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Strategic Limitation of Market Size and Product Quality for Entry

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Abstract This paper derives an entrant's optimal choice of quality and market size. We derive a subgame perfect equilibrium of two stage games in which the entrant chooses its product quality and product market size in the first stage and competes with the incumbent in price in the second stage. We find that the entrant's optimal strategy in the first stage involves only the limitation of market size. However, when quality is given, an entrant with lower quality than a critical level does not need to limit market size but an entrant with higher quality than the critical level needs to limit market size. When market size is given, an entrant with a smaller market than a critical size does not need to limit quality but an entrant with a bigger market than the critical level needs to limit quality.

Keywords product quality, market size, entry

JEL Classification D43, L13, L15

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1. INTRODUCTION

Shaked and Sutton (1982, 1983) show that entrants to a market have incentives to choose to engage in vertical differentiation to relax price competition.[1] Huang and Schmit (1988, 1992), Donnenfeld and Weber (1992, 1995) and Noh and Moschini (2006) consider sequential entry in a vertically differentiated market and investigate strategic choice of the incumbent and the entrant for their product qualities.

Gelman and Salop (1983) show that the entrant's strategic limitation of capacity can avoid severe price competition with the incumbent. Spence (1977), Dixit (1980) and Allen et al. (2000) show the incumbent's strategic choice of capacity to deter entry. However, market size has not been considered as much as capacity in economic models of entry and/or entry deterrence. Commitment of market size may not seem as natural as commitment of capacity or product quality; however, restricting business territory, targeting a special group of consumers and/or limiting the match rate of products can be examples of the entrant's limitation of its market size and it can be committed because changing these limitations requires as much time as changing capacity and/or product quality.[2, 3] Moreover, in literatures of marketing, market segmentation, targeting and positioning are three key strategic choices.

We investigate the entrant's optimal choice of quality and market size for entry into a monopoly market. For the investigation, we derive a subgame perfect equilibrium of two stage games in which the entrant chooses its product quality and product market size in the first stage and competes with the incumbent in price in the second stage. We find that the entrant's optimal strategy in the first stage involves only the limitation of market size. When quality is given, an entrant with lower quality than a critical level does not need to limit market size but an entrant with higher quality than the critical level needs to limit market size. When market size is given, an entrant with a smaller market than a critical

[1] Vertical differentiation model was first developed by Mussa and Rosen (1978) as a monopoly model and by Gabszewicz and Thisse (1979) as a duopoly model.

[2] Farrell (1986) and Milgrom and Roberts (1986) introduced the matching model in which a product gives a consumer positive utility only when the product matches preference of the consumer. Bae (2012) shows that limitation of match rate is a better strategy for entry than limitation of product quality.

[3] Though limitation of capacity and limitation of market size have some common features, they also have different aspects in practice and theory. Limitation of the entrant's market size leaves some exclusive market to the incumbent while limitation of the entrant's capacity does not. Moreover, commitment power of capacity and commitment power of market size differ, and which one is more powerful differs across industries.

size does not need to limit quality but an entrant with a bigger market than the critical level needs to limit quality.

The rest of this paper is organized as follows. Section 2 introduces a two stage game in which the entrant chooses its product quality in the first stage and competes with the incumbent in price in the second stage and derives a Nash equilibrium of the second stage subgames. In section 3, we derive the entrant's optimal choice of product quality and market size. Finally, section 4 concludes the paper.

2. PRICE COMPETITION IN A SEMI-SEGMENTED MARKET

We consider a situation in which an entrant chooses its product quality q and market size r in the first stage and competes with the incumbent in price in the second stage. We assume that the production cost of both firms is 0 and products are indivisible.[4] There is a continuum of consumers with measure 1 and each consumer is indexed by v. We assume that v is uniformly distributed in the interval $[0, 1]$. In the second stage, the entrant can sell its product to only r consumers whose index v is also uniformly distributed in the interval $[0, 1]$ while the incumbent can sell to all the consumers.[5] We refer to the r consumers to whom both the incumbent and the entrant can sell as the common market.

Each consumer purchases at most one product and at most one unit. Let $u(v)$ denote the utility of consumer v. If consumer v does not purchase any product, $u(v) = 0$. If consumer v purchases the incumbent's product at price p, $u(v) = v - p$. If consumer v in the common market purchases the entrant's product at price p, $u(v) = qv - p$. Each firm sets price to maximize its expected profit and each consumer decides whether or not to purchase and whose product to purchase to maximize his utility.[6] We derive a subgame perfect equilibrium of this two stage game.

