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Household Utility Function in the Computable Overlapping Generations Model*

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Abstract We consider the specification of the household utility function in the overlapping generations model. We apply the two criteria for the selection of utility function: the zero long-run elasticity of labor supply and the hump-shaped consumption profile. We identify some appropriate utility functions that satisfy both criteria. We compare the two utility functions through the simulations of both the household behavior and the macroeconomic behavior in overlapping generations model with aging population. The Kimball and Shapiro (KS) utility function seems to be a promising one for the simulation of household behavior and overlapping generations model.

Keywords zero long-run elasticity of labor supply, hump-shaped consumption profile, intertemporal elasticity of substitution

JEL Classification E21, C68

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1. INTRODUCTION

The overlapping generations (OG) model is one of the main workhorses of modern economic analysis. It was originated by Samuelson (1958) and further developed by Diamond (1965). The computable overlapping generations model is developed by Auerbach and Kotlikoff (1987). It has been widely used in the analysis of public finance issues such as the efficiency of tax system and the sustainability of pension system.

In this paper, we examine the choice of household utility function in the overlapping generations model. Shin and Choi (2007) studied the utility function used by Auerbach and Kotlikoff and conducted simulations under various values of parameters such as the elasticity of intertemporal substitution and the elasticity of substitution between consumption and leisure. However, the utility function used by Auerbach and Kotlikoff is not compatible with the steady state unless the elasticity of substitution between consumption and leisure is one. We investigate alternative functional forms for utility function which satisfy two criteria. One is the hump-shapedness of consumption profile and the other is the zero long-run elasticity of substitution. We find two utility functions which satisfy both criteria: namely, the Auerbach and Kotlikoff (AK) utility function with the unit elasticity of substitution between consumption and leisure and the Kimball and Shapiro (KS) utility function. Then we identify the most promising utility function via simulation of both the household behavior and the overlapping generations model. It seems that the Kimball and Shapiro (KS) utility function is a promising one for the household model and overlapping generations model.

The paper consists of 5 sections. The section 2 explains the stylized facts regarding the consumption and labor supply. Section 3 examines various forms of household utility function. Section 4 presents simulation results on the economic effects of population aging using an overlapping generations model. Section 5 summarizes and concludes this paper.

2. STYLIZED FACTS WITH REGARD TO THE LIFE-CYCLE CONSUMPTION AND LABOR SUPPLY

The traditional life-cycle model predicts that the consumption increases at a constant rate during the lifetime. However, the empirical studies on lifecycle behavior show that both the consumption and income profiles along the age exhibit hump-shapedness. In addition, the two variables show some degree of comovement (Thurow, 1969; Ghez and Becker, 1975, chapter 2; De Nardi et al., 2001; Browning and Crossley, 2001). For this phenomenon, there are three explanations. One is to explain it by the existence of liquidity constraint (Thurow, 1969). Another is to explain it by the income uncertainty and precautionary savings (Nagatani, 1972; Carroll, 1997; Gourinchas and Parker, 2002). The other is to explain it by the hump-shapedness of labor productivity and the substitution between consumption and leisure (Heckman, 1974).

In Korea, Park and Jun (2009) conducted an empirical study on the behavior of household using the Korean Labor and Income Panel Study by Korea Labor Institute and the Household Survey by National Statistical Office. The study shows the hump-shaped consumption and income profile.

Another stylized fact related with labor supply is that the labor supply does not change much in response to the long term increase in the real wage rate. This suggests that the long-run elasticity of labor supply is zero (Kimball and Shapiro, 2008). This is also a necessary condition for the existence of steady state or balanced growth path (King, Plosser and Rebelo, 2002).

3. FORMS OF HOUSEHOLD UTILITY FUNCTION

3.1. OVERLAPPED CES UTILITY FUNCTION OF AUERBACH AND KOTLIKOFF (1987)

Each household perform economic activity during T years. The lifetime utility function has the following overlapped CES form.

$$U = \max_{c_i, l_i} \left[\sum_{i=1}^{T} \beta_i x_i^{1-1/\gamma} + \theta \beta_T b^{(1-1/\gamma)} \right]^{1/(1-1/\gamma)}, \quad \beta_i = \frac{\psi_i}{(1+\rho)^i} \tag{1a}$$

where x_i is the composite good produced by combining the consumption good and leisure at age i, b is the amount of bequest, and the parameter $\theta, \psi_i, \rho, \gamma$ represents the weight on bequest, the survival probability at age i, the time preference rate, and the intertemporal elasticity of substitution respectively. The composite good x_i is produced by the consumption good and leisure using a CES technology.

$$x_i = (c_i^{1-1/\varepsilon} + \alpha l_i^{1-1/\varepsilon})^{1/(1-1/\varepsilon)} \tag{1b}$$

where the parameter ε represents the elasticity of substitution between consumption c_i and leisure l_i.

Household has time endowment $e_i = 1$ which is used for leisure or labor. Thus the lifetime budget constraint of household is as follows:

$$\sum_{i=20}^{80} d_i c_i + d_T b \leq \sum_{i=20}^{80} d_i h_i w_i (1-l_i) + d_{20} s, \; d_i = 1/\prod_{k=1}^{i} (1+r_k) \tag{2}$$

where h_i denotes the human capital or productivity of labor, and w_i is the efficiency wage rate, r_k is the interest rate in period k, and s is the amount of bequest received from the parents.

The solution of the above problem is as follow:

$$\begin{aligned} c_j &= \left(\frac{p_j}{1}\right)^{\varepsilon} \frac{\beta_j^{\gamma} I}{(d_j p_j)^{\gamma} \cdot p^{1-\gamma}} = \left(\frac{p_j}{1}\right)^{\varepsilon} \left(\frac{\beta_j p}{d_j p_j}\right)^{\gamma} \left(\frac{I}{p}\right) \\ l_j &= \left(\frac{\alpha}{w_j*}\right)^{\varepsilon} c_j = \left(\frac{\alpha p_j}{w_j*}\right)^{\varepsilon} \left(\frac{\beta_j p}{d_j p_j}\right)^{\gamma} \left(\frac{I}{p}\right) \\ &= \alpha^{\varepsilon} \left(\frac{\beta_j}{d_j}\right)^{\gamma} \left(\frac{p_j}{w_j*}\right)^{\varepsilon-1} \left(\frac{p_j}{p}\right)^{1-\gamma} \left(\frac{I}{w_j*}\right) \end{aligned} \tag{3}$$

where w_j^* is the opportunity cost of leisure at age j, $p_j \equiv (1+\alpha^{\varepsilon} w_j^{*1-\varepsilon})^{1/(1-\varepsilon)}$ and

$$p \equiv \left[\sum_{j=1}^{T} \beta_j^{\gamma} (d_j p_j)^{1-\gamma} + (\theta \beta_T)^{\gamma} d_T^{1-\gamma}\right]^{1/(1-\gamma)}.$$

As we can see in the above equation related to the leisure, as the technology progresses, the productivity of labor increases and the real wage rate increases. This increase in wage rate has both the income effect and the substitution effect. When $\varepsilon < 1$, the income effect approximately dominates the substitution effect and the demand for leisure increases unlimited as real wage rate increases.[1] This increase in leisure is incompatible with steady state equilibrium since the endowment of time is limited. When $\varepsilon > 1$, the demand for leisure decreases down to 0 as technology progresses. This decrease in leisure is unrealistic in light of the historical trend that supports the somewhat constant household labor supply.

