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Coarse Information Leads to Less Effective Signaling*

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Abstract This study considers firms' coarse information about a worker's possible types in Spence's (1973) job market signaling model. Using incentive compatibility constraints appropriate to coarse information, we derive perfect Bayesian equilibria, which are refined into a unique equilibrium by invoking an extension of Cho and Kreps' (1987) Intuitive Criterion. In the unique refined equilibrium, a high-type worker may acquire a higher education level with a lower wage than in Spence's (1973) model. This implies that education signaling may be less effective signal when firms have coarse information about a worker's possible types compared to that in Spence (1973).

Keywords Job market signaling, Coarse information, Perfect Bayesian equilibrium, Extended Intuitive Criterion

JEL Classification C72, D82, D83, J30

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1. INTRODUCTION

One of the most important issues facing firms is hiring the "right" people. In labor markets, a worker's education level can be a useful signal to attract firms. Spence (1973) proposes a job market signaling model, in which firms cannot directly observe the productivities of workers prior to hiring and they are able to signal the productivities by their education level. Spence (1973) shows that the high-type and the low-type workers can be separated by the firms if they pick an appropriate cutoff education level.

In the previous literature on job market signaling, it is assumed that firms have exact information about a worker's possible types (e.g., abilities or productivities) but faces uncertainty about his true types. However, we think this assumption is quite strong in that, in the real world, it is very difficult for firms to exactly know a worker's possible productivities. Then it is natural to ask what happens if firms may have *coarse information* about a worker's possible types. For instance, even if firms believe that a worker has high ability, they may not exactly know what his true ability is but rather only know that it belongs to a *high-type set* which contains the worker's true ability (see Figure 1 in Section 2). In this sense, we say that firms have *coarse information* about the worker's possible types. Incorporating firms' coarse information into Spence's (1973) job market signaling model, we derive perfect Bayesian equilibria and refine them into a unique separating equilibrium by invoking an extension of Cho and Kreps' (1987) Intuitive Criterion. In the refined equilibrium, a high-type worker may acquire a higher education level with a lower wage than in the model of Spence (1973). In this sense, education signaling may be less effective when firms have coarse information about the worker's possible productivities than otherwise.[1]

Related Literature

Since the seminal works of Spence (1973, 1974), the literature on job-market signaling model (more generally, signaling game) has significantly grown and gained broad acceptance (see Kreps and Sobel (1994) and Riley (2001) for lit-

[1] To the best of our knowledge, there is no job-marketing signaling model à la Spence (1973) where firms face coarse information about a worker's possible types.

erature review). Unfortunately Spence's job-market signaling model is plagued with a continuum of equilibria. To rule out unreasonable equilibria of the Spence model, many refinement concepts have been developed in game theory.[2] Among others, the intuitive criterion of Cho and Kreps (1987) eliminates all separating and pooling equilibria but a unique least-cost separating equilibrium (aka. Riley equilibrium after Riley (1979)).[3] As such, the intuitive criterion plays a role of cornerstone in the analysis of signaling games including the Spence model.

However, their result is vulnerable to dynamic modifications of the Spence model. Weiss (1983) and Admati and Perry (1987) point out that if workers cannot commit to an education level and firms can make wage offers before workers complete their education, the Spence's results do not hold. Reflecting their criticisms in a dynamic model with a sequence of wage offers (public or private), Noldeke and van Damme (1990) and Swinkels (1999) find that the conclusion of Cho and Kreps (1987) no longer hold.[4] This problem also permeate into a dynamic Spence model with learning. Assuming that the employer learns about the worker's true productivity from on-the-job performance, Alós-Ferrer and Prat (2012) conclude that the Riley equilibrium still remains as the only separating equilibrium surviving the intuitive criterion, whereas there may be a plethora of pooling equilibria which survive the intuitive criterion.[5] This leads them to conjecture that any attempt to further refine the equilibrium set using stronger refinement criteria such as divinity, D1, D2, universal divinity, and NWBR must impose additional restrictions. In a job-market signaling game with

[2]These include, among others, divinity and universal divinity Banks and Sobel (1987), intuitive criterion, D1, and D2 of Cho and Kreps (1987), NWBR (never-a-weak-best-response) criterion and strategic stability of Kohlberg and Mertens (1986), perfect sequentiality of Grossman and Perry (1987), and undefeated criterion of Mailath et al. (1993).

[3]If there are more than two types of the worker, D1 instead of the intuitive criterion should be applied to obtain the Riley equilibrium (see Cho and Sobel (1990)).