Now, we derive the Nash equilibrium of the second stage price competition subgame between the incumbent and the entrant. The incumbent's product and

[4] The significance and due consequences of this assumption will be discussed after deriving our main results.

[5] If the entrant limits its market size by focusing on consumers with some range of v, limitation of market size has similar effects as limitation of quality. Therefore, in order to contrast limitation of market size with limitation of quality, we assume that the entrant limits its market size uniformly across v.

[6] Suppose that the incumbent can discriminate prices between the common market and the exclusive market. Then, limitation of market size does not reduce the competition between the incumbent and the entrant in the common market, and hence the entrant does not have incentive to limit its market size. Therefore, we do not consider price discrimination by the incumbent.

the entrant's product are identical and hence the second stage subgame is a discontinuous game if $q = 1$ while they are differentiated products hence the second stage subgame is a continuous game if $q < 1$. Suppose that $q < 1$ until we later consider the case in which $q = 1$.

Let p_I and p_E denote the prices charged by the incumbent and the entrant, respectively, and let $d_I(p_I, p_E)$ and $d_E(p_I, p_E)$ denote the quantities demanded for the incumbent's product and the entrant's product, respectively, when the firms charge prices p_I and p_E. Consumer v purchases the incumbent's product if $v - p_I > \max\{0,\ qv - p_E\}$ and the entrant's product if $qv - p_E > \max\{0, v - p_I\}$. Therefore,

$$d_I(p_I, p_E) = \begin{cases} 1 - p_I & \text{if } p_I \leq \frac{p_E}{q} \\ (1-r)(1-p_I) + r\left(1 - \frac{p_I - p_E}{1-q}\right) & \text{if } \frac{p_E}{q} \leq p_I \leq p_E + 1 - q \\ (1-r)(1-p_I) & \text{if } P_E + 1 - q \leq p_I \leq 1 \\ 0 & \text{if } p_I \geq 1 \end{cases}, \tag{1}$$

$$d_E(p_I, p_E) = \begin{cases} r\left(1 - \frac{p_E}{q}\right) & \text{if } p_E \leq p_I - 1 + q \\ r\left(\frac{p_I - p_E}{1-q} - \frac{p_E}{q}\right) & \text{if } p_I - 1 + q \leq p_E \leq q p_I \\ 0 & \text{if } p_E \geq q p_I \end{cases}, \tag{2}$$

Lastly, let $\pi_I(p_I, p_E)$ and $\pi_E(p_I, p_E)$ denote profits of the incumbent and the entrant, respectively, when the firms charge prices p_I and p_E. That is, $\pi_I(p_I, p_E) \equiv p_I d_I(p_I, p_E)$ and $\pi_E(p_I, p_E) \equiv p_E d_E(p_I, p_E)$.

We categorize the Nash equilibria of the second stage subgames into the joint market equilibrium and exclusive market equilibrium according to the characteristics of the equilibrium outcomes. The equilibrium in which the incumbent and the entrant shares the common market is referred to as the joint market equilibrium, and the equilibrium in which the entrant monopolizes the common market is referred to as the exclusive market equilibrium. Any pure strategy equilibrium is either a joint market equilibrium or an exclusive market equilibrium because there is no equilibrium in which the entrant does not sell at all. Proposition 1 summarizes the joint market equilibrium and exclusive market equilibrium.

Propostion 1. *There exists the unique joint market equilibrium* $\left(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq}\right)$ *if and only if q and r satisfy* $\pi_I\left(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq}\right) \geq \frac{1-r}{4}$. *There exists the unique exclusive market equilibrium* $\left(\frac{1}{2}, q - \frac{1}{2}\right)$ *if and only if q and r satisfy* $\pi_I\left(\frac{1}{2} - \frac{r}{4(1-q+rq)}, q - \frac{1}{2}\right) \leq \frac{1-r}{4}$. *There does not exist values of q and r* *with which both a joint market equilibrium and an exclusive market equilibrium exist.*

Proof. See Appendix A. □

Because $\pi_I(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq}) \geq \frac{1-r}{4}$ if and only if $q \leq F(r) \equiv \frac{20-12r-4\sqrt{1+3r}}{24-33r+9}$, there exists a joint market equilibrium if and only if $q \leq F(r)$, where $F(0) = \frac{2}{3}$, $F^{'}(r) > 0$ and $F(1) = 1$. There exists a non-empty set of (q,r) such that if the entrant chooses (q,r) in the set, then the Nash equilibrium of the second stage subgame is an exclusive market equilibrium because for any $q \in \left(\frac{2}{3}, 1\right)$, there exists an exclusive market equilibrium if r is arbitrarily small. A non-binding upper bound of r for an exclusive market equilibrium is $\frac{1}{6}$ because $\pi_I\left(\frac{1}{2}-\frac{r}{4(1-q+rq)}, q-\frac{1}{2}\right) > \frac{1-r}{4}$ if $r \geq \frac{1}{6}$.