Auerbach and Kotlikoff (1987) set the value of $\varepsilon = 0.8$. As a consequence, the demand for leisure increases unlimited as the generations proceed. To overcome this difficulty, they assume that the time endowment of a generation increases at the same rate of technical progress as generations go on (Altig et. al. (2001)). This assumption seems to be unnatural. To avoid this awkwardness, we can assume that the coefficient α representing the leisure preference decreases as generations go on. That is, we can assume that the coefficients of leisure changes as the productivity changes: $\alpha' = \alpha h_1^{1-1/\varepsilon}$. For instance, for $\varepsilon = 0.8$ and $h_1 = 1.2$, $h_1^{1-1/\varepsilon} = 1.2^{(-0.25)} \approx 0.96$. This implies that, the more productive the generation, the less is the leisure preference of the generation. Anyway,

[1]This relation holds approximately since the boundary condition may be binding in the old.

the assumption seems to be not so natural. Thus we consider several alternative functional forms for household utility function in the next subsections.

3.2. UTILITY FUNCTIONS FREQUENTLY USED IN MACROECONOMICS

Several utility functions are frequently used in macroeconomic models. We consider the following (period) utility functions.

1. $u(c_t, l_t) = lnc_t + \alpha_t \frac{l_t^{1-1/\eta}}{1-1/\eta}$

1-1. $u(c_t, n_t) = lnc_t - B\frac{n_t^{1+1/\eta}}{1+1/\eta}$

2. $u(c_t, n_t) = \frac{c_t^{1-1/\gamma}}{1-1/\gamma} - B\frac{n_t^{1+1/\eta}}{1+1/\eta}$

3. $u(c_t, n_t) = \frac{1}{1-1/\gamma}(c_t - B\frac{n_t^{1+1/\eta}}{1+1/\eta})^{1-1/\gamma}$

   (Greenwood-Hercowitz-Huffman (GHH) preferences)

where c_t represents the consumption at age t, l_t represents the leisure at age t, and n_t represents the labor supply at age t.

King, Plosser and Rebelo (1988) proposed a class of utility functions, KPR utility function, which is compatible with the steady state of a growing economy. That is, the KPR utility function satisfies the zero long-run elasticity of labor supply. It is of the form as follows.

$$u(c_t, l_t) = \frac{c_t^{1-1/\gamma}}{1-1/\gamma}v(l_t)$$
$$u(c_t, l_t) = lnc_t + v(l_t)$$

The first two utility functions, numbered 1 and 1-1, satisfy the zero long-run elasticity of labor supply since it is a KPR utility function with $\gamma = 1$. The problem of these two utility functions is that the lifetime profile of consumption do not exhibit the hump-shapedness. This is evident by the following equation obtained from the first order conditions (For the derivation, refer to the Appendix):

$$\frac{c_{i+1}}{c_i} = \frac{\beta_{i+1}}{d_{i+1}}\frac{d_i}{\beta_i} = \frac{1+r_{i+1}}{1+\rho}$$

This is due to the additive separability between consumption and labor in the utility function. The additive separability makes the complementarity between consumption and labor zero. Note that the real wage rate of labor is hump-shaped along the age. In the Auerbach and Kotlikoff model, the hump-shape of wage along the age transmits into consumption as the household substitutes leisure by consumption when the opportunity cost of leisure increases.

The second function numbered 2 in the above does not belong to the class of KPR utility functions. Thus the long-run elasticity of labor supply is not equal to zero. This function does not exhibit the hump-shapedness of consumption profile either. The third function numbered 3 in the above does not belong to the class of KPR utility functions. It exhibits, however, some hump-shapedness of consumption. From these discussions, we can notice that none of the above utility functions satisfy both the zero elasticity of labor supply and the hump-shaped consumption profile.

3.3. UTILITY FUNCTIONS PROPOSED BY KIMBALL AND SHAPIRO

Kimball and Shapiro (2008) proposed the following utility function which belongs to the class of KPR utility function.

$$u(c_t,n_t) = \frac{c_t^{1-1/\gamma}}{1-1/\gamma}[A+(1-\gamma)B\frac{n_t^{1+1/\eta}}{1+1/\eta}]^{1/\gamma}$$

This function, we call it KS utility function, allows the substitution between consumption and leisure. Indeed, the parameter $1-\gamma$ represents the substitutability between consumption and leisure. $\gamma = 1$ corresponds to additive separability while $\gamma = 0$ corresponds to perfect substitutability between consumption and leisure (Kimball and Shapiro (2008) p. 8). The γ also reflects the intertemporal elasticity of substitution as can be seen in the following equation derived from (13) in the Appendix.

$$\frac{c_{t+1}}{c_t} = \left(\frac{d_{t+1}}{d_t}\frac{\beta_t}{\beta_{t+1}}\right)^{-\gamma}\frac{\left[A+(1-\gamma)B\frac{n_{t+1}^{1+1/\eta}}{1+1/\eta}\right]}{\left[A+(1-\gamma)B\frac{n_t^{1+1/\eta}}{1+1/\eta}\right]}$$

This utility function satisfies the zero long-run elasticity of labor supply. The simulation shows that this utility function also satisfies the hump-shapedness of consumption profile.

So far, we checked whether the above utility functions satisfy the two criteria, namely, the hump-shaped consumption profile and the zero long-run elasticity of labor supply. Table 1 shows the result of check via the two criteria. Two utility functions satisfy both the zero long-run elasticity of labor supply and the hump-shaped consumption profile: the AK utility function with $\varepsilon = 1$ and the KS utility function.

