Abstract I construct a model of a speculation for an economy with boundedly rational agents. Speculation is defined as a trade for gains from price change while investment is defined as a trade for consumption. In a model without boundedly rational agents, the equilibrium price exactly reflects the underlying value of the good traded whether the agents trade for price gain or for their own consumption. However a model of speculation with boundedly rational agents produces fundamentally different equilibrium from that of investment. The price rises higher than the fundamental value of the asset. Moreover the price may rise even higher than the level which equals the expectation of boundedly rational agents since the extra premium can be justified by the higher rate of price rise. In particular the paper shows that rational agents are responsible for the amplification of the price bubble since the price rises higher than the expectation of boundedly rational agents alone. Hence price bubble in the asset market occurs due to the cooperation of rational agents once there exists uncertainty as to the existence of boundedly rational agents.

Keywords speculation, investment, boundedly rational agents.

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1. INTRODUCTION

The recent global financial crises which have originated from the U.S. subprime mortgage market had a devastating effect on the world economy. House prices in many countries have risen to a level which many have argued was much higher than a reasonable level. Even during the period the price was rising, many people have warned that the price went much higher than it should. Nevertheless buyers in the housing market kept buying houses at already high prices. The precipitous fall of house prices subsequently brought about a market-wide collapse.

It is not true that the whole market was completely ignorant of the possiblity that the house price was much higher than it should be. Rather the market participants cooperated together to raise the price to such a high level although many recognized the possibility of a bubble in the house price. This paper attempts to explain how such an outcome may occur in an economy which has boundedly rational agents as well as rational agents.

Standard economic theory predicts that price of assets like houses should reflect their fundamental value which depends on how much utility users obtain from their consumption. In a market for non-durable commodities, it is easy to see why the price should be equal to the value of the commodities when consumed; otherwise the buyer would purchase them paying more than they can get.

In the case of durable goods like houses which can be resold in the future periods, the current price may go higher as long as the future resale price is also higher. Buyers of durable goods should be willing to pay the price which is equal to the sum of the future consumption values since it is ultimately the consumption which gives the good its value. Hence the price of assets like houses equals the discounted sum of consumption values over their life.

However when there may be buyers who do not subscribe to the standard economic theory, the evolution of asset prices changes fundamentally. Some buyers may simply project the past performance of the prices to the future and expect that the price rise may continue indefinitely. When this possibility of boundedly rational market participants exists, other market participants start to behave entirely different from the prediction of the standard economic theory even if they understand the standard theory and they, in fact, subscribe to it.

Rational agents, when they purchase assets, are willing to pay the price which can be justified by the value from its consumption or the resale value. In fact rational agents rationally take account of the fact that they may have the opportunity to trade with boundedly rational agents who are willing to pay a price higher than the standard economic theory predicts. It follows that a market full of rational agents may produce the price path of a bubble from the anticipation that boundedly rational agents may turn up in the future.

The paper constructs a model which has boundedly rationl agents as well as rational agents. They trade a unit of house over two periods, after which the consumption of the house takes place.

The main result of the analysis of the market equilibrium is under what condition the price path follows that of a bubble. We define the bubble as a market outcome where the price rises higher than justified by the fundamental. When boundedly rational agents may trade over the evolution of the market, the price should reflect the possibility of trading against them. A bubble in the model means a price evolution higher than the level justified by the simple possibility of trading against boundedly rational agents.

We find that the extent of bubble depends on the chance of trading with boundedly rational agents. However there is a secondary effect which amplifies the price adjustment. When the future price is expected to be higher by a certain proportion, the current price may go even higher to be consistent with the price change. The size of bubble depends on the likelihood of trading against boundedly rational agents.

The standard economic theory has difficulties in explaining why boundedly rational agents exist in the first place. In the real world, bounded rationality can be explained by factors like the lack of relevant information, the limitation of computational capacity, bounded memory, and myopia. It is beyond the scope of the current paper to construct the bahavior of boundedly rational agents using more fundamental factors.

Once we drop the assumption of common knowledge of the rationality of agents, it is possible to explain how even rational agents may act like boundedly rational agents. While the presense of boundedly rational agents is an assumpition imposed exogenously, we can construct a story about a situation where common knowledge of rationality fails. Indeed even a small possibility of boundedly rational behavior induces other rational agents to act similar to boundedly rational agents.