If there does not exist a pure strategy equilibrium, then there exists a mixed strategy equilibrium because the second stage subgame is continuous when $q < 1$. Proposition 2 gives a non-degenerate mixed strategy equilibrium

Propostion 2. *Suppose that $F(r) < q < 1$ and $\pi_I\left(\frac{1}{2}-\frac{r}{4(1-q+rq)}, q-\frac{1}{2}\right) > \frac{1-r}{4}$. Then, there exists a Nash equilibrium in which the incumbent and the entrant set prices p_I and p_E randomly with the probability distribution functions $F_I(p_I)$ and $F_E(p_E)$, respectively. The support of $F_I(p_I)$ is $\left[\frac{1-\sqrt{r}}{2}, \frac{2q-1}{2q}\right] \cup \left\{\frac{1}{2}\right\}$, $F_I(p_I) \equiv H_I(p_I)$ for $p_I < \frac{2q-1}{2q}$, $F_I(p_I) \equiv 1 - \frac{(1+\sqrt{r})(2q-(1+\sqrt{r}))}{2q-1}$ for $\frac{2q-1}{2q} \leq p_I < \frac{1}{2}$ and $F_I(p_I) \equiv 1$ for $p_I \geq \frac{1}{2}$, and $H_I(p_I)$ is solution of the following first order differential equation with initial condition $H_I(\frac{1-\sqrt{r}}{2}) = 0$:*

$$
\begin{aligned}
&Arp_E\left(1-\frac{p_E}{q}\right)+Brp_E\left(\frac{b_I-p_E}{1-q}-\frac{p_E}{q}\right) \\
&-\left(1-A-B-G_I\left(\frac{p_E}{q}\right)\right)\frac{rp_E^2}{q(1-q)}+\frac{rp_E}{1-q}\int_{\frac{p_E}{q}}^{b_I} p_I dH_I(p_I) \\
&=\frac{r(1+\sqrt{r})\,(2q-(1+\sqrt{r}))}{4q},
\end{aligned}
$$

where $A \equiv \frac{(1+\sqrt{r})(2q-(1+\sqrt{r}))}{2q-1}$ and $B \equiv \frac{q^2(1-q)(1-2b_E)(1-F_I(b_I))}{b_E}$.[7]

[7] $F_I(p_I)$ has an atom with measure A at $p_I = \frac{1}{2}$ and another atom with measure B at $p_I = \frac{2q-1}{2q}$.

$F_E(p_E)$ is a solution of the following first order differential equation with initial condition $F_E(\frac{q(1-\sqrt{r})}{2}) = 0$:

$$F_E\left(qp_I\right)\left(\left(1-r\right)p_I\left(1-p_I\right)+rp_I\left(1-\frac{p_I}{1-q}\right)\right)$$
$$+\frac{rp_I}{1-q}\int_{a_E}^{qp_I} p_E\, dF_E\left(p_E\right)+\left(1-F_E\left(qp_I\right)\right)p_I\left(1-p_I\right)=\frac{1-r}{4}.$$

Expected profits of the incumbent and the entrant in the Nash equilibrium are $\frac{1-r}{4}$ and $\frac{r(1+\sqrt{r})(2q-(1+\sqrt{r}))}{4q}$, respectively.

Proof. See Appendix B. □

Now, we consider the Nash equilibrium of the second stage game when the entrant chooses $q = 1$ in the first stage so that its product becomes identical to the incumbent's product. Then, if $r = 1$, the unique Nash equilibrium is the Bertrand-Nash equilibrium. Hereafter, we assume that $r < 1$. We also assume that when $p_I = p_E$, αr of the r consumers with $v \geq p_1$ purchase the entrant's product and rest of those consumers with $v \geq p_1$ purchase the entrant's product for some α satisfying $0 < \alpha < 1$. Then,

$$d_I(p_I,p_E) = \begin{cases} 1-p_I & \text{if } p_I < p_E \\ (1-\alpha r)(1-p_I) & \text{if } p_I = p_E \\ (1-r)(1-p_I) & \text{if } p_I > p_E, \end{cases} \tag{3}$$

$$d_E(p_I,p_E) = \begin{cases} r(1-p_E) & \text{if } p_E < p_I \\ \alpha r(1-p_E) & \text{if } p_E = p_I \\ 0 & \text{if } p_E > p_I. \end{cases} \tag{4}$$

When a firm charges a positive price, its rival always has incentive to undercut its price. Therefore, there is no pure strategy Nash equilibrium in which a firm charges positive price. However, both firms' setting prices to zero is not a Nash equilibrium because the incumbent can sell at some positive price to consumers whose preferences are matched only by its product. Therefore, the second stage subgame does not have a pure strategy Nash equilibrium.