[4]With making public wage offers, Noldeke and van Damme (1990) show that, for a given positive period length Δ between wage offers, there is a unique sequential equilibrium surviving NWBR, which converges to the Riley equilibrium as $\Delta \to 0$. Considering private wage offers and applying no refinements, Swinkels (1999) obtains the unique sequential equilibrium for a given positive Δ and a unique pooling equilibrium at no education as $\Delta \to 0$.

[5] Our model can be connected with Alós-Ferrer and Prat (2012) in that firms have coarse information in signaling stage, which afterwards gives room for learning about the worker's true productivity.

grades, Daley and Green (2014) face a similar issue and show that, when grades are RC-informative, D1 rules out all but one equilibrium, which is, however, not the Riley equilibrium that fails D1 under RC-informativeness.

Feltovich et al. (2002) incorporate extra noisy information on types into the Spence model with three types, and show that high types may choose to not signal (or countersignal) for differentiating themselves from median types.[6] This idea is generalized by Araujo et al. (2007), who allows for a two-dimensional type space (cognitive and noncognitive types) with a single signaling instrument. In multidimensional signaling setting, Kohlleppel (1983) gives an example where complete separation is not possible, while Quinzii and Rochet (1985) provides sufficient conditions for a separating equilibrium to exist.[7]

2. THE MODEL

We allow for coarse information in the model of Spence (1973). There is a risk-neutral worker who can be one of two types: high type or low type. The low-type (hight-type) worker has productivity θ_L (θ_H, respectively) with $\theta_L < \theta_H$. The worker has private information about his own ability and then, to signal his own type, he acquires education level e with cost e/θ_i for $i = L, H$. As in Spence (1973), we assume that the worker's ability does not depend on his education level.[8]

There are two risk-neutral firms (employers) that can hire the worker, and the worker's ability is not known to the firms. The firms only observe the worker's education level. Each firm produces output θ in monetary terms employing only labor if it employs a worker with productivity θ. If the firms competitively offer wages to the worker, the worker accepts the higher of the two wage offers with the usual random tie-breaking decision. Bertrand competition between the two firms will drive down expected profits to zero. For simplicity, they are henceforth

[6]Hertzendorf (1993) considers a signaling model with two signal, one of which is noisy, and two types of senders, which precludes countersignaling.

[7] In a market with asymmetric information, Engers (1987) obtains weaker sufficient conditions for the same result.

[8]The "dependent" case (e.g., Spence, 1974) can be easily accommodated in our model without changing the main implication.

represented by a single firm with zero profit.

To reflect coarse information about the worker's possible types, we assume that the worker's productivity lies in the set $\Theta \equiv \Theta_L \cup \Theta_H$, where $\theta_L \in \Theta_L = [\underline{\theta}_L, \bar{\theta}_L]$ and $\theta_H \in \Theta_H = [\underline{\theta}_H, \bar{\theta}_H]$ with $\underline{\theta}_L < \bar{\theta}_L < \underline{\theta}_H < \bar{\theta}_H$.[9] The firm only perceives that low-type (high-type) θ_L (θ_H) belongs to *low-type set* Θ_L (*high-type set* Θ_H, respectively), so that it regards Θ_L and Θ_H as the worker's possible "types." In this sense, the firm has *coarse information* about the worker's possible types. Thus, the worker and the firm have different type spaces T_ω and T_f for the worker, respectively, where $T_\omega = \{\theta_L, \theta_H\} \neq \{\Theta_L, \Theta_H\} = T_f$.[10] This assumption is the main difference from those of Spence (1973). The firm has prior belief μ about the worker's types, which is a *uniform distribution* on Θ with $\mu(\Theta_L) = q$. The worker strategically decides education level $e \in [0, E]$ and after observing e, the firm offers him a wage $w \in [0, W]$, where E and W are sufficiently large numbers.

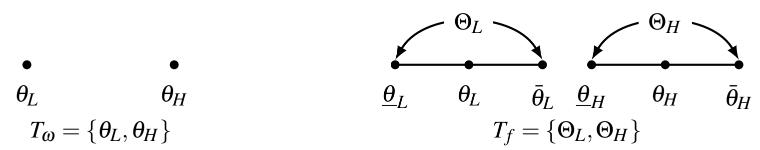

Figure 1: Type spaces of the worker and the firm

Now we define the (expected) payoff functions of the agents. When type-θ worker acquires education level e and receives wage w, his utility is given by

$$U(\theta, e, w) = w - \frac{e}{\theta}.$$

After observing the worker's education level e, the firm updates its belief about the worker's types and has (interim) expected profit:

$$V(e, w) = \sum\nolimits_{t \in T_f} \mu(t|e) \mathbb{E}_\mu[\theta|t] - w.$$

[9] One may consider the case where $\bar{\theta}_L = \underline{\theta}_H$. However, it does not change our main implication.