3.4. SIMULATION OF HOUSEHOLD BEHAVIOR

We simulate the household behavior with two utility functions, namely, the Auerbach and Kotlikoff (AK) utility function with $\varepsilon = 1$, and the Kimball and Shapiro (KS) utility function.[2]

The choice of parameters is geared to generating similar characteristics which was recorded in the study of Korea household behavior in Park and Jun (2009). The characteristics we focused on are as follows. First, the income profile is hump-shaped and has a peak around the age 53. Second, the consumption profile is hump-shaped and has a peak around the age 58. Third, the consumption profile and the income profile crosses around the age 61.

We assumed the technical progress rate which affects the labor productivity to be 0.02 at annual rate. We assume that the value of α is varying with age in the AK utility function. We assume that the values of A and B are varying with age in KS utility function. In the simulation, we added the bequest in the model to reflect the fact that the savings before retirement is very large according to the empirical study. The values of main parameters are shown in Table 3 and the simulation result is illustrated in Figures 1 and 2. The simulation shows that the two household models accord with the empirical result in Park and Jun (2009). The two models replicates the hump-shaped consumption and income profile with corresponding peaks in Park and Jun (2009).

The KS utility function involves parameters ρ, θ, A, B, η while the AK utility function involvles parameters $\rho, \theta, \alpha, \gamma$. Thus the KS utility function has one more parameter than the AK utility function. This makes the KS function more flexible in replicating the household behavior than the AK function. More specifically, the parameter A affects the consumption profile while the parameter B determines the shape of labor income profile.

In addition, the KS utility function has also computational advantage in that the computing time is far less than that in the AK utility function.

[2]The AK utility function with is important in that it is frequently used in the literatures of macroeconomics and public finance, though the assumption of seems to be strong.

4. SIMULATION WITH OVERLAPPING GENERATIONS MODEL

POPULATION PROJECTION

We use the population projection (2005–2100) made by National Statistical Office in 2006 and extended by the Committee on the Estimation of National Pension Finance in 2008. According to the projection, the Korean population increases from 48.1 million in 2005 to 49.3 million in 2018. Then the population decreases to 42.3 million in 2050 and to 21.0 million in 2100. The ratio of the old aged more than 64 to the young aged 20–64 increases monotonically from 0.14 in 2005 to 1.00 in 2068 and to 1.12 in 2100.

THE OG MODEL

We use the overlapping generations model which is a modified version of the one used in Choi and Shin (2009) and Shin and Choi (2010) in that the household utility function is different and only the household sector and the firm sector exist: The representative firm produces the good with Cobb-Douglas production function. The households supply labor and capital in the factor markets. The households consume the goods produced by the firm sector. The sum of labor supplies by households is the aggregate labor supply in the economy. The sum of household assets coincides with the capital stock of the economy.[3] We assume that the exogenous rate of technological progress is 2% in the period of 1945-2005 and then slowly declines to 1% in the year 2100 and stays the level of 1% after 2100.

THE SIMULATION RESULT

We consider the economic impact of the aging process in Korea using a simple overlapping generations model. And we compare the simulation results under the two forms of household utility function: the Auerbach and Kotlikoff (AK) utility function and the Kimball and Shapiro (KS) utility function.

Our simulation shows that the population aging has the following effects on the economy.

First of all, it decreases the growth rate of labor supply (See Figure 4). This brings forth the rise of wage rate (Figure 5). The rise of wage rate increases the lifetime income and savings. This increase in savings mitigates the adverse

[3]For more detailed structure of the model, please refer to Choi and Shin (2009) or Shin and Choi (2010).

effect on capital accumulation of the diminishing population. Thus the growth rate of capital stock is always larger than that of labor supply (Figure 6).[4]

Second, the capital population ratio increases as the population diminishes and the capital stock increases (Figure 7). This causes the per capita income increase. But its increasing rate decreases as the growth rate of capital stock decreases faster than that of population (Figure 8). Since the growth rate of both the effective labor supply and capital stock decreases, the growth rate of GDP decreases (Figure 9).

Third, the capital-labor ratio increases since the growth rate of capital is larger than that of the effective labor supply (Figure 10). The increases in capital-labor ratio accords with the decrease in the interest rate and the increase in the wage rate (Figure 11).

The two household model (AK model and KS model) shows similar trends in most variables. One of the differences lies in the size of the growth rates of capital stock and labor. The growth rate of capital stock in KS model is always larger than that in AK model. This seems to be due to the fact that the positive effect on household savings of the rising wage rate in the KS model is larger than that in the AK model.

The growth rate of labor supply in KS model is also larger than that in AK model. Note that the difference of the capital growth rate between the two models is larger than that of the growth rate of labor supply. This leads to the higher growth rate of capital/population, per capita income, and the higher capital/labor, capital/output ratios, and lower interest rate in KS model.

5. CONCLUDING REMARKS

The household utility function plays central role in the overlapping generations model. In order to determine the right form of the household utility function, we investigated various forms of household utility function with the two criteria: the zero long-run elasticity of labor supply and the hump-shaped consumption profile. We identified two forms of utility function that satisfy both criteria from the solution of household utility maximization: the AK utility function with $\varepsilon = 1$ and the KS utility function.

The simulation exercise suggests that the KS utility function is more appropriate than the AK utility function. Household simulation shows that the two utility functions can replicate the consumption and income profiles which appear

[4]Note that the individual labor supply does not change in response to the rise in wage rate due to the zero long-run elasticity of labor supply.

in the empirical study. The KS utility function, however, seems to be superior to the AK utility function in two aspects. First, the KS utility function provides flexibility in replicating the profiles since it has one more parameter to adjust to the data. Second, the KS model involves less computing time due to its algorithmic simplicity.

Macroeconomic simulation shows that the broad picture of the trend in various aggregate variables is similar in both models: the AK model and the KS model. The KS model is accompanied by larger capital accumulation and larger capital/labor ratio and faster growth in per capita income than the AK model. Thus the negative effect of population aging on economic growth in the KS model is smaller than that in the AK model.

The two models are complementary in that they reveal slightly different results in the macroeconomic simulation. On the other hand, the microeconomic simulation shows that the KS model is superior to the AK model. Therefore the KS utility function seems to be a promising candidate for the simulation of household model and overlapping generations model.