Section 2 reviews existing research which attempts to explain bubbles in a financial markets. Section 3 constructs a model of house market in which trades take place among various types of agents. Section 4 shows that bubble does not exist even if agents recognize the future opportunities for resale of assets, unless there exist boundedly rational agents who will buy the asset at a high price. Section 5 provides the main analysis that the presence of boundedly rational agents in the future changes the equilibrium behavior of rational agents. Section

6 explains how investors may behave as boundedly rational agents and Section 7 concludes.

2. LITERATURE

There is a huge set of research papers in the general area of investments and speculation in economics and finance. Since it is impossible to cover them all here, we review mainly the approaches which employ the existence of bounded rationality.

Kindleberger and Aliber (2005) provide a comprehensive explanation of the asset market movements over price cycles. Their description includes explanations of how agents form expectations about the future evolution of the asset price and how the market works on the expectations to generate price paths which depart from the standard prediction of economic theory.

Frankel and Froot (1990) provide an empirical account of the role of chartists and fundamentalists in the foreign exchange market, who can be identified as boundedly rational agents and rational agents in the present paper.

Tirole's paper (1982) is a good starting point for a theoretic analysis, since he shows that full rationality rules out speculative gain in the financial market. While he did not construct a model of speculative gain, his result provides a clear direction for producing bubble, which is the assumption of bounded rationality.

Milgrom and Stokey(1982) produces a similar analysis in that under common knowledge of rationality, agents may not even trade while the price adjusts exactly for any attempt to gain from information which only a subset of market participants have. The main intuition behind the result is that any offer to trade is interpreted as an attempt to exploit the informational advantage.

Tirole (1982) and Milgrom and Stokey (1982) can be understood as providing the reference model under full rationality while the current model starts with the failure of rationality to explain price bubbles in the asset market.

De Long *et al.* (1990) is an early attempt to explicitly introduce bounded rationality to construct a bubble. In their analysis positive-feedback investment strategy plays the same role as boundedly rational investment behavior in the curent paper. They show that the size of speculation depends on the uncertainty in the market. Palomino (1996) and Gaunersdorfer (2000) follow De Long *et al.* (1990) and produce bubbles in various settings from heterogeneity among investors.

Hommes (2006) reviews models of heterogeneous agents in economics and finance, which covers not only the heterogeneity approach but also the bounded rationality approach to explain trading bahavior mainly from a computational perspective.

Chiarella, He, and Zheng (2011) develop a model with chartists and fundamentalists to privide explanation of the asset market movements which cannot be accounted for by the models with heterogeneous agents. Similarly Chiarella, Iori, and Perello (2009) focus on the trading mechanisms employed in the exchange market to explain how large price changes occur in a model with chartists and fundamentalists.

While these papers suggest various ways through which price bubbles may occur in the asset market, the present paper attempts to focus on the role of rational agents to amplify the uncertainty due to the possibility of boundedly rational agents.

Dufy (2001) and Hommes *et al.* (2008) provide experimental research outcomes in the laboratory environments. While these papers do not explain the occurrence of bubbles in the financial markets, they suggest the mechanism through which boundedly rational agents work.

The short list of papers cited above constitutes the set of approaches which try to reconcile the economic theory with the market phenomena unaccounted by the standard models.

3. MODEL

We construct a model of house market with 3 types of agents: first, residential buyers, who buy the house for residential use, second, rational investors, who buy the house purely for resale, and third, boundedly rational investors, who buy the house at a fixed rate of price rise. The boundedly rational investors expect the price of the house will rise by β in the next period regardless of its fundamental value and thus pay the high price currently: if the current price of the house is p, then they expect its price will be $(1+\beta)p$ in the next period.[1]

The agents of the first type are those who have residential need and the house market is occupied by this group in the standard economic theory, where trades take place among those who consume it. The agents of the second type are rational investors who make investments for the purpose of resale. They have no residential demand for the house (probably because they already own a house in

[1]The determination of the size of the factor β can be explained in a few ways including the extrapolation of the past price movements, which is the behavior chartist investors subscribe to. In particular boundedly rational agents definitely suffer losses from the trade, whose explanation is beyond the scope of the current paper. We provide a short explanation of the formation of the expectation in Section 6.

which they live) but nevertheless participate in the house market for gains from price change of the house. The agents of the third type are boundedly rational agents who buy the house at a premium regardless of the intrinsic value or the expected future price at which they can resell it.