[10] In Spence (1973), the firm has exact information about the worker's possible types and so $T_\omega = T_f = \{\theta_L, \theta_H\}$.

where $\mathbb{E}_\mu[\cdot|t]$ is a conditional expectation operator given $t \in T_f$ under probability measure ν.

3. PERFECT BAYESIAN EQUILIBRIUM

Adopting perfect Bayesian equilibrium (PBE) as a solution concept in the signaling game between the worker and the firm, we derive separating and pooling PBEs.[11] We define perfect Bayesian equilibrium below when the firm has coarse information about the worker's possible types.

Definition 1. *Let μ is a prior on T_f such that $\mu(\Theta_L) = q$. A profile of pure strategies $(\tilde{e}, \tilde{w})$ and a system of posterior beliefs $\mu(\cdot|e)$ is a perfect Bayesian equilibrium (PBE) if it satisfies the following:*

(i) $\forall t \in T_\omega$, $\tilde{e}(t) \in \operatorname{argmax}_{e\in[0,E]} \tilde{w}(e) - c(e,t)$.

(ii) $\forall e \in \mathbb{R}_+$, $\tilde{w}(e) \in \operatorname{argmax}_{w\in[0,W]} \sum_{t\in T_f} \mu(t|e)\mathbb{E}_\mu[\theta|t] - w$.

(iii) If $\sum_{t'\in T_f(e)} \mu(t') > 0$ *where* $T_f(e) = \{t \in T_f : \tilde{e}(t) = e\}$, *posterior beliefs are given by*

$$\mu(t|e) = \frac{\mu(t)}{\sum_{t'\in T_f(e)} \mu(t')}, \forall t \in T_f(e)$$

and if $\sum_{t'\in T_f(e)} \mu(t') = 0$, *then* $\mu(\cdot|e)$ *is any probability distribution on* $T_f(e)$.

Note that $T_\omega \neq T_f$. If $T_\omega = T_f := T = \{\theta_L, \theta_H\}$, Definition 1 reduces to the standard perfect Bayesian equilibrium.[12]

3.1. SEPARATING PBE

In a separating PBE, the type-θ_i worker chooses education level $\tilde{e}(\theta_i) = e_i^*$ with $e_H^* \neq e_L^*$ for $i = L,H$. If the firm observes the worker's education level e_i^*, then it believes that the worker's type belongs to Θ_i and offers him wage $\tilde{w}(e_i^*) = w_i^*$ for $i = L,H$. The zero expected profit condition for the firm implies

$$V(e_i^*, w_i^*) = \mathbb{E}_\mu[\theta|\Theta_i] - w_i^* = 0, \quad \text{for} \quad i = L,H, \tag{1}$$

[11] For simplicity, we focus only on pure strategy PBEs.

[12] More precisely, putting $T_\omega = T_f = T = \{\theta_L, \theta_H\}$ and replacing $\mathbb{E}_\mu[\theta|t]$ in (*ii*) and $T_f(e)$ in (*iii*) by t and $T(e) := \{t \in T : \tilde{e}(t) = e\}$, respectively, we obtain the standard PBE.

which yields

$$w_L^* = \frac{\underline{\theta}_L + \bar{\theta}_L}{2} \quad \text{and} \quad w_H^* = \frac{\underline{\theta}_H + \bar{\theta}_H}{2}. \tag{2}$$

It is clear that $\tilde{e}(\theta_L) = e_L^* = 0$ and the firm takes education level e_H^* as the cutoff education level for the high type, which satisfies the following incentive compatibility constraints:

$$\begin{aligned} U(\theta, e_H^*, w_H^*) = w_H^* - \frac{e_H^*}{\theta} \le w_L^* = U(\theta, 0, w_L^*),\ \forall \theta \in \Theta_L, \\ U(\theta, e_H^*, w_H^*) = w_H^* - \frac{e_H^*}{\theta} \ge w_L^* = U(\theta, 0, w_L^*),\ \forall \theta \in \Theta_H, \end{aligned} \tag{3}$$

which are equivalent to

$$\begin{aligned} U(\bar{\theta}_L, e_H^*, w_H^*) = w_H^* - \frac{e_H^*}{\bar{\theta}_L} \le w_L^*, \\ U(\underline{\theta}_H, e_H^*, w_H^*) = w_H^* - \frac{e_H^*}{\underline{\theta}_H} \ge w_L^*. \end{aligned}$$

This implies that $(w_H^* - w_L^*)\bar{\theta}_L \le e_H^* \le (w_H^* - w_L^*)\underline{\theta}_H$.[13] Then, we obtain the following result.