APPENDIX: THE SOLUTION OF HOUSEHOLD PROBLEM OF UTILITY MAXIMIZATION

The first function numbered 1 in the text belongs to the class of KPR utility functions where $\gamma = 1$. The household problem is to choose the consumption and leisure in order to maximize the lifetime utility.

$$\max \sum_{i=1}^{T} \beta_i \left[\ln c_i + \alpha_i \frac{l_i^{1-1/\eta}}{1-1/\eta} \right]$$
$$s.t. \sum_{i=1}^{T} d_i c_i = \sum_{i=1}^{T} d_i h_i w_i (1-l_i), \;\; l_i \leq 1, \;\; i = 1, \ldots, T,$$

where $\beta_i = 1/(1+\rho)^i$ and $d_i = 1/\prod_{k=1}^{i}(1+r_k)$.

The Lagrangian and the first order condition is as follows.

$$L = \sum_{i=1}^{T} \beta_i \left[\ln c_i + \alpha_i \frac{l_i^{1-1/\eta}}{1-1/\eta} \right] + \lambda \left[\sum_{i=1}^{T} d_i h_i w_i (1-l_i) - d_i c_i \right] + \sum_{i=1}^{T} \mu_i (1-l_i)$$

$$L_{c_i} = \frac{\beta_i}{c_i} - \lambda d_i = 0 \;\; \rightarrow \;\; c_i = \beta_i/(d_i \lambda) \qquad \text{(A-1)}$$

$$
\begin{aligned}
L_{l_i} &= \beta_i \alpha_i l_i^{-1/\eta} - \lambda d_i h_i w_i - \mu_i = 0 \\
&\to \beta_i \alpha_i l_i^{-1/\eta} = \lambda d_i w_i^*, w_i^* = h_i w_i + \frac{\mu_i}{\lambda d_i}
\end{aligned} \tag{A-2}
$$

$$
\begin{aligned}
L_\lambda &= \sum_{i=1}^{T} d_i h_i w_i (1 - l_i) - d_i c_i = 0 \\
&\to \sum_{i=1}^{T} d_i w_i^* (1 - l_i) - d_i c_i = 0
\end{aligned} \tag{A-3}
$$

$$
\begin{aligned}
L_{\mu_i} &= 1 - l_i \geq 0, \mu_i \geq 0, (1 - l_i)\mu_i = 0 \\
&\to (l_i = 1, \mu_i \geq 0) \text{ or } (l_i < 1, \mu_i = 0)
\end{aligned} \tag{A-4}
$$

Using (3) for i and $i+1$, we obtain the following.

$$
\frac{c_{i+1}}{c_i} = \frac{\beta_{i+1}}{\beta_i} \frac{d_i}{d_{i+1}}, \; c_i = \frac{\beta_i}{\beta_1} \frac{d_1}{d_i} c_1
$$

Using (4) for i and $i+1$, we obtain the following.

$$
\frac{\beta_{i+1}}{\beta_i} \left(\frac{l_{i+1}}{l_i} \right)^{-1/\eta} = \frac{d_{i+1} w_{i+1}^*}{d_i w_i^*}, \; \frac{l_{i+1}}{l_i} = \left(\frac{\beta_{i+1} d_i}{\beta_i d_{i+1}} \frac{w_i^*}{w_{i+1}^*} \right)^{\eta}
$$

Using (3) and (4), we obtain the following.

$$
l_i = \left(\frac{\alpha_i}{w_i^*} c_i \right)^{\eta} = \left(\frac{\alpha_i}{w_i^*} \frac{\beta_i d_1}{\beta_1 d_i} c_1 \right)^{\eta}
$$

In the case of the utility function 1-1 at the beginning of Section 3.2, we obtain a closed-form solution. The household problem with this utility function is as follows.

$$
\begin{aligned}
&\max \sum_{i=1}^{T} \beta_i \left[\ln c_i - B \frac{n_i^{1+1/\gamma}}{1+1/\gamma} \right] \\
&s.t. \sum_{i=1}^{T} d_i c_i = \sum_{i=1}^{T} d_i h_i w_i n_i
\end{aligned}
$$

where $\beta_i = 1/(1+\rho)^i$, and $d_i = 1/\prod_{k=1}^{i}(1+r_k)$.

The solution of the problem is as follows.

$$d_i = 1/\prod_{k=1}^{i}(1+r_k)$$

$$c_i = \left(\frac{\beta_i}{d_i}\right)\left(\frac{\sum_{k=1}^{T}\frac{(d_k h_k w_k)^{1+\eta}}{(B\beta_k)^{\eta}}}{\sum_{k=1}^{T}\beta_k}\right)^{\frac{1}{1+\eta}} = B^{-\frac{\eta}{1+\eta}}\left(\frac{\beta_i}{d_i}\right)\left(\frac{\sum_{k=1}^{T}\frac{(d_k h_k w_k)^{1+\eta}}{\beta_k^{\eta}}}{\sum_{k=1}^{T}\beta_k}\right)^{\frac{1}{1+\eta}},$$

$$\begin{aligned} n_i &= \left(\frac{d_i h_i w_i}{B\beta_i}\left(\frac{\sum_{k=1}^{T}\beta_k}{\sum_{k=1}^{T}\frac{(d_k h_k w_k)^{1+\eta}}{(B\beta_k)^{\eta}}}\right)^{\frac{1}{1+\eta}}\right)^{\eta} \\ &= \left(B^{-\frac{1}{1+\eta}}\frac{d_i h_i w_i}{\beta_i}\left(\frac{\sum_{k=1}^{T}\beta_k}{\sum_{k=1}^{T}\frac{(d_k h_k w_k)^{1+\eta}}{\beta_k^{\eta}}}\right)^{\frac{1}{1+\eta}}\right)^{\eta}, \quad i = 1,\ldots,T. \end{aligned}$$

The utility maximization problem of household with KS utility function is as follows.