The agents of the first two types exist in fixed proportions throughout the model:[2] the proportion of the first type is ρ, and the proportion of the second type is $(1-\rho)$. The third type may exist only in the second period with probability ε and there may be no third type agents with probability $(1-\varepsilon)$. We assume that there are more than two agents of the third type in the market if they are present.[3] When agents trade in the house market, they expect to meet each type with the probabilities equal to the proportions. Moreover they expect that with probability $(1-\varepsilon)$ they will trade in a market with only residential buyers and rational agents and with probability ε they will trade in a market with boundedly rational agents as well as the other two types .

There is 1 unit of house available, which is traded over 2 periods in the house market and consumed in the third period. Initially the house is supplied from outside the market, which can be interpreted as being sold by the developer. In the first period there are only residential buyers and rational investors who compete to buy the house put for sale. In the seond period the agent who has purchased the house in the first period may put it for sale. With probability ε all 3 types of agents compete to buy it while with probability $(1-\varepsilon)$ only the first two types compete. In the third period the consumption of the house takes place in the sense that the owner of the house lives in the house getting utility from it. Only the residential buyers get a positive utility, $\hat{u}$, while the rational investors and boundedly rational investors get 0 utility. Agents do not discount the future.

In the first two periods when trade takes place over the ownership of the house, the buyers submit sealed-bids and the winner whose bid is the highest wins the house and pays the second highest bid, that is, the house is sold using the second-price-sealed-bid auction. In the third period, there is no further trade and the owner who has the house in the period has to consume it.[4]

[2]Each type exists in continuum so that there are infinitely many agents of each type whose proportions are given as in the text.

[3]The assumption that there are more than two agents of the third type is made to utilize the market mechanism of the second-price-sealed-bid auction for the determination of the equilibrium price. In particular when there are more than two of the third type agents in the market, the bid of the third type is the price at which the trade takes place. This artificial assumption is made to simplify the determination of the price in the model.

[4]We can allow further trading to the residential buyers who will make consumption in the third period without major change in the analytic results. While trading and consumption of houses take place simultaneously in the real world, allowing both activities in the model prevents the

The focus of the analysis in the model is on the equilibrium price of the house, particularly in the first period when the residential buyers and rational investors compete expecting a capital gain due to the presence of the boundedly rational investors in the second period.[5] In the analysis we denote the price of the house in period t by p_t.

We assume that the structure of the model including the proportions of agent types and their preference is common knowledge. The equilibrium employed in the analysis is the subgame perfect equilibrium in which agents have a correct prediction of future equilibrium play at each decision time.

4. IMPOSSIBILITY OF SPECULATION IN PERFECTLY RATIONAL ECONOMY

In this section we establish the standard result in the theory of investment where, in the absence of boundedly rational investors, the price of assets must be equal to its fundamental value. We assume that there is no boundedly rational investors who expect the price will rise by β: $\varepsilon = 0$.

Proposition 1. *Suppose that $\varepsilon = 0$. Then, the equilibrium price sequence is given by: $p_t^* = \hat{u}$, for $t = 1, 2$.*

Proof. When $\varepsilon = 0$, there is no boundedly rational investors and this information is known to other market participants. Hence they cannot expect to sell the house at a higher price than its fundamental value. Since we solve for the subgame perfect equilibrium, we solve the game backward from the last period.

In period 3, only the residential buyers get positive utility from the ownership of the house, which equals $\hat{u}$. Hence the price of the house in period 2 cannot be higher than $\hat{u}$. But the residential buyers get a positive surplus if the house purchased at any price strictly less than $\hat{u}$. The rational investors will submit a bid which equals 0 since they get 0 utility from the house in the third period. However, since there is a continuum of residential buyers who in the second-price-sealed-bid auction will submit a bid which equals $\hat{u}$, the price of the house in the second period is given by $p_2^* = \hat{u}$.

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demonstration of the effect of bounded rationality in a simple model. Hence we make simplifying assumption that there is no further trading in the third period.

[5]Since we are interested explaining the role of rational agents in amplifying the uncertainty due to the boundedly rational agents, allowing trading by boundedly rational agents in the first period removes the window to separate the effects. For this reason we rule out the boundedly rational agents in the first period trading.