The second stage subgame has a unique mixed strategy Nash equilibrium. The incumbent will not charge prices outside the interval $[\frac{1-\sqrt{r}}{2}, \frac{1}{2}]$ because charging prices outside the interval is strictly dominated by charging the price of $\frac{1}{2}$. After deleting the incumbent's strictly dominated strategies, the entrant will not charge price outside the interval $[\frac{1-\sqrt{r}}{2}, \frac{1}{2}]$ because charging prices outside the interval is strictly dominated by charging the price of $\frac{1-\sqrt{r}}{2}$. For the entrant to choose a price slightly lower than $\frac{1}{2}$, there must be an atom at price $\frac{1}{2}$ in the incumbent's mixed strategy. This situation is very similar to the Edgeworth – Bertrand game with asymmetric capacities, whose unique mixed strategy equilibrium has been derived by Levitan and Shubik (1972) for special cases, and by Osborne and Pitchik (1986) for more general cases. The unique Nash equilibrium and the two firms' expected profits in the Nash equilibrium are presented in Proposition 3.

Propostion 3. *Suppose that $q = 1$ and $r < 1$. Then, there exists the unique Nash equilibrium in which the incumbent and the entrant set prices p_I and p_E randomly with the probability distribution functions $G_I(p_I)$ and $G_E(p_E)$, respectively, where $G_I(p_I) \equiv \frac{p_I(1-p_I)-\frac{1-r}{4}}{p_I(1-p_I)}$ for $p_I < \frac{1}{2}$, $G_I(p_I) \equiv 1$ for $p_I \geq \frac{1}{2}$, and $G_E(p_E) \equiv \frac{p_E(1-p_E)-\frac{1-r}{4}}{rp_E(1-p_E)}$.[8] The common support of $G_I(p_I)$ and $G_E(p_E)$ is $[\frac{1-\sqrt{r}}{2}, \frac{1}{2}]$. Expected profits of the incumbent and the entrant in the Nash equilibrium are $\frac{1-r}{4}$ and $\frac{r(1-r)}{4}$, respectively.*

Proof. See Appendix C. □

3. ENTRANT'S STRATEGIC CHOICE OF PRODUCT QUALITY AND MARKET SIZE

We partition the set of (q,r) into four sets JME, EME, MSE and BNE in which the Nash equilibrium of second stage subgame is the joint market equilibrium, exclusive market equilibrium, mixed strategy equilibrium and Bertrand-Nash equilibrium, respectively. Let $\pi_I^*(q,r)$ and $\pi_E^*(q,r)$ denote the expected profit of the incumbent and the entrant, respectively in the Nash equilibrium of the second stage subgame when the entrant has chosen the value (q,r) in the first

[8] $G_I(p_I)$ has an atom $1-r$ at $p_I = \frac{1}{2}$.

stage.[9] Then,

$$\pi_I^*(q,r) = \begin{cases} \frac{4(1-q)(1-q+rq)}{(4(1-q)+3rq)^2} & \text{if } (q,r) \in \text{JME} \\ \frac{1-r}{4} & \text{if } (q,r) \in \text{EME} \\ \frac{1-r}{4} & \text{if } (q,r) \in \text{MSE} \\ 0 & \text{if } (q,r) \in \text{BNE}, \end{cases} \tag{5}$$

$$\pi_E^*(q,r) = \begin{cases} \frac{rq(1-q)}{(4(1-q)+3rq)^2} & \text{if } (q,r) \in \text{JME} \\ \frac{r(2q-1)}{4q} & \text{if } (q,r) \in \text{EME} \\ \frac{r(1+\sqrt{r}(2q-(1+\sqrt{r}))}{4q} & \text{if } (q,r) \in \text{MSE} \\ 0 & \text{if } (q,r) \in \text{BNE}. \end{cases} \tag{6}$$