$$\max_{\{c_t\}\{n_t\}}\sum_{t=0}^{T}\beta_t\left(\frac{c_t^{1-1/\gamma}}{1-1/\gamma}\left[A_t+(1-\gamma)B_t\frac{n_t^{1+1/\eta}}{1+1/\eta}\right]^{1/\gamma}\right)+\beta_T\theta\frac{b^{1-1/\gamma}}{1-1/\gamma}$$

$$s.t.\sum_{t=0}^{T}d_t c_t + d_T b \le \sum_{t=0}^{T}d_t(h_t w_t n_t + s_t)$$

The Lagrangian and the first order conditions are as follows:

$$\begin{aligned} L = &\sum_{t=0}^{T}\beta_t\left(\frac{c_t^{1-1/\gamma}}{1-1/\gamma}[A_t+(1-\gamma)B_t\frac{n_t^{1+1/\eta}}{1+1/\eta}]^{1/\gamma}\right) \\ &+\beta_T\theta\frac{b^{1-1/\gamma}}{1-1/\gamma}+\lambda[\sum_{t=0}^{T}(d_t(h_t w_t n_t + s_t)-d_t c_t)-d_T b] \end{aligned}$$

$$\begin{aligned} L_{c_t} =& \beta_t c_t^{-1/\gamma}[A_t+(1-\gamma)B_t\frac{n_t^{1+1/\eta}}{1+1/\eta}]^{1/\gamma}-\lambda d_t = 0 \\ &\rightarrow \beta_t c_t^{-1/\gamma}\left[A_t+(1-\gamma)B_t\frac{n_t^{1+1/\eta}}{1+1/\eta}\right]^{1/\gamma} = \lambda d_t \qquad \text{(A-5)} \end{aligned}$$

$$L_{n_t} = -\beta_t c_t^{1-1/\gamma}[A_t + (1-\gamma)B_t \frac{n_t^{1+1/\eta}}{1+1/\eta}]^{\frac{1}{\gamma}-1} B_t n_t^{1/\eta} + \lambda d_t h_t w_t = 0$$

$$\rightarrow \beta_t c_t^{1-1/\gamma}[A_t + (1-\gamma)B_t \frac{n_t^{1+1/\eta}}{1+1/\eta}]^{\frac{1}{\gamma}-1} B_t n_t^{1/\eta} = \lambda d_t h_t w_t \tag{A-6}$$

$$L_\lambda = \sum_{t=0}^{T}(d_t(h_t w_t n_t + s_t) - d_t c_t - d_T b) = 0 \tag{A-7}$$

$$L_b = \beta_T \theta b^{-1/\gamma} - \lambda d_T = 0 \tag{A-8}$$

From (A-5) and (A-6), we obtain the following.

$$\frac{1}{c_t B_t n_t^{1/\eta}}[A_t + (1-\gamma)B_t \frac{n_t^{1+1/\eta}}{1+1/\eta}] = \frac{1}{h_t w_t} \tag{A-9}$$

$$\rightarrow [A_t + (1-\gamma)B_t \frac{n_t^{1+1/\eta}}{1+1/\eta}] = \frac{c_t B_t n_t^{1/\eta}}{h_t w_t}$$

We also obtain the following.

$$\frac{1}{c_t B_t n_t^{1/\eta}}[A_t + (1-\gamma)B_t \frac{n_t^{1+1/\eta}}{1+1/\eta}] = \frac{1}{h_t w_t} \tag{A-10}$$

$$\rightarrow \frac{c_t}{n_t} = [A_t + (1-\gamma)B_t \frac{n_t^{1+1/\eta}}{1+1/\eta}] \frac{h_t w_t}{B_t n_t^{1+1/\eta}}$$

$$\rightarrow c_t = [\frac{A_t}{B_t n_t^{1/\eta}} + (1-\gamma)\frac{n_t}{1+1/\eta}] h_t w_t$$

On the other hand, from the equation (A-5) applied to period t and $t+1$, we obtain the following.

$$\frac{\beta_{t+1}}{\beta_t}\left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\gamma}}\left(\frac{[A_{t+1} + (1-\gamma)B_{t+1}\frac{n_{t+1}^{1+1/\eta}}{1+1/\eta}]}{[A_t + (1-\gamma)B_t \frac{n_t^{1+1/\eta}}{1+1/\eta}]}\right)^{1/\gamma} = \frac{d_{t+1}}{d_t} \tag{A-11}$$

Replacing [ ] with $\frac{B_t c_t n_t^{1/\eta}}{h_t w_t}$ in the above equation, we obtain the following.

$$\frac{n_{t+1}}{n_t} = \left(\frac{\beta_t d_{t+1}}{\beta_{t+1} d_t}\right)^{\gamma\eta}\left(\frac{h_{t+1} w_{t+1} B_t}{h_t w_t B_{t+1}}\right)^{\eta}$$

$$= \left(\frac{d_{t+1}/\beta_{t+1}}{d_t/\beta_t}\right)^{\gamma\eta}\left(\frac{h_{t+1} w_{t+1} B_t}{h_t w_t B_{t+1}}\right)^{\eta} \tag{A-12}$$

From the equations (A-5) and (A-8), we obtain the following.

$$b^{-1/\gamma} = (1/\theta)c_T^{-1/\gamma}[A_T + (1-\gamma)B_T \frac{n_T^{1+1/\eta}}{1+1/\eta}]^{1/\gamma} \tag{A-13}$$

$$\rightarrow b = c_T \theta^{\gamma}[A_T + (1-\gamma)B_T \frac{n_T^{1+1/\eta}}{1+1/\eta}]$$

Thus, given n_0, n_t is determined by equation (A-12). And then c_t is determined by (A-10). And b is determined by the equation (A-13). Finally, n_0 is determined by the budget constraint.

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Table 1: Properties of several utility functions used in macroeconomic models

	hump-shaped consumption profile	zero long-run elasticity of labor supply
Auerbach and Kotlikoff (AK)	O	X ($\varepsilon \neq 1$) O ($\varepsilon = 1$)
KPR with $\gamma = 1$: $u(c_t, l_t) = \ln c_t + \dfrac{\alpha l_t^{1+1/\eta}}{1+1/\eta}$	X	O
$u(c_t, n_t) = \frac{c_t^{1-1/\gamma}}{1-1/\gamma} - B\frac{n_t^{1+1/\eta}}{1+1/\eta}$	X	X
GHH: $u(c_t, n_t) = \frac{1}{1-1/\gamma}\left[c_t - B\frac{n_t^{1+1/\eta}}{1+1/\eta}\right]^{1-1/\gamma}$	O	X
KS: $u(c_t, n_t) = \frac{c_t^{1-1/\gamma}}{1-1/\gamma}\left[A + (1-\gamma)B\frac{n_t^{1+1/\eta}}{1-1/\eta}\right]^{1/\gamma}$	O	O

Table 2: Values of Main Parameters

Parameters	Auerbach and Kotlikoff (1987) with $\varepsilon = 1$	Kimball and Shapiro (2008)
ρ	0.001	0.001
r	0.05	0.05
w	1	1
θ	30	30
γ	0.15	0.7
α	varying with age*	-
A	-	varying with age**
B		varying with age***
η	-	0.5

* The value of α decreases from 1 at age 20 by 0.16 every year until the age 47, decreases by 0.1 every year until the age 52, and stays constant for six years and then increases by 0.15 every year.
** The value of A increases from 1 at age 20 by 0.04 every year until the age 53 and stays constant for six years and then decreases by 0.06 every year.
*** The value of B decreases from 1 at age 20 by the rate of 3% every year until the age 47 and stays constant for six years and then increases by the rate of 5% every year.