In the first period, all market participants expect correctly that the price of the house in the second period is $\hat{u}$. In this period rational investors will submit a positive bid since they know that they can with certainty sell the house back in the second period at $p_2^* = \hat{u}$. Moreover they will get a positive surplus if the house can be purchased at any price strictly less than $\hat{u}$. Hence they will submit a bid which is equal to $\hat{u}$. The residential buyers make similar calculation except they can hold the house without any loss until the third period even if the house is not sold in the second period. Hence the equilbrium price in the first period is $p_1^* = \hat{u}$.

It follows that the equilibrium price of the house is given by $p_t^* = \hat{u}$, for $t = 1, 2$. The proof is complete. $\square$

The intuition behind the proposition is easy to see. Since only the residential buyers get utility $\hat{u}$ in the third period, the fundamental value of the house is also $\hat{u}$. Although rational investors are willing to submit a positive bid in the first period with a plan to sell it back in the second period, the maximum bid they submit cannot be higher than $\hat{u}$ since that is the equilibrium price of the house in the second period. Since the house is sold through the second-price-sealed-bid auction, the optimal bid cannot be less than $\hat{u}$. It follows that the equilibrium house is exactly the same as the fundamental value of the house.

The proposition shows that the future opportunity to trade alone does not induce the agents to bid higher than the fundamental value. Since all market participants know correctly that the price of the house in the last trade round is equal to the fundamental value, they are not willing to submit a bid higher than it in the period before. This result continues to hold even if we allow more trading rounds as long as the absence of boundedly rational investors in the market is common knowledge. Hence speculation does not occur in perfectly rational market regardless of whether there are agents who trade for a gain from trades.

5. SPECULATION IN BOUNDEDLY RATIONAL ECONOMY

Before the global financial crises, many people including economists like Shiller and Rubini warned that the house price has risen above the level justified by the fundamental value. Nevertheless market participants appeared to have ignored the warning. The statement made by the CEO of the Citigroup, C. Prince summarizes the conflicting situation nicely. Prince wrote "When the music stops, in terms of liquidity, things will get complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing." right before the breakout of global financial crises. (Financial Times, July 9, 2007) People like

Prince were aware that a bubble is developing in the market. However there was no alternative to participating in the bubble; if then do not participate, they will lose the opportunity to make money in any case. Our analysis of the model produces a price path consistent with the statement made by Prince.

In the present section, we assume that there is a positive probability that boundedly rational agents may participate in the future trading round: $\varepsilon > 0$. When there is a positive probability that boundedly rational agents participate in the future trading, rational agents and residential buyers take it into account and raise their bids in the previous round accordingly. We start with a lemma which shows that the bids submitted by rational agents and residential buyers are identical in the first period although they have different consumption valuations of the house in the third period. However their bids are different in the second period, after which the consumption of the house takes place.

Lemma 1. *Suppose that $\varepsilon > 0$. Let b_t^r and b_t^f be the the bids submitted in period t by the rational agents and the residential buyers, respectively. The bids submitted in the first period by the rational agents and the residential buyers are identical, $b_1^r = b_1^f$, and they are different in the second period, $b_2^r < b_2^f$.*

Proof. It is well known that in the second-price-sealed-bid auction, the bidding strategy where the bid equals the valuation of the bidder is a weakly dominant strategy.

In the second period, the rational agent has 0 value from the consumption while the residential buyer has the value equal to $\hat{u}$ since after the second period, the owner has to consume the house; the rational agent has no utility from the consumption of the house while the residential buyer gets the utility $\hat{u}$ from the consumption. It follows that in the second period, the rational agent is willing to bid 0 for the house and the residential buyer is willing to bid up to $\hat{u}$. Since $\hat{u} > 0$, it follows that $b_2^r < b_2^f$.

In the first period market participants take account of the price they can get from the trade in the second period. If they purchase the house in the first period, they can sell it back in the second period if they do not want to own it until the third period when the consumption takes place. Since they can expect to trade against the boundedly rational agents with positive probability, they are willing to submit the bid which equals the expected price they can get from the sales. In case there is no boundedly rational agents, they can sell it to residential buyers if they do not want to keep it for their own consumption. Since there are continuum of the residential buyers in the second period market, even the rational agents who do not get positive utility from the consumption can expect to recoup the equilbrium price in the second period. Hence the expectation of the second period price is identical for the rational agents and the residential buyers. Thus the agents of the both types submit the same bid in the first period.