In (5) and (6), $\pi_I^*(q,\ r)$ is continuous but $\pi_E^*(q,\ r)$ has a discontinuity on the graph of $q = F(r)$ which is the boundary of JME. Both $\pi_I^*(q,\ r)$ and $\pi_E^*(q,\ r)$ are continuous in JME$\cup$EME because a mixed strategy equilibrium near the boundary of JME concentrates probability in a small neighborhood of a near exclusive market equilibrium on the boundary. In contrast, a mixed strategy equilibrium near the boundary of JME has totally different support with a near joint market equilibrium at the boundary. However, $\pi_I^*(q,\ r)$ is continuous because the incumbent is indifferent between staying in the joint market equilibrium and raising price to $\frac{1}{2}$ while $\pi_E^*(q,r)$ is discontinuous on the boundary of JME because the entrant's profit jumps up when the incumbent raises its price to $\frac{1}{2}$. In EME$\cup$MSE, $\pi_I^*(q,\ r)$ is constant over q because the equilibrium expected profit of the incumbent's does not depend on q, but $\pi_E^*(q,\ r)$ strictly increases in respect to q.

There can be situations in which the entrant can choose only one of the two variables or can choose the variables in a subset of $(0,\ 1]^2$. Let $q^*(r)$ denote the value of q which maximizes $\pi_E^*(q,r)$ for given r, and $r^*(q)$ denote the value of r which maximizes $\pi_E^*(q,r)$ for a given q. Proposition 4 gives $q^*(r)$ and $r^*(q)$.

Propostion 4. *(i)* $q^*(r) = \begin{cases} 1 & \text{if } r < \frac{3+\sqrt{6}}{6} \\ \left\{1, \frac{4}{4+3r}\right\} & \text{if } r = \frac{3+\sqrt{6}}{6} \\ \frac{4}{4+3r} & \text{if } r > \frac{3+\sqrt{6}}{6} \end{cases}$.

[9] We have not proved that there does not exist a non-degenerate mixed strategy equilibrium when there is a joint market equilibrium or an exclusive market equilibrium. Neither have we proved uniqueness of the mixed strategy equilibrium in Proposition 2. We assume the uniqueness of Nash equilibrium in the second stage subgames.

(ii) $r^*(1) = \frac{1}{2}$ *and* $r^*(q) = \begin{cases} 1 & \text{if } q \le \frac{4}{7} \\ \frac{4(1-q)}{3q} & \text{if } \frac{4}{7} \le q \le \frac{2}{3} \end{cases}$.

Proof. See Appendix D. □

Proposition 4 (i) does not mean $\pi_E^*(q,r)$ monotonically increase in q when $r < \frac{3+\sqrt{6}}{6}$. When the entrant chooses q from $(0,Q]$ for some $Q < 1$, it may not be optimal for the entrant to choose Q even when $r \le \frac{3+\sqrt{6}}{6}$. Proposition 4(i) shows that $q^*(1) = \frac{4}{7}$ as derived in Choi and Shin (1992).

By (i) of Proposition 4,

$$\pi_E^*(q^*(r),r) = \begin{cases} \frac{r(1-r)}{4} & \text{if } r \le \frac{3+\sqrt{6}}{6} \\ \frac{1}{48} & \text{if } r \ge \frac{3+\sqrt{6}}{6} \end{cases}. \quad (7)$$

Because $\pi_E^*(q^*(r),r)$ is maximized when $r = \frac{1}{2}$, Proposition 5 follows.

Propostion 5. *In the subgame perfect equilibrium of the two stage game, the entrant chooses* $q = 1$ *and* $r = \frac{1}{2}$ *in the first stage.*

Proof. Omitted. □

Limitation of market size with full quality is optimal entry strategy because entry with low quality shrinks market size as well as valuation. But the result depends on the assumption that cost is independent of quality. If higher quality incurs more cost, optimal entry strategy may become mixture of market size limitation and quality limitation or even limitation of quality with full market size. However, our result under independent cost assumption serves as a benchmark for more realistic analyses.

4. CONCLUSION

We have investigated an entrant's optimal choice of quality and market size by deriving a subgame perfect equilibrium of two stage games in which the entrant chooses its product quality and product market size in the first stage and competes with the incumbent in price in the second stage. We find that the entrant's optimal strategy in the first stage involves only the limitation of market size. When quality is given, an entrant with lower quality than a critical level does not need to limit market size but an entrant with higher quality than the critical level needs to limit market size. When market size is given, an entrant with a smaller market than a critical size does not need to limit quality but an entrant with a bigger market than the critical level needs to limit quality.