Proposition 2. *There are separating PBEs such that, in one of them, the education levels of type θ_L and type θ_H are*

$$\tilde{e}(\theta_L) = e_L^* = 0, \quad \textit{and} \quad \tilde{e}(\theta_H) = e_H^* \in [\underline{e}_H, \bar{e}_H],$$

where $\underline{e}_H = (w_H^* - w_L^*)\bar{\theta}_L$ and $\bar{e}_H = (w_H^* - w_L^*)\underline{\theta}_H$. The firms's posterior belief and offered wage are

$$\mu(\Theta_L|e) = \begin{cases} 1 & \textit{if } e < e_H^*, \\ 0 & \textit{if } e \ge e_H^*, \end{cases} \quad \textit{and} \quad \tilde{w}(e) = \begin{cases} w_L^* & \textit{if } e < e_H^*, \\ w_H^* & \textit{if } e \ge e_H^*, \end{cases} \tag{4}$$

with

$$w_L^* = \frac{\underline{\theta}_L + \bar{\theta}_L}{2} \quad \textit{and} \quad w_H^* = \frac{\underline{\theta}_H + \bar{\theta}_H}{2}.$$

[13] If $\bar{\theta}_L = \underline{\theta}_H$, there exists the unique separating PBE since the only education level $e_H^* = \bar{\theta}_L = \underline{\theta}^H$ satisfies incentive compatibility constraints (3), in which case, two shaded areas become tangential and the red line on the x-axis reduces to a single point in Figure 2 below. On the other hand, if $\bar{\theta}_L > \underline{\theta}_H$, separating PBE does not exist since there is no education level satisfying (3), in which case, two shaded areas become overlapped in Figure 2 below.

In Figure 2, we illustrate a separating equilibrium of Proposition 2. The left and right shaded areas indicate all possible indifference curves for $\theta \in \Theta_L$ and $\theta \in \Theta_H$, respectively. The firm picks cutoff education level e_H^* on the red line for the high type, which contains education levels satisfying incentive compatibility constraints (3). That is, the firm treats the worker whose education level is at least e_H^* as the high type and offers wage w_H^* and whose education level is less than e_H^* as the low type and offers wage w_L^*. This strategy is described by the blue step function. Therefore, in equilibrium, the low type chooses no education while the high type chooses e_H^*.

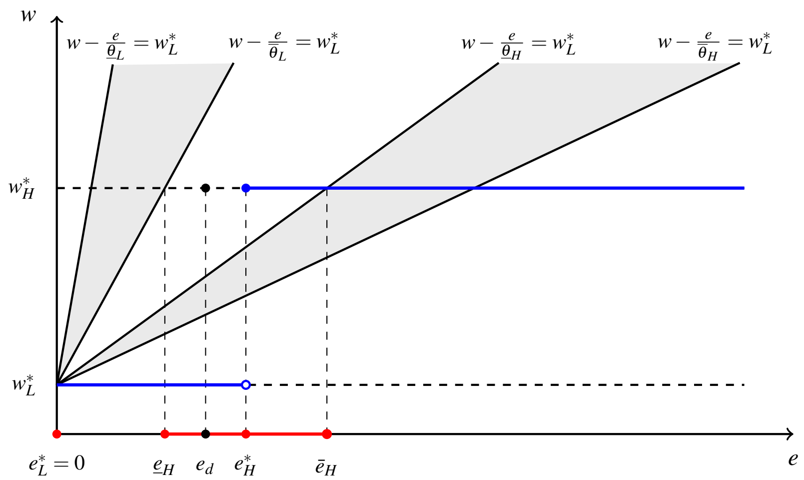

Figure 2: Education-wage pairs $\{(e_L^*, w_L^*), (e_H^*, w_H^*)\}$ depict a separating PBE in Proposition 2