The proof is complete. □

Next we characterize the subgame perfect equilibrium of the house market. The equilibrium consists of price sequence over two periods during which trades take place. In the second period, there are two possible cases depending on whether there are boundedly rational agents. In the first period market participants take account of the two possible cases in determining their optimal bidding strategies.

Proposition 2. *Suppose that* $\frac{1}{1+\beta} > \varepsilon > 0$. *The subgame perfect equilibrium of the market consists of the following price sequence. The house price in the first period is*

$$p_1 = \frac{(1-\varepsilon)\hat{u}}{1-\varepsilon(1+\beta)}. \tag{1}$$

The house price in the second period is

$$p_2^B = \frac{(1+\beta)(1-\varepsilon)\hat{u}}{1-\varepsilon(1+\beta)} \tag{2}$$

if there are boundedly rational agents in the market and

$$p_2^R = \hat{u} \tag{3}$$

if there are no boundedly rational agents in the market.

Proof. We solve for the subgame perfect equilibrium using backward induction for this model. It should be remembered the two periods are interrelated since the boundedly rational agents are willing to submit the bid which equals the first period price times a certain factor, $(1+\beta)$: boundedly rational agents perdict that the price of the house will rise by the factor without any logical justification and thus submit the bid consistent with the expectation.

If there are no boundedly rational agents in the second period, the house price equals $\hat{u}$ since there are only rational agents and residential buyers: $p_2^R = \hat{u}$. The rational agents submit the bid of 0 while the residential buyers submit the bid of $\hat{u}$. It follows that one of the residential buyers purchase the house at the price of $\hat{u}$. (or the current owner keeps the house if it was one of the residential buyers, which is equivalent to a trade at the price of $\hat{u}$.)

On the other hand, if there are boundedly rational agents in the second period, the house price equals $(1+\beta)p_1$ where p_1 is the equilibrium price of the house in the first period. As before, the rational agents submit the bid of 0 and the residential buyers submit the bid of $\hat{u}$. But in this case boundedly rational agents submit the bid which equals $(1+\beta)p_1$ since they expect that the house price will rise by the factor $(1+\beta)$. Since there are more than two boundedly rational agents if they exist, the winner of the second-price-sealed-bid auction wins the house at the price $(1+\beta)p_1$.

In the first period, market participants correctly predict the future evolution of the house price and submit their bids consistent with the expectation. The bids of the rational agents and the residential buyers are identical from Lemma 1 and equal the expected value of the second period equilibrium price.

The second period price equals $(1+\beta)p_1$ with probability ε and $\hat{u}$ with probability $(1-\varepsilon)$. It follows that the first period price should satisfy the following equation.

$$p_1 = \varepsilon(1+\beta)p_1 + (1-\varepsilon)\hat{u}. \tag{4}$$

Solving for p_1 in the equation above, we obtain

$$p_1 = \frac{(1-\varepsilon)\hat{u}}{1-\varepsilon(1+\beta)}. \tag{5}$$

We need the assumtion that $\frac{1}{1+\beta} > \varepsilon$ to guarantee that p_1 has a positive value, since otherwise the solution has a negative value.[6]

It remains to compute the second period price in the case when there are boundedly rational agents from the first period price. Since it equals the first period price times $(1+\beta)$, the second period price when there are boundedly rational agents is

$$p_2^B = \frac{(1+\beta)(1-\varepsilon)\hat{u}}{1-\varepsilon(1+\beta)}. \tag{6}$$

The proof is complete. □

The proposition shows how the equilibrium price sequence in the house market changes when there is a positive probability of trading against boundedly rational agents in the future period. When there is a positive probability of trading against boundedly rational agents who would be willing to pay a price which is not consistent with the fundamental value, the equilibrium price changes more than the rate of price change expected by the boundedly rational agents, i.e., β.

[6]The model does not have a valid solution for a big noise which satisfies $\varepsilon \geq \frac{1}{1+\beta}$. While we cannot provide an intuitive meaning of the assumption, the requirement can be interpreted as a condition under which rational agents can calculate a price consistent with the equilibrium condition of the asset market.

Rather the possibility of the gains from speculation may amplify the initial deviation due to the presence of boundedly rational agents as will be shown below.