APPENDIX A

Proof of Propostion 1. (i) The only candidate for joint market equilibrium is $(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq})$ which is the unique Nash equilibrium if the demand functions of the incumbent and the entrant, respectively, are

$$d_I(p_I, p_E) = (1-r)(1-p_I) + r(1 - \frac{p_I - p_E}{1-q}),$$
$$d_E(p_I, p_E) = r\left(\frac{p_I - p_E}{1-q} - \frac{p_E}{q}\right).$$

The entrant does not have incentive to deviate from $(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq})$. The incumbent does not have incentive to deviate from $(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq})$ if and only if $\pi_I(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq}) \geq \frac{1-r}{4}$.

(ii) The incumbent does not have incentive to charge a price higher than $\frac{1}{2}$ in any case. Moreover, in an exclusive market equilibrium, the incumbent does not have incentive to charge a price lower than $\frac{1}{2}$ because it does not sell in the common market. Therefore, in an exclusive market equilibrium, the incumbent charges price $\frac{1}{2}$ and hence the entrant charges price $q - \frac{1}{2}$ beacuse $q - \frac{1}{2}$ is less than $\frac{q}{2}$ which the profit maximizing price of the entrant when it monopolizes the common market. Because the entrant does not have incentive to deviate from $(\frac{1}{2}, q-\frac{1}{2})$, $(\frac{1}{2}, q-\frac{1}{2})$ is the unique exclusive market equilibrium if and only if the incumbent does not have incentive to deviate from $(\frac{1}{2}, q-\frac{1}{2})$. If the incumbent does not have incentive to deviate from $(\frac{1}{2}, q-\frac{1}{2})$ by chaging the price to either $\frac{2q-1}{2q}$ or $\frac{1}{2} - \frac{r}{4(1-q+rq)}$, it does not have incentive to deviate from $(\frac{1}{2}, q-\frac{1}{2})$ at all. However, $\pi_I(\frac{2q-1}{2q}, q-\frac{1}{2}) > \frac{1-r}{4}$ whenever $\pi_I(\frac{2q-1}{2q}, q-\frac{1}{2}) \geq \pi_I(\frac{1}{2} - \frac{r}{4(1-q+rq)}, q-\frac{1}{2})$, that is, there is no exclusive market equilibrium when $\frac{2q-1}{2q}$ is best response of the incumbent to the entrant's price $q-\frac{1}{2}$. Therefore, $(\frac{1}{2}, q-\frac{1}{2})$ is the unique ve market equilibrium if and only if $\pi_I(\frac{1}{2} - \frac{r}{4(1-q+rq)}, q-\frac{1}{2}) \leq \frac{1-r}{4}$.

(iii) Whenever $\pi_I(\frac{1}{2} - \frac{r}{4(1-q+rq)}, q-\frac{1}{2}) \leq \frac{1-r}{4}$, $q-\frac{1}{2} > \frac{q(1-q)}{4(1-q)+3rq}$ and hence $\pi_I(\frac{2q-1}{2q}, q-\frac{1}{2}) > \pi_I(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq})$ and $\pi_I(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq}) < \frac{1-r}{4}$. In words, the entrant's price is lower in the only candidate of a joint market equilibrium $(\frac{2(1-q)}{4(1-q)+3rq}, \frac{q(1-q)}{4(1-q)+3rq})$ than in an exclusive market equilibrium, and hence the incumbent has incentive to deviate from the candidate by charging a price of $\frac{1}{2}$ whenever there exists an exclusive market equilibrium. Therefore, there does not exist values of q and r with which both a joint market equilibrium and an exclusive market equilibrium exist. □