3.2. POOLING PBE

In a pooling PBE, both types of the worker choose the same education level $\tilde{e}(\theta_L) = \tilde{e}(\theta_H) = e_p^*$ and the firm cannot distinguish the worker's types. Thus, keeping its prior belief, the firm offers the same wage $\tilde{w}(e_p^*) = w_p^*$ to both types. The zero expected profit condition implies

$$V(e_p^*, w_p^*) = \mu(\Theta_L)\mathbb{E}_\mu[\theta|\Theta_L] + \mu(\Theta_H)\mathbb{E}_\mu[\theta|\Theta_H] - w_p^* = 0,$$

which, by (3.1), yields

$$w_p^* = qw_L^* + (1-q)w_H^*.$$

To pool the worker's types, the firm selects cutoff education level e_p^*, which satisfies the following incentive compatibility constraints:

$$\begin{aligned} U(\theta, e_p^*, w_p^*) &= w_p^* - \frac{e}{\theta} \geq w_L^* = U(\theta, 0, w_L^*), \ \forall \theta \in \Theta_L, \\ U(\theta, e_p^*, w_p^*) &= w_p^* - \frac{e}{\theta} \geq w_L^* = U(\theta, 0, w_L^*), \ \forall \theta \in \Theta_H, \end{aligned} \tag{5}$$

which are equivalent to

$$U(\underline{\theta}_L, e_p^*, w_p^*) = w_p^* - \frac{e_p^*}{\underline{\theta}_L} \geq w_L^*.$$

This implies that $0 \leq e_p^* \leq (w_p^* - w_L^*)\underline{\theta}_L$. Then, we obtain pooling PBEs as follows.

Proposition 3. *There are pooling PBEs such that, in one of them, the worker's education level is given by*

$$\tilde{e}(\theta_L) = \tilde{e}(\theta_H) = e_p^* \in [0, \bar{e}_p]$$

where $\bar{e}_p = (w_p^* - w_L^*)\underline{\theta}_L$ *and the firm's posterior belief and offered wage are given by*

$$\mu(\Theta_L|e) = \begin{cases} 1 & \text{if } e < e_p^*, \\ q & \text{if } e \geq e_p^*, \end{cases} \quad \text{and} \quad \tilde{w}(e) = \begin{cases} w_L^* & \text{if } e < e_p^*, \\ w_p^* & \text{if } e \geq e_p^*, \end{cases} \tag{6}$$

with

$$w_p^* = qw_L^* + (1-q)w_H^*.$$

In Figure 3, we illustrate a pooling equilibrium of Proposition 3. Both shaded areas are identical to Figure 2. Similar to the separating equilibrium, to pool both types, the firm picks cutoff education level e_p^* on the red line, which satisfies incentive compatibility constraints (5). The firm's strategy is described by the blue step function. Consequently, in equilibrium, both types choose education level of e_p^*.

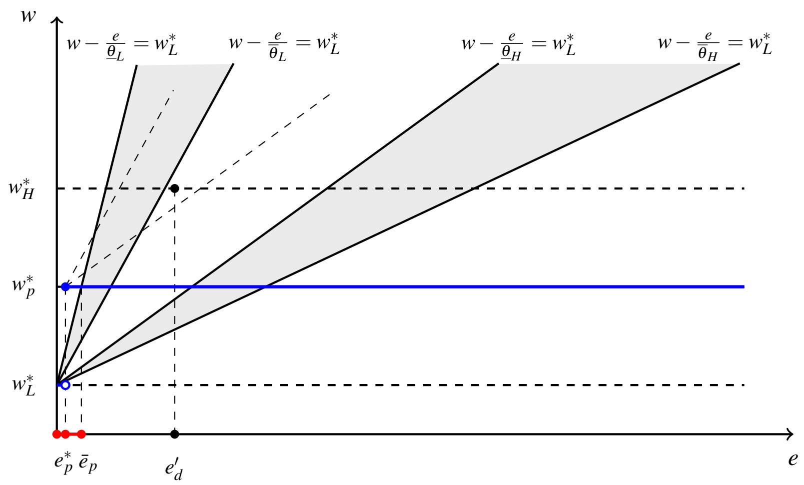

Figure 3: An education-wage pair (e_p^*, w_p^*) represents a pooling PBE in Proposition 3