The equilibrium price depends only on the probability of trading against boundedly rational agents, ε. In particular the number of boundedly rational agents when they exist does not affect the equilibrium price as long as more than two of them exist in the market. This unusual feature follows from the modelling approach using the second-price-sealed-bid auction. Hence we should not extend the result to predict that the number of boundedly rational agents does not matter in the real speculative market. In general we expect that the number of boundedly rational agents induce a greater degree of competition among bidders, which will make the price close to the price derived in the current model with the second-price-sealed-bid auction since the second-price-sealed-bid auction provides the most competitive market environment.

The equilibrium price is independent of the proportion of rational agents against that of residential buyers. In fact they behave identically for the analysis of the model except that only residential buyers will be willing to submit a positive bid in the second period. Because we assume that there is a continuum of residential buyers and 1 unit of house, the result does not depend on the number of residential buyers.

Rational agents and the residential buyers are willing to pay more than the fundamental value since there is a positive probability of gains from speculation. In fact they correctly expect the probability of loss from the speculation as well as gain. However the first period price cannot be lower than the fundamental value. Indeed it is easy to confirm that

$$\frac{(1-\varepsilon)}{1-\varepsilon(1+\beta)} > 1 \tag{7}$$

when $\beta > 0$ and $\frac{1}{1+\beta} > \varepsilon > 0$.

The following proposition shows that when the probability of trading against boundedly rational agents is big enough, the first period price may rise even more than the factor by which boundely rational agents expect the house prise to rise.

Proposition 3. *If* $\frac{1}{1+\beta} > \varepsilon \geq \frac{1}{2+\beta}$, *then* $p_1 \geq (1+\beta)\hat{u}$.

Proof. From Proposition 2, the equilibrium house price in the first period equals

$$p_1 = \frac{(1-\varepsilon)\hat{u}}{1-\varepsilon(1+\beta)}. \tag{8}$$

Hence $p_1 \geq (1+\beta)\hat{u}$ if

$$\frac{(1-\varepsilon)}{1-\varepsilon(1+\beta)} \geq (1+\beta). \tag{9}$$

Rearranging the terms yields that $p_1 \geq (1+\beta)\hat{u}$ if $\varepsilon \geq \frac{1}{2+\beta}$. □

Notice that the equilibrium price in the market without boundedly rational agents is $\hat{u}$, which is equal to the fundamental value of the house. One would expect that when boundedly rational agents participate in the market, the house price may rise at the maximum to the extent that boundedly rational agents expect to rise. Moreover there is uncertainty as to whether boundedly rational agent may be present; when they do not exist in the future trading round, the rational agents and the residential buyers may have to suffer a loss, which should be reflected in the equilibrium price.

However the equilibrium in a market with boundedly rational agents yields a price sequence which may go even higher than the factor by which boundedly rational agents expect the price to rise. In particular it is surprising that the equilibrium price in the first period may rise more than the factor, β, which boundedly rational agents expect the price to rise. To have this outcome, we need the probability of trading against boundedly rational agents big enough. Once there is enough chance that boundedly rational agents may pay a high price, rational agents and residential buyers coordinate to raise the price even higher. To understand this outcome, we need to consider the mechanism which determines the equilirbium.

When there is a positive probability of trading against boundedly rational agents, rational agents and residential buyers are willing to pay as much as the expected extra gain due to the uncertainty of price, which is the product of probability of trading against boundedly rational agents and the size of price change. However once this uncertainty is taken into account, additional price change can be justified since this additional price rise is also amplified by the same factor, β. To have this outcome, the initial uncertainty of trading against boundedly rational agents, ε, should be bigger than some function of the factor, β. The bigger the factor, β, is, the smaller the initial uncertainty need to be in order to support a higher price.

6. WHO ARE BOUNDEDLY RATIONAL AGENTS?

The model introduces boundedly rational agents without explaining who they are and why they exist. Indeed the existence of boundedly rational agents cannot be explained endogenously in the model. However we can tell a story how even rational agents may act like boundedly rational in the asset market. The critical ingredient for the story is the failure of common knowledge of rationality of all agents.

In an early paper, Kreps *et al.* (1982) explained how the failure of common knowledge of rationality of all agents may generate cooperation in a finitely repeated prisoners' dilemma. The game of prisoners' dilemma has a unique equilibrium in dominant strategies, which is to confess about a crime agents committed even if they can achieve a better payoffs by not confessing. Moreover the same equilibrium remains a subgame perfect equilibrium for any finitely repeated game of the stage game of prisoners' dilemma. The cooperation can be attained only for an infinitely repeated environment.