APPENDIX B

Proof of Propostion 2. When $F(r) < q < 1$ and $\pi_I(\frac{1}{2}-\frac{r}{4(1-q+rq)}, q-\frac{1}{2}) > \frac{1-r}{4}$, there does not exist a pure strategy equilibrium and hence there exists a mixed strategy equilibrium because the second stage subgame is continuous when q < 1. The incumbent would not charge prices above $\frac{1}{2}$ because charging the prices is strictly dominated by charging the price $\frac{1}{2}$. Consider the two firms' reactions starting from the incumbent's charging the price $\frac{1}{2}$. If the incumbent charges price $\frac{1}{2}$, the entrant would charge $\widehat{p}_E(\frac{1}{2}) = \frac{2q-1}{2}$ and then, the incumbent charges $\widehat{p}_I(\widehat{p}_E(\frac{1}{2})) = \frac{\widehat{p}_E(\frac{1}{2})}{q} = \frac{2q-1}{2q}$. As the entrant lowers its price from $q-\frac{1}{2}$ to some price p the incumbent also lowers its price from $\frac{2q-1}{2q}$ to $\widehat{p}_I(p) = \frac{p}{q}$ until the entrant charges $\frac{q(1-\sqrt{r})}{2}$ at which the incumbent is indifferent in lowering its price to $\frac{1-\sqrt{r}}{2}$ and raising its price to $\frac{1}{2}$. This situation is very similar to the Edgeworth – Bertrand game. Because the incumbent's profit is $\frac{1-r}{4}$ when it charges price $\frac{1}{2}$ and any price the entrant would charge in the mixed strategy equilibrium, its expected profit must be $\pi_I^m \equiv \frac{1-r}{4}$. Let $F_I(p_I)$ and $F_E(p_E)$ denote the incumbent's mixed strategy and the entrant's strategy, respectively, with supports $[a_I, b_I] \cup \{\frac{1}{2}\}$ and $[a_E, b_E]$ denote the supports of $F_I(p_I)$ and $F_E(p_E)$, respectively, in the mixed strategy equilibrium, where $a_I = \frac{1-\sqrt{r}}{2}$, $b_I = \frac{2q-1}{2q}$, $a_E = \frac{q(1-\sqrt{r})}{2}$ and $b_E = \frac{2q-1}{2}$.

The entrant's expected profit in the mixed strategy is $\pi_E^m \equiv \frac{r(1+\sqrt{r})(2q-(1+\sqrt{r}))}{4q}$ because when the entrant charges $\frac{q(1-\sqrt{r})}{2}$ it monopolizes the common market with probability 1. Therefore, $F_I(p_I)$ has an atom with measure $A \equiv (1+\sqrt{r}) \times\frac{(2q-(1+\sqrt{r}))}{2q-1}$ at $p_I = \frac{1}{2}$ for the entrant's expected profit to be $\frac{r(1+\sqrt{r})(2q-(1+\sqrt{r}))}{4q}$ when the entrant chages price $\frac{2q-1}{2}$. For the entrant to lower its price from b_E, $F_I(p_I)$ has another atom at b_I. When the entrant decrases its price by a small positive amount dp from b_E and the incumbent wuuses the mixed strategy $F_I(p_I)$, the change in the entrant's expected profit is $d\pi_E \equiv -r((1-2b_E)dp + dp^2) \times(1-F_I(b_I)) + \int_{b_I-\frac{dp}{q}}^{b_I} \frac{rb_E dp}{q(1-q)}\, dF_I(p_I)$ for the ent rant to charge price $b_I - dp$ as well as b_I for arbitrarily small positive dp we need $\lim_{dp\downarrow 0}\frac{d\pi_E}{dp} = 0$, that is, we need $F_I(p_I)$ has an atom of measure $B \equiv \frac{q^2(1-q)(1-2b_E)(1-F_I(b_I))}{b_E}$. Therefore,

$$F_I = \begin{cases} G_I(p_I) & \text{if } p_I < b_I \\ 1-A & \text{if } b_I \le p_I < \frac{1}{2} \\ 1 & \text{if } p_I \ge b_I \end{cases},$$

where $\frac{G_I(p_I)}{1-A-B}$ is a probability distribution function whose support is $[a_I, b_I]$.

Because the incumbent must be indifferent in charging prices in the support of $F_I(p_I)$,

$$\int_{a_E}^{qp_I} \left((1-r)\, p_I\,(1-p_I) + rp_I \left(1 - \frac{p_I - p_E}{1-q}\right)\right) dF_E(p_E) + \int_{qp_I}^{b_E} p_I\,(1-p_I) dF_E(p_E) = \pi_I^m, \tag{B1}$$

for all $p_I \in [a_I, b_I] \cup \left\{\frac{1}{2}\right\}$.

Similarly, we have

$$A\ r\ p_E \left(1 - \frac{p_E}{q}\right) + B\ r\ p_E \left(\frac{b_I - p_E}{1-q} - \frac{p_E}{q}\right) + \int_{\frac{p_E}{q}}^{b_I} r\ p_E \left(\frac{p_I - p_E}{1-q} - \frac{p_E}{q}\right) dG_I(p_I) = \pi_E^m, \tag{B2}$$

for all $p_E \in [a_E, b_E]$.