Table 3: Accordance of household models with the empirical result

	Park and Jun (2009)	AK utility function	KS utility function
age of income peak	53	52	53
age of consumption peak	58	58	58
age of consumption overtaking income	around 61	61	61

Table 4: Forecasts of Growth Rate

	GDP growth rate		Per capita GDP growth rate	
	AK	KS	AK	KS
2006	0.0299	0.0337	0.0184	0.0292
2030	0.0080	0.0103	0.0090	0.0190
2050	-0.0014	0.0010	0.0077	0.0162
2070	-0.0046	-0.0022	0.0098	0.0147
2100	-0.0072	-0.0048	0.0061	0.0081

Table 5: Forecasts of the growth rates of labor supply and capital stock

	Growth rate of labor supply		Growth rate of capital stock	
	AK	KS	AK	KS
2006	0.0282	0.0311	0.0351	0.0415
2030	0.0062	0.0081	0.0133	0.0169
2050	-0.0022	-0.0004	0.0012	0.0051
2070	-0.0050	-0.0032	-0.0031	0.0008
2100	-0.0078	-0.0059	-0.0052	-0.0016

Table 6: Forecasts of capital/labor ratio, wage rate, and interest rate

	Capital/Labor		wage rate		interest rate	
	AK	KS	AK	KS	AK	KS
2005	7.8	7.5	1.00	1.00	0.043	0.044
2030	9.1	9.5	1.04	1.06	0.038	0.037
2050	10.2	11.0	1.07	1.10	0.035	0.033
2070	10.8	12.0	1.09	1.12	0.034	0.031
2100	11.6	13.6	1.10	1.16	0.032	0.028

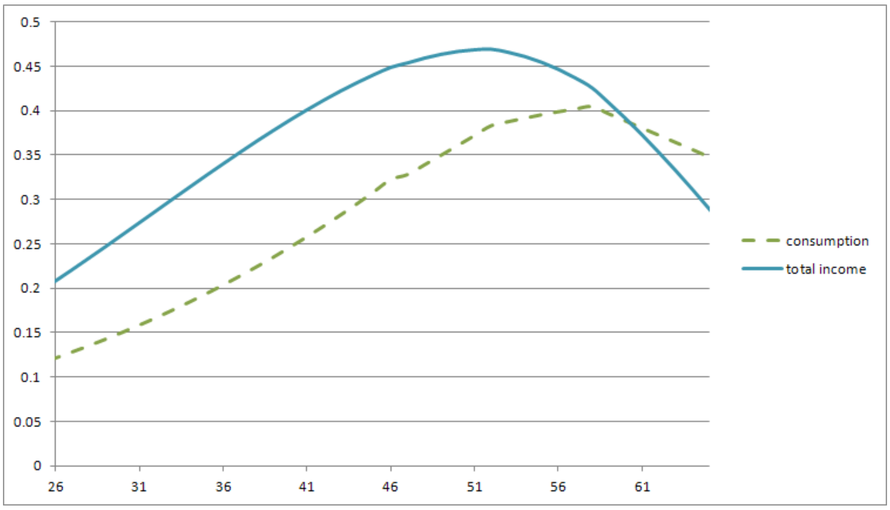

Figure 1: The profile of income and consumption: Auerbach and Kotlikoff (AK) utility function ($\varepsilon = 1$)

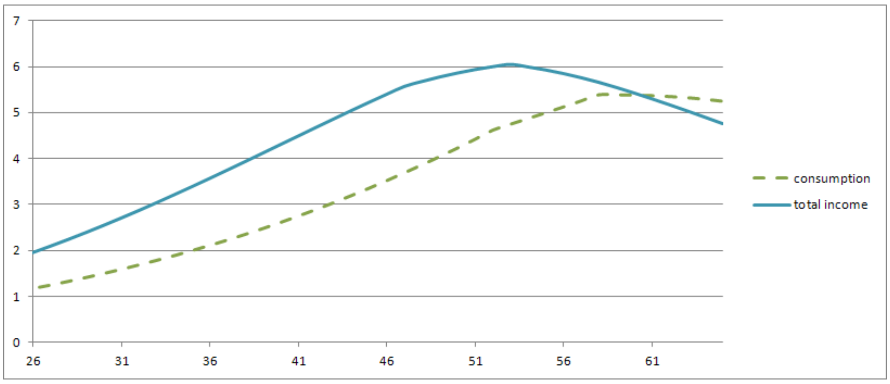

Figure 2: The profile of income and consumption: Kimball and Shapiro (KS) utility function