4. REFINEMENTS OF THE PERFECT BAYESIAN EQUILIBRIA

Now we refine the PBEs obtained in Section 3 by invoking an extension of Cho and Kreps' (1987) Intuitive Criteria (CKIC). For $T' \subset T_f$, let $\mathrm{BR}(T', e)$ be the set of all pure-strategy best responses of the firm to education level e under belief $\mu(\cdot|e)$ such that $\mu(T'|e) = 1$:

$$\mathrm{BR}(T', e) = \bigcup\nolimits_{\mu:\mu(T'|e)=1} \mathrm{BR}(\mu, e),$$

where

$$\mathrm{BR}(\mu, e) = \mathrm{argmax}_{w\in[0,W]} \sum\nolimits_{t\in T_f} \mu(t|e)\mathbb{E}_\mu[\theta|t] - w.$$

Let $U^*(\theta)$ be the utility of type-θ worker in a PBE. Since $T_\omega = \{\theta_L, \theta_H\} \neq \{\Theta_L, \Theta_H\} = T_f$ in our model, we define an extension of the CKIC in the following way.

Definition 4. (Extended Intuitive Criterion) *A PBE fails Extended Intuitive Criterion* **(HKIC)** *if there exists $t \in T_f \setminus D(e)$ with some e such that for some $\theta \in t$,*

$$U^*(\theta) < \min\nolimits_{w\in BR(T_f\setminus D(e),e)} U(\theta, e, w), \tag{7}$$

where

$$D(e) \equiv \left\{ t \in T_f \,\middle|\, U^*(\theta) > \max_{w \in BR(T_f, e)} U(\theta, e, w), \ \forall \theta \in t \right\}. \tag{8}$$

Similarly to the CKIC, our HKIC proceeds in the following way. First, if the firm identifies a type (i.e., Θ_L or Θ_H) in T_f such that *every worker with a productivity within that type* cannot beat the equilibrium utility by acquiring off-the-equilibrium education level e even when the firm offers him the most favorable wage, then it considers set $D(e)$ of such types and restricts its beliefs to $T_f \setminus D(e)$. Second, if the firm finds a type in $T_f \setminus D(e)$ such that *a worker with some productivity within that type* has greater utility at off-the-equilibrium education level e than the equilibrium utility even when the firm offers him the most unfavorable wage, then the original equilibrium is vulnerable to e and fails the HKIC.

It is worth noting that the HKIC is a generalized form of the CKIC since, in the perspective of the firm, the HKIC treats a type as an interval of productivities, whereas the CKIC considers a type as a single productivity. Indeed, when T_f is the set of singletons (i.e., $\Theta_L = \{\theta_L\}$ and $\Theta_H = \{\theta_H\}$), the HKIC reduces to the CKIC.

We obtain a unique PBE below by invoking the HKIC, which eliminates all of the separating PBEs but the least-cost one, as well as all of the pooling PBEs.[14]

Theorem 5. *Under the HKIC, there is a unique (separating) PBE in which the education levels of type θ_L and type θ_H are*

$$\tilde{e}(\theta_L) = e_L^* = 0, \quad \text{and} \quad \tilde{e}(\theta_H) = e_H^* = \underline{e}_H \equiv (w_H^* - w_L^*)\bar{\theta}_L$$

and the firm's posterior belief and offered wage are

$$\mu(\Theta_L|e) = \begin{cases} 1 & \text{if } e < e_H^*, \\ 0 & \text{if } e \geq e_H^*, \end{cases} \quad \text{and} \quad \tilde{w}(e) = \begin{cases} w_L^* & \text{if } e < e_H^*, \\ w_H^* & \text{if } e \geq e_H^*, \end{cases} \tag{9}$$

with

$$w_L^* = \frac{\underline{\theta}_L + \bar{\theta}_L}{2} \quad \text{and} \quad w_H^* = \frac{\underline{\theta}_H + \bar{\theta}_H}{2}.$$

[14] In Figures 2 and 3, each given PBE fails the HKIC with off-the-equilibrium education level e_d.

In the separating equilibrium of Proposition 2, the high type has incentive to reduce education cost as long as the firm rightly perceives his type. In Figure 2, observing off-the-equilibrium education level $e_d \in [\underline{e}_H, e^*_H)$, the firm believes that the sender is the high type since only the high type can be better off by sending e_d. Thus, the original separating equilibrium education level $e^*_H \in (\underline{e}_H, \bar{e}_H]$ does not survive the HKIC and only cost-minimizing $\underline{e}_H$ remains. In the pooling equilibrium of Proposition 3, the high type has incentive to reveal his type by choosing a higher education level. In Figure 3, observing off-the-equilibrium education level e'_d which makes the only high type better off, the firm considers that e'_d between two dashed indifference curves in Figure 3 is sent by the high type. In this way, all the pooling equilibria are do not survive the HKIC and thus are eliminated. The refined equilibrium is illustrated in Figure 4. Interestingly, this separating PBE does not hinge on any value of θ_L or θ_H as long as $\theta_L \in \Theta_L$ and $\theta_H \in \Theta_H$.