(B1) and (B2) can be rewritten as:

$$F_E(qp_I)\left((1-r)\, p_I\,(1-p_I) + rp_I\left(1 - \frac{p_I}{1-q}\right)\right) + \frac{rp_I}{1-q}\int_{a_E}^{qp_I} p_E\ dF_E(p_E) + (1 - F_E(qp_I))\, p_I\,(1-p_I) = \pi_I^m, \text{ for all } p_I \in [a_I, b_I] \cup \{\tfrac{1}{2}\}; \tag{B3}$$

$$A\ r\ p_E\left(1 - \frac{p_E}{q}\right) + B\ r\ p_E\left(\frac{b_I - p_E}{1-q} - \frac{p_E}{q}\right) - \left(1 - A - B - G_I\left(\frac{p_E}{q}\right)\right)\frac{r{p_E}^2}{q(1-q)} + \frac{rp_E}{1-q}\int_{\frac{p_E}{q}}^{b_I} p_I dH_I(p_I) = \pi_E^m, \text{ for all } p_E \in [a_E, b_E]. \tag{B4}$$

Finally, it can be easily checked, by reasoning, that the incumbent does not have incentive to charge a price outside the support of $F_I(p_I)$ when the entrant uses the mixed strategy $F_E(p_E)$, and the entrant does not have incentive to charge a price outside the support of $F_E(p_E)$ when the incumbent uses the mixed strategy $F_I(p_I)$. □

APPENDIX C

Proof of Propostion 3. Suppose that the entrant uses the mixed strategy $G_E(p_E)$. Then, the incumbent's expected profit is $\frac{1-r}{4}$ when the incumbent charges price in the interval $[\frac{1-\sqrt{r}}{2}, \frac{1}{2}]$ while it is less than $\frac{1-r}{4}$ if the incumbent charges price outside the interval. Now, suppose that the incumbent uses the mixed strategy $G_I(p_I)$. Then, the entrant's expected profit is $\frac{r(1-r)}{4}$ when the entrant charges price in the interval $[\frac{1-\sqrt{r}}{2}, \frac{1}{2})$ while it is less than $\frac{r(1-r)}{4}$ if the entrant charges price outside the interval. Therefore, $G_I(p_I)$ and $G_E(p_E)$ constitute a Nash equilibrium. We omit the proof for the uniqueness of Nash equilibrium in the second stage price setting game because it can be constructed by following Osborne and Pitchik (1986). □

APPENDIX D

Proof of Propostion 4. (i) When $r \leq \frac{5}{12}$, $q^*(r) = 1$ because $\pi_E^*(q,r)$ is strictly increacsing in q. When $\frac{5}{12} \leq r < \frac{3+\sqrt{6}}{6}$, $q^*(r) = 1$ because $\pi_E^*(q,r) \leq \frac{1}{48}$ if $(q,r) \in$ JME; $\pi_E^*(q,r)$ strictly increases in q if $(q,r) \notin$ JME; and $\pi_E^*(1,r) > \frac{1}{48}$ if $\frac{3-\sqrt{6}}{6} < r < \frac{3+\sqrt{6}}{6}$. When $r > \frac{3+\sqrt{6}}{6}$, $q^*(r) = \frac{4}{4+3r}$ because $\pi_E^*(\frac{4}{4+3r}, r) = \frac{1}{48}$ if $r \geq \frac{5}{12}$; $\pi_E^*(q,r) < \frac{1}{48}$ if $(q,r) \in$ JME and $q \neq \frac{4}{4+3r}$; $\pi_E^*(q,r)$ strictly increases in q if $(q,r) \notin$ JME; and $\pi_E^*(1,r) < \frac{1}{48}$ if $r > \frac{3+\sqrt{6}}{6}$. Finally, when $r = \frac{3+\sqrt{6}}{6}$, $q^*(r) = \frac{4}{4+3r}$ and 1 because $\pi_E^*(\frac{4}{4+3r}, r) = \frac{1}{48}$ if $r = \frac{3+\sqrt{6}}{6}$; $\pi_E^*(q,r) < \frac{1}{48}$ if $(q,r) \in$ JME and $q \neq \frac{4}{4+3r}$; $\pi_E^*(q,r)$ strictly increases in q if $(q,r) \notin$ JME; and $\pi_E^*(1,r) = \frac{1}{48}$ if $r = \frac{3+\sqrt{6}}{6}$. (ii) Because $\pi_E^*(1,r) = \frac{r(1-r)}{4}$, $r^*(1) = \frac{1}{2}$. The remaining holds because $\pi_E^*(q,r) = \frac{rq(1-q)}{(4(1-q)-3rq)^2}$ if $q \leq \frac{2}{3}$. □

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