Kreps *et al.* construct a model of a finitely repeated setting and assume, in addition, that one of the agents may be boundedly rational with a small probability. In contrast to a model of full rationality, the small probability of boundedly rational behavior creates a big room for cooperation in a finitely repeated prisoners' dilemma.

Initially the rational agent starts by testing the opponent's rationality and chooses a cooperative action. In response to this attempt, the opponent pretends he is indeed a boundedly rational agent although he is fully rational. The rational agent in turn responds by continuing with the cooperative action choice. The cooperation continues until there remains a small number of periods to gain from cooperation. The cooperation benefits both agents since they can enjoy the Pareto optimal payoff until one of them deviates to the stage game equilibrium action, which is to confess about the crime.

The current model may be supported by a similar story. Initially rational agents and residential buyers know that there is no possibility of gain from trading if the price is higher than the fundamental value unless some boundedly rational agents may pay the high price in the future. However once there is a small probability of trading against boundedly rational agents, they may pretend to be boundedly rational even if they behave totally different under full rationality. What matters is whether the price can be justified by the future gain. Moreover the future gain amplifies the initial uncertainty since an additional premium in the price can be also justified by future price change.

To have the speculative outcome, it is important that agents buy the house for resale and not solely for consumption. In a standard economic theory, prices of commodities are determined by the willingness to pay of buyers, which depends on the consumption values of the commodities. However agents buy commodities not just for their own consumption. In particular, assets such as houses generate utility from the consumption and at the same time may be resold to other agents in the future. The agents who buy the assets in the future may buy them for their own consumption as well as further resale in the future. While the consumption value of the commodities may be fixed, the equilibrium price also depends on the future price the owner can secure from the resale. The resale price is only realized in the future and thus it depends on the expectation of the agents. If they expect the price to rise above the fundamental value in the future, the agents are willing to pay a high price since they can secure surplus from the future resale.

Extending the model for the possibility of speculative gain does not work if everyone knows that everyone is rational. However once there is a positive probability of boundedly rational behavior in the future, rational agents respond to the change in the environment by paying higher price. The significance of our result in the previous section lies in that the first period price rises to a high level even if the market in the first period has no boundedly rational agents.

7. CONCLUSION

The global financial crises showed that the existing economic theory falls short of providing a proper explanation of the working mechanism of the financial markets, not to mention a comprehensive framenwork for a sound regulation. While the standard economic theory rules out the possibility of bubbles such as the one based on Ponzi scheme, the scandalous case of Madoff demonstrated that Ponzi scheme has been played in the most developed financial market of the U.S.

It is not true that economists have absolutely no idea of how to explain a bubble in the financial markets. Instead the standard set of assumptions employed in the economic models rules out the possibility of developing a reasonably rich story for the explanation. The critical component we need for the explanation is the failure of rationality. The assumption of rationality is very convenient in making the logical structure of economic models complete. The price to be paid for the logical completeness is the lack of relevance of the economic theory for the explanation of the many phenomena in the real world.

The current paper provides a simple example of how to construct a model of speculation once we drop the assumption of rationality. By allowing for the possibility of boundedly rational behavior in the financial market, we are able to construct an equilibrium in which the price of an asset goes well above the fundamental value of the asset.

In the financial markets, investors are indistinguishable from speculators. Agents participate in the market to gain surplus from the trades. The first type of gain is to buy the asset below the consumption value, which is determined by the preference of the market participants. There is a second type of gain, which comes purely from the price change. Since the price change takes place in the future after the trades, the expectation of the price change should reflect the uncertainty in the future. As long as the market participants expect the future price to be higher, it is totally rational to bid consistent with the expectation.

While the standard economic theory pins down the future expectation by the consumption value of the assets, those agents who buy them in early periods may anticipate that they can sell them to other agents before the eventual discovery of the consumption value. Once they take account of this possibility, the equilibrium price departs from the standard prediction with no possibility of bubble.

The model we employed in the current paper is extremely simple, yet we can produce a fairly large deviation of the equilibrium price from the one in the standard model with full rationality. It would be interesting to see how the prediction changes when we develop richer trading environments including more trading rounds.

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