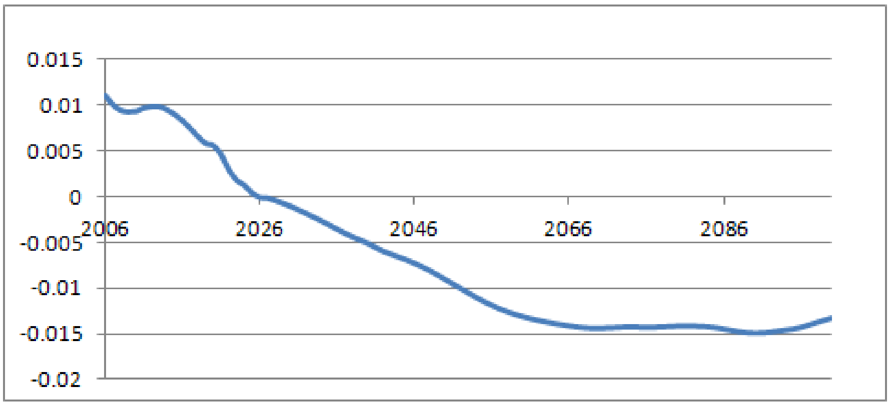

Figure 3: Change rate of population

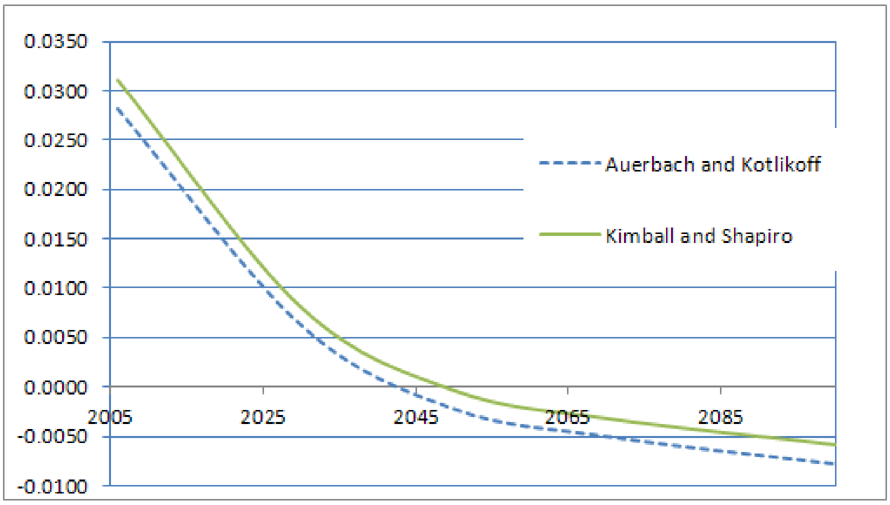

Figure 4: Change rate of labor supply

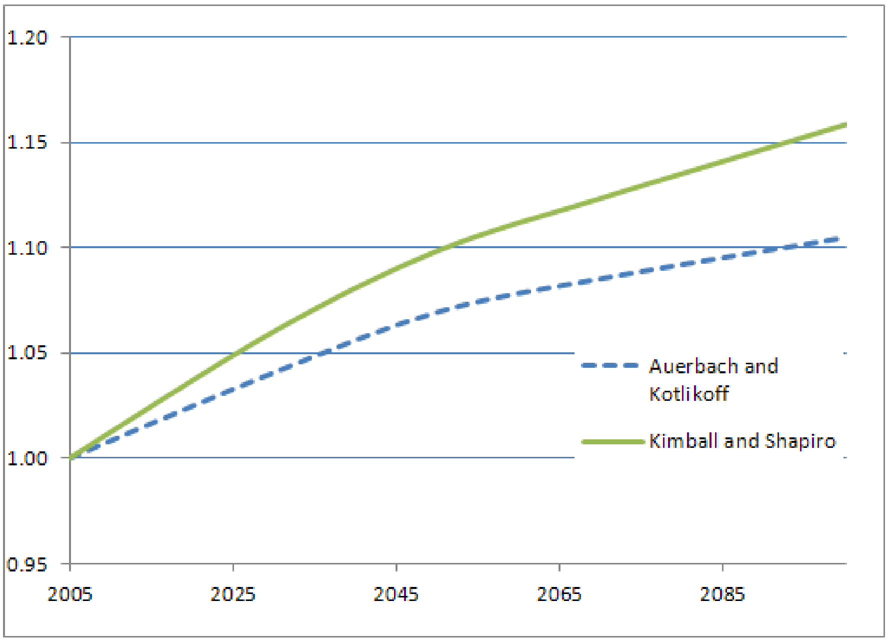

Figure 5: Trend of efficiency unit wate rate

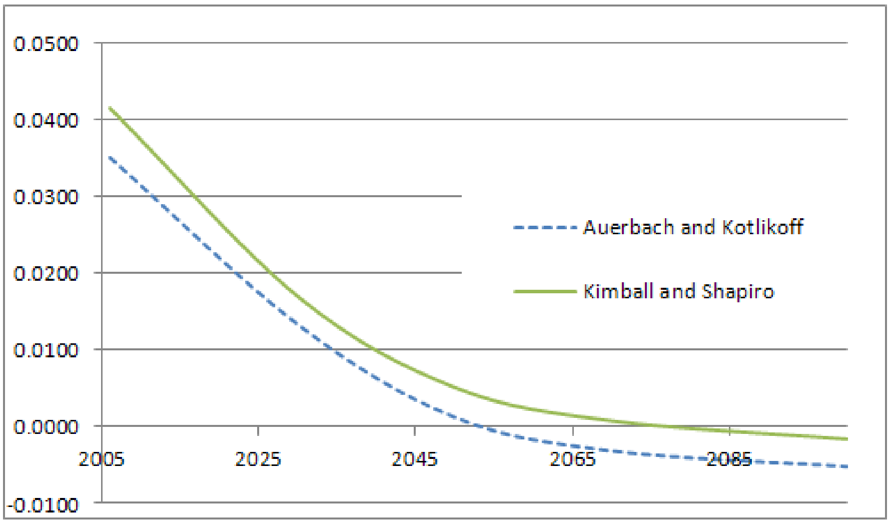

Figure 6: Growth rate of capital stock

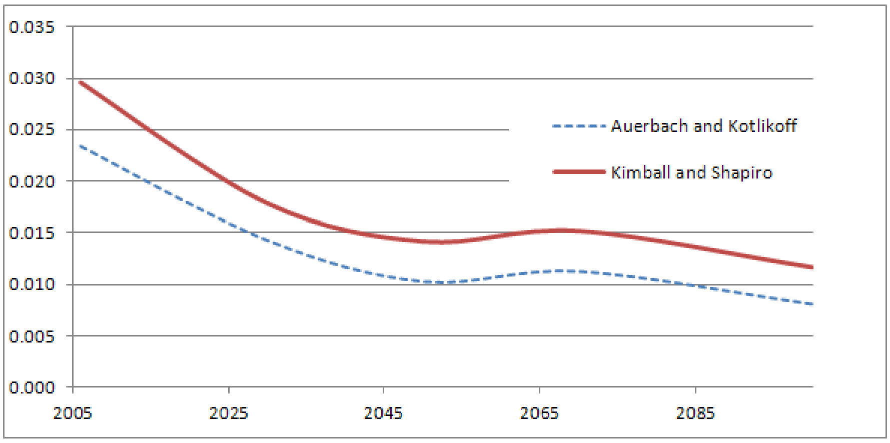

Figure 7: Growth rate of capital/population