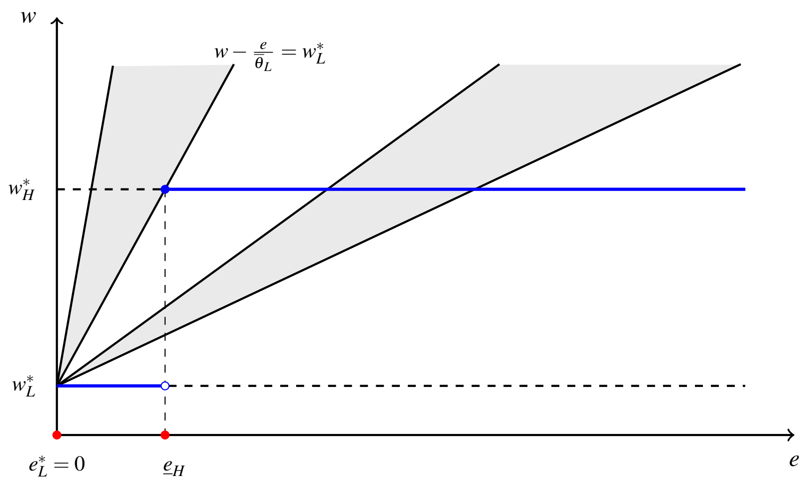

Figure 4: The education-wage pair $(\underline{e}_H, w^*_H)$ of the refined PBE in Theorem 5

5. MAIN IMPLICATIONS

In this section, we provide the main implications of the unique refined PBE. One may believe that the worker can take advantage of the firm's coarse information and thus the high type spends less money to signal his type and obtains more wage compared to the benchmark. However, Theorem 6 shows that this cannot be the case. When the firm has coarse information about the worker's possible types, a high-type worker may acquire a higher education level with a lower wage than in Spence's (1973) model (benchmark). This implies that education may be less effective signal than in the benchmark.

To proceed, let

$$\hat{\theta} := \frac{\bar{\theta}_L}{\theta_L}\left(\frac{\underline{\theta}_H + \bar{\theta}_H}{2} - \frac{\underline{\theta}_L + \bar{\theta}_L}{2}\right) + \theta_L,$$

which is the high type whose equilibrium education level in the benchmark is $\underline{e}_H$. As long as $(\underline{\theta}_L + \bar{\theta}_L)/2 \leq \theta_L$, we see that

$$\frac{\underline{\theta}_H + \bar{\theta}_H}{2} < \min\{\hat{\theta},\ \bar{\theta}_H\}.$$

Now we state the main result.

Theorem 6. *Suppose that the low-type worker's productivity is not overvalued (i.e., $(\underline{\theta}_L + \bar{\theta}_L)/2 \leq \theta_L$). If the high-type worker's productivity θ_H falls in the interval $((\underline{\theta}_H + \bar{\theta}_H)/2, \min\{\hat{\theta}, \bar{\theta}_H\})$, then he should acquire a higher education level with a lower wage than in the benchmark.*

PROOF : Let us consider the benchmark where $\theta_H \in \big((\underline{\theta}_H + \bar{\theta}_H)/2, \min\{\hat{\theta},\ \bar{\theta}_H\}\big)$. Recall that, in the unique cost-minimizing separating PBE of the benchmark, the wage and effort of the high type are given by[15]

$$w_H^{\circ} = \theta_H, \quad e_H^{\circ} = (\theta_H - \theta_L)\theta_L.$$

Since $w_H^* = (\underline{\theta}_H + \bar{\theta}_H)/2$, in the refined equilibrium, it is obvious that $w_H^* = (\underline{\theta}_H + \bar{\theta}_H)/2 < \theta_H = w_H^{\circ}$. Since $\theta_H < \min\{\hat{\theta},\ \bar{\theta}_H\}$,

$$e_H^{\circ} < (\hat{\theta} - \theta_L)\theta_L = \left[\frac{\bar{\theta}_L}{\theta_L}\left(\frac{\underline{\theta}_H + \bar{\theta}_H}{2} - \frac{\underline{\theta}_L + \bar{\theta}_L}{2}\right) + \theta_L - \theta_L\right]\theta_L = (w_H^* - w_L^*)\bar{\theta}_L = e_H^*,$$