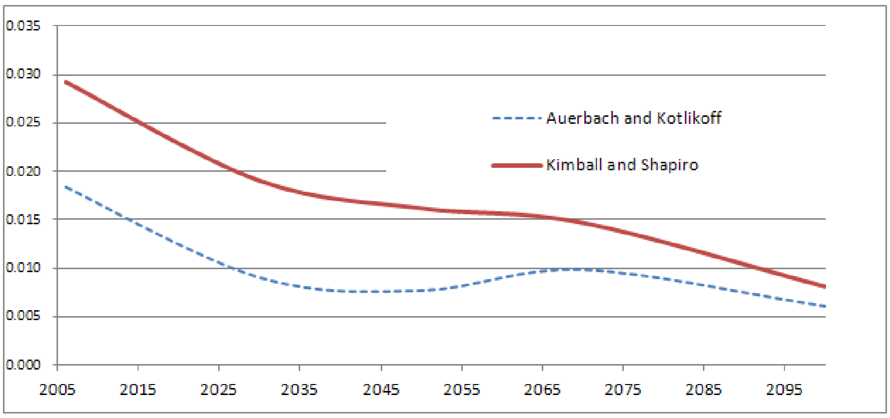

Figure 8: Growth rate of per capita income

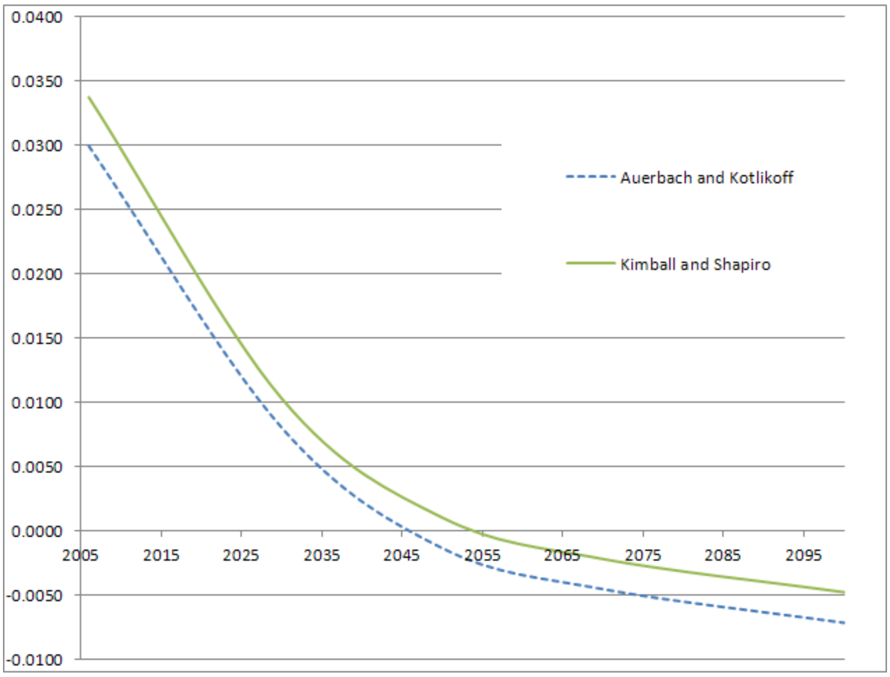

Figure 9: GDP growth rate

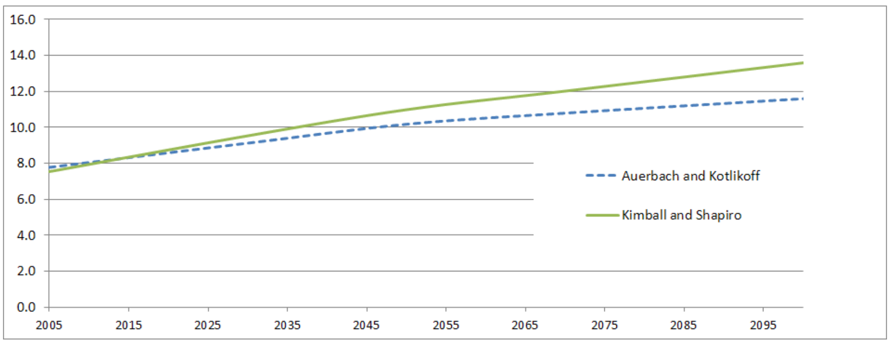

Figure 10: Capital Labor Ratio

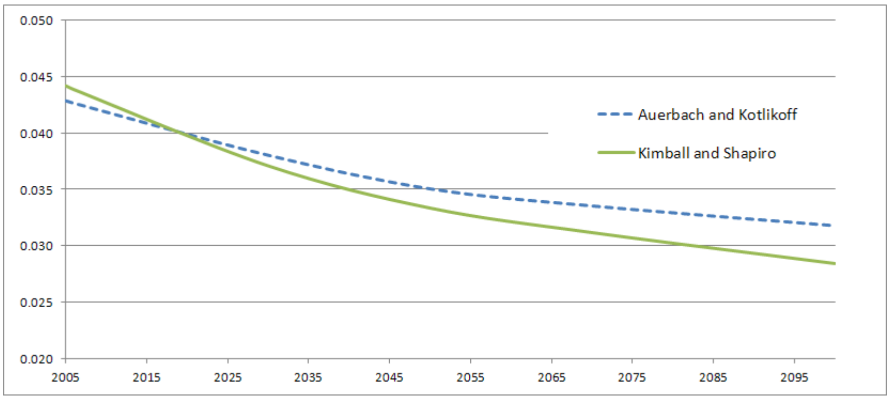

Figure 11: Trend of Interest Rate

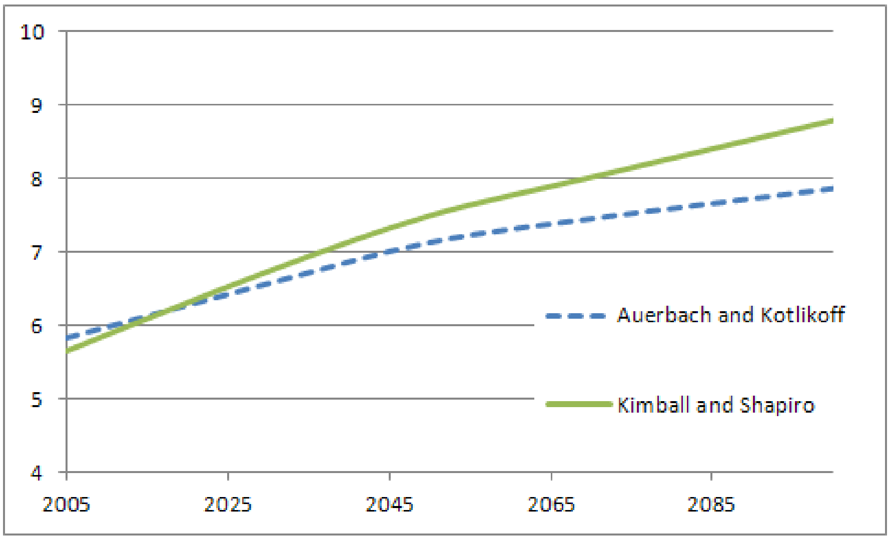

Figure 12: Trend of capital-output ratio