[15] We will use "∘" for the cost minimizing separating PBE of the benchmark.

which implies that $e_H^\circ < e_H^*$. Hence, we conclude that $w_H^* < w_H^\circ$ but $e_H^* > e_H^\circ$ for $\theta_H \in \big((\underline{\theta}_H + \bar{\theta}_H)/2,\, \min\{\hat{\theta}, \bar{\theta}_H\}\big)$. ■

Figure 5 illustrates how the signal becomes less effective due to the firm's coarse information. The red open interval $((\underline{\theta}_H + \bar{\theta}_H)/2, \hat{\theta})$ on the vertical axis indicates the set of θ_H's for which Theorem 5.1 holds when $\hat{\theta} < \bar{\theta}_H$. Note that this interval corresponds to the open-ended thick part on the red indifference curve of the low type, which is the locus of cost-minimizing PBEs for such θ_H's in the benchmark. When the firm has coarse information about the worker's true ability, the high-type worker may choose education level $e_H^* = \underline{e}_H > e_H^\circ$ for wage $w_H^* < w_H^\circ$, where he chooses education level e_H° for wage w_H° in the benchmark.

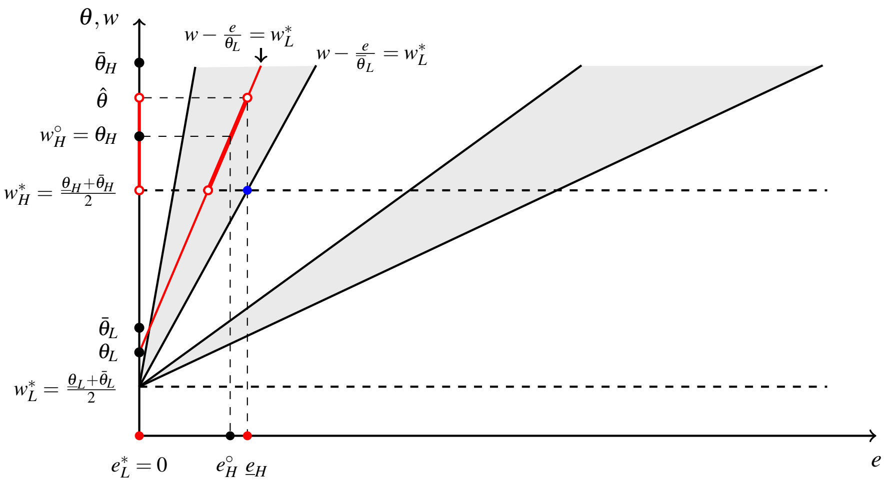

Figure 5: Comparison of the equilibrium education-wage pair of the high type under coarse information with that in the benchmark

The following example shows that Theorem 6 may not be valid when the conditions are violated.

Example: We consider the case where

$$(\underline{\theta}_L, \theta_L, \bar{\theta}_L) = (2, 2.6, 3) \quad \text{and} \quad (\underline{\theta}_H, \theta_H, \bar{\theta}_H) = (4, 4.95, 5),$$

which imply that

$$\frac{\underline{\theta}_L + \bar{\theta}_L}{2} = 2.5 < 2.6 = \theta_L$$

and

$$\hat{\theta} = \frac{3}{2.6}\left(\frac{5+4}{2} - \frac{3+2}{2}\right) + 2.6 = 4.908 < 4.95 = \theta_H.$$

Thus, in Theorem 6, the first condition holds, but the second one is violated. However, we have

$$w_H^{\circ} = 4.95 > 4.5 = w_H^{*} \quad \text{and} \quad e_H^{\circ} = 6.11 > 6 = e_H^{*},$$

and therefore, the worker chooses a lower education level for a lower wage compared to the benchmark. Hence, Theorem 6 is not valid in this example. □

6. CONCLUDING REMARKS

In this study, we consider firms' coarse information about a worker's possible types in Spence's (1973) job market signaling model. With incentive compatibility constraints for coarse information, we find PBEs and refine them into a unique PBE by invoking the HKIC. In the refined equilibrium, a high-type worker may incur more cost for education to be perceived as the high type than in the benchmark even when he is paid a lower wage. In the future research, one can investigate the case where education is productive signal as in Spence (1974) and extend our model to include learning